A two-generation model with altruism for reverse mortgage demand
Supplementary material

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Appendix A. Two-Generation Model with Parent Bequest

Previous studies have found home equity release products such as reverse mortgages to be welfare-enhancing by generating higher consumption for the parent at the expense of bequests (Hanewald et al., 2016, Andreasson and Shevchenko, 2021, Koo et al., 2022). However, these studies have not considered the financial impact on future generations, which we aim to capture with our new two-generation model with altruism. This section defines a more traditional utility framework, common in past studies. In this framework, the parent derives a bequest utility only in one period, upon death. We compare the two-generation model with parent bequest with our two-generation model with parent altruism, defined in Section 2.1.

A. Parent Bequest

Without altruism the parent no longer cares about the child's consumption and housing outcomes at each time t, and instead $U^P(t)$ satisfies

$$U^P(t) = I^{P,A}(t) \cdot \big(U(C^P(t), H^P(t))\big) + I^{P,D}(t) \cdot \theta \cdot B(W^P(t)). \tag{A.1}$$

This replaces $U^P(t)$ in equation 1, with the rest of Section 2.1 remaining the same. In this case, θ acts as a preference parameter for terminal bequest, for both generations. Our results show that such

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a model underestimates the welfare gains from the early bequest and fails to capture the parent's happiness from providing their child with a better future. Furthermore, given fixed weights ω_p and ω_c in equation 8, there is no parameter to control for the parent's preference to help their child through inter-generational wealth transfers made during retirement.

In contrast, in the presence of parent altruism, the child's current wellbeing, modelled by their consumption and housing utility $U(C^C(t), H^C(t))$, flows from child to parent at each time t the parent lives. Furthermore, gifts made during the parent's retirement, that can further impact the child's future wellbeing when they themselves retire, can be captured by replacing the parent's bequest utility with a more 'pure' form via the term $\rho \cdot V^C(T^P)$. This results in $U^P(t)$ for our two-generation model with parent altruism. This model allows us to further analyse the impact that parent altruism has on welfare gains from home equity release products such as reverse mortgages. In particular, focusing on major financial decisions for the family, where the 'Bank of Mum and Dad' provides their children with a lump sum gift for their first home deposit.

Appendix B. Age Pension

The Australian Age Pension is a means-tested public pension designed to provide a safety net to Australians who have limited means to support themselves in retirement. As of March 2023, 2.6 million elderly Australians (58% of those aged 65 and over) received a full or part Age Pension[1]. Therefore, modelling Age Pension income is an essential component of lifecycle models. Since July 1, 2023, the qualifying age for the Age Pension is 67 years. We apply the official income and asset means tests to compute the Age Pension entitlement that the parent receives, where the pension payment is based on the lowest entitlement calculated under each test. Moreover, as we model the ability of reverse mortgages to provide children with an early bequest, we consider the implications of 'gifts' on Age Pension payments. The allowable gift amount for Age Pensioners is $10,000 in one financial year and $30,000 over five financial years. Any gifts that exceed the allowable amount are counted as financial assets under the means-tests for up to 5 years. If the parent takes a reverse mortgage for personal income, we assume it is consumed immediately in the same year that it is paid, and thus is excluded from means-testing.

For the assets test, to be eligible to receive the full Age Pension $AP^{full}(t)$, the individual's assessable assets must be less than the asset threshold of $T^{asset}(t)$ (which differs by homeownership status). If the individual's asset value exceeds $T^{asset}(t)$, then there is a $3 reduction per fortnight in the Age Pension payment for every $1,000 of assets over $T^{asset}(t)$. Hence, we can describe the individual Age

[1]Australian Institute of Health and Welfare (31 March 2023). Income support for older Australians. Available at https://www.aihw.gov.au/reports/australias-welfare/income-support-for-older-australians

Pension entitlement under the asset test in year t as

$$AP^{asset}(t) = \begin{cases} \max(0, AP^{full}(t) - 0.003 \cdot (S(t) + FOA(t) - T^{asset}(t))), & \text{if } W(t) + S(t) \geqslant T^{asset}(t) \\ AP^{full}(t), & \text{otherwise,} \end{cases}$$

where $S(t)$ and $FOA(t)$ represent the superannuation and 'financial and other assets' at time t, respectively.

