Term Annuity Bonds - the Ideal Pension Investment?

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Too many individuals will retire poor!

The individual is responsible for her/his DC pension,

and often financially not enough sophisticated.

Annuities too expensive!
(Conflicts of interest, Stiglitz (1998) and Merton (2014).)

Annuity type instruments **must become less expensive** to become widely used in pensions!

Main Question for Retirement Funding:

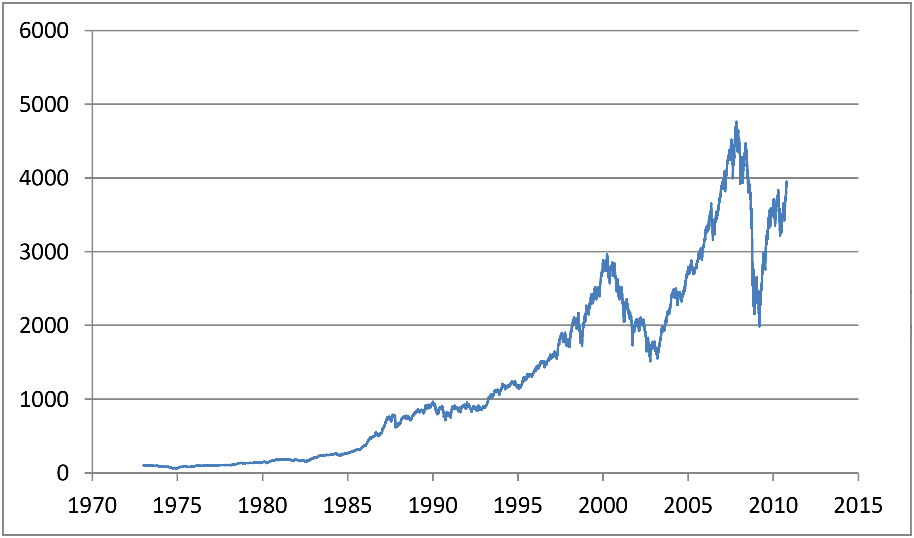
Can one generate from the pension contributions

higher risk-less pension payouts

than currently produced?

Benchmark approach
Pl. & Heath (2006), Pl. (2024)

MSCI S_t


6000
5000
4000
3000
2000
1000
0
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015

Inexpensive Approximation of Zero-Coupon Bond:

How expensive would have been at time t

an inexpensive approximation $P(t, T)$

of a ZC-bond with maturity T?

Inexpensive Approximation of ZC-Bond $P(t, T)$

Pays **one savings account unit** at time T:

1. Pl. & Heath (2006):

$$P(t, T) = 1 - \exp\left\{-\frac{S_t}{2(e^{\tau_T} - e^{\tau_t})}\right\}$$

Activity time τ_t, **diversified stock portfolio** S_t.

2. Fergusson & Pl. (2022), Barone Adesi, Pl. & Sala (2024):

$$P(t, T) \approx 1 - \exp\left\{-\frac{S_t}{2(e^{\bar{\tau}_T} - e^{\bar{\tau}_t})}\right\}$$

Trendline $\bar{\tau}_t$.

3. Pl. (2024):

$$P(t, T) \approx 1 - \exp\left\{-\frac{S_t}{2(e^{\bar{\tau}_T} - e^{\tau_t})}\right\}.$$

