

Update on R Packages: Retirement Income Toolkit and Affine Mortality Models

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 - **Economic Scenario Generator:** generates scenarios for main economic variables, such as equity returns, dividend yields, GDP and so on;
 - **Cash Flow Simulation and Pricing:** uses the output from the other modules to simulate the cash-flows from a wide range of products, supporting the research about product comparison.

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```
install.packages("devtools")  
library(devtools)  
devtools::install_github("https://github.com/print-hi/rit")
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where

- $\Delta \in \mathbb{R}^{M \times M}$ is the mean reversion matrix;
- $\theta^Q \in \mathbb{R}^M$ is the long term mean of the process;
- $\Sigma \in \mathbb{R}^{M \times M}$ is the volatility matrix;
- $W^Q(t)$ is a standard Brownian motion;
- $D(X(t), t)$ is a diagonal matrix;

The affine framework (Duffie and Kan (1996))

- Let $\mu(t) = X_1(t) + \dots + X_M(t)$;
- The survival probability of newborn in year t until time T , $S(t, T)$, is modelled as an exponentially affine function of $X(t)$:

$$\begin{aligned} S(t, T) &= \mathbb{E} \left[\exp \left(- \int_t^T \mu(t, s) ds \right) \mid \mathcal{F}_t \right] \\ &= \exp [A(t, T) + B(t, T)' X(t)] \end{aligned} \quad (2)$$

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- The factor loading $B(t, T)$ and $A(t, T)$ depend on the mortality dynamics specified for $X(t)$;

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```
library(devtools)
install_github("ungolof/AffineMortality")
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- Analysis of age-period models;

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- Improve the analysis techniques to account for incomplete cohort datasets;

References

- Y. Shen, M. Sherris, A. M. Villegas, J. Ziveyi et. al. (2023), *Modelling Retirement Income Risks and Solutions with rit: A Retirement Income Toolkit in R*, Working Paper;
- F. Ungolo, L.P.D.M. Garces, M. Sherris, Y. Zhou (2023a), *Estimation, Comparison and Projection of Multi-factor Age-Cohort Affine Mortality Models*, to appear on the North American Actuarial Journal;
- F. Ungolo, L.P.D.M. Garces, M. Sherris, Y. Zhou (2023b), *AffineMortality: An R package for estimation, analysis and projection of affine mortality models*, CEPAR Working Paper;