Dynamic Hedging of Longevity Risk in Group Self-Annuity Portfolios

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32nd Colloquium on Pensions and Retirement Research

27 November 2024

Research motivation

Background

- Longevity risk-pooling products[1] are increasingly popular:
  - No risk premium is charged.
  - No capital is required.
  - Reduce adverse selection and increase post-retirement utility (Valdez et al., 2006; Hanewald et al., 2013).
  - Existing products: QSuper Lifetime Pension[2]; GuardPath Modern Tontine[3].
- The **systematic longevity risk** undermines the effectiveness of the risk-pooling products:
  - Members prefer a stable and high level of survival benefit.
  - Benefits **reduce** when members live longer than expected.
  - Benefits are **volatile** due to the uncertainty of the level of longevity.
  - Undiversifiable.

[1] For example, group self-annuity (Piggott et al., 2005), pooled annuity fund (Stamos, 2008; Donnelly et al., 2014), tontine (Milevsky and Salisbury, 2015) among others.

[2] https://qsuper.qld.gov.au/our-products/superannuation/lifetime-pension

[3] https://www.guardiancapital.com/investmentsolutions/guardpath-modern-tontine-trust/

Longevity risk transfer solutions

- Annuity providers and defined benefit pension plans.
- Hedge adverse financial effect of longevity risk.

	Customised	**Index-based**
Counterparty	A third party	Capital market investors
Trading frequency	Static	Dynamic
Underlying	The book population	A whole population
Population basis risk	No	Yes
Effectiveness	More effective	Less effective
Cost	More expensive	Cheaper

Table 1: A comparison of longevity risk transfer solutions.

Longevity swap trading volume

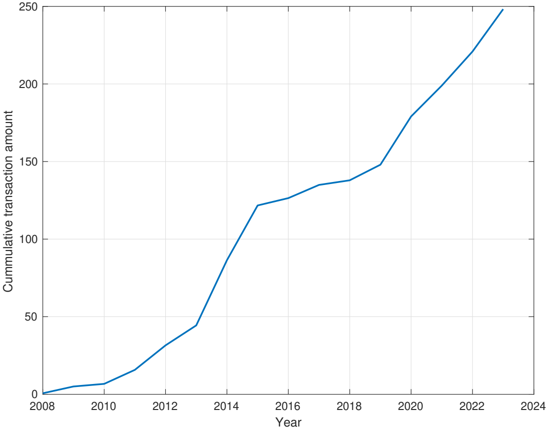


Figure 1: Cumulative amount of longevity swap transactions in US$ billion since the transaction of the q-forward in January 2008 between J.P.Morgan and Lucida. Data source: https://www.artemis.bm/longevity-swaps-and-longevity-risk-transfers/.

Research objective

- Research gap: The application of index-based longevity securities in longevity pooling products remains unexplored.
- We propose an innovative dynamic hedging framework to reduce the volatility of the GSA survival benefit:
  - Hedges the systematic longevity risk.
  - Resorts to the capital market using standardised longevity securities.
  - Allows for population basis risk.
  - In a discrete-time setting for practical implementation purposes.

Methodology

Mortality model

Two populations:

- Population F: The GSA fund population.
- Population R: The reference population of the longevity securities.

Augmented Common Factor (ACF) mortality model (Li and Lee, 2005):

$$\log\left(m_{x,t}^{(i)}\right) = a_x^{(i)} + \underbrace{G_x K_t}_{\text{common factor}} + \underbrace{g_x^{(i)} k_t^{(i)}}_{\text{population-specific factor}} + \epsilon_{x,t}^{(i)}, \text{ for } i \in \{F, R\}, \quad (1)$$

where K_t follows a random walk with drift and $k_t^{(i)}$ follows an AR(1) process.