Activity time τ_t

$$\tau_t = \ln([\sqrt{S_\cdot}]_{t_0,t} + e^{\tau_{t_0}})$$

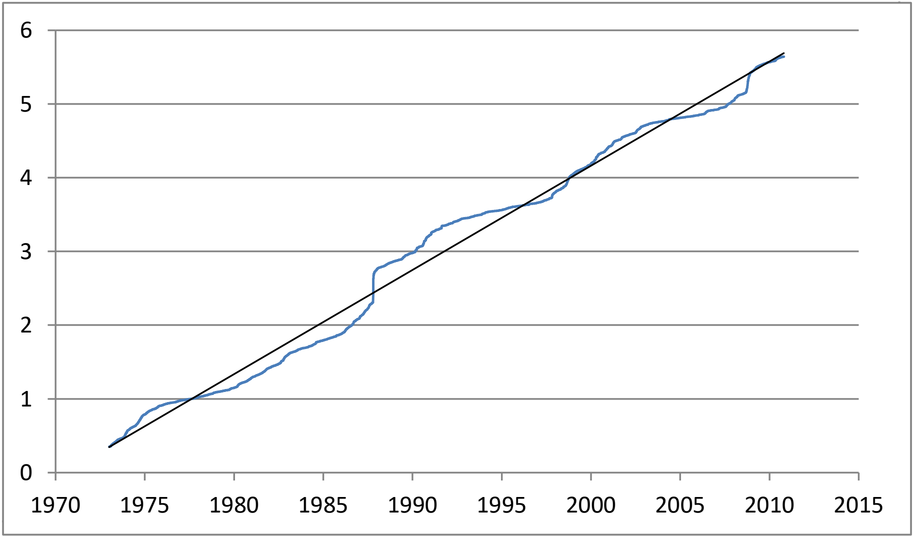
Quadratic variation:

$$[\sqrt{S_\cdot}]_{t_0,t_n} \approx \sum_{i=1}^{n} (\sqrt{S_{t_i}} - \sqrt{S_{t_{i-1}}})^2$$

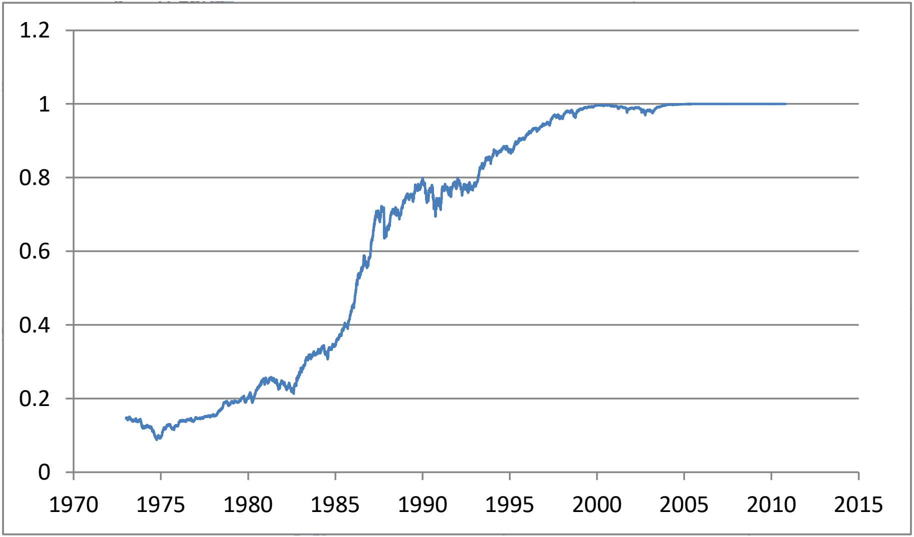
One observes approximately **linear activity time**:
Maximize R^2 of trendline $\rightarrow$
activity times $\tau_{t_0}, \tau_{t_1}, ..., \tau_{t_n}$,

Trendline $\bar{\tau}_{t_i} = \bar{\tau}_{t_0} + \bar{a} t_i$.

Activity time $\tau_t = \ln([\sqrt{S_\cdot}]_{t_0,t} + e^{\tau_{t_0}})$ and Trendline $\bar{\tau}_t$