For the income test, an individual's eligibility for the Age Pension depends on her total assessable income, $I(t)$. Income from financial assets such as savings, shares and superannuation is deemed. Deeming assumes that financial investments earn a given rate of return, regardless of the amount of income they actually earn. We assume that both the parent and adult child stop receiving income from employment or other sources after retirement, and only receive income from financial assets that they either consume or save, depending on their consumption target. Thus, we assume total assessable income is equal to total deemed income, $I(t) = I^{deeming}(t)$. Total financial assets below the deeming rule threshold $T^{deeming}(t)$ contribute to the assessable income at a lower deeming rate, DR_1, while financial assets over the threshold are deemed at a higher rate defined as deeming rate, DR_2. Hence, the total deemed income from superannuation savings $S(t)$ and FOA $FOA(t)$ at time t, satisfies

$$I^{deeming}(t) = \max(S(t) + FOA(t), T^{deeming}(t)) * DR_1 + \max(0, S(t) + FOA(t) - T^{deeming}(t)) * DR_2.$$

The Age Pension entitlement under the income test is determined by comparing total assessable income with the income threshold, $T^{income}(t)$. The full Age Pension entitlement is reduced by 50 cents for every dollar above the income threshold. Hence, we can describe the individual Age Pension entitlement under the income test in year t as

$$AP^{income}(t) = \max\Big(0, AP^{full}(t) - 0.5 \cdot \max(0, I(t) - T^{income}(t))\Big).$$

The actual Age Pension amount is calculated as the minimum of the entitlement under both means-tests,

$$P(t) = \min(AP^{asset}(t), AP^{income}(t)).$$

Table B.10: Age Pension parameters (2022)

Parameter	Value	Description
$AP^{\text{full}}(t)$	\$25,677.6	Full Age Pension entitlement
$T^{\text{asset}}(t)$	\$280,000	Assets test threshold (Homeowner)
$T^{\text{asset}}(t)$	\$504,500	Assets test threshold (Non-Homeowner)
$T^{\text{deeming}}(t)$	\$56,400	Deeming threshold
$T^{\text{income}}(t)$	\$4,940	Income test threshold
DR_1	0.25%	Deeming rate 1
DR_2	2.25%	Deeming rate 2

Appendix C. Long-term Care Costs

We model long-term care (LTC) costs in retirement, which depend on the health state of the parent and adult child after the age of 67. The five health states, $G(t) \in \{1, 2, 3, 4, 5\}$, are healthy (state 1), mildly disabled at home needing care at cost LTC_2 (state 2), severely disabled at home needing care at cost LTC_3 (state 3), in residential aged care needing care at cost LTC_4 (state 4), and deceased (state 5). The health states are defined by the number of activities of daily living (ADLs) individuals cannot perform and whether they reside at home or in residential aged care. The six ADLs are eating, dressing, bathing, toileting, continence and mobility. States 1, 2, and 3 are defined by experiencing 0, 1, and 2-6 ADL difficulties, respectively.

The Australian government offers a Home Care Package (HCP) to help older persons access affordable care services. These services include support for activities of daily living (ADLs) such as hygiene, food preparation, and transportation. There are different levels of HCPs available based on individual needs, and the government subsidises most of the costs. However, older persons must pay basic and care fees. The HCP level is determined by the number of ADLs an individual can perform and their physical and mental health. In this study, we assume that in state 2, the retiree receives a level 2 (low care) HCP package, and in state 3, a level 4 (high care) HCP. State 4 is associated with the costs of the parent or adult child moving into residential aged care, based on means-tested care fees.

A. LTC costs for States 1, 2, 3, and 5

For both the healthy (state 1), $G(t) = 1$ and deceased (state 5), $G(t) = 5$, the individual does not require long-term care, and the LTC cost remains zero, $LTC(t) = 0$. In state 2 ($G(t) = 2$), individuals are provided with a Level 2 Home Care Package. The basic daily fee for this care level is set at \$10.66. In state 3 ($G(t) = 3$), individuals are provided with a Level 4 Home Care Package. The basic daily fee for this care level is set at \$11.26. The basic daily fees for all care levels are provided in Table C.11. In addition to the basic daily fees, the income tested care fee is calculated based on total testable income, including deemed income from financial assets. For our simulations, the only source of income is the Age Pension (if eligible), combined with income from superannuation and deemed income from FOA. All calculations are based on the schedule of fees and charges for residential and home care from 1 July 2022, published by the Australian Department of Health. We also show calculations for a single person, due to the assumption that the parent is a single female homeowner and the adult child is a single female non-homeowner at the start of time $t = 0$.

