Risk-sharing for Pooled Annuities with Stochastic and Correlated Mortality Rates

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Outline

Background

Mutual mortality pooling products:

- Reduce idiosyncratic mortality risks of retirees.
- Require less capital than annuity products:
  - Benefits potential buyers by a lower loading and thus price.
  - Benefits the issuer by lower financial and longevity risks in the portfolio.

However, current studies on pooling products mostly assume constant mortality rates.

- Mortality rates are stochastic and correlated random variables.

Research Objectives

In this research, we aim to:

- Study pooling products with different cohorts whose mortality rates are stochastic and correlated.
  - Extend current risk-sharing rules to adopt stochastic mortality rates.
  - Propose and test new risk-sharing rules that distribute total mortality credit based on expected value, volatility, and correlation of mortality rates.
- Include death benefit in fair risk-sharing.
- Study the effect of pool size, mix of ages, initial contributions, amount and length of death benefits, and volatilities and correlations of mortality rates on fund balances and benefit payments.

Properties of Mortality-pooling Products

Heterogeneity of fund members:

- Whether members of different genders, ages, initial contributions, and investment returns are allowed to enter the pool.

Capability to allow new entrants:

- If new members are allowed to join in the subsequent years after initial establishment.

Fairness:

- The expected fund value after risk sharing should be equal to the initial fund value.

Sustainability:

- Whether external fund is needed.

Payment level, volatility, and duration:

- The amount, stableness, and length of benefit payments.

Mutual Mortality Pooling Retirement Products

Tontines:

- Sabin (2010): Fair tontine and fair tontine annuity.
- Milevsky and Salisbury (2015): Modern Tontines.

Pooled annuities:

- Piggott et al. (2005): Payment adjustments for empirical mortality and interest rate.

Risk-sharing with decumulation strategies:

- Donnelly and Young (2017), Fullmer and Sabin (2018), and Denuit and Robert (2021).

Risk-sharing Rules

The share of total mortality credit is determined by:

- Proportional: Capital at risk which depends on the fund balance and the probability of death.
- Proposed: Expectation, variance, and covariance between the mortality rates of cohorts in the pool.
- Regression: Covariance between the capital at risk and the total mortality credit.
- Alive-only: Only members alive will share the total mortality credit.

Fund Operation

Between two benefit payments, the fund operates as the following:

1. Accumulation of investment return.
2. Experience survivorship.
3. Risk-sharing and distributing the total mortality credit.
4. Benefit paying from the balance after risk-sharing.

Preliminary Results: Assumptions on the Pool

Assumptions:

- 600 members in the pool.
- 200 members aged 65, 190 members aged 70, 150 members aged 80, and 60 members aged 90.
- Age 65: initial balance of 6 or 4.
- Age 70: initial balance of 5 or 3.
- Age 80: initial balance of 3 or 2.
- Age 90: initial balance of 2 or 1.

The results presented in the slides are intended solely for the purpose of illustrating the idea and are considered preliminary. Results will differ with more refined assumptions and more younger age groups.

Preliminary Results: Assumptions on Mortality Rates

Table 1: Mean and Standard Deviation of Mortality Rates at Different Ages.

qx	65	70	80	90
μ	0.0301	0.0456	0.1101	0.2260
σ	0.0005	0.0009	0.0033	0.0117

Table 2: Correlation of Mortality Rates at Different Ages 65, 70, 80, and 90.

corr	65	70	80	90
65	1	0.9747	0.5552	0.2824
70	0.9747	1	0.7270	0.4896
80	0.5552	0.7270	1	0.9546
90	0.2824	0.4896	0.9546	1