$$P(t,T) = 1 - \exp\left\{-\frac{S_t}{2(e^{\bar{\tau}T} - e^{\tau t})}\right\}$$



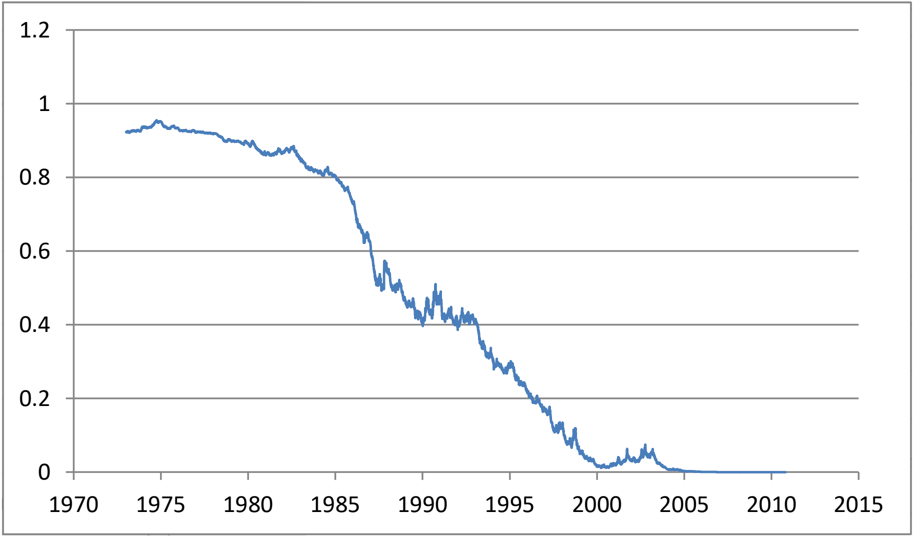
Hedge Portfolio V_t

$$V_{t_i} = V_{t_{i-1}} \left(1 + \pi_{t_{i-1}} \left(\frac{S_{t_i}}{S_{t_{i-1}}} - 1\right)\right)$$

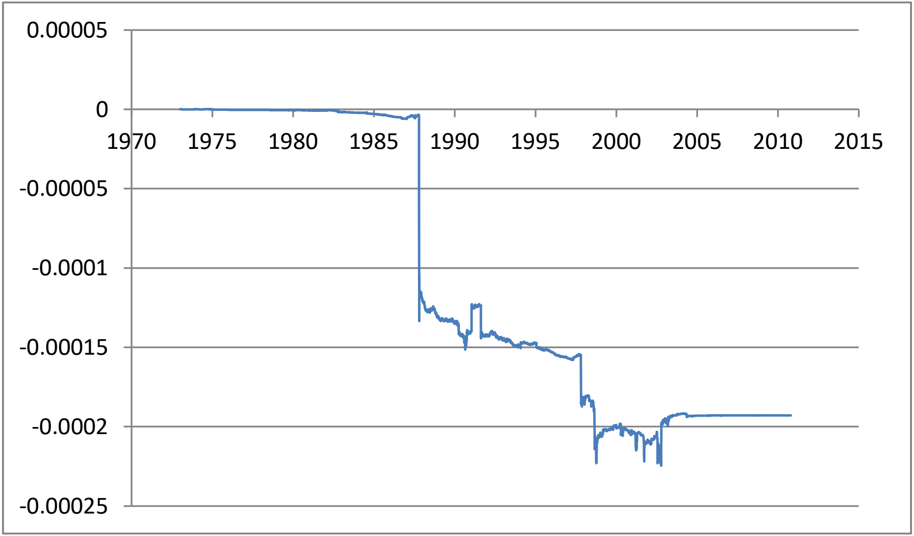
Dynamically invested in diversified stock portfolio S_t with weight:

