Target Volatility Strategies for Group Self-Annuity Portfolios

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Outline

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Motivation

- Global shift from DB to DC schemes implies that both individual longevity and investment risk have to be managed by the individual, or transferred to a private providers.
- Among private post-retirement solutions, it is well documented that standard annuity products offer optimal decumulation strategy (Yaari 1965).
- Thin market for annuities, contributing factors to annuity puzzle include high loadings and bequest motives (Brown 2009).
- Standard annuities are considered to be inflexible and illiquid assets by many individuals, as they imply an irreversible decision (Pitacco 2016).
- Need for innovative decumulation products capable of mitigating longevity risk whilst preserving stable income for retirees.

Outline

The GSA fund dynamics

- Consider a homogeneous GSA pool with N_0 individuals aged x joining the fund at time 0, each of them injecting initial capital amount c.
- GSA fund, F_t, evolves according to return on investment and benefit payouts.
- Benefit payouts also depend on mortality experience and investment returns.
- Transition intensity is denoted as μ_{x+t}.
- GSA scheme pays living benefits to surviving members; can incorporate possibility of paying out death benefits.

The GSA fund dynamics cont...

- The total living benefit amount paid by the GSA scheme at time t is

$$B_t = \frac{F_t}{\overline{a}_{x+t}}. \tag{1}$$

- Corresponding individual benefit payment

$$L_t = \frac{B_t}{N_t} \tag{2}$$

with N_t being the number of survivors at time t.

- Future risk not factored in $\overline{a}_{x+t}$ but is accounted in the living benefits assessed at time t;
- This is a natural choice within a GSA arrangement as risks are retained by surviving participants.

The GSA fund dynamics and Components

- Upon one member's death at time t, an amount

$$D_t = \beta \frac{F_t}{N_{t^-}}. \quad (3)$$

is paid to member's beneficiaries.

- Equity process is denoted as S_t and r risk-free asset with w_t the proportion of fund invested in equity.
- The GSA fund dynamics can then be described as

$$dF_t = \left[w_t \frac{dS_t}{S_t} + (1 - w_t) r dt\right] F_t - B_t dt + D_t dN_t. \quad (4)$$

- The GSA fund can be split into three components as follows:
  1. principal, F_t^1;
  2. investment returns, F_t^2;
  3. mortality credits, F_t^3,
- The fund value at time t is then

$$F_t = F_t^1 + F_t^2 + F_t^3. \quad (5)$$

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Heston Stochastic Volatility Model

- The dynamics of the equity process

$$dS_t = \mu S_t dt + \rho\sqrt{v_t}S_t dW_t^1 + \sqrt{1-\rho^2}\sqrt{v_t}S_t dW_t^2, \quad (6)$$
$$dv_t = \kappa(\theta - v_t)dt + \sigma_u\sqrt{v_t}dW_t^1, \quad (7)$$

- At each instant, weights are dynamically rebalanced to maintain a target volatility.
- Equity weights as rebalanced by setting

$$w_t' = \min\left(\frac{TV}{\sqrt{\hat{\sigma}^2(t)}}, 1\right).$$

- Volatility track through exponentially weighted moving average (EWMA)

$$\hat{\sigma}^2(t+\Delta t) = \lambda \cdot \hat{\sigma}^2(t) + \frac{(1-\lambda)}{\Delta t}\left(\frac{dS_t}{S_t}\right)^2, \quad (8)$$

Illustration of Target Volatility

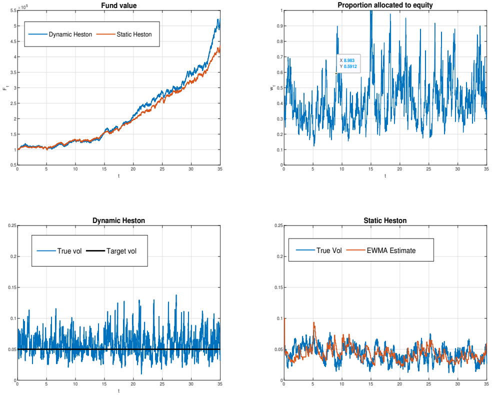

Figure 1: Illustrative simulation: Monthly target volatility rebalancing. Rebalancing decisions are based on EWMA estimates of the volatility, since the true volatility is not observable in the market.

