Designing Sustainable, Stable and Equitable Payouts under Sociodemographic Longevity Inequality

Presenter: Gayani Thalagoda
Joint work with: Katja Hanewald, Andrés Villegas, Jonathan Ziveyi

School of Risk and Actuarial Studies, UNSW Sydney

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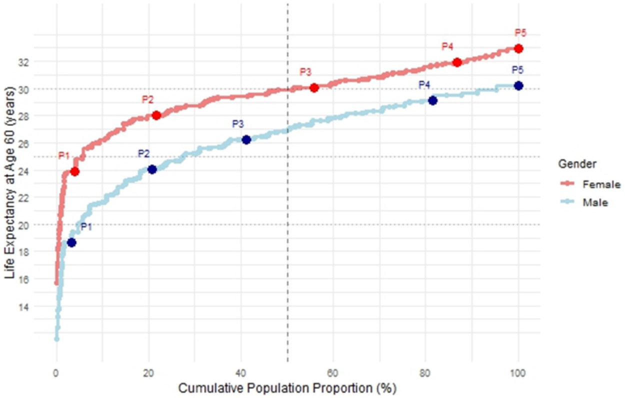
Longevity inequalities are well known

Australian period life expectancy during 2016-2017 at age 60 (Source: Huang et al., 2024)



Unpriced inequalities → Fairness (equitability) issues in Longevity Risk-Sharing arrangements

Longevity risk-sharing (LRS) arrangements

- Provide income for life by pooling and mutually sharing longevity risk
- Defining feature: At least part of the longevity risk remains uninsured

  ART Lifetime (AU), UniSuper Lifetime (AU), Digital Tontine (Sweden), TontineIRA (US)
- Wealth of short-lived subsidize the long-lived
- Not underwritten, age-based uniform pricing

When same-aged participants are heterogeneous,

unpriced inequalities $\rightarrow$ Fairness (equitability) issues in LRS arrangements

Related insights

Extensive research on risk-sharing designs: GSA, Tontines, Pooled Annuity Overlays

Piggott et al. (2005); Sabin (2010); Donnelly et al. (2014); Milevsky & Salisbury (2015); Qiao and Sherris (2013); Sabin and Forman (2016); Fullmer and Sabin (2019); Olivieri et al. (2022); Bégin and Sanders (2024); Hieber and Lucas (2022).

- **Homogeneous participants**: payout design is well understood
  Denuit et al. (2022); Jiao et al. (2022); Feng & Liu (2024)
- **Heterogeneous participants**: challenges remain
  - Actuarial fairness for all cohorts cannot be achieved simultaneously, only collective fairness
    Milevsky and Salisbury (2016); Chen et al. (2023); Bernard et al. (2025)
  - Even actuarially fair designs can create inequitable cross-subsidization, leading to selection
    Moenig & Zhu (2024); Thalagoda et al. (2025); Milevsky et al. (2025)
  - Joint wealth-longevity heterogeneity: alternative sharing rules depend on social cohesion
    Denuit et al. (2022); Milevsky & Dhaene (2024)

Importance of equitability in LRS

Result: Misalignment between longevity risk profile and expected benefit consumption profile

Differences in lifespan distribution by sociodemographic status

$\Longrightarrow$ Unpriced longevity inequalities (Uniform payout rule)

$\Longrightarrow$ Some may overpay relative to their risk, while others underpay (ex-ante)

$\Longrightarrow$ Implicit wealth transfers that disproportionately favor longer-lived groups

$\Longrightarrow$ Inequitable distribution of lifetime benefit consumption

Implications of inequitable LRS

Differences in lifespan distribution by sociodemographic status

$\implies$ Unpriced longevity inequalities (Uniform payout rule)

$\implies$ Some participants may overpay relative to their risk, while others underpay (ex-ante)

$\implies$ Implicit wealth transfers that disproportionately favour longer-lived groups

$\implies$ Inequitable distribution of lifetime benefit consumption

$\implies$ Perceived as 'unfair' by groups with shorter expected lifetimes

$\implies$ Reduced willingness to participate

$\implies$ Adverse selection

Our work: Enforce equitability as a counteraction (Fehr & Schmidt, 1999)