Notations: Death and survival probabilities

- One-year death probability:

$$q_{x,t}^{(i)} \approx 1 - \exp(-m_{x,t}^{(i)}). \tag{2}$$

- T-year survival probability:

$$S_{x,t}^{(i)}(T) := \prod_{s=1}^{T} \left(1 - q_{x+s-1,t+s}^{(i)}\right). \tag{3}$$

- Best estimate of T-year survival probability given current information:

$$p_{x,u}^{(i)}(T, K_t, k_t^{(i)}) := \mathbb{E}\left[S_{x,u}^{(i)}(T)|\mathcal{F}_t\right] = \mathbb{E}\left[S_{x,u}^{(i)}(T)|K_t, k_t^{(i)}\right]. \tag{4}$$

The GSA fund process

The GSA fund evolves as follows:

$$\text{Number of survivors: } N_t = \underbrace{N_{t-1} S^{(F)}_{x+t-1,t-1}(1)}_{\text{Ignore small sample risk}}, \tag{5}$$

$$\text{Investment return: } F_t^- = F_{t-1}^+ (1+r), \tag{6}$$

$$\text{Survival benefit: } B_t = \frac{F_t^-}{\ddot{a}^e_{x+t,t} N_t}, \tag{7}$$

$$\text{Death benefit: } D_t = \beta \frac{F_t^-}{N_{t-1}}, \quad 0 \leq \beta < 1, \tag{8}$$

$$\text{Fund value after payment: } F_t^+ = F_t^- - B_t N_t - D_t \Delta N_t, \tag{9}$$

where

$$\ddot{a}^e_{x+t,t} = \sum_{s=0}^{+\infty} (1+r)^{-s} p^{(F)}_{x+t,t}\left(s, K_t, k_t^{(F)}\right), \quad \Delta N_t = N_t - N_{t-1}. \tag{10}$$

S-forwards

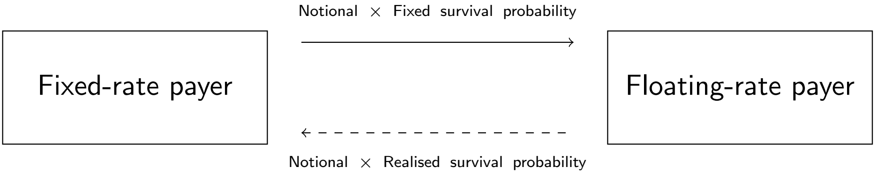


Figure 2: Settlement of an S-forward contract at maturity.

A yearly rolling hedging strategy

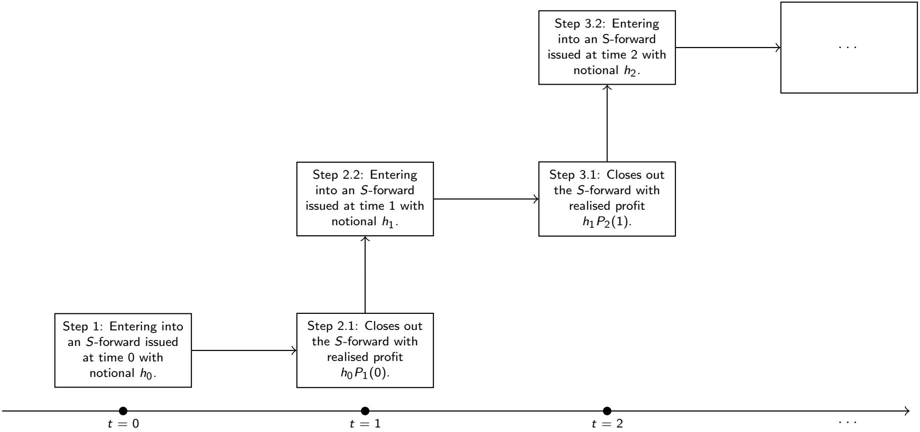


Figure 3: The yearly rolling hedging strategy. **The GSA fund is the fixed-rate payer.** $P_{t+1}(t)$ is the hedging profit or loss per \$1 notional.

