



**UNIL** | Université de Lausanne

HEC Lausanne

## **The Role of Tontines in Retirement Decumulation**

**Peter Hieber, HEC Lausanne, Switzerland,**

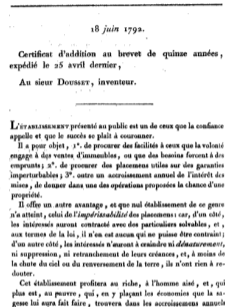
8th International Pension Research Association (IPRA),  
Conference, Paris, 14.06.2023

# Agenda

1. Motivation: Mutual (life/pension) insurance products = tontines
2. How to design **heterogeneous** mutual (**life/pension**) insurance
  - a. Share common cash-flow
  - b. Combine actuarially fair individual accounts
3. Extensions and interesting research questions

## Past and future of tontines?! . . .

- ▶ **Mutual insurance** = Risks are shared within a pool.  
*Tontines, pooled annuities, survivor funds, group self annuitization* (used as synonyms).
- ▶ **Dominant insurance** several hundred years ago.
- ▶ **Similar popularity soon!?** (digitalization, risk regulation).
- ▶ Li, Y., & Rothschild, C. (2020). Selection and redistribution in the Irish tontines of 1773, 1775, and 1777. **Journal of Risk and Insurance**, 87(3), 719-750.
- ▶ Milevsky, M. A. (2015). King William's tontine: Why the retirement annuity of the future should resemble its past. Cambridge University Press.



... they start a revival today:

The New York Times

## *When Others Die, Tontine Investors Win*



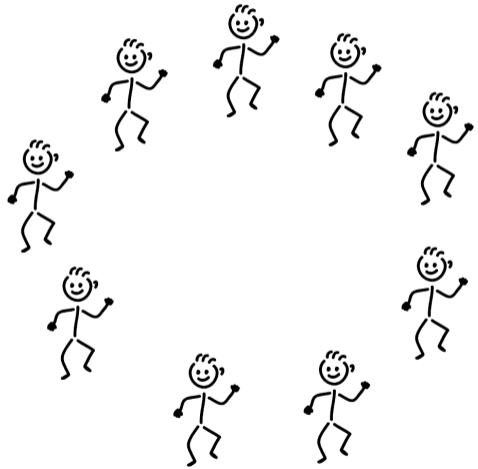
By Tom Verde

March 24, 2017

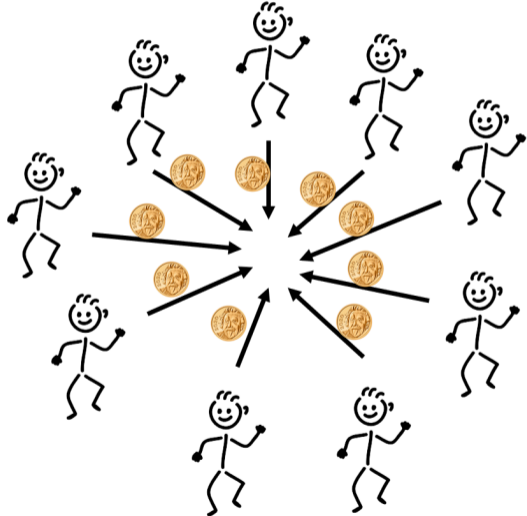
Living a long life is its own reward. But when you invest in a [tontine](#), there's an added benefit: You collect money that would have gone to people who have died.

That is part of the macabre appeal of the tontine, a 350-year-old investment vehicle that fell into disfavor more than a century ago but is now getting fresh consideration as a way to help people receive steady income in retirement.