$\implies$ Improve perceived fairness

$\implies$ More willingness to participate

$\implies$ Reduced adverse selection

Our contribution: Menu of sustainable, stable and equitable payout rules

$$\min_{B\in\mathcal{C}} \left(\lambda_S\, S(B),\ \lambda_E\, E(B)\right) \quad \text{s.t.} \quad \int_0^{\omega-x} e^{-\delta t}\, S(t)\, B(t)\, dt = W_0,$$

$$B^\star \in \arg\max_{B\in\mathcal{P}} \sum_{k=1}^{K} w_k\, \mathbb{E}[U_k(b_k)].$$

$B(t)$: fund-level payout at time t,

$b_k(t) = \dfrac{p_k W_k}{\sum_{i=1}^{K} p_i W_i} B(t)$: average benefit per surviving member of group k,

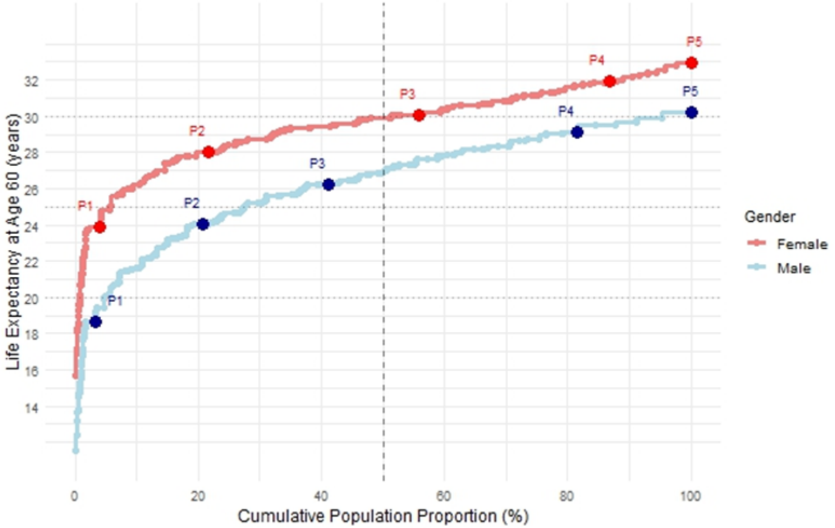
$E(B) = \sum_{k<h} \frac{p_k\, W_k\, p_h\, W_h}{(\sum_{i=1}^{K} p_i\, W_i)^2} (\mathcal{B}_{hk} - \mathcal{B}_{kh})$: measures equitability loss (Thalagoda et al., 2025),

$S(B) = \displaystyle\int_0^{\omega-x} \left(\dot{B}(t)\right)^2 dt$: measures stability loss,

$\displaystyle\int_0^{\omega-x} e^{-\delta t}\, S(t)B(t)\, dt = W_0$: ensures sustainability.

Numerical Application:
Australian Socio-economic Profiles

Australian period life expectancy during 2016-2017 at age 60 (Source: Huang et al., 2024)