$$\pi_t = (1 - P(t,T)^{-1}) \ln(1 - P(t,T))$$

Weight π_t invested in MSCI

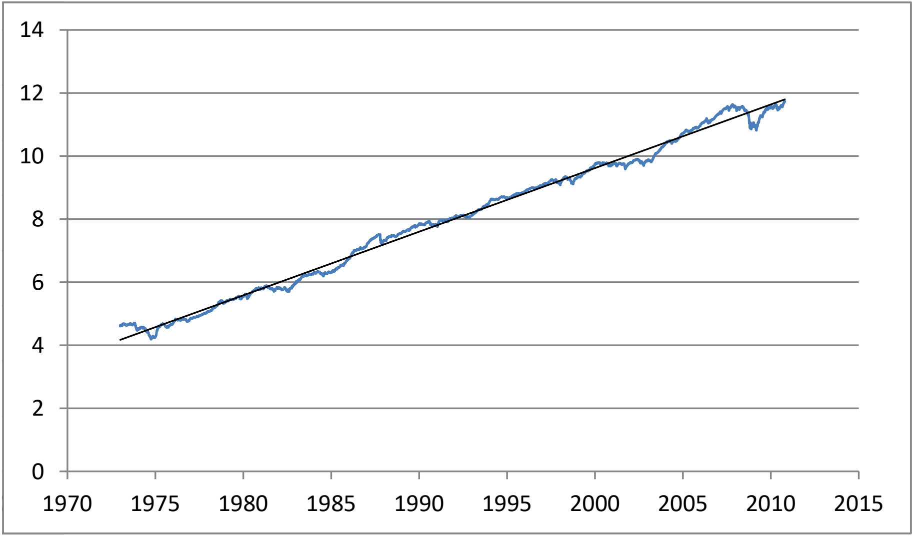


Hedge error $V(t) - P(t,T)$ with MSCI



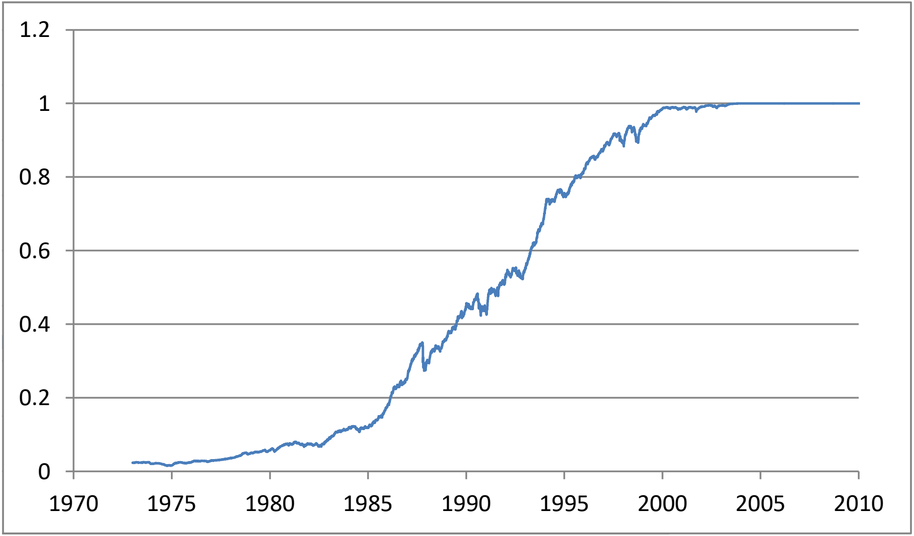
Logarithm of discounted EWI114

Pl. & Rendek (2012, 2020), $\approx$ 20% growth rate

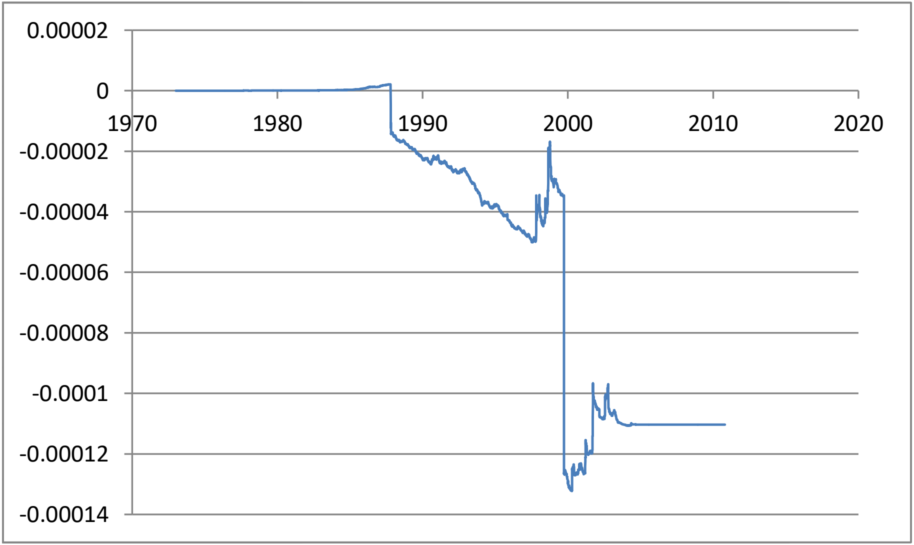


Inexpensive Approximated ZC-Bond

EWI114 as benchmark $\rightarrow P(t_0, T) \approx 0.023$



Hedge Error with EWI114



Benchmark Approach:

Pl.& Heath (2006).