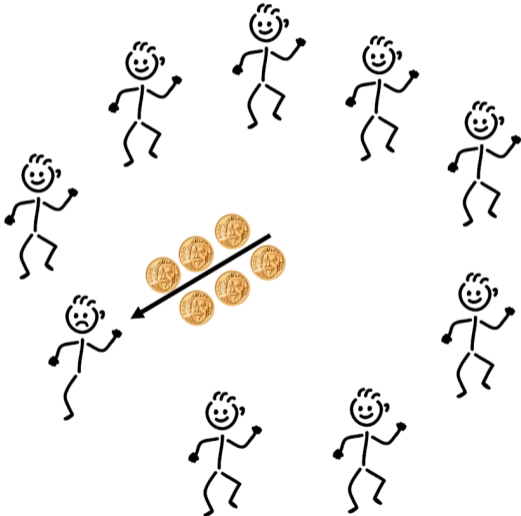
# Mutual insurance: Premium payments



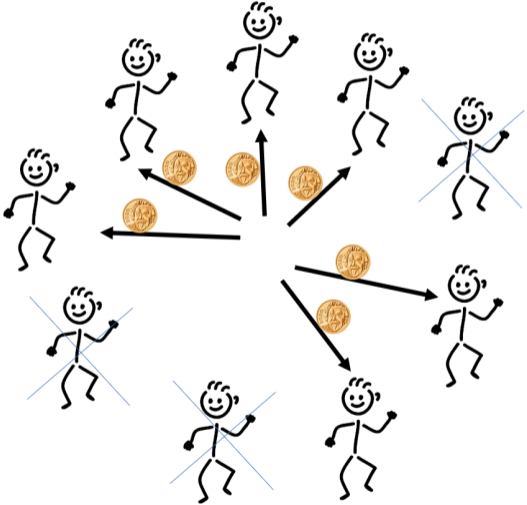
# Mutual insurance: Premium payments



# Mutual insurance: Disability benefit

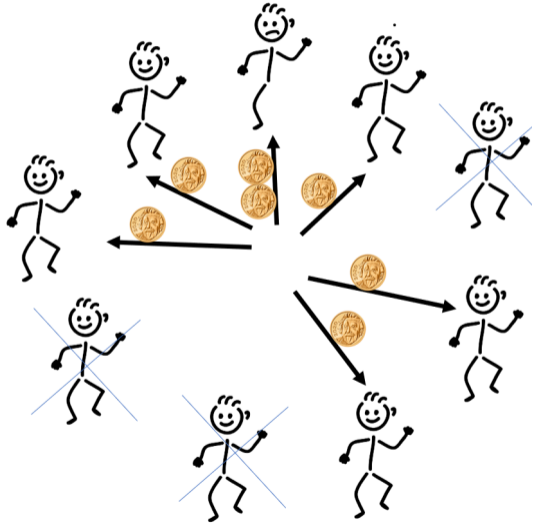


# Mutual insurance: Modern tontine





# Mutual insurance: Modern Life-Care tontine



## Trends in life and pension products

World-wide trend for life and pension products:

- ▶ Transparent.
- ▶ Cost-efficient.
- ▶ Defined-contribution (**mutual insurance instead of guarantees**).
- ▶ Participate in capital markets (stock investments).

See also **pan-European Personal Pension Product (PEPP)** (EU) 2019/1238.

Examples follow on next slides.

## Many initiatives to move to tontine-like products

- ▶ **Superannuation** fund: [Australian](#) retirement trust (formerly QSuper).
- ▶ **Variable payment life annuities** (VPLAs) in [Canada](#).
- ▶ **CPF LIFE Scheme** in [Singapore](#).
- ▶ **Le Conservateur** in [France](#).
- ▶ **Tontine Trust** in [Ireland](#).
- ▶ ...

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## Designing **heterogeneous** mutual insurance schemes

We want to:

- ▶ Design single-premium mutual insurance schemes that pay **1 for life**.
- ▶ Pool members are **heterogeneous** (by age, health).
- ▶ This is a **multi-period scheme**.
- ▶ (For now), we **ignore financial risk** and consider **mortality risk of individuals** as **independent**.

## Designing heterogeneous mutual insurance schemes

There are three (similar but different) approaches:

(a) **Share common cash-flow in a survivor pool**

special case: “**natural tontine**” ([Milevsky, Salisbury \[2015\]](#))

(b) **Combine actuarially fair individual tontine accounts** (see, e.g., [Sabin, Fullmer \[2010, 2018\]](#), [Donnelly, Guillén, Nielsen \[2013, 2014\]](#), [Denuit \[2019\]](#), [Hieber, Lucas \[2022\]](#), [Denuit, Hieber, Robert \[2022\]](#) and many more).

(c) **Adjust realized vs. projected mortality**

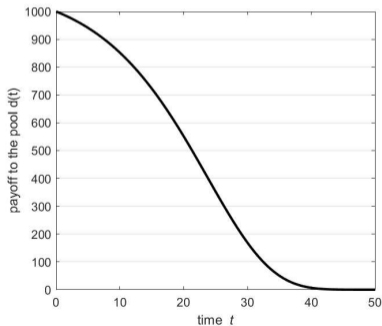
(see, e.g. [Piggott, Valdez, Detzel \[2005\]](#), [Qiao, Sherris \[2013\]](#)).