Table C.11: Basic Daily Fee ($) – Home Care and Residential Care

Service	Health State $G(t)$	Basic Daily Fee
Home care - level 1 package		\$10.08
Home care - level 2 package	State 2	\$10.66
Home care - level 3 package		\$10.97
Home care - level 4 package	State 3	\$11.26
Residential care	State 4	\$54.69

Let $I^{fee}(t)$ be the annual fee from income tested care fees, $I^{test}(t)$ be the total testable income and AP(t) be Age Pension entitlement at time t. If the individual receives full Age Pension entitlement, see Appendix B, or is single and has an annual income less than $T_1 = \$29,234.40$, then the income tested fees are zero, $I^{fee}(t) = 0$. A second higher income threshold is set at $T_2 = \$56,295.20$, after which a higher daily and annual cap for the income tested fees applies. Here we set $I^{test}(t) = AP(t) + I^{deemeed}(t)$, based on the assumptions of our model, with deemed income $I^{deemeed}(t)$ previously defined in Appendix B.

For every dollar that the total testable income $I^{test}(t)$ exceeds the income free threshold of T_1, 50 cents is added to the annual income tested care fee. If $I^{test}(t)$ is below T_2, then there is a daily cap of $D_1 = \$16.15$[2]. If $I^{test}(t)$ exceeds T_2, then the daily cap becomes $D_2 = \$32.30$. Thus,

$$I^{fee}(t) = \begin{cases} 0, & I^{test}(t) \leqslant T_1 \text{ or } AP(t) = AP^{full}(t), \\ \max(0.5 \cdot (I^{test}(t) - T_1), D_1 \cdot 364), & T_1 \leqslant I^{test}(t) \leqslant T_2, \\ D_1 + \max(0.5 \cdot (I^{test}(t) - T_2), D_2 \cdot 364), & I^{test}(t) \geqslant T_2. \end{cases}$$

Thus for state 2 ($G(t) = 2$), where individuals are provided with a Level 2 Home Care Package, the associated annual long-term care cost is $LTC_2 = 10.66 \cdot 364 + I^{fee}(t)$. Likewise, for state 3 ($G(t) = 3$), where individuals are provided with a Level 4 Home Care Package, the associated annual long-term care cost is $LTC_3 = 11.26 \cdot 364 + I^{fee}(t)$. All means-tested fees in states 2-4 have a lifetime cap of $LCAP = \$70,558.66$, after which only basic or additional services fees apply (including accommodation costs).

[2]We assume the annual caps are satisfied based on these daily caps.

Table C.12: Income tested fees ($) for Home Care Packages

Description	Variable	Rate
Income free threshold (single)	T_1	$29,234.40
Couple illness separated (single rate)	-	$28,662.40
Couple living together (single rate)	-	$22,679.80
First income fee daily cap	D_1	$16.15
Second Income threshold (single)	T_2	$56,295.20
Second income fee daily cap	D_2	$32.30
Lifetime cap	LCAP	$70,558.66

B. LTC cost for State 4

State 4 ($G(t) = 4$) pertains to individuals in residential aged care. Here, the base daily care fee is $54.69. However, the total cost might be augmented by a means-tested care fee, which is calculated based on the individual's income and assets. The income tested fee for residential care is again calculated based on the amount that total testable income $I^{test}(t)$ exceeds the income free threshold of T_1. Thus, $I^{fee,RC}(t)$ the annual income tested fee for residential care satisfies

$$I^{fee,RC}(t) = \max(0.5 \cdot (I^{test}(t) - T_1), 0).$$

The asset test for residential care considers total testable assets, similar to the calculations for Age Pension entitlement, however, residential care means-testing includes a portion of the primary home value as testable assets. The home exception cap in 2022 is $178,839.20, implying home equity that exceeds this amount is not tested. In our simulations, total testable assets $W^{test}(t)$ satisfies,

$$W^{test}(t) = S(t) + FOA(t) + \max(H(t) - L(t), 178,839.20),$$

where $S(t)$ is superannuation, $FOA(t)$ is 'financial and other assets', $H(t)$ is the house value, and $L(t)$ is the loan against the home, at time t. For the parent in Scenario 2 and 3, additional assets from gifting in excess of $10,000 are added to both total testable assets and also deemed for additional income, only for the first 5 years, $t < 5$.