Comparison of Return Rates from Mortality Credit

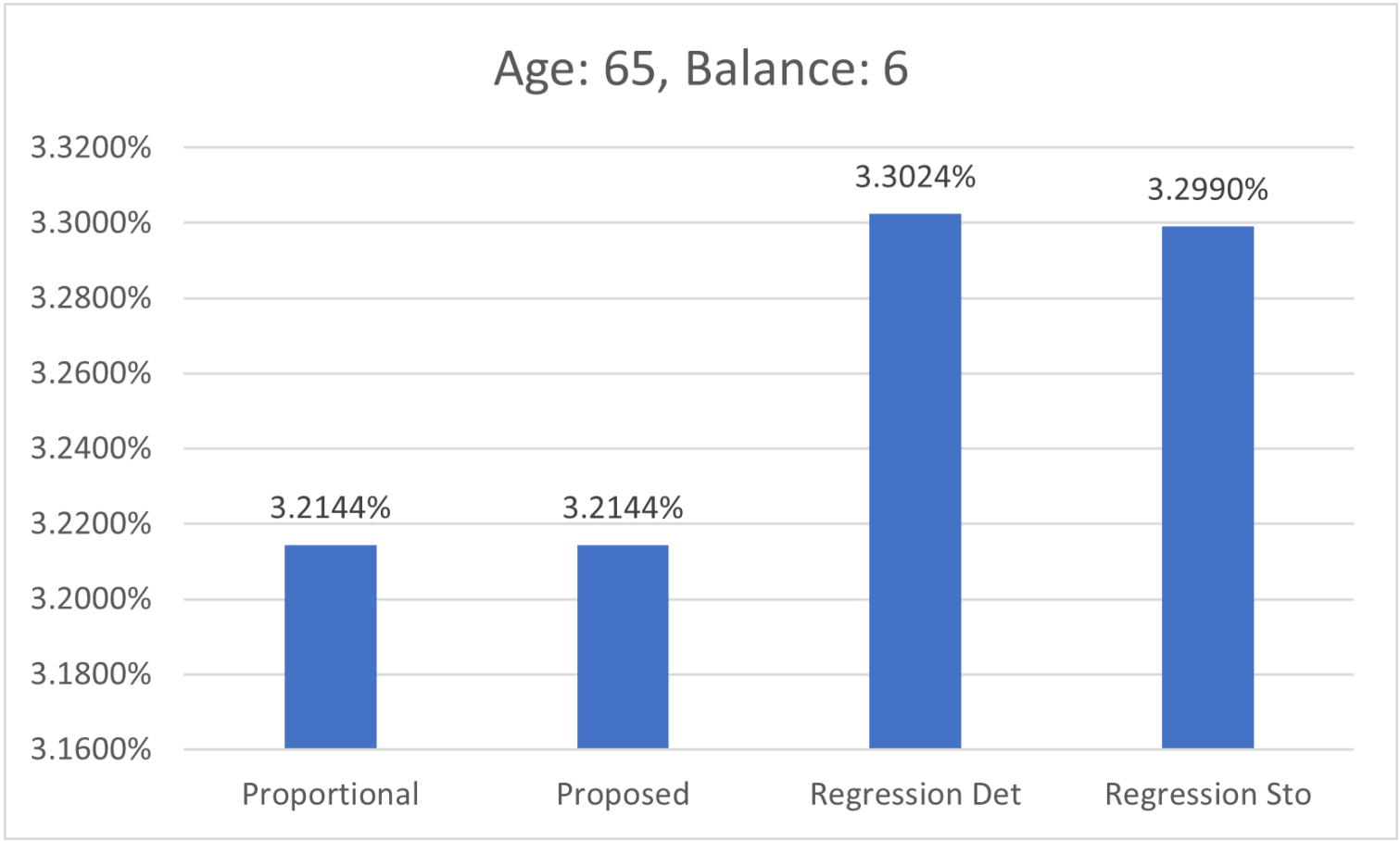

Figure 1: Comparison of Return Rates from Mortality Credit between Risk-sharing Rules (Given Survival).