## (a) Share common cash-flow

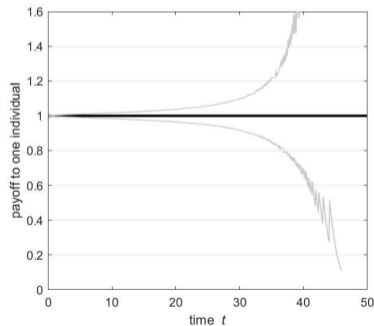
Introduce “**natural tontine**”: homogeneous pool of  $n$  receives ( $n = 1\,000$ ,  $x = 65$ )

$$d(t) = n \cdot {}_t p_x . \quad (\text{pool payoff})$$

**pool**



**individual**

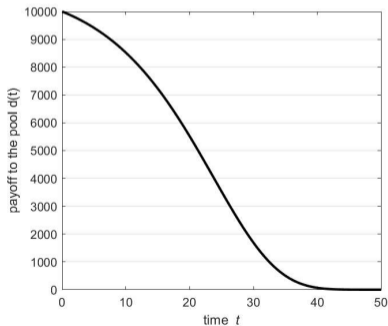


## (a) Share common cash-flow

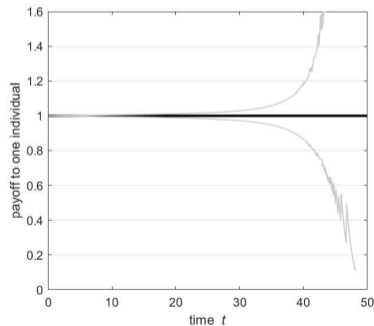
Introduce “**natural tontine**”: homogeneous pool of  $n$  receives ( $n = 10\,000$ ,  $x = 65$ )

$$d(t) = n \cdot {}_t p_x . \quad (\text{pool payoff})$$

**pool**



**individual**





## (a) Share common cash-flow

- ▶ Payoff  $d(t)$  is a purely financial payoff, no mortality risk!
- ▶ Payoff to individual is risky.
- ▶ Extension to heterogeneous pools (for example by age) is possible (Milevsky, Salisbury [2016]).
- ▶ This is a closed-pool: Decreasing pool-size leads to high volatility for high ages. ( $\implies$  **ton(tine-ann)uity.**)

Chen, A., Hieber, P., Klein, J. K. (2019). *Tonuity: A novel individual-oriented retirement plan*. **ASTIN Bulletin**, 49(1), 5-30.

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## (b) Combine actuarially fair individual accounts

- ▶ Pool members  $\mathcal{L}_0 = \{1, 2, \dots, n\}$ . Time in periods  $t = 0, 1, 2, \dots$
- ▶ The financial market is deterministic at an annual return  $\delta_t$ .
- ▶ Individual  $j \in \mathcal{L}_0$  contributes **single premium**  $c_j(0)$  at time 0.
- ▶ Each year  $t = 1, 2, \dots$ , individual withdraws  $s_j(t)$  from its individual account  $c_j(t)$ :

$$c_j(t) = e^{\int_{t-1}^t \delta_s ds} c_j(t-1) - s_j(t).$$

- ▶ The account is lost upon death.

## (b) Combine actuarially fair individual accounts

We sum over the **accounts of deceased** in  $(t - 1, t]$  (**mortality credits**):

$$X(t) = \sum_{j \in \mathcal{D}_t} e^{\int_{t-1}^t \delta_s ds} c_j(t-1).$$

A **survivor**  $j \in \mathcal{L}_t$  receives in  $t$ :  $W_j(t) = s_j(t) + \beta_j(X(t))$ , where:

- $s_j(t)$ : individual, **fixed withdrawal amount**,
- $\beta_j(X(t))$ : **collective part** of the benefits, i.e. the mortality credits.