Hedging mechanism

The total benefit $B_{t+1}^{(H)}$ is determined as follows:

$$B_{t+1}^{(H)} := \underbrace{B_{t+1}}_{\text{unhedged benefit}} + \underbrace{\frac{h_t P_{t+1}(t)}{N_t S_{x+t,t}^{(F)}(1)}}_{\text{distributed hedging profit or loss}}. \quad (11)$$

At time $t+1$, the hedging profit (or loss) of the **fixed rate payer** is:

$$P_{t+1}(t) = (1+r)^{-(T^*-1)} \left[\underbrace{\mathbb{E}_{t+1}\left[S_{x^f,t}^{(R)}(T^*) \right]}_{\text{Expectation at } t+1} - \underbrace{p_{x^f,t}^{(R)}\left(T^*, K_t, k_t^{(R)} \right)}_{\text{Expectation at } t} \right]. \quad (12)$$

$$N_{t+1} \nearrow \Rightarrow B_{t+1} \searrow; \mathbb{E}_{t+1}\left[S_{x^f,t}^{(R)}(T^*) \right] \nearrow \Rightarrow \text{Hedging profit}$$

The mean-variance optimisation problem

Given the information at time t, fund members aim to solve the following mean-variance optimisation problem (Wong et al., 2017):

$$\min_{h_t \in \mathbb{R}} \left\{ V_t(h_t | \mathcal{F}_t) := \mathrm{Var}_t \left[B_{t+1}^{(H)} \right] - 2\phi_t \left(\mathbb{E}_t \left[B_{t+1}^{(H)} \right] - \mathbb{E}_t \left[B_{t+1} \right] \right) \right\}, \quad (13)$$

where $V_t(h_t)$ is the objective function, the parameter $\phi_t (\geq 0)$ controls the mean-variance trade-off.

$\phi_t \nearrow \Rightarrow$ Put more weights on the mean $\Rightarrow$ Less risk-averse

Risk preference

Risk preference	**Hedge ratio**	**Role in the S-forwards**
Risk-averse	$h_t > 0$	Fixed-rate payer
Risk-neutral	$h_t = 0$	-
Risk-seeking	$h_t < 0$	Floating-rate payer

Table 2: Hedge ratio and risk preference.

The mean-variance set

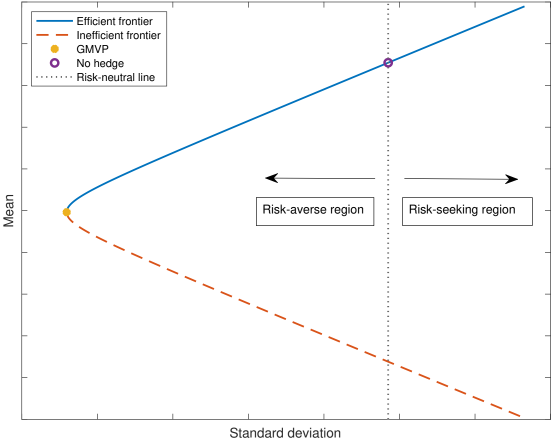


Figure 4: The mean-variance set of longevity hedge using S-forwards for the GSA fund members. The mean-variance set shows a parabolic shape. GMVP: The global minimum variance point.

Measurements of hedge effectiveness

- Variance reduction ratio (VRR):

$$\text{VRR}_t(\phi_t) := 1 - \frac{\mathrm{Var}_t\left[B_{t+1}^{(H)}\right]}{\mathrm{Var}_t\left[B_{t+1}\right]}, \quad \text{for } \phi_t \leq \phi_t^{(RN)}, \tag{14}$$

  where $\phi_t^{(RN)}$ is the risk-neutral (mean-variance) trade-off parameter.
- Optimal objective function value: $V_t(h_t^*)$.