Minimal assumptions $\rightarrow$ **mathematical statements**!

Benchmark S_t - diversified stock portfolio,

e.g.: ASX200, DAX, MSCI, EWI114

Risk management designed around **diversified stock portfolio**,
$\rightarrow$ **higher long-term payouts** than classical theory.

First Assumption:

Growth optimal portfolio (GOP) S_t^* exists!

$\rightarrow$ **Real-world pricing formula:**

$$H_t = \mathbf{E}_t\left(\frac{S_t^*}{S_T^*} H_T\right)$$

minimal possible price

$\mathbf{E}_t\,(.)$ conditional expectation under real-world probability measure given the information at t.

Second Assumption:

Diversified stock portfolio S_t
has in activity time τ_t
entropy-maximized dynamics.

Minimal market model Pl. (2001).
Branching process!
Independent 'birth and death' of wealth units.

Natural dynamics of diversified wealth!

Pl. (2024)

Minimal Market Model in Activity Time

Pl. (2001), Pl. 2024 $\rightarrow$ S_t **squared Bessel process**

observed quadratic variation:

$$\left[\sqrt{S_\cdot}\right]_{t_0,t} = \int_{t_0}^{t} e^{\tau_s} d\tau_s = e^{\tau_t} - e^{\tau_{t_0}}$$

$\rightarrow$ **activity time**

$$\tau_t = \ln\left(\left[\sqrt{S_\cdot}\right]_{t_0,t} + e^{\tau_{t_0}}\right)$$

Risk-Neutral Pricing:

Savings account S_t^0 as numéraire,

$\frac{S_t^0}{S_t^*}$ **is not a martingale**

$\rightarrow$ **Risk-neutral prices more expensive!**

Pl. (2024)

Benchmark-Neutral Pricing

Takes diversified stock portfolio S_t as a numéraire

$\frac{S_t}{S_t^*}$ martingale $\rightarrow$ numéraire change possible $\rightarrow$

Benchmark-neutral pricing formula:

$$H_t = \mathbf{E}_t^{Q_S}\left(\frac{S_t}{S_T} H_T\right)$$

$\mathbf{E}_t^{Q_S}(.)$ conditional expectation
under benchmark-neutral probability measure Q_S,
Pl. (2024)

→ Inexpensive Approximate ZC-Bond

$$P(t, T) = \mathbf{E}^{Q_S}\left(\frac{S_t}{S_T}|\mathcal{F}_t\right) = 1 - \exp\left\{-\frac{S_t}{2(e^{\tau_T} - e^{\tau_t})}\right\}$$

Minimal possible price

The initial value of the savings account is fixed as the value of one unit of the currency at an initial time $t_0 \leq T_1 < T_2$.

Term Annuity Bond (TAB)

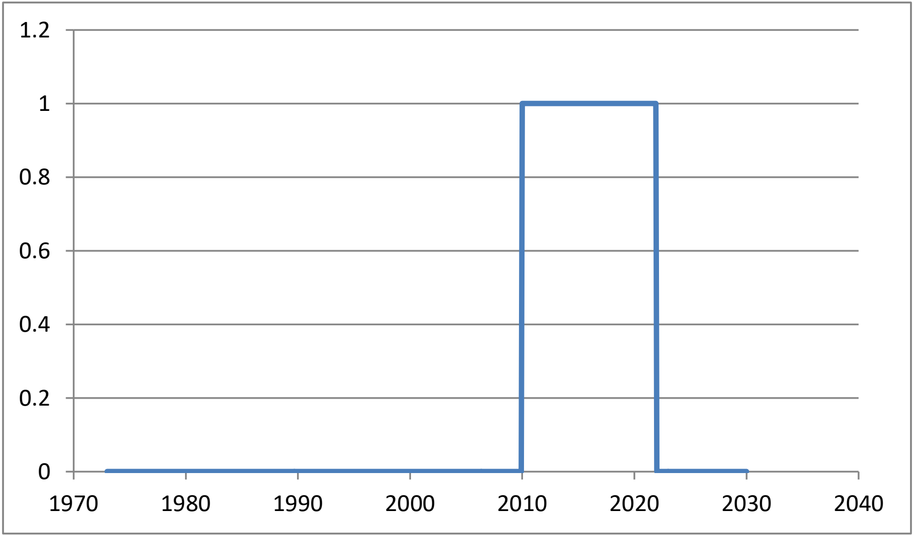
(T_1-T_2)-TAB:

First payment date on 1 January in the year T_1.

