Mortality Sharing in a Multi-state Model of Functional Disability and Health Status

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Background

- Multi-state models for pricing LTC policies and annuity products.
- Individual’s (healthy) life expectancy is determined by many variables: age, gender, health and functional disability statuses etc.
- For example, mortality and disability risks increase with age.
- The transition rates to and from disability depend on an individual’s health status, such as chronic illness (Brown and Warshawsky (2013)).

Some models analyse the dynamics of functional disability and health status [Sherris and Wei, 2021].

Other models incorporate systematic trends and uncertainties [Li et al., 2017, Sherris and Wei, 2021, Fu et al., 2021].
Literature Gaps

- No previous literature on mortality sharing (mutual insurance) designs across functional disability states and chronic illness status.
- Prior research mostly focuses on functional disability, do not fully separate health status from disability [Hieber and Lucas, 2020, Chen et al., 2021].
- Disability models ignore recovery probabilities, trends and uncertainties in LTC and mortality risks.
Motivation

Research Goal

▶ Design a framework for pooling different health and mortality risks: ‘pooled health care annuity product’.

Why mortality risk sharing in a multi-state setting?

▶ Pooling heterogeneous lives is attractive to individuals in good and poor health, less adverse selection costs [Valdez et al., 2006].
▶ Mutual risk sharing products e.g pooled annuities and tontines provide enhanced annuity benefits through mortality and morbidity credits [Piggott et al., 2005, Qiao and Sherris, 2013].
Multi-state Model Setup

Figure 1: A Five-State Transition Model
Multi-state Pooling

Figure 2: A Two-State Transition Model

\[ F_0 = l_x \ddot{a}_x B_0, \]
\[ F_1 = (F_0 - l_x B_0) \ast (1 + R_1), \]
\[ B_1 = \frac{F_1}{l_{x+1}^{*} \ddot{a}_{x+1}}, \]
\[ B_{t+1} = B_t \ast \left( \frac{1 + R_t}{1 + r} \right) \left( \frac{p_{x+t}}{p_{x+t}^{*}} \right). \]
Multi-state Pooling

Figure 3: A Three-State Model with Recovery

\[
F^h_0 = l^*_x (\ddot{a}^{hh} B^h_0 + \ddot{a}^{hf} B^f_0),
\]
\[
F^f_0 = l^*_x (\ddot{a}^{fh} B^h_0 + \ddot{a}^{ff} B^f_0),
\]
\[
\begin{bmatrix}
F^h_0 \\
F^f_0
\end{bmatrix} =
\begin{bmatrix}
l^*_x \\
\end{bmatrix} \otimes
\begin{bmatrix}
\ddot{a}^{hh} & \ddot{a}^{hf} \\
\ddot{a}^{fh} & \ddot{a}^{ff}
\end{bmatrix}
\begin{bmatrix}
B^h_0 \\
B^f_0
\end{bmatrix},
\]
\[
F_0 = L_x \otimes (A_x B_0),
\]
\[
F_t = L_{x+t} \otimes (A_{x+t} B_t).
\]
Multi-state Pooling

\[ \mathbf{F}_{t+1} = \mathbf{L}_{x+t+1} \otimes \left( (\mathbf{P}_{x+t}^*)^{-1} \left( \frac{\mathbf{F}_t - \mathbf{L}_{x+t} \otimes \mathbf{B}_t}{\mathbf{L}_{x+t}} \right) (1 + R_t) \right), \]

\[ \mathbf{F}_{t+1} = \mathbf{L}_{x+t+1} \otimes (\mathbf{A}_{x+t+1} \mathbf{B}_{t+1}), \]  \hspace{1cm} (1)

\[ \mathbf{P}_{x+t}^* = \begin{bmatrix} p_{x+t}^{hh*} & p_{x+t}^{hf*} \\ p_{x+t}^{fh*} & p_{x+t}^{ff*} \end{bmatrix}, \]

\[ \mathbf{L}_{x+t+1} = \begin{bmatrix} l_{x+t+1}^h \\ l_{x+t+1}^f \end{bmatrix}. \]
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- Estimate the three-state model based on the HRS survey data.
- Set $B_0^h = $ 12,000 and $B_0^f = $ 36,000 per year at the start.
- The initial pool size 850 healthy and 250 disabled 65-old males.
- Idiosyncratic risk: 1000 paths Multinomial distribution.
- Annual pricing rate: $r = 3\%$. 
Fitted Transition Rates