Comparison of Return Rates from Mortality Credit

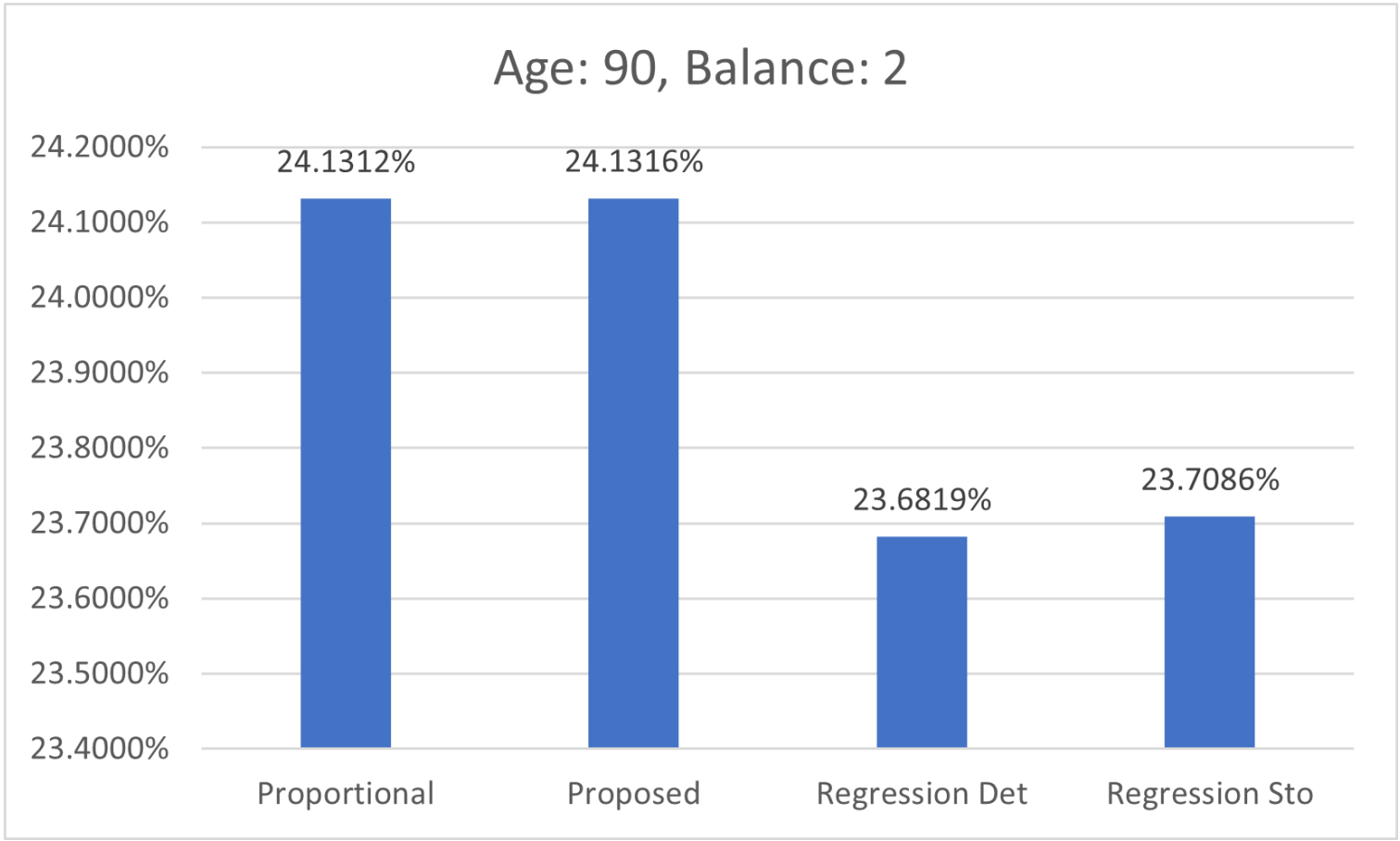

Figure 2: Comparison of Return Rates from Mortality Credit between Risk-sharing Rules (Given Survival).