Let $A^{fee,RC}(t)$ be the annual asset tested fee for residential care. If total testable assets $W^{test}(t)$ are below the asset free threshold $AT_0 = \$52,500$, then the asset tested fee is zero, $A^{fee,RC}(t) = 0$. There are then progressive rates based on the amount that total testable assets exceed two thresholds, the first asset threshold set at $AT_1 = \$178,839.20$ and the second asset threshold at $AT_2 = \$431,517.60$. Total testable assets between AT_0 and AT_1 are multiplied by asset rate 1, $r_1^{asset} = 17\%$. Total testable assets between AT_1 and AT_2 are multiplied by asset rate 2, $r_2^{asset} = 1\%$. Total testable assets exceeding AT_2 are multiplied by asset rate 3, $r_3^{asset} = 2\%$. Thus $A^{fee,RC}(t)$ satisfies

$$\begin{aligned} A^{fee,RC}(t) = r_1^{asset} \cdot \max(0, \min(W^{test}(t), AT_1) - AT_0), & \quad \text{(C.1)} \\ + r_2^{asset} \cdot \max(0, \min(W^{test}(t), AT_2) - AT_1), & \quad \text{(C.2)} \\ + r_3^{asset} \cdot \max(0, W^{test}(t) - AT_2). & \quad \text{(C.3)} \end{aligned}$$

The total means-tested fee for residential aged care, $MT^{fee}(t)$, is calculated based on first considering the amount paid in accommodation fees compared to the total fees paid due to both the income and asset tested fees for residential care, $I^{fee,RC}(t)+A^{fee,RC}(t)$. The cap on the daily cost of accommodation fees is \$60.47, thus the annual accommodation fee $AC^{fee}(t)$ satisfies

$$AC^{fee}(t) = \min(I^{fee,RC}(t) + A^{fee,RC}(t), 60.47 \cdot 364).$$

The excess fees due to means-testing define the total means-tested fee for residential care, which must satisfy both an annual cap of $ACAP = \$29,399.40$ and a lifetime cap of $LCAP = \$70,558.66$. Thus, $MT^{fee}(t)$ satisfies

$$MT^{fee}(t) = \min(\max(I^{fee,RC}(t) + A^{fee,RC}(t) - AC^{fee}(t), 0), LCAP). \tag{C.4}$$

The lifetime cap is controlled by considering the cumulative amount of means-tested fees paid up until time t. Once the lifetime cap is met, $MT^{fee}(t) = 0$ for the remaining years but basic, accommodation and additional service fees must still be paid. The final long-term care costs for state 4, $(G(t) = 4)$, can now be calculated by including the daily cost, accommodation and means-tested fee $LTC_4 = \$54.69 \cdot 364 + AC^{fee}(t) + MT^{fee}(t)$. In our model, we do not consider the daily fee of \$54.69 as a cost, but rather a living expense in residential care that covers the costs of daily living, such as meals, cleaning, laundry, heating and cooling. It is thus treated as consumption, a part of the consumption target, rather than an additional expense for long-term care. Thus in our model, we let $LTC_4 = AC^{fee}(t) + MT^{fee}(t)$, just covering accommodation and means-tested fees.

Table C.13: Means-tested fees for residential care as of 1 July 2022

Description	Variable	Rate
Income free threshold (single)	T_1	\$29,234.40
Home exemption cap	-	\$178,839.20
Asset free threshold	AT_0	\$52,500
First asset threshold	AT_1	\$178,839.20
Second asset threshold	AT_2	\$431,517.60
First asset test fee rate	r_1^{asset}	17%
Second asset test fee rate	r_2^{asset}	1%
Third asset test fee rate	r_3^{asset}	2%
Annual cap	ACAP	\$29,399.40
Lifetime cap	LCAP	\$70,558.66

Appendix D. Tax

Australia has a progressive personal income tax system, see Table D.14, that we apply to both the parent and child. When the parent is in retirement, she pays no tax on investment returns from her superannuation if it has been converted to an account-based pension and follows the minimum withdrawal rates set by the Australian Government, so we assume that this is the case. Investment returns from FOA are taxed based on the retired parents' tax bracket, accounting for means-tested

welfare payments from the Age Pension (not taxed, but included to determine the relevant tax bracket).

For the adult child, the progressive personal income tax schedule is applied to all income from employment, government welfare payments and FOA returns before retirement. Returns from superannuation are taxed at 15% in the hands of the superannuation fund[3]. After reaching the retirement age of 67, the adult child follows the same tax rules that the parent experienced during her retirement, immediately converting all superannuation savings into an account-based pension to gain tax benefits.