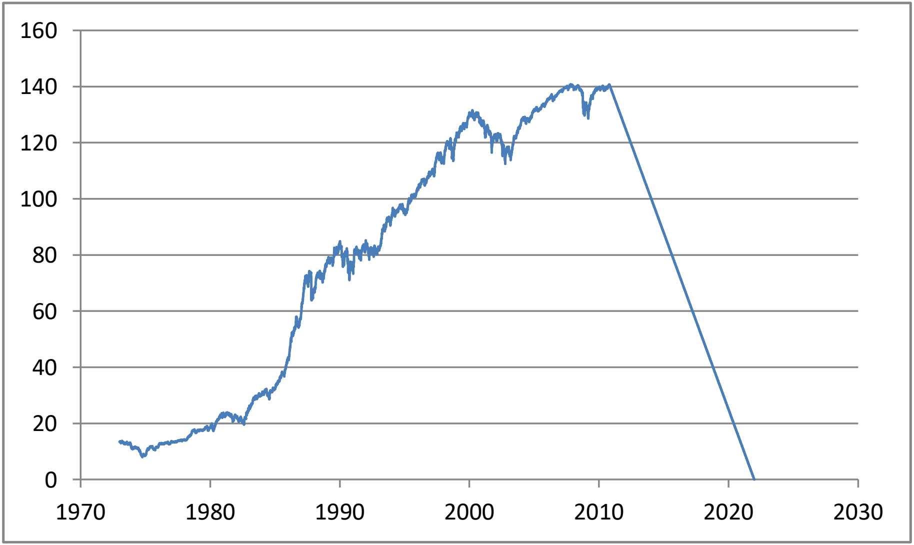
Last payment date 1 December in the year $T_1 + x$, e.g., $x = 12$.

Pays every month one unit of the savings account.

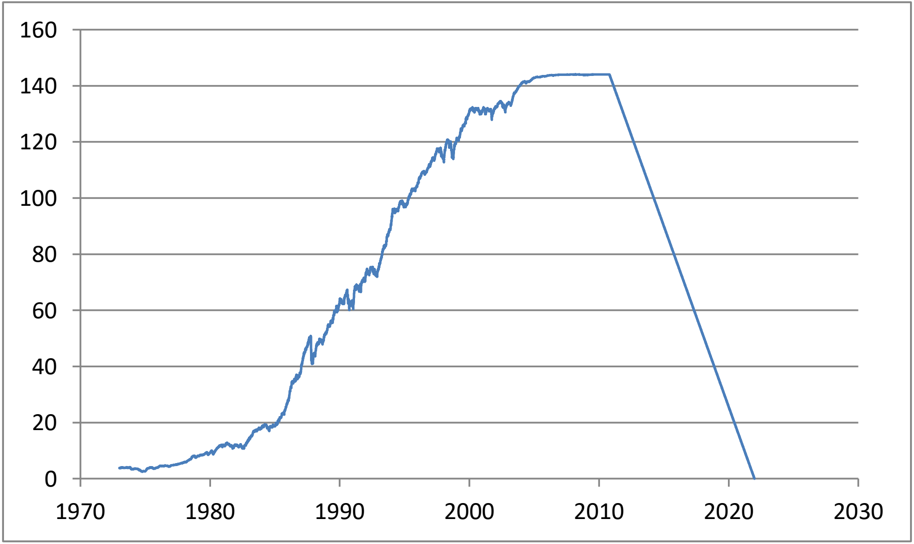
Payments of (2010-2021)-TAB



(2010-2021)-TAB with MSCI



(2010-2021)-TAB with EWI114



Summary

1. One can produce higher pension payouts than currently practiced!

2. The production method is mathematically sound and empirically extremely accurate!

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