Scenarios-based portfolio management: from theory to practice

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Hypothetical scenario

March 2020:

→ COVID-19 fear and equity markets have fallen by 40%
→ But central banks are intervening in markets and providing liquidity and support
→ A portfolio manager identifies two possible scenarios for the next 12 months:
  1. Equities fall another 20% (30% likelihood)
  2. Equities rally 50% (70% likelihood)
Incorporating scenarios into portfolio construction

How can insight into market scenarios be efficiently incorporated into portfolio management decisions?
1. We explore how academic research techniques address this problem
2. We consider the practical challenges regarding industry application
Relevant literature

1. Single-period mean-variance efficient portfolio construction
   ➔ Markowitz (1952), Tobin (1958), Sharpe (1963)

2. Multi-period utility maximization
3. Regime-switching

4. Parameter uncertainty

5. Robust portfolio construction
   → Peijnenburg (2011)
Explore the effectiveness of different approaches to acknowledging scenarios.

Problem setting:

— Seek to maximise utility of terminal wealth assuming CRRA preferences ($\alpha_R = 5$)
— Institutional constraints: no leverage or net short positioning
— Two assets, each with normal i.i.d. returns

<table>
<thead>
<tr>
<th></th>
<th>‘Bad’ Scenario</th>
<th></th>
<th>‘Good’ Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exp. Return</td>
<td>Volatility</td>
<td>Exp. Return</td>
</tr>
<tr>
<td>Cash</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Stocks</td>
<td>-20%</td>
<td>18%</td>
<td>40%</td>
</tr>
</tbody>
</table>
Maximising utility

Detail objective function

\[ U(W_T) = \frac{W_T^{1-A_R}}{(1 - A_R)} \]

Solution technique:
- Determine distribution to be sampled from
- Sample from this distribution to approximate the distribution of outcomes
- Consider different weights and compare which one maximises expected utility
Alternative portfolio construction approaches

Four alternative portfolio construction approaches

<table>
<thead>
<tr>
<th>Description</th>
<th>Approach 1</th>
<th>Approach 2</th>
<th>Approach 3</th>
<th>Approach 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Use the most likely scenario</td>
<td>Take a mean of the expectations</td>
<td>Account for parameter uncertainty in expected returns</td>
<td>Sample from both distributions</td>
</tr>
<tr>
<td>Mean</td>
<td>50%</td>
<td>29%</td>
<td>29% (p/u: 32.1%)</td>
<td>N/A</td>
</tr>
<tr>
<td>Volatility</td>
<td>18%</td>
<td>18%</td>
<td>18%</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Simulated distributions

Simulated Return Distributions

- **Approach 1** - "Most likely"
- **Approach 2** - "Weighted average"
- **Approach 3** - "Parameter uncertainty"
- **Approach 4** - "Sample from both scenarios"
## Sample summary statistics

### Comparison of sample summary statistics

<table>
<thead>
<tr>
<th>Approach</th>
<th>Description</th>
<th>Mean</th>
<th>Volatility</th>
<th>Volatility (sampled)</th>
<th>Volatility (sampled)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Approach 1</td>
<td>Use the most likely scenario</td>
<td>50%</td>
<td>18%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Approach 2</td>
<td>Take a mean of the expectations</td>
<td>29%</td>
<td>18%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Approach 3</td>
<td>Account for parameter uncertainty in expected returns</td>
<td>29% (p/u: 32.1%)</td>
<td>18%</td>
<td>36.8%</td>
<td></td>
</tr>
<tr>
<td>Approach 4</td>
<td>Sample from both distributions</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Sample from both distributions**
## Optimal allocations

<table>
<thead>
<tr>
<th>Description</th>
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<th>Approach 3</th>
<th>Approach 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use the most likely scenario</td>
<td>Take a mean of the expectations</td>
<td>Account for parameter uncertainty in expected returns</td>
<td>Sample from both distributions</td>
<td></td>
</tr>
<tr>
<td>Optimal allocation</td>
<td>100%</td>
<td>100%</td>
<td>42.5%</td>
<td>40%</td>
</tr>
</tbody>
</table>
Efficiency assessment

We use CEW (certainty equivalent wealth) to compare outcomes.

<table>
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<th>Approach 3</th>
<th>Approach 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEW</td>
<td>-80.8%</td>
<td>-80.8%</td>
<td>-0.04%</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Discussion of results

→ Ignoring information about expected returns is inefficient (Approach 1)

→ Ignoring information about the variability in outcomes is inefficient (Approach 1 and Approach 2)

→ In this case, using parameter uncertainty appears a reasonable proxy to sampling from both distributions

→ Sampling from both distributions is most efficient (if applying a sampling-based solution technique)
In practice

→ It appears intuitive to make full use of scenario-based information when constructing portfolios.

→ In practice there are a range of challenges:

1. Large universe of assets
2. Potentially more than two scenarios
3. Correlation structure required (and possibly correlation scenarios)
4. Parametrisation challenges – need to derive well-formed parameters for each asset / scenario combination
5. Computational challenges – curse of dimensionality
In practice

—Nonetheless, some interesting reflections:

—Use scenarios, where available, to sense-check parameter estimates

—Acknowledge that there can be great uncertainty in estimates of expected return
Conclusion

→ The academic literature has considered techniques for accommodating scenario-based views on assets
→ Ignoring the uncertainty created by divergent scenarios can generate significant utility cost
→ In practice there are many factors which make a pure scenarios-based approach difficult
→ But there is the possibility to integrate some of the academic techniques into industry practice