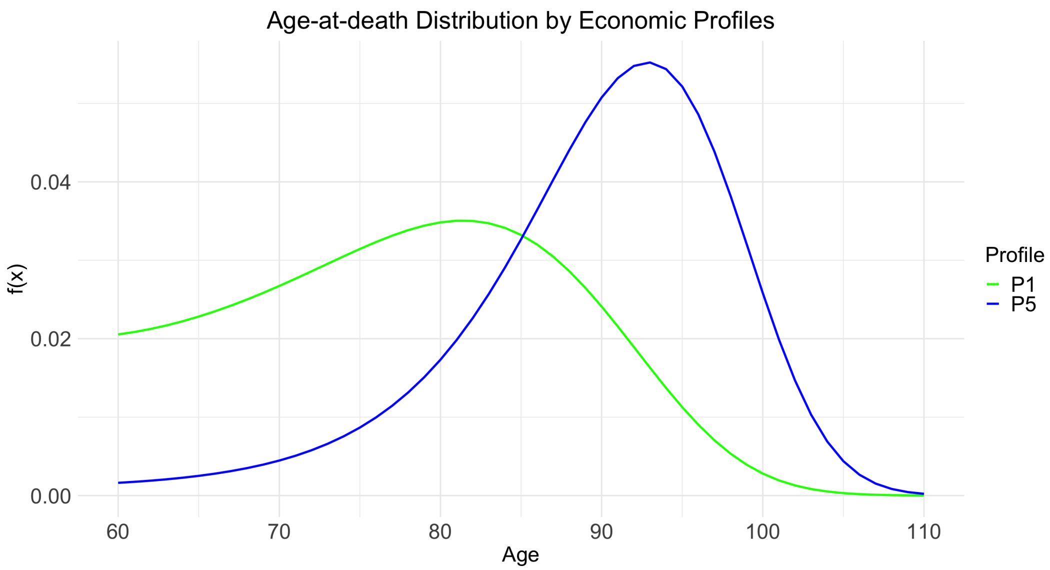
Pool Setup

- Age: 60-old males
- Groups:

  P5: Married, homeowner, $>$ \$1,000, D10-IRSAD: Tend to live longer and collect more payouts

  P1: Single, non-homeowner, $<$\$499, D1-IRSAD: Tend to leave early and subsidize the long-lived group

- Pool composition: (50%, 50%)
- Initial investment: (\$1,000k, \$1,000k)
- Closed pool


Age-at-death Distribution by Economic Profiles
0.04
0.02
0.00
f(x)
60
70
80
90
100
110
Age
Profile
P1
P5

Design challenge: Menu of sustainable, stable and equitable payout rules

Find $B(t)$ that minimizes stability loss and equitability loss subject to

$$\int_0^\infty e^{-\delta t} S^{\text{Realised}}(t) B(t) dt = \text{Initial Wealth}$$

Note: The payout at time t of the risk sharing product is given by:

$$B(t) = B(0) \frac{S^{\text{Pricing}}(t)}{S^{\text{Realised}}(t)}$$

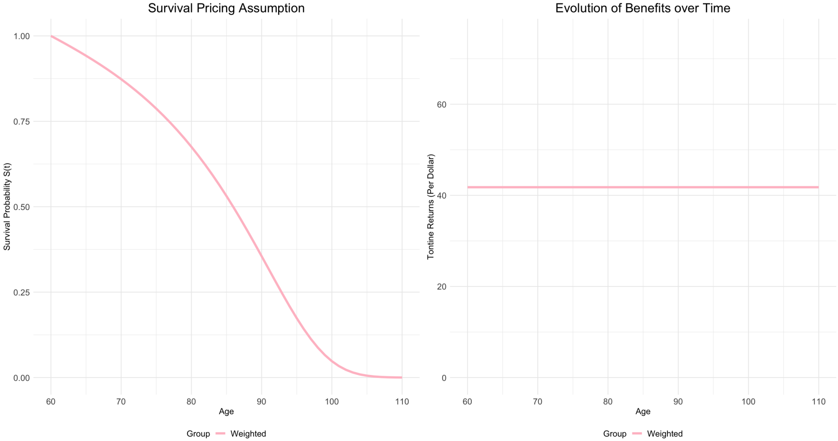
.

$$\boxed{\text{How to choose } S^{\text{Pricing}}(t)?}$$

Payout Stability

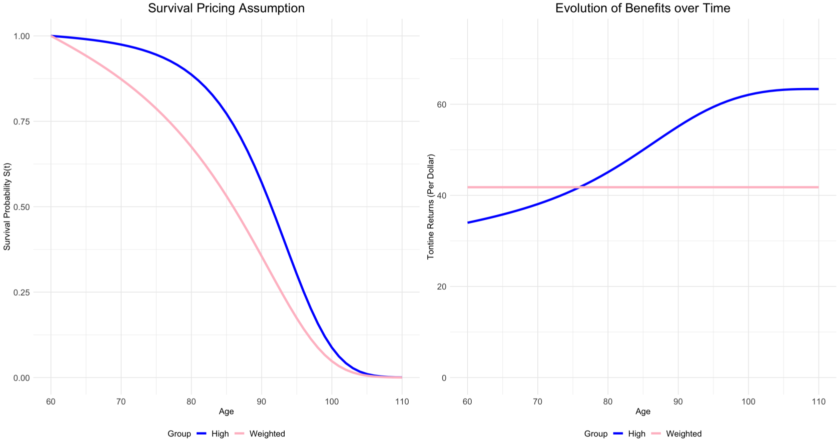
Payout design challenge: How to choose $S^{\text{Pricing}}(t)$?