Australia does not impose death or estate taxes.

Table D.14: Australian residents tax rates for 2022-23

Taxable income ($)	Tax on this income ($)
0 – $18,200	Nil
$18,201 – $45,000	19c for each $1 over $18,200
$45,001 – $120,000	$5,092 plus 32.5c for each $1 over $45,000
$120,001 – $180,000	$29,467 plus 37c for each $1 over $120,000
$180,001 and over	$51,667 plus 45c for each $1 over $180,000

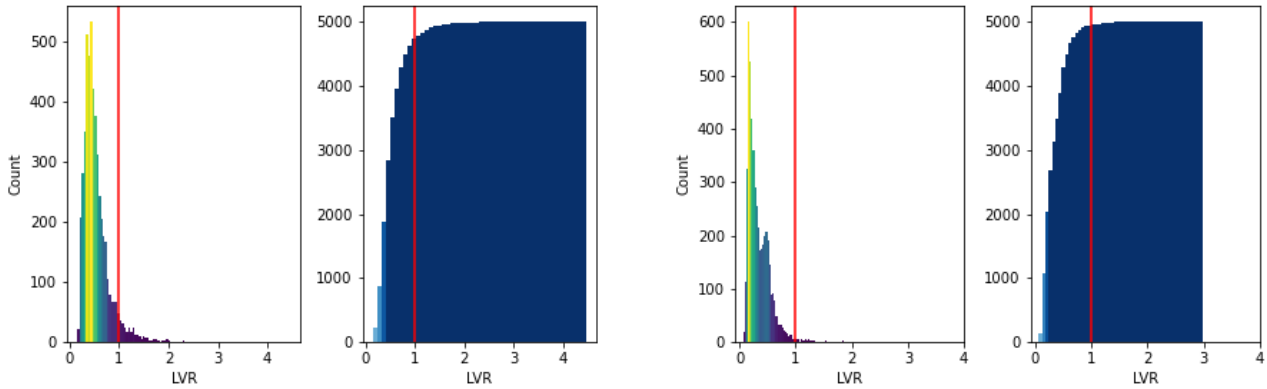
Appendix E. Reverse Mortgage

The parent has the option to take out a reverse mortgage. We make the following assumptions. First, we only consider parents whose primary residence is used as security for a reverse mortgage, which is consistent with the participation eligibility of most providers in Australia. Second, when used to supplement income, the parent is assumed to immediately consume the reverse mortgage income in the year it was borrowed, so that it does not affect Age Pension entitlement. Additionally, if the parent chooses to take out a lump sum reverse mortgage and gift the amount to their adult child, any amount above the allowable gifting limit of $10,000 per financial year, or $30,000 over a rolling five-year period, will be considered a deprived asset. This means the excess amount will still be counted in the asset test for five years and can affect the Age Pension due to the deemed income from the deprived asset.

The parent's final LVR when the home is sold in Scenario 3 is shown in Figure E.1 when both the parent and child are in Q1 (left) and Q4 (right). Lower starting wealth (Q1) results in a higher proportion of parents hitting the maximum LVR ratio, where 312 (6.24%) parents, based on 5,000 simulations, have an LVR greater than 1. The average LVR for parents in Q1 is 0.55, with a median of 0.48 and a standard deviation of 0.29.

[3]This is based on superannuation contributions below the annual concessional contributions cap, currently set at $27,500 (FY2023/24). We do not consider the tax benefits from imputation credits in Australia which could further reduce tax.

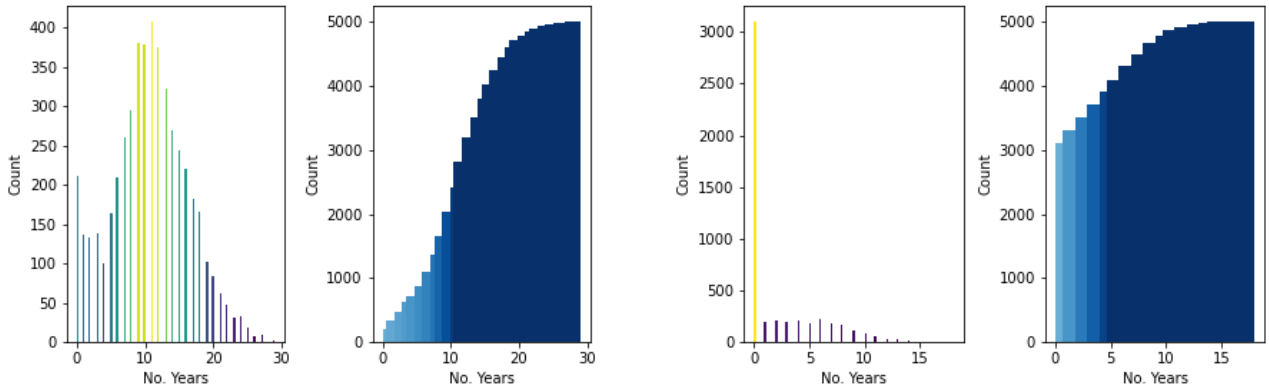
Figure E.1: Parents' reverse mortgage LVR