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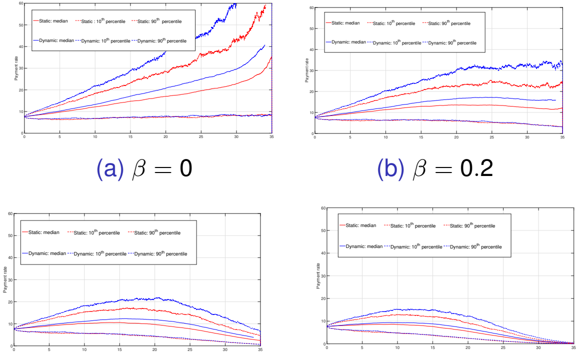
Benchmark case with 70% equity allocation

- Benchmark case involves 70% equity allocation at $t = 0$ and associated target volatility of 12% pa.



(a) $\beta = 0$ (b) $\beta = 0.2$

(c) $\beta = 0.5$ (d) $\beta = 1$

Figure 2: Living benefit payment quantiles for various death benefit proportions.

Benchmark case cont...

- The dynamic target case consistently pays out higher living benefits compared to the static volatility strategy which does not adjust the equity and cash composition due to changing market conditions.
- Figures 2 present varying cases of β which relates to proportional being paid as death benefits.
- Death benefit payments result in reduced living benefit payments through time as the fund value will be proportionally reduced due to lower mortality credits, hence compromising its performance.
- When $\beta \geq 50\%$, living benefit payments may fall below initial payments as revealed in Figures 2(c) and 2(d).

Comparison of living benefits for varying equity compositions

- Analysis performed relative to 70% equity strategy with corresponding target volatility of 12% pa.
- When initial equity composition is 90% , the median and 90^{th} percentile are superior to those for the 70% initial equity strategy.
- Strategies with less equity holdings have low payout structures due to the limited performance of the underlying fund.

Initial Equity Quantile	0% TVol	Static	50% TVol	Static	90% TVol	Static
0.1	1.0334	1.0673	1.0214	1.0414	0.98089	0.95122
0.5	0.59768	0.62887	0.89837	0.89141	1.0858	1.1172
0.9	0.3384	0.40804	0.78174	0.78109	1.1376	1.2684

Table 1: Relative individual living benefits at Age 75 for varying initial allocations and $\beta = 0$.

Comparison of living benefits - varying age

- For a 50-50 initial equity-cash holding, living benefit decrease with age.
- As highlighted above, this is due to limited performance resulting from less equity exposure.
- This is the case for the two investment strategies under consideration.
- The general finding is that less equity, implies less volatility in living benefit resulting in decreasing benefits with age.

Age	75		80		85	
Quantile	Tvol	Static	Target Vol	Static	Target Vol	Static
0.1	1.0214	1.0414	1.0032	1.0074	0.97553	0.98871
0.5	0.89837	0.89141	0.84934	0.83602	0.79279	0.80119
0.9	0.78174	0.78109	0.73913	0.72292	0.67854	0.67

Table 2: Relative individual living benefits for 50% initial equity allocation and $\beta = 0$.

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Conclusions

- This paper devises target volatility strategies for GSA portfolios.
- Compares benefit profiles emerging under static and dynamic underlying fund investment strategies.
- Benefit profiles are assessed by analysing various quantiles and alternative strategies involving varying equity compositions are presented.
- Overall, higher living benefit profiles are obtained under a dynamic target volatility strategy.
- A trade-off between equity proportion and the impact on lower quantile of living benefits emerges, which suggests an optimal proportion of equity composition.

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