Figure 4: The fitted transition rates of the static model and the crude transition rates by age for females and males from the HRS sample.
Figure 5: t-year fitted transition probabilities of a 65 year old male from the static model.
Figure 6: The number of survivors, pool fund value, individual fund value and annuity benefits for healthy and disabled pool participants.
Main Results

Table 1: Pooled Health Care Annuity Payouts

<table>
<thead>
<tr>
<th>Annuity Payments</th>
<th>Age 75</th>
<th></th>
<th></th>
<th>Age 95</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5%</td>
<td>Mean</td>
<td>95%</td>
<td>5%</td>
<td>Mean</td>
<td>95%</td>
</tr>
<tr>
<td>Static</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Healthy</td>
<td>$10,830</td>
<td>$11,926</td>
<td>$12,864</td>
<td>$1,632</td>
<td>$10,756</td>
<td>$18,148</td>
</tr>
<tr>
<td>Disabled</td>
<td>$29,811</td>
<td>$36,619</td>
<td>$44,746</td>
<td>$21,056</td>
<td>$44,033</td>
<td>$84,594</td>
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<tr>
<td>Trend</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Healthy</td>
<td>$10,544</td>
<td>$11,746</td>
<td>$12,731</td>
<td>$5,427</td>
<td>$9,936</td>
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<tr>
<td>Disabled</td>
<td>$31,030</td>
<td>$38,368</td>
<td>$47,595</td>
<td>$18,875</td>
<td>$34,928</td>
<td>$61,170</td>
</tr>
</tbody>
</table>
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- Pooling mortality and disability risk improves the payouts based on individual’s statuses while reducing costs.
- Systematic trends and uncertainties significantly impact the annuity benefits: mortality improvements, a decreasing trend for healthy.
- Morbidity compression: increased disability benefits at more advanced ages.
Future Work

- Incorporate equity funding.
- Compare the pooled annuity benefits with the standard life care product.
- Compare the pooled annuity benefits in the 3-state model with a 5-state.
Thanks! Questions/comments?


References I
References II


References III


Appendix

Model Estimation

- The multi-state latent factor intensity model proposed in Li et al. (2017) to estimate the transition rates.
- The transition intensity for transition type s for an individual j at time t is assumed to be of the form

\[ \lambda_{j,s}(t) = \exp \left( \beta_s + \gamma'_s w_j(t) + \alpha_s \psi(t) \right). \]  

(2)

- \( \beta_s \): baseline log-intensity for transition type s
- \( w_j(t) \): vector with the observed predictors for individual j
- \( \psi(t) \): stochastic latent process for systematic uncertainties
- \( \gamma_s \) and \( \alpha_s \): measure sensitivities of logarithm of \( \lambda_{j,s}(t) \) w.r.t \( w_j(t), \psi(t) \)
Appendix

1. Static model: the transition rate $\lambda_{j,s}(t)$ is assumed to be dependent on age and gender only

$$\ln(\lambda_{j,s}(t)) = \beta_s + \gamma_{s}^{\text{age}}x_j(t) + \gamma_{s}^{\text{female}}G_j. \quad (3)$$

2. Trend model: the systematic time trend/linear time index is included

$$\ln(\lambda_{j,s}(t)) = \beta_s + \gamma_{s}^{\text{age}}x_j(t) + \gamma_{s}^{\text{female}}G_j + \gamma_{s}^{\text{time}}t. \quad (4)$$

3. Frailty model: time trend and the latent factor $\psi(t)$ are included, to account for systematic uncertainty

$$\ln(\lambda_{j,s}(t)) = \beta_s + \gamma_{s}^{\text{age}}x_j(t) + \gamma_{s}^{\text{female}}G_j + \gamma_{s}^{\text{time}}t + \alpha_s\psi(t), \quad (5)$$

$\alpha_s$ measures the sensitivity of $\ln(\lambda_{j,s}(t))$ w.r.t. the latent factor.