$$B(t) = B(0)\frac{S^{\text{Pricing}}(t)}{S^{\text{Realised}}(t)}$$



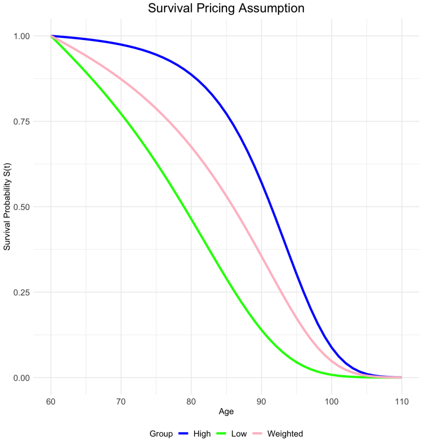
Payout design challenge: How to choose $S^{\text{Pricing}}(t)$?

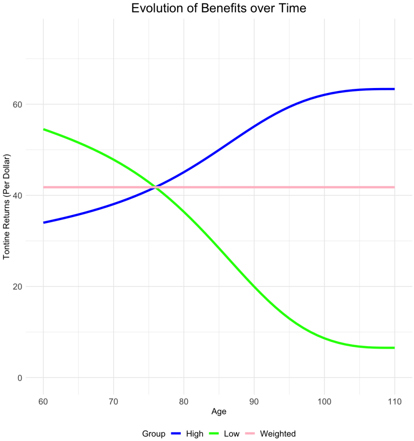
$$B(t) = B(0)\frac{S^{\text{Pricing}}(t)}{S^{\text{Realised}}(t)}$$



Payout design challenge: How to choose $S^{\text{Pricing}}(t)$?

$$B(t) = B(0)\frac{S^{\text{Pricing}}(t)}{S^{\text{Realised}}(t)}$$





Payout Equitability

Benefit Equitability (Thalagoda et al., 2025)

$$\underbrace{\Delta\mathcal{B}}_{\text{total inequality}} = \underbrace{\sum_{k=1}^{K}\left(\frac{p_k W_k}{\left(\sum_{i=1}^{K} p_i W_i\right)^2}\right)^2 \Delta\mathcal{B}_{kk}}_{\text{within}} + \underbrace{\sum_{k=2}^{K}\sum_{h=1}^{k-1} 2\,\frac{p_k W_k\, p_h W_h}{\left(\sum_{i=1}^{K} p_i W_i\right)^2}\,\Delta\mathcal{B}_{kh}}_{\text{between}}$$

$$= \sum_{k=1}^{K}\left(\frac{p_k W_k}{\left(\sum_{i=1}^{K} p_i W_i\right)^2}\right)^2 \Delta\mathcal{B}_{kk} + \sum_{k=2}^{K}\sum_{h=1}^{k-1} \frac{p_k W_k\, p_h W_h}{\left(\sum_{i=1}^{K} p_i W_i\right)^2}\,\underbrace{\left(\mathcal{B}_{hk} + \mathcal{B}_{kh}\right)}_{\text{inequitability}}$$

Quantifying Equitability (Per Dollar Invested)

