A Universe of Individual-Level Mortality Models

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33rd Colloquium on Pensions and Retirement Research

24 November 2025

Introduction

- **Context & Motivation**
  - Increasing importance of individual-level mortality modelling for insurers, pension funds and longevity researchers
  - Need for methods that integrate age and covariate effects
- **Research Gap**
  - Ng (2023) provided a taxonomy of mortality models based on characteristics such as parametric nature
  - Separated statistical from machine learning models, overlooking potential combinations
- **Objective**
  - Present a modular framework and introduce novel combined models (e.g., survival tree partitioning, mob tree partitioning)

Individual-Level Mortality Modelling

- **Data Structure:** Survival data with age at entry, age at exit, death indicator, and covariates. (mortality rate is not observable)

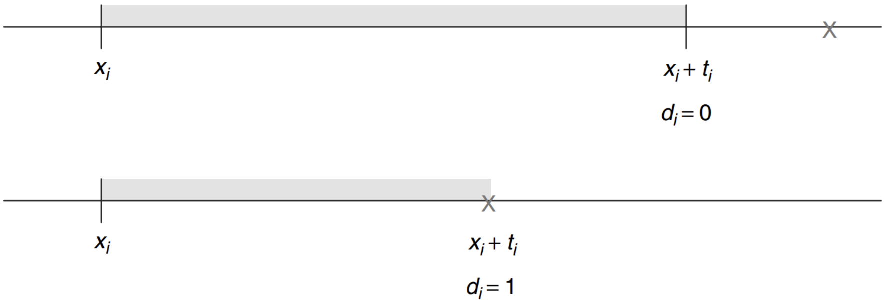
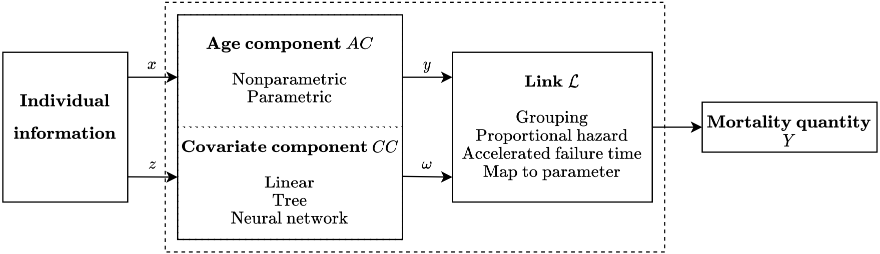


Figure 1: Diagram of survival model setup (Richards 2012)

Universe of Individual-level Mortality Models



Individual-level mortality function

Estimation of AC, CC parameters can be simultaneous or sequential

Age Component

- **Purpose:** Define the distributional form of age effect
- **Two Approaches:**
  - **Non-Parametric Age Component:**
    - Models y empirically with a step function
    - Example: Kaplan-Meier / Nelson-Aalen estimators
  - **Parametric Age Component:**
    - Assumes age at death follows a specific distribution
    - Common models include Gompertz, Makeham, Perks, Beard, Kannisto, and Thatcher laws
    - Enables interpolation/extrapolation and likelihood-based estimation

Covariate Component

- **Role:** Captures how non-age information (e.g., demographics, health measures) affects mortality
- **Modelling Options:**
  - **Linear:** $\omega = \mathcal{B}z$
  - **Tree-Based:** Non-linear mapping via binary splits; provides grouping of individuals
  - **Neural Network:** More flexible, continuous mapping suited for high-dimensional data

