# Mortality Sharing in a Multi-state Model of Functional Disability and Health Status

Doreen Kabuche[1,2], Michael Sherris[1,2], Andrés M. Villegas[1,2] and Jonathan Ziveyi[1,2]

[1]School of Risk and Actuarial Studies, UNSW Sydney
[2]ARC Centre of Excellence in Population Ageing Research (CEPAR)

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# Outline

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# Background

- Multi-state models for pricing LTC policies and annuity products.
- Individual's (healthy) life expectancy is determined by many variables: age, gender, health and functional disability statuses etc.
- For example, mortality and disability risks increase with age.
- The transition rates to and from disability depend on an individual's health status, such as chronic illness (Brown and Warshawsky (2013)).

# Previous Literature

- Many studies propose multi-state models for disability: Olivieri and Pitacco (2001), Rickayzen and Walsh (2002), Leung (2006), Fong et al., (2015) etc.
- Some models analyse the dynamics of functional disability and health status [Sherris and Wei, 2021].
- Other models incorporate systematic trends and uncertainties [Li et al., 2017, Sherris and Wei, 2021, Fu et al., 2021].

# Literature Gaps

- Life care annuity: [Brown and Warshawsky, 2013, Sherris and Wei, 2021].
- No previous literature on mortality sharing (mutual insurance) designs across functional disability states and chronic illness status.
- Prior research mostly focuses on functional disability, do not fully separate health status from disability [Hieber and Lucas, 2020, Chen et al., 2021].
- Disability models ignore recovery probabilities, trends and uncertainties in LTC and mortality risks.

# Motivation

Research Goal

- Design a framework for pooling different health and mortality risks: 'pooled health care annuity product'.

Why mortality risk sharing in a multi-state setting?

- Pooling heterogeneous lives is attractive to individuals in good and poor health, less adverse selection costs [Valdez et al., 2006].
- Mutual risk sharing products e.g pooled annuities and tontines provide enhanced annuity benefits through mortality and morbidity credits [Piggott et al., 2005, Qiao and Sherris, 2013].

# Outline

# Multi-state Model Setup

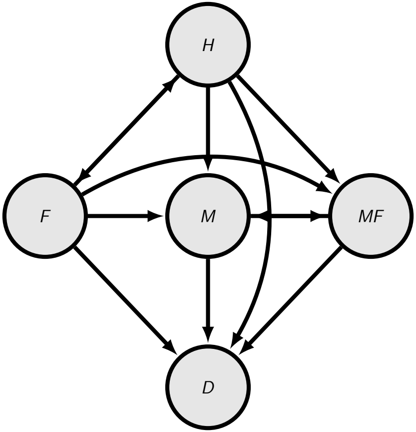


Figure 1: *A Five-State Transition Model*

# Multi-state Pooling

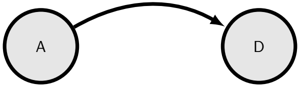


Figure 2: *A Two-State Transition Model*

$$F_0 = l_x^* \ddot{a}_x B_0,$$

$$F_1 = (F_0 - l_x^* B_0) * (1 + R_1),$$

$$B_1 = \frac{F_1}{l_{x+1}^* \ddot{a}_{x+1}},$$

$$B_{t+1} = B_t * \left(\frac{1 + R_t}{1 + r}\right)\left(\frac{p_{x+t}}{p_{x+t}^*}\right).$$

# Multi-state Pooling

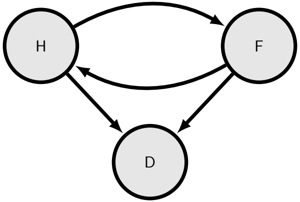


Figure 3: *A Three-State Model with Recovery*

$$F_0^h = l_x^{h*}\left(\ddot{a}_x^{hh}B_0^h + \ddot{a}_x^{hf}B_0^f\right),$$
$$F_0^f = l_x^{f*}\left(\ddot{a}_x^{fh}B_0^h + \ddot{a}_x^{ff}B_0^f\right),$$

$$\begin{bmatrix} F_0^h \\ F_0^f \end{bmatrix} = \begin{bmatrix} l_x^{h*} \\ l_x^{f*} \end{bmatrix} \otimes \left( \begin{bmatrix} \ddot{a}_x^{hh} & \ddot{a}_x^{hf} \\ \ddot{a}_x^{fh} & \ddot{a}_x^{ff} \end{bmatrix} \begin{bmatrix} B_0^h \\ B_0^f \end{bmatrix} \right),$$