## (b) Combine actuarially fair individual accounts

Key assumptions on the sharing rule  $\beta_j$ :

- ▶ **Self-sufficiency property:**  $\sum_{j \in \mathcal{L}_{t-1}} \beta_j(X(t)) = X(t)$ .
- ▶ **Actuarial fairness property:**

$$\mathbb{E}_{t-1}[\beta_j(X(t))] = \underbrace{q_{x_j+t-1}}_{\text{probability to die in } (t-1, t]} \cdot \underbrace{e^{\int_{t-1}^t \delta_{sds}} c_j(t-1)}_{\text{amount at risk at time } t}. \quad (1)$$

Denuit, M. (2020). Investing in your own and peers' risks: The simple analytics of P2P insurance. *European Actuarial Journal*, 10(2), 335-359.

Hieber, P., & Lucas, N. (2022). Modern life-care tontines. *ASTIN Bulletin*, 52(2), 563.

## Example: Sharing rule

Share linearly according to (1) **death probability** and (2) **amount invested**.

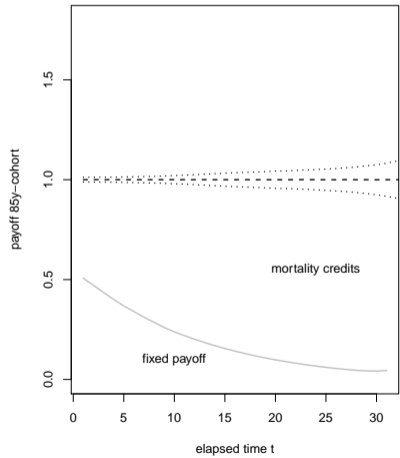
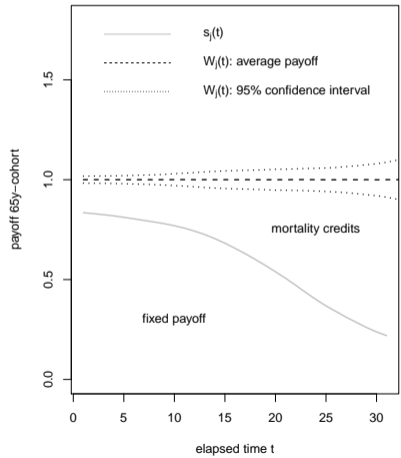
### Example (Linear risk sharing rule)

At time  $t$ , each individual  $j \in \mathcal{L}_{t-1}$  receives the mortality credit:

$$\beta_j(X(t)) = \frac{q_{x_j+t-1} \cdot c_j(t-1)}{\sum_{j \in \mathcal{L}_{t-1}} q_{x_j+t-1} \cdot c_j(t-1)} \cdot X(t). \quad (2)$$

(see, e.g., [Donnelly, Guillén, Nielsen \[2013, 2014\]](#), [Schumacher \[2018\]](#))

# Numerical example, two groups/cohorts



## Actuarial fairness: Insurer's view

For each  $t = 0, 1, \dots$ , the premium equivalence holds: (**pool view**)

$$\underbrace{\sum_{j=1}^n c_j(t)}_{\text{total account values}} = \sum_{j=1}^n \underbrace{\sum_{s=t+1}^{\omega-x_j} e^{-\int_t^s \delta_u du} W_j(s)}_{\text{discounted future benefits individual } j} . \quad (3)$$

- ▶ Right hand side: **random** (big letter!)
- ▶ Left hand side: **deterministic**. (this determines mutual insurance!)



## Actuarial fairness: Individual's view

For each  $t = 0, 1, \dots$ , the contract is fully-funded: (individual view)

$$\underbrace{c_j(t)}_{\text{retrospective reserve}} = \mathbb{E}_t \left[ \underbrace{\sum_{s=t+1}^{\omega-x_j} e^{-\int_t^s \delta_u du} W_j(s)}_{\text{prospective reserve}} \right]. \quad (4)$$

Expected present value of future benefits equals the current account value.