VRR $\nearrow \Rightarrow$ More effective
Objective function value $\nearrow \Rightarrow$ Less effective

Numerical results

Data and assumption

Age of the GSA members at the inception of the fund	x	65
Initial contribution per member	c	$10,000
GSA payment frequency		Yearly
Risk-free interest rate	r	3% per annum
Fund population		EW population[4]
Reference population		UK total population
Reference age of the S-forwards	x_f	75
Time-to-maturity of the S-forwards	T^*	10 years

Table 3: Baseline assumptions in the numerical study.

[4]England and Wales population.

Risk-neutral trade-off parameter

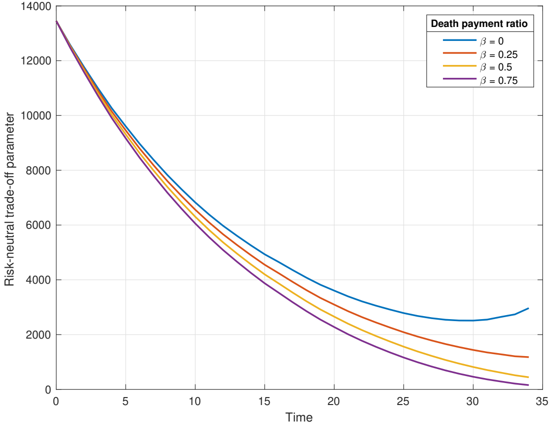
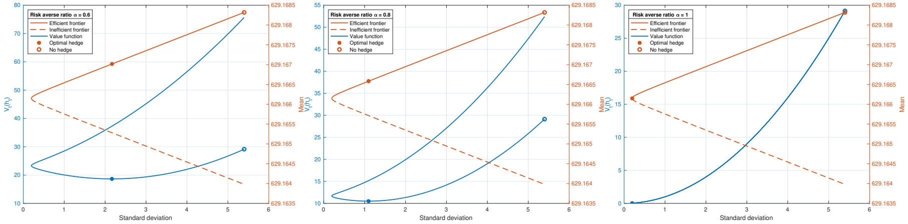


Figure 5: The mean of the risk-neutral trade-off parameter $\phi_t^{(RN)}$.

$\phi_t^{(RN)} \nearrow \Rightarrow$ More willing to hedge systematic longevity risk

An example at $t = 0$



(a) $\alpha = 0.6$ (b) $\alpha = 0.8$ (c) $\alpha = 1$

Figure 6: The mean-variance set and the objective function value at time $t = 0$. We assume that $\phi_t = (1-\alpha)\phi_t^{(RN)}$, where $\alpha \in [0, 1]$ is the risk-averse ratio.

$\alpha \nearrow \Rightarrow$ More risk-averse

Hedge effectiveness: Variance reduction ratio

Age	Mean	Minimum	Maximum	95% confidence interval
65	99.87%	99.85%	99.88%	(99.86%, 99.87%)
70	99.90%	99.87%	99.92%	(99.89%, 99.91%)
75	99.93%	99.90%	99.95%	(99.92%, 99.94%)
80	99.88%	99.85%	99.90%	(99.87%, 99.89%)
85	98.59%	97.64%	99.20%	(98.16%, 98.95%)
90	87.22%	83.16%	90.84%	(85.41%, 88.92%)
95	50.46%	46.18%	54.17%	(48.62%, 52.21%)

Table 4: The impact of longevity hedge on the variance reduction ratio (the risk-averse ratio $\alpha = 1$ and the death payment ratio $\beta = 0$).