$$\mathbf{F}_0 = \mathbf{L}_x \otimes (\mathbf{A}_x \mathbf{B}_0),$$

$$\mathbf{F}_t = \mathbf{L}_{x+t} \otimes (\mathbf{A}_{x+t} \mathbf{B}_t).$$

# Multi-state Pooling

$$\mathbf{F}_{t+1} = \mathbf{L}_{x+t+1} \otimes \left( \overset{\substack{\text{Mortality/morbidity credits} \\ \uparrow}}{(\mathbf{P}^*_{x+t})^{-1}} \left( \frac{\mathbf{F}_t - \mathbf{L}_{x+t} \otimes \mathbf{B}_t}{\mathbf{L}_{x+t}} \right) \overset{\substack{\text{Financial credits} \\ \uparrow}}{(1 + R_t)} \right),$$

$$\mathbf{F}_{t+1} = \mathbf{L}_{x+t+1} \otimes (\mathbf{A}_{x+t+1}\mathbf{B}_{t+1}), \tag{1}$$

$$\mathbf{P}^*_{x+t} = \begin{bmatrix} p^{hh^*}_{x+t} & p^{hf^*}_{x+t} \\ p^{fh^*}_{x+t} & p^{ff^*}_{x+t} \end{bmatrix},$$

$$\mathbf{L}_{x+t+1} = \begin{bmatrix} l^{h^*}_{x+t+1} \\ l^{f^*}_{x+t+1} \end{bmatrix}.$$

# Outline

# Numerical Example

- Estimate the three-state model based on the HRS survey data.
- Set $B_0^h = \$\ 12,000$ and $B_0^f = \$\ 36,000$ per year at the start.
- The initial pool size 850 healthy and 250 disabled 65-old males.
- Idiosyncratic risk: 1000 paths Multinomial distribution.
- Annual pricing rate: $r = 3\%$.

# Fitted Transition Rates

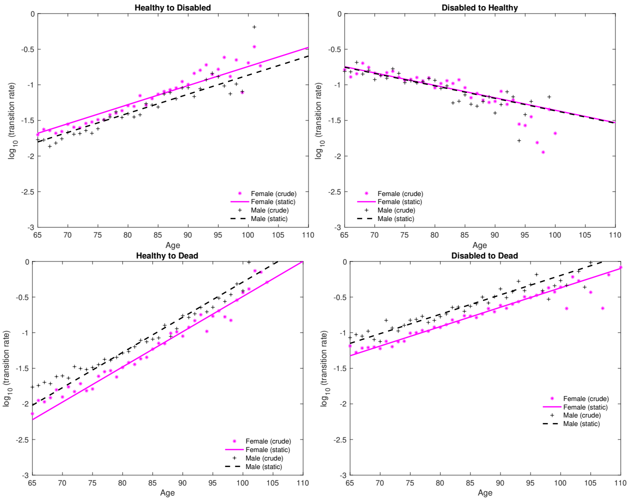


Figure 4: The fitted transition rates of the static model and the crude transition rates by age for females and males from the HRS sample.

# Fitted Transition Probabilities

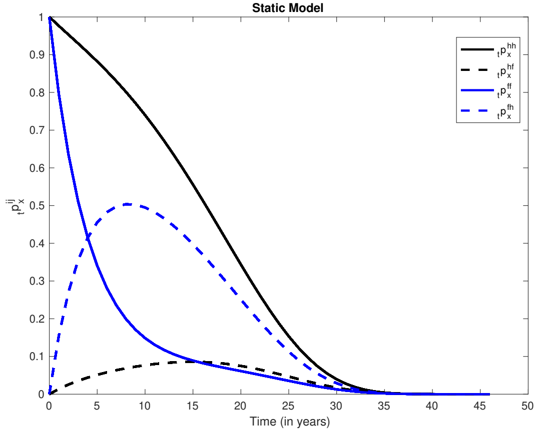


Figure 5: t-year fitted transition probabilities of a 65 year old male from the static model.