Link Function

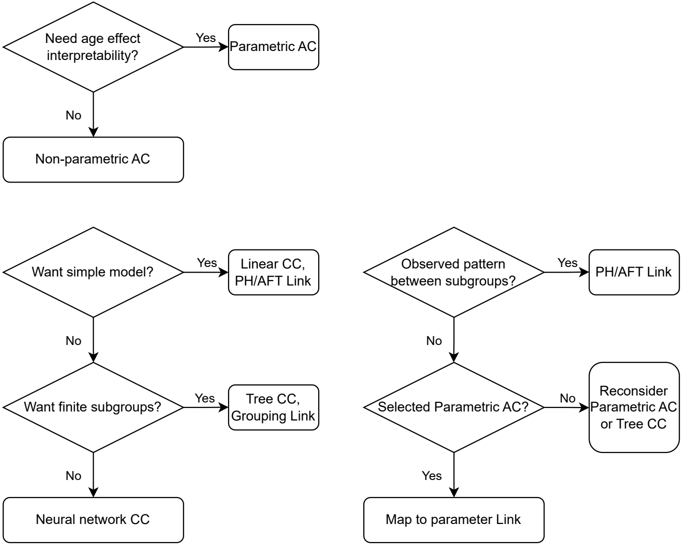
- **Function of $\mathcal{L}$**: Integrate the outputs of age and covariate components to yield the overall mortality output Y
- **Different Forms:**
  - **Grouping Link:** $Y = \sum_{i=1}^{k} y_i \, 1\{\omega = i\}$
  - **Proportional Hazards (PH) Link:** $Y = \mu = \mu_0 \exp(\omega)$
  - **Accelerated Failure Time (AFT) Link:**

$$X \stackrel{d}{=} \exp(\omega) X_0$$

$$Y = \mu(x) = \exp(\omega)\mu_0(\exp(\omega)x), \omega \in \mathbb{R}$$

  - **Map-to-Parameter Link:** $Y = y|(\theta = \omega), \omega \in \Theta$
    - Example: $AC =$ Gompertz$(\alpha, \beta), CC : z \to \theta = (\alpha, \beta)$

Component Selection Guide



Case Study: CLHLS Dataset

- Chinese Longitudinal Healthy Longevity Survey (CLHLS) with seven waves spanning 1998–2018
- Includes data on oldest-old (aged $80+$) with over 59,000 observations
- Rich covariate information (demographics, health, cognitive abilities, lifestyle)
- Left-truncated and right-censored survival data

Combined Models

- **Survival Tree:** $AC =$ non-parametric, $CC =$ Tree-based, $L =$ Grouping
- **Survival Tree Partitioning (STP):** Estimate survival tree and then apply a parametric model to each terminal node
- **Mob Tree Partitioning (MTP):** Estimate AC and CC simultaneously to maximize the likelihood
- **Proportional Hazards (PH) Model:** Extends the Cox model with parametric age component

Evaluation Metrics

Out of sample metrics:

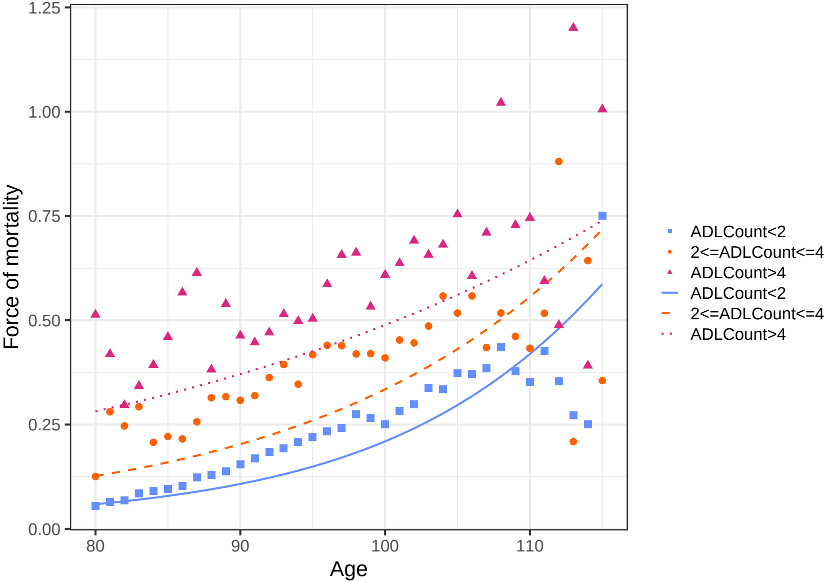
- Integrated Brier Score (IBS):

$$IBS = \int \left(\hat{S}(t|x) - S(t|x) \right)^2 dt$$

- Log Score: log likelihood evaluated on test/validation set

$$\mathcal{S}_L(S(x|z), (L, C, \delta)) = \sum_i [\delta_i \log \mu(C_i) - H(C_i) + H(L_i)]$$