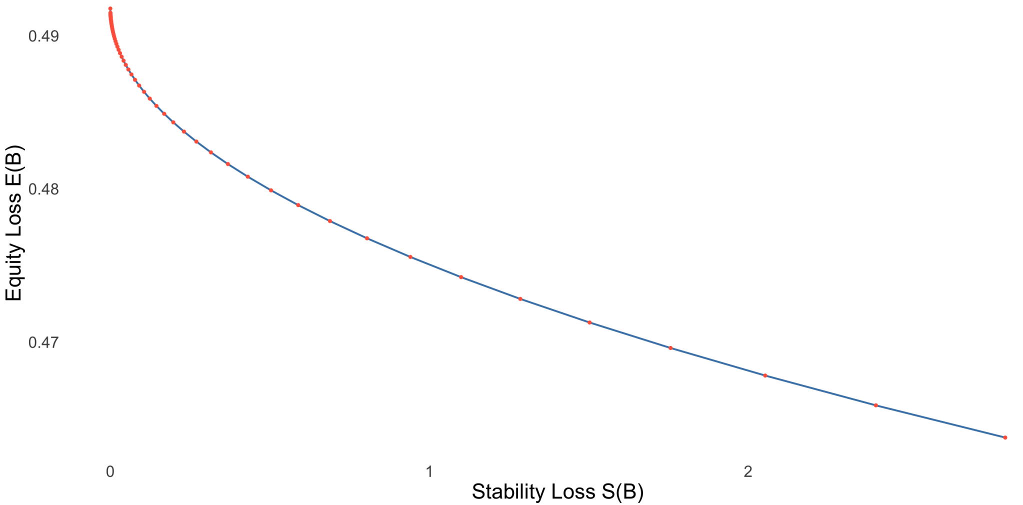
$$\Delta\mathcal{B} = \frac{1}{4}\,\Delta\mathcal{B}_{P1,P1} + \frac{1}{4}\,\Delta\mathcal{B}_{P5,P5} + \frac{1}{4}\,\mathcal{B}_{P5,P1} + \frac{1}{4}\,\mathcal{B}_{P1,P5}$$

Pricing Assumption	$\Delta\mathcal{B}_{P1,P1}$	$\Delta\mathcal{B}_{P5,P5}$	$\mathcal{B}_{P5,P1}$	$\mathcal{B}_{P1,P5}$	$(\mathcal{B}_{P5,P1} - \mathcal{B}_{P1,P5})$
Weighted	\$0.47	\$0.38	\$0.54	\$0.06	-\$0.48
Low-Income	\$0.41	\$0.21	\$0.39	\$0.04	-\$0.35
High-Income	\$0.51	\$0.48	\$0.63	\$0.07	-\$0.56

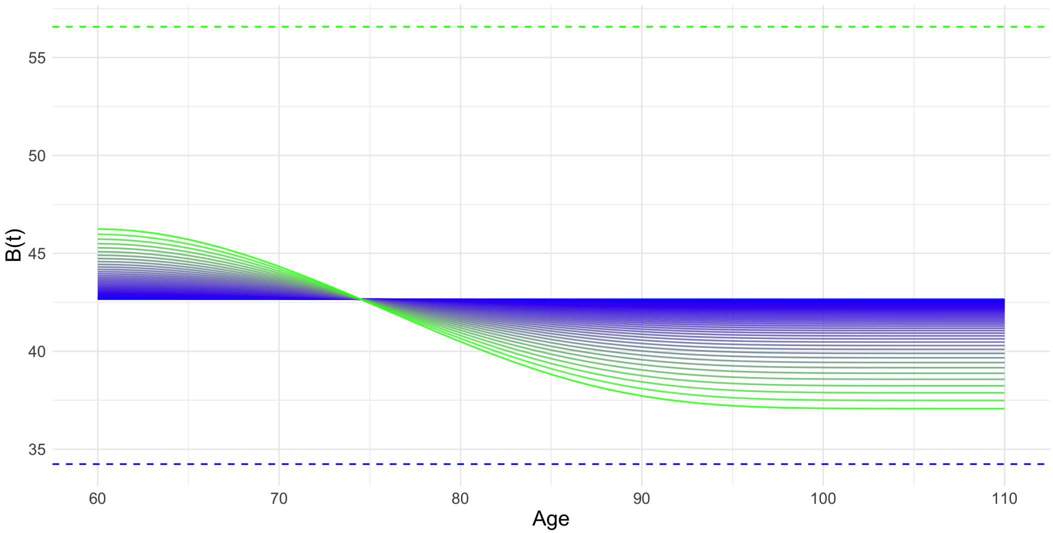
In the presence of unpriced risk heterogeneity, different payout rules lead to different levels of in-equitability and stability → Trade-off.