Hedge effectiveness: Optimal objective function value

Age	Mean	Minimum	Maximum	95% confidence interval
Optimal longevity hedge				
65	0.0380	0.0358	0.0402	(0.0370, 0.0391)
70	0.0310	0.0132	0.0754	(0.0201, 0.0463)
75	0.0241	0.0101	0.0694	(0.0147, 0.038)
80	0.0433	0.0147	0.1884	(0.0240, 0.0739)
85	0.5459	0.1667	1.7258	(0.2872, 0.9583)
90	5.57	1.72	19.21	(3.09, 9.57)
95	31.66	12.56	107.91	(19.17, 51.74)
Without longevity hedge				
65	28.27	26.47	29.90	(27.49, 29.08)
70	31.58	15.92	58.54	(22.74, 42.66)
75	33.43	17.73	72.39	(23.54, 46.34)
80	37.20	14.03	124.02	(21.97, 59.13)
85	37.91	20.72	73.35	(27.31, 52.17)
90	42.96	18.77	117.79	(27.70, 66.01)
95	63.79	25.51	201.19	(39.05, 102.34)

Table 5: The impact of longevity hedge on the optimal objective function value.

Population basis risk index

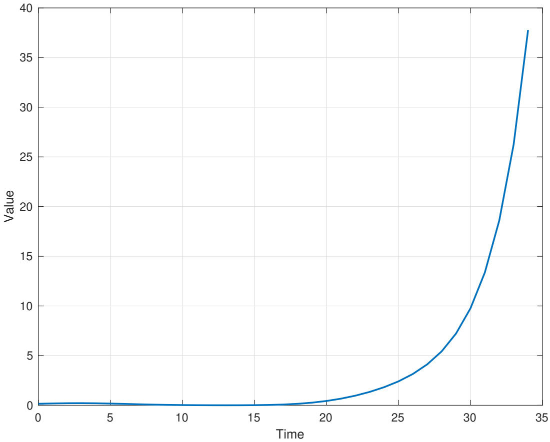
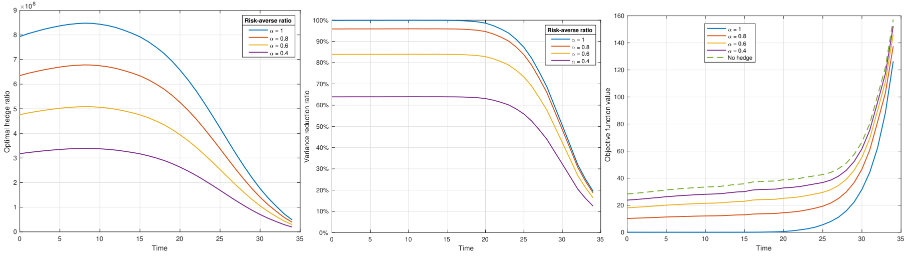


Figure 7: The value of the population basis-risk index $I_t := \frac{\mathrm{Var}_t\left(g^{(F)}_{x+t} k^{(F)}_{t+1}\right)}{\mathrm{Var}_t(G_{x+t} K_{t+1})}$ for $t = 0, 1, \cdots, 34$.

Hedge effectiveness: Risk-averse ratio



(a) Mean of h_t^* (b) Mean of VRR_t (c) Mean of $V_t(h_t^*)$

Figure 8: The impact of risk-averse ratio α on the optimal hedge ratio h_t^*, VRR and $V_t(h_t^*)$ (the death payment ratio $\beta = 0$).

Conclusion

Conclusion

We propose a dynamic systematic longevity risk hedging framework for the GSA in the presence of the population basis risk:

- The mean-variance set is a tool for hedging strategy selection.
- The framework is practical:
  - Provides a semi-closed-form solution of the optimal hedge ratio.
  - In a discrete-time setting.
  - Does not require the hedging instrument to have a long time-to-maturity.
- Increases the effectiveness of a GSA.
- Applies to other risk-pooling products.
- The framework is robust (see the appendix):
  - The S-forwards' time-to-maturity and reference age.
  - The hedger's population.
  - Interest rate risk.
  - The size of the GSA pool.

Thank you!