Effect of Balance on Return Rate

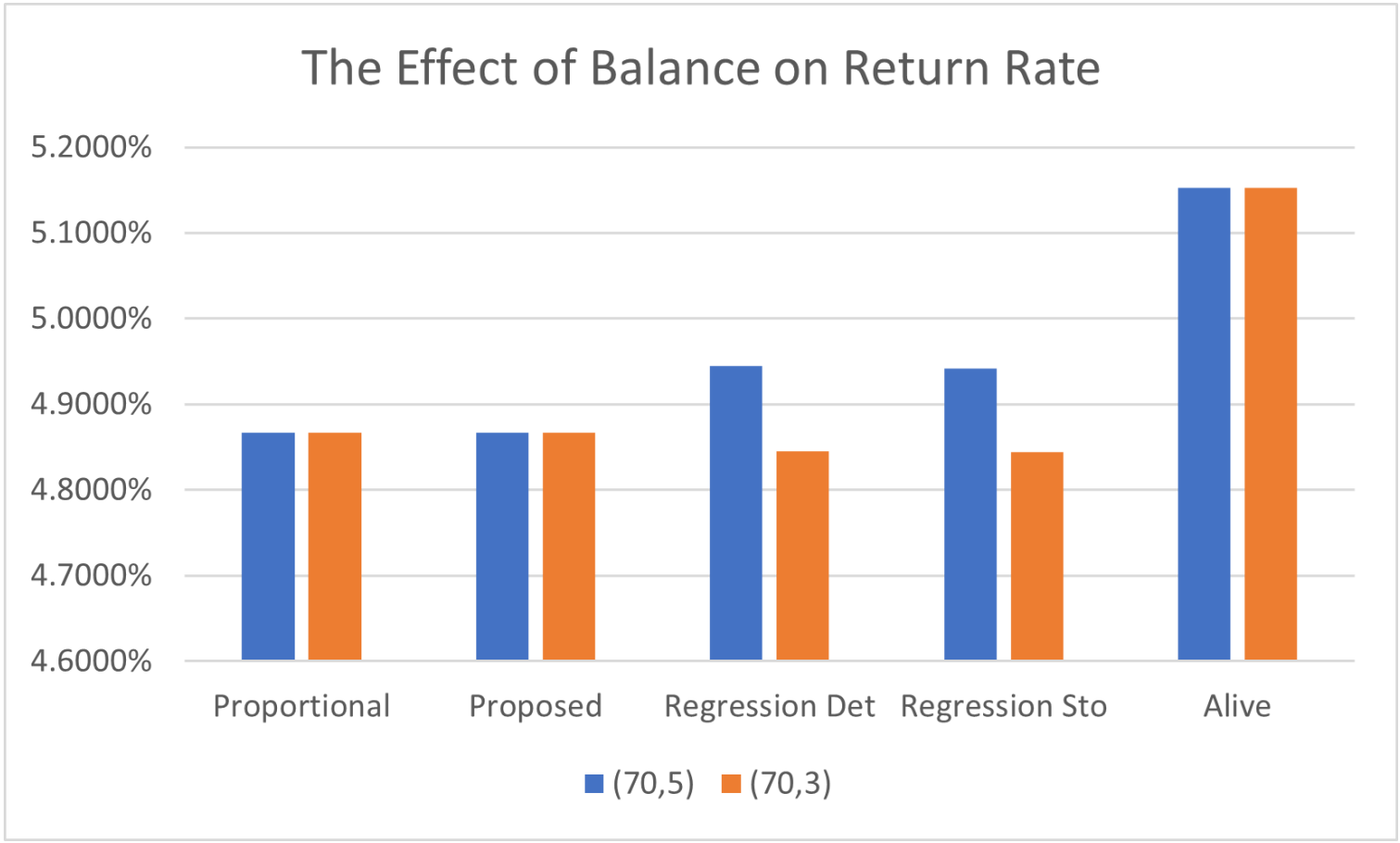

Figure 3: Comparison of Return Rate at Age 70 with Balances of 3 and 5.