Designing Sustainable, Stable and Equitable Payouts

Pareto Frontier (λ_E/λ_S: 0 → large)



Actuarially Feasible Payout Paths on the Pareto Frontier



Collective Choice – Case 1 (No Inequity Aversion)

Lifetime Utility per Dollar Invested:

$$E[U_k] = \int_0^{\infty} e^{-\rho_k t} S_k(t)\, u_k(b_k(t))\, dt$$

CRRA Utility:

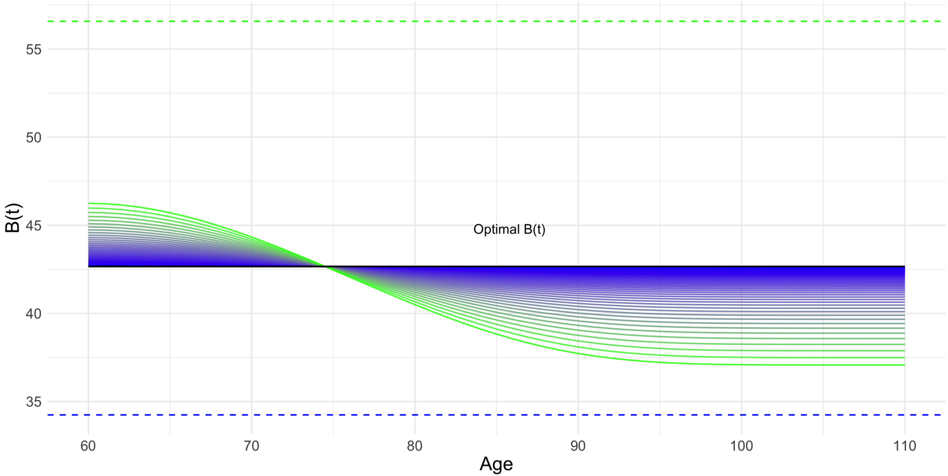
$$u_k(b) = \begin{cases} \dfrac{b^{1-\gamma_k}}{1-\gamma_k}, & \gamma_k \neq 1, \\ \log(b), & \gamma_k = 1 \end{cases}$$

Social Welfare:

$$\mathcal{W} = \sum_{k \in K} w_k\, E[U_k], \quad \sum_{k \in K} w_k = 1$$

- $\rho_k > 0$: subjective time–preference (utility discount) rate for group k,
- $S_k(t)$: survival probability for group k to age t,
- $b_k(t)$: consumption / benefit at time t per dollar invested,
- γ_k: coefficient of relative risk aversion for group k.

Case 1: Optimal B*(t)



Collective choice - Case 2 (With inequity aversion)

Expected utility with inequity aversion (Fehr & Schmidt, 1999):

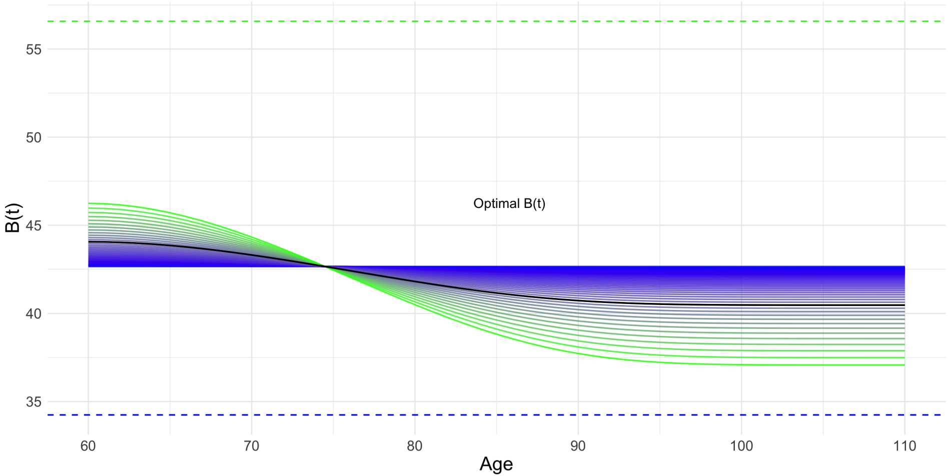
$$E[U_k] = \int_0^\infty e^{-\rho_k t} S_k(t)\Big[u_k(b_k(t)) - \underbrace{\alpha_k \sum_{h \neq k} \max(f(b_h(t)) - f(b_k(t)), 0)}_{\text{Disadvantageous inequity (envy)}} - \underbrace{\beta_k \sum_{h \neq k} \max(f(b_k(t)) - f(b_h(t)), 0)}_{\text{Advantageous inequity (guilt)}}\Big]\, dt$$