Partial Dependence Plot



Application to Life Expectancy

- **Using the Compound Gompertz Model:**
  - Life expectancy is derived by integrating the force of mortality
  - Example equation using the exponential integral:

$$e_G(x) = \frac{1}{b} \exp\left(\frac{a}{b} e^{bx}\right) E_1\left(\frac{a}{b} e^{bx}\right)$$

- **Findings:**
  - Life expectancy differs substantially based on covariate strata (e.g., ADL Count, Interviewer Rated Health)
  - E.g., at age 80, overall life expectancy is around 8 years, but varies significantly among groups

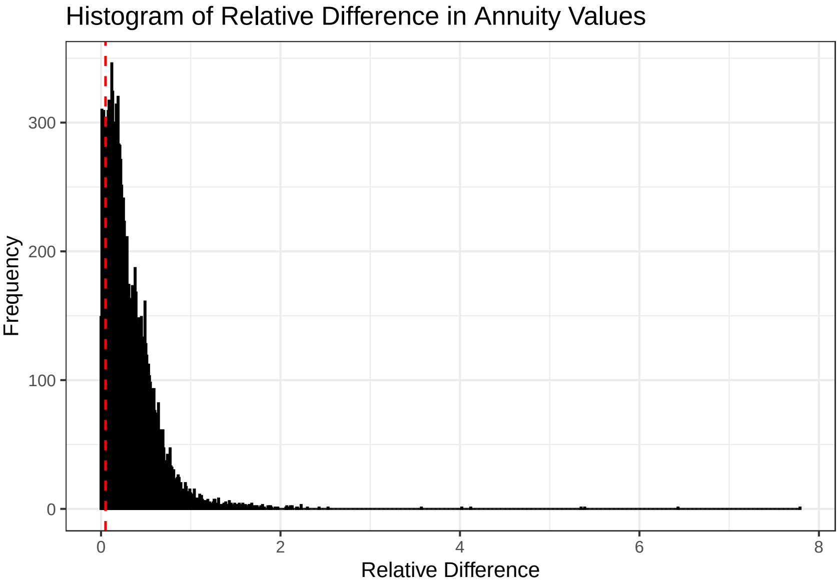
Life Expectancy by ADL Count

Table 1: Life expectancy in years for each cohort at age 80, 90, 100, 110.

	80	**90**	**100**	**110**
Age-only model (Gompertz)	8.0 (5.7)	4.7 (3.8)	2.7 (2.3)	1.4 (1.3)
ADLCount<2	10.0 (6.9)	6.3 (4.8)	3.7 (3.1)	2.1 (1.9)
2<=ADLCount<=4	6.1 (4.9)	4.1 (3.5)	2.6 (2.4)	1.7 (1.6)
ADLCount>4	3.3 (3.0)	2.5 (2.4)	1.9 (1.8)	1.5 (1.4)
Interviewer Rated Health=1	11.0 (7.2)	6.8 (5.0)	3.9 (3.2)	2.1 (1.9)
Interviewer Rated Health=2	9.7 (6.7)	6.0 (4.6)	3.6 (3.0)	2.0 (1.8)
Interviewer Rated Health>2	6.0 (4.8)	4.0 (3.4)	2.6 (2.3)	1.6 (1.5)
Interviewer Rated Health is missing	9.9 (6.8)	6.2 (4.7)	3.6 (3.0)	2.0 (1.8)

Note:
Life expectancy is rounded to 1 decimal place (1 day is 0.0027).

Annuity Factor Differences



Conclusion

- **Summary of Contributions:**
  - Introduced a novel, modular framework for individual-level mortality modelling
  - The framework unifies age and covariate effects through distinct components
  - Developed new combined models with promising empirical performance
- **Insights from CLHLS:**
  - Covariates such as ADL count and Interviewer rated health significantly influence mortality
  - MTP and PH models achieve excellent prediction accuracy using the CLHLS dataset

Future Works

- Give guidance on comparing and selecting models
- Use neural networks to utilise high-dimensional data
- Extend the framework to more complex outcomes (competing risks, multi-state models)
- Consider time-varying covariates, unobserved heterogeneity, and covariate uncertainty
- Apply the methodology to pricing and retirement planning

Thank you!