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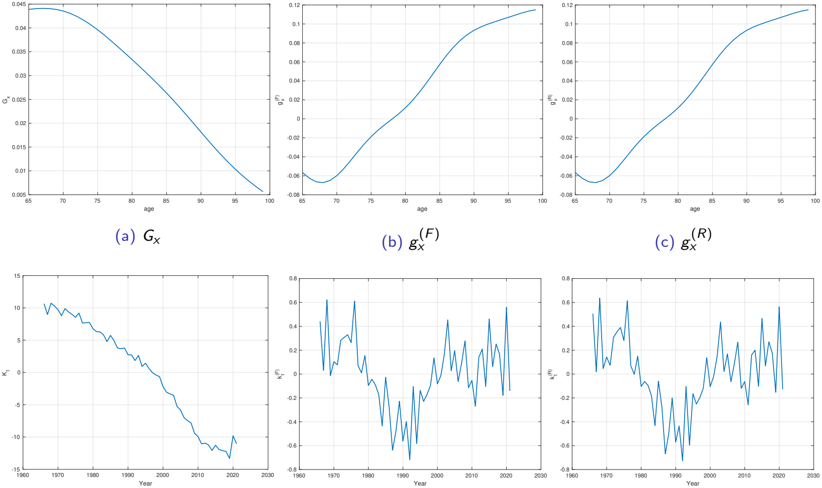
Appendix

Estimates of parameters in the ACF model

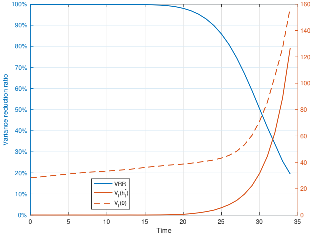


(a) G_x (b) $g_x^{(F)}$ (c) $g_x^{(R)}$

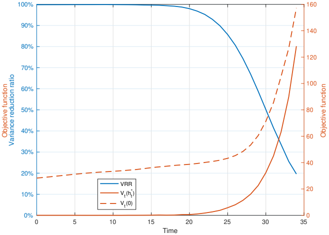
(d) K_t (e) $k_t^{(F)}$ (f) $k_t^{(R)}$

Figure 9: Estimates of parameters in the ACF model. The ACF model parameters are calibrated to the mortality data of the population aged from 65 to 99 over the period from 1966 to 2019.

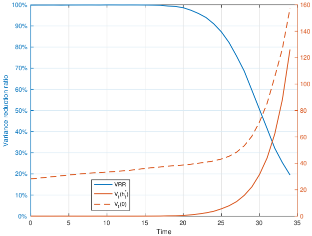
Robustness: The time-to-maturity of the S-forwards



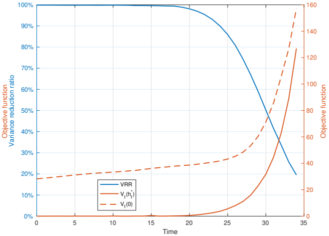
(a) $T^* = 4$



(b) $T^* = 8$



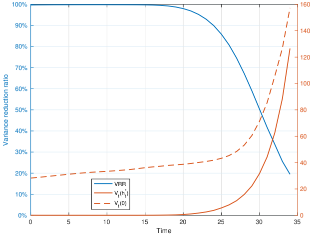
(c) $T^* = 12$



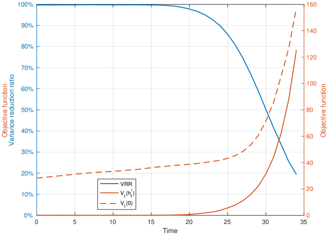
(d) $T^* = 16$

Figure 10: The impact of time-to-maturity on hedge effectiveness.