Effect of Age on Return Rate

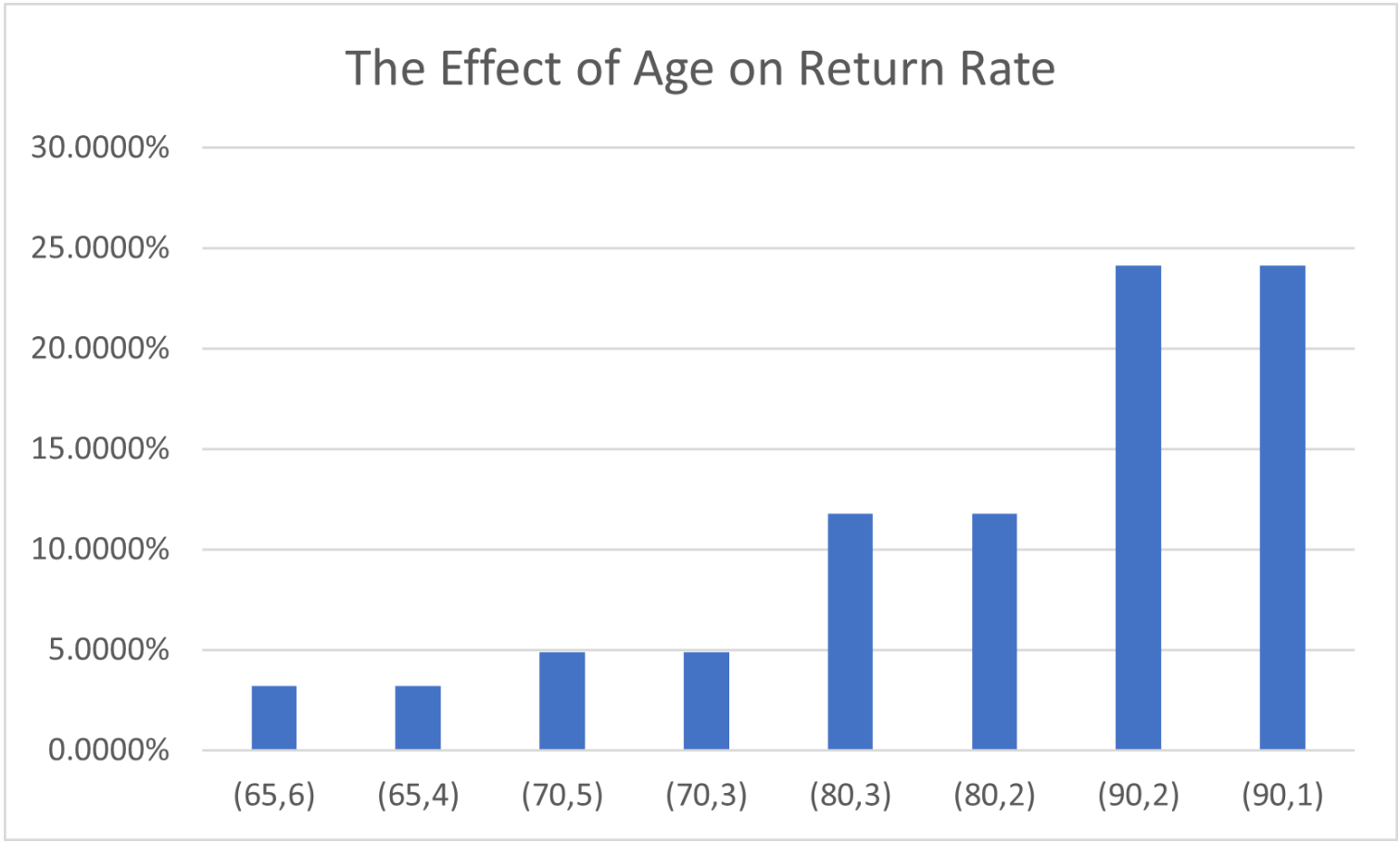

Figure 4: The Effect of Age on Return Rate with Proposed Rule.

Conclusions

- We derive the extended regression rule and the proposed rule to adopt stochastic and correlated mortality rates.
- We use the correlation between mortality rates calibrated in our previous work:
  Zhou, Y., Garces, L. P., Shen, Y., Sherris, M., & Ziveyi, J. (2023). Age-Dependent Multi-Cohort Affine Mortality Model with Cohort Correlation. Available at SSRN 4456316.
- We study how the selection of risk-sharing rule, the inclusion of volatility and correlation of mortality rates, account balance, and age affect return rate.
- The effect of including stochasticity and correlation in the risk-sharing rule is subject to the assumptions on the pool size, mix of ages, and mortality rates. Results will differ with more refined assumptions and more younger age groups.

Thank You!

References I

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