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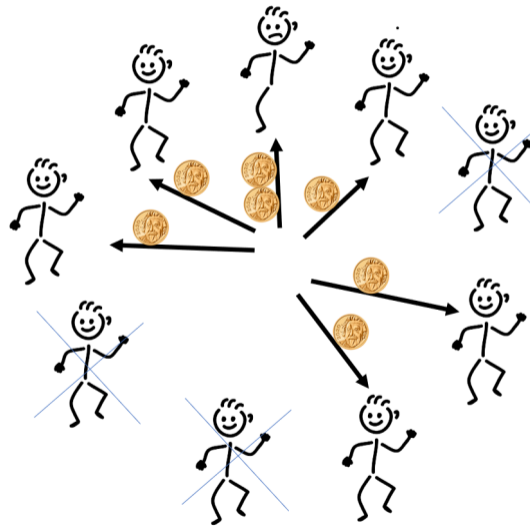
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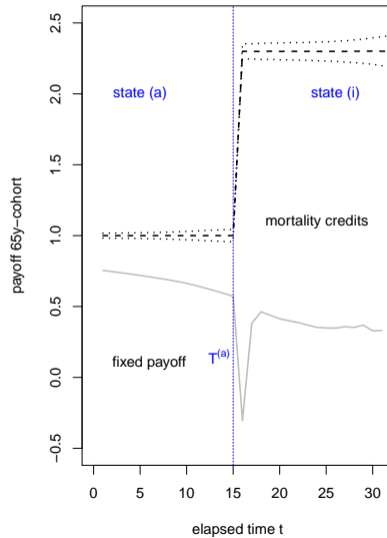
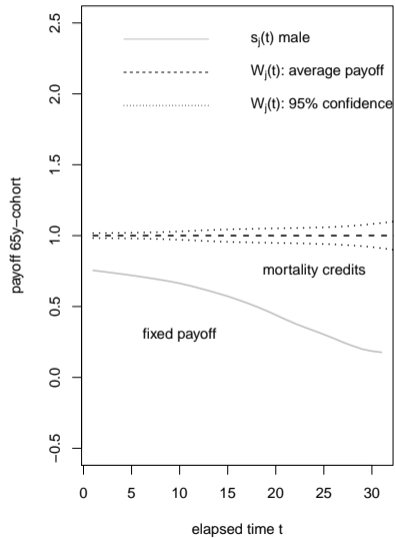
## Modern Life-Care tontine

- ▶ An ageing population faces **longevity risk**, **demographic risk** but also **morbidity risks** (long-term care).
- ▶ **Idea**: Offer a combined product with higher payments in dependency.
- ▶ **Why?**:
  - Mortality and morbidity risks are *negatively correlated*.
  - Cost reduction due to **reduced adverse selection!**
- ▶ **How?**: Mortality rates account for dependency  
⇒ Higher mortality credits for dependent people.

Hieber, P., & Lucas, N. (2022). Modern life-care tontines. **ASTIN Bulletin**, 52(2), 563.

# Modern Life-Care tontine





## Some comments:

- ▶ There is need to extend this also to **financial risk** (random return  $\delta_t$ ). (see **Donnelly, Guillén, Nielsen [2014]**).
- ▶ Actuarial fairness at all times allows people to **join later**.
- ▶ Mortality tables are used to **fairly assign mortality credits**. If there is agreement, the tables can be **updated over time**.
- ▶ It is possible to design a tontine with **premium refund option**. This simply reduces risk sharing. (*important element of QSuper Australian pension fund*)

## Thank you!

(b) individual accounts, (a) shared cash-flow

- (b) Hieber, P., Lucas, N. (2022). Modern life-care tontines. **ASTIN Bulletin**, 52(2), 563.
- (b) Denuit, M., Hieber, P., and Robert, C. Y. (2022). Mortality credits within large survivor funds. **ASTIN Bulletin**, 52(3), 813-834.
- (b) Denuit, M. (2019). Size-biased transform and conditional mean risk sharing, with application to P2P insurance and tontines. **ASTIN Bulletin**, 49(3), 591-617.
- (b) Donnelly, C., Guillén, M., and Nielsen, J. P. (2014). Bringing cost transparency to the life annuity market. **Insurance: Mathematics and Economics**, 56, 14-27.
- (a) Milevsky, M. A., and Salisbury, T. S. (2016). Equitable retirement income tontines: Mixing cohorts without discriminating. **ASTIN Bulletin**, 46(3), 571-604.
- (a) Chen, A., Hieber, P., and Klein, J. K. (2019). Tonuity: A novel individual-oriented retirement plan. **ASTIN Bulletin**, 49(1), 5-30.
- (a) Chen, A., Hieber, P., & Rach, M. (2021). Optimal retirement products under subjective mortality beliefs. **Insurance: Mathematics and Economics**, 101.

# Questions? Comments?





## Modern Life-Care Tontine

Using real mortality/disability (France), we compute the quotient

$$\alpha(T^{(a)}) = \frac{\ddot{a}_x^{\text{active}}}{\ddot{a}_x^{\text{disabled}}},$$

as a function of the time until disability  $T^{(a)}$ :