Aggregate social welfare: With welfare weights $\{w_k\}_{k \in K}$ and $\sum_{k \in K} w_k = 1$,

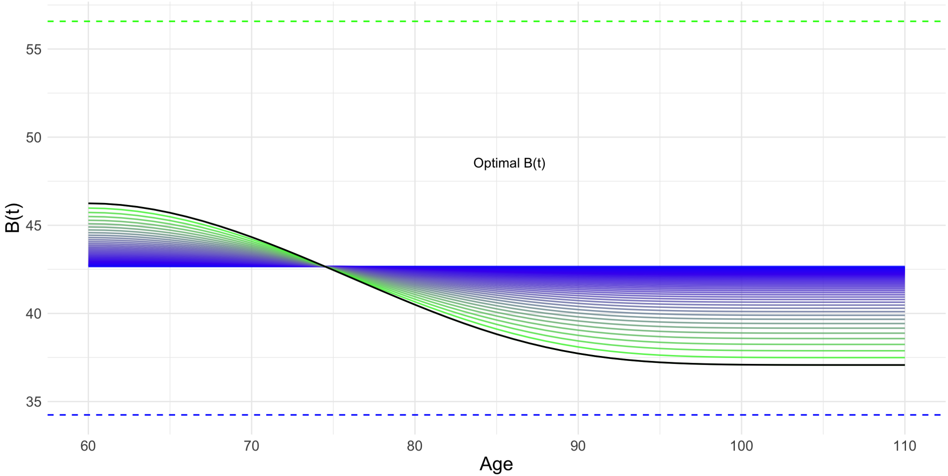
$$\mathcal{W} = \sum_{k \in K} w_k \, E[U_k].$$

- $u_k(\cdot)$: CRRA utility,
- α_k: *envy parameter* - aversion to disadvantageous inequity ($b_h(t) > b_k(t)$),
- β_k: *guilt parameter* - aversion to advantageous inequity ($b_k(t) > b_h(t)$).

Case 2: Optimal B*(t)



Case 2: Optimal B*(t)



Equitable Longevity Risk-Sharing: Conclusion

Age-based pricing: not reflective of longevity inequalities $\rightarrow$ perceived as 'inequitable'

Our contribution: quantitative framework to design stable, equitable, sustainable payouts

Mechanism: detect $\rightarrow$ design $\rightarrow$ decide

Beneficiaries:

- Retirees: Equitable payouts $\rightarrow$ perceived fairness
- Funds: Broader participation $\rightarrow$ Larger, more stable pools
- Policy: Supports SDG 10.4 $\rightarrow$ Reduces inequality in lifetime income
- Policy (Australia): to deliver income for a dignified retirement, alongside government support, in an equitable and sustainable way (*Superannuation Objective Act 2024*.)

Thank you!

Gayani Thalagoda
g.thalagoda@unsw.edu.au

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Benefit Equitability (Thalagoda et al., 2025)

$$\underbrace{\Delta B}_{\text{total inequality}} = \underbrace{\sum_{k=1}^{K}\left(\frac{p_k W_k}{\left(\sum_{i=1}^{K} p_i W_i\right)^2}\right)^2 \Delta B_{kk}}_{\text{within}} + \underbrace{\sum_{k=2}^{K}\sum_{h=1}^{k-1} 2\,\frac{p_k W_k\, p_h W_h}{\left(\sum_{i=1}^{K} p_i W_i\right)^2}\,\Delta B_{kh}}_{\text{between}}$$

$$= \sum_{k=1}^{K}\left(\frac{p_k W_k}{\left(\sum_{i=1}^{K} p_i W_i\right)^2}\right)^2 \Delta B_{kk} + \sum_{k=2}^{K}\sum_{h=1}^{k-1} \frac{p_k W_k\, p_h W_h}{\left(\sum_{i=1}^{K} p_i W_i\right)^2}\,\underbrace{\left(B_{hk} + B_{kh}\right)}_{\text{inequitability}}$$

$$= \frac{1}{4}\Delta B_{Low,Low} + \frac{1}{4}\Delta B_{High,High} + \underbrace{\frac{1}{4} B_{Low,High} + \frac{1}{4} B_{High,Low}}_{\text{inequitability}}$$