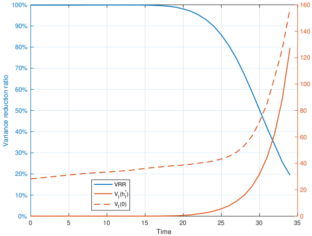
Robustness: The reference age of the S-forwards



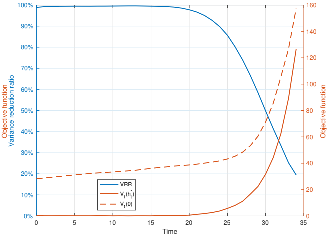
(a) $x_f = 65$



(b) $x_f = 70$



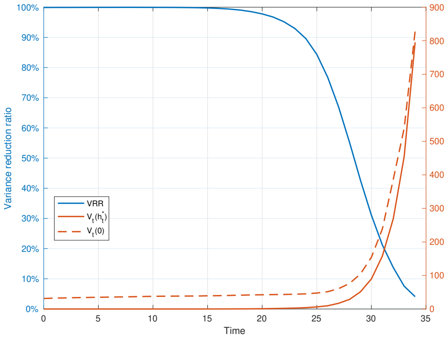
(c) $x_f = 80$



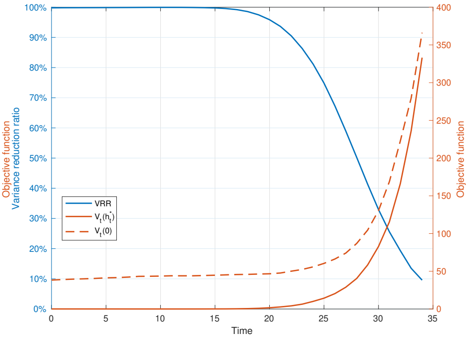
(d) $x_f = 85$

Figure 11: The impact of reference age on hedge effectiveness.

Robustness: The hedger's population



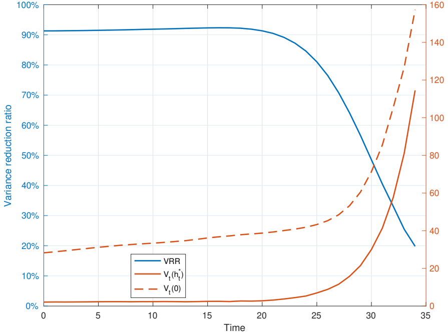
(a) Northern Ireland



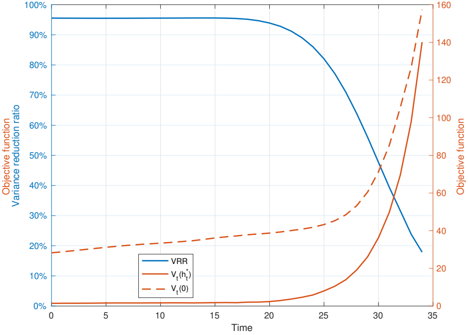
(b) Scotland

Figure 12: The impact of hedger's population on hedge effectiveness.

Robustness: Interest rate risk



(a) Realised risk-free interest rate is 1%



(b) Realised risk-free interest rate is 5%

Figure 13: The impact of interest rate risk on hedge effectiveness.

Robustness: The GSA pool size

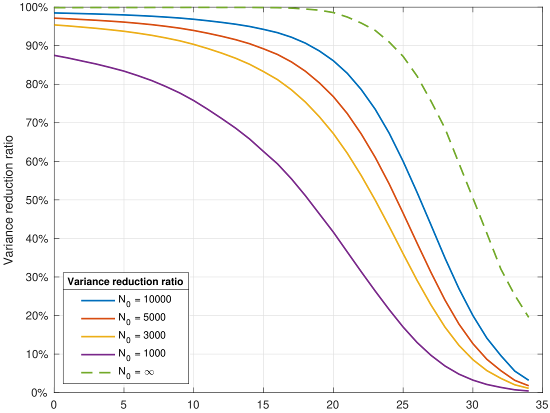


Figure 14: The impact of pool size on hedge effectiveness for a finite pool size with $N_0 =$ 10,000, 5,000, 3,000, and 1,000. The dashed line represents the case with no small sample risk.