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Link to paper:
https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5256615

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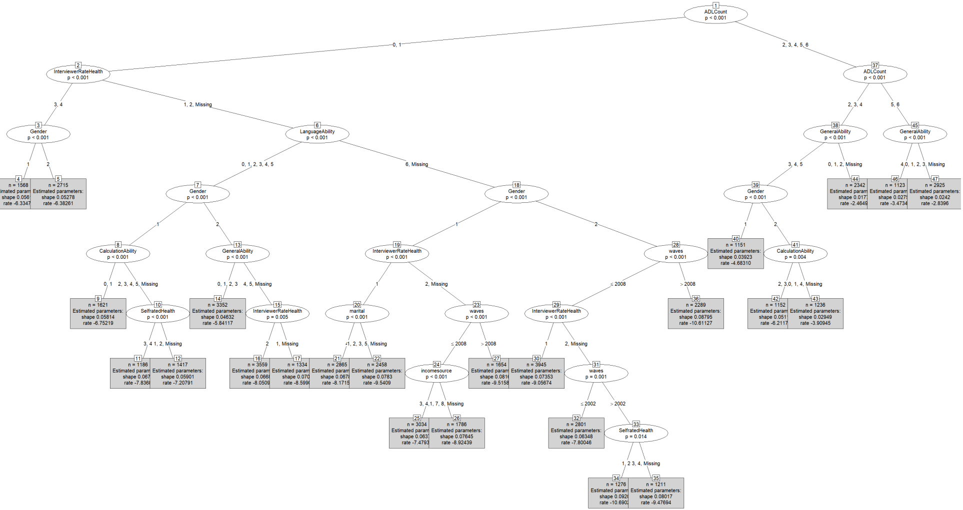
Mob Tree Partitioning results



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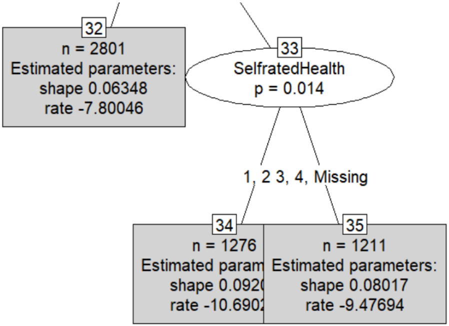

Model Comparison (IBS)

Table 2: Test IBS for each model in the 8 samples.

	Gompertz	Beard	STP_Gompertz	STP_Beard	MTP	GPH
1998-2000	0.033	0.033	0.034	0.035	0.034	0.034
2000-2002	0.03	0.03	0.031	0.031	0.031	0.031
2002-2005	0.037	0.036	0.037	0.037	0.038	0.038
2005-2008	0.037	0.036	0.038	0.037	0.038	0.038
2008-2011	0.041	0.04	0.041	0.04	0.041	0.041
2011-2014	0.041	0.038	0.038	0.036	0.038	0.04
2014-2018	0.051	0.049	0.052	0.05	0.052	0.054
1998-2018	0.039	0.038	0.039	0.038	0.039	0.04

Model Comparison (log Score)

Table 3: Test log score for each model in the 8 samples.

	Gompertz	Beard	STP_Gompertz	STP_Beard	MTP	GPH
1998-2000	-1562.7	-1562.6	-1513.1	-1513.9	-1517.4	-1502.7
2000-2002	-1741.7	-1740	-1681.5	-1680.9	-1687.6	-1669.5
2002-2005	-2912.1	-2908.8	-2866.9	-2868.9	-2877.3	-2855.1
2005-2008	-2250.2	-2243	-2207.1	-2199.9	-2210	-2200.9
2008-2011	-2701.2	-2693.1	-2615.5	-2613.7	-2615.8	-2610.9
2011-2014	-1752.5	-1736.5	-1636	-1631	-1637.1	-1643.6
2014-2018	-1692.5	-1677.8	-1626.2	-1612.7	-1624.4	-1619.5
1998-2018	-15235	-15173.3	-14773.3	-14747.9	-14719.4	-14734.2