Note: Histogram and CDF for parents' LVR when both parent and child start in Q1 (left) and Q4 (right) for Scenario 3.

We also compare the number of years the parents use the reverse mortgage for personal income when their liquid assets run out, and they need to access home equity to reach their consumption target. Figure E.2 shows the number of years when the parent and child are both in Q1 (left) and Q4 (right). These results are only for Scenario 1 when the parent does not gift and uses the reverse mortgage only for personal income. We compare this to the results of Scenario 3, in Figure E.3, as now the parent gifts their child a lump sum equal to 20% of their first home deposit, significantly reducing the number of years the parent can use the reverse mortgage for personal income. When the parent and child are both in Q1, the parent can use a reverse mortgage for income for 10.7 years on average in Scenario 1, compared to only 4.1 years in Scenario 3. Whilst overall, there is a utility gain in aggregate total lifetime utility, the parent is sacrificing personal income for the future wellbeing of the child.

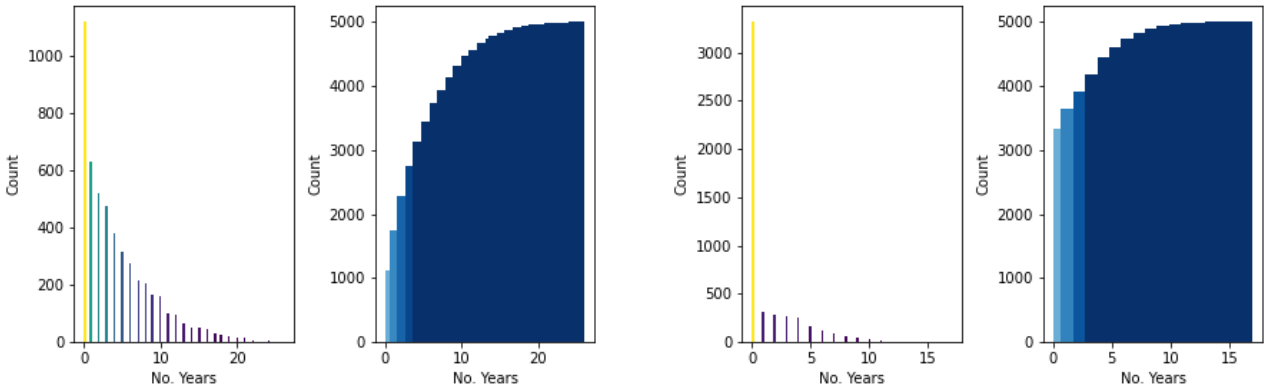
Figure E.2: Number of years parent uses reverse mortgage for personal income in Scenario 1



Note: Histogram and CDF for number of years the parent uses reverse mortgage for personal income in Scenario 1. The parent and child are assumed to both be in Q1 (left) and Q4 (right).

Table E.15 reports the maximum loan-to-value (LVR) ratios used for the commercial reverse mortgage products in all scenarios, based on the ratios advertised by Heartland, Australia's largest reverse mortgage provider.

Figure E.3: Number of years parent uses reverse mortgage for personal income in Scenario 3



Note: Histogram and CDF for number of years the parent uses reverse mortgage for personal income in Scenario 3. The parent and child are assumed to both be in Q1 (left) and Q4 (right).

Table E.15: Reverse mortgage LVR

Age	LVR	Age	LVR	Age	LVR	Age	LVR	Age	LVR
67	27%	72	32%	77	37%	82	42%	87	47%
68	28%	73	33%	78	38%	83	43%	88	48%
69	29%	74	34%	79	39%	84	44%	89	49%
70	30%	75	35%	80	40%	85	45%	90	50%
71	31%	76	36%	81	41%	86	46%	90+	50%

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