# Main Results

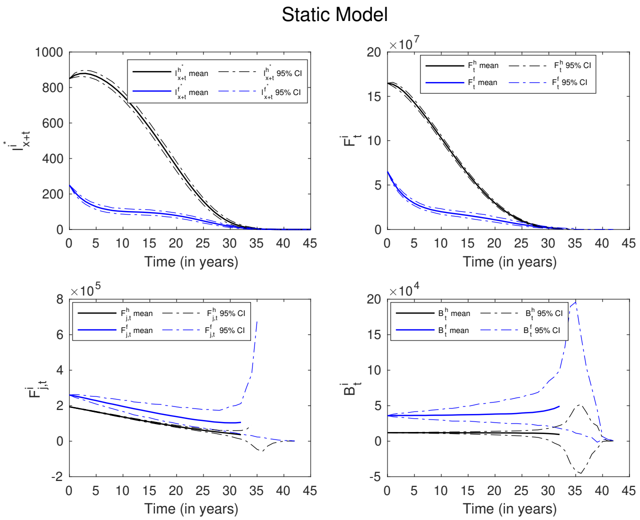


Figure 6: The number of survivors, pool fund value, individual fund value and annuity benefits for healthy and disabled pool participants.

# Main Results

Table 1: Pooled Health Care Annuity Payouts

| Annuity Payments | | Age 75 | | | Age 95 | | |
|---|---|---|---|---|---|---|---|
| | | 5% | Mean | 95% | 5% | Mean | 95% |
| Static | Healthy | $10,830 | $11,926 | $12,864 | $1,632 | $10,756 | $18,148 |
| | Disabled | $29,811 | $36,619 | $44,746 | $21,056 | $44,033 | $84,594 |
| Trend | Healthy | $10,544 | $11,746 | $12,731 | $5,427 | $9,936 | $14,113 |
| | Disabled | $31,030 | $38,368 | $47,595 | $18,875 | $34,928 | $61,170 |

# Outline

# Conclusion

- Pooling mortality and disability risk improves the payouts based on individual's statuses while reducing costs.
- Systematic trends and uncertainties significantly impact the annuity benefits: mortality improvements, a decreasing trend for healthy.
- Morbidity compression: increased disability benefits at more advanced ages.

# Future Work

- Incorporate equity funding.
- Compare the pooled annuity benefits with the standard life care product.
- Compare the pooled annuity benefits in the 3-state model with a 5-state.

Thanks! Questions/comments?

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# Appendix

Model Estimation

- The multi-state latent factor intensity model proposed in Li et al. (2017) to estimate the transition rates.
- The transition intensity for transition type $s$ for an individual $j$ at time $t$ is assumed to be of the form

$$\lambda_{j,s}(t) = \exp\left(\beta_s + \gamma_s' w_j(t) + \alpha_s \psi(t)\right). \tag{2}$$

- $\beta_s$: baseline log-intensity for transition type $s$
- $w_j(t)$: vector with the observed predictors for individual $j$
- $\psi(t)$: stochastic latent process for systematic uncertainties
- $\gamma_s$ and $\alpha_s$: measure sensitivities of logarithm of $\lambda_{j,s}(t)$ w.r.t $w_j(t)$, $\psi(t)$

# Appendix

1. Static model: the transition rate $\lambda_{j,s}(t)$ is assumed to be dependent on age and gender only

$$\ln(\lambda_{j,s}(t)) = \beta_s + \gamma_s^{\text{age}} x_j(t) + \gamma_s^{\text{female}} G_j. \tag{3}$$

2. Trend model: the systematic time trend/ linear time index is included

$$\ln(\lambda_{j,s}(t)) = \beta_s + \gamma_s^{\text{age}} x_j(t) + \gamma_s^{\text{female}} G_j + \gamma_s^{\text{time}} t. \tag{4}$$

3. Frailty model: time trend and the latent factor $\psi(t)$ are included, to account for systematic uncertainty

$$\ln(\lambda_{j,s}(t)) = \beta_s + \gamma_s^{\text{age}} x_j(t) + \gamma_s^{\text{female}} G_j + \gamma_s^{\text{time}} t + \alpha_s \psi(t), \tag{5}$$

$\alpha_s$ measures the the sensitivity of $\ln(\lambda_{j,s}(t))$ w.r.t the latent factor.