Frailty and Risk Classification for Life Annuities

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Introduction & motivation

The importance of private pension solutions is expected to increase

However:

The life annuity market is still underdeveloped

To expand their business, insurers have started offering higher annuity rates to those whose health conditions are critical

- The portfolio can become larger, but also more heterogeneous
- How to justify from a theoretical point of view the different annuity rates?
- Trade-off between portfolio size and heterogeneity in respect of the insurer's risk profile

Heterogeneity in life insurance – I

Heterogeneity of a population in respect of mortality is due to:

- Biological and physiological individual features (Age, gender)
- Living environment (Climate, pollution, nutrition standards)
- Occupation
- Individual lifestyle
- Current health condition, personal and/or family medical history
- ...

The effect of some of these features is unobservable

Heterogeneity in life insurance – II

Heterogeneity in life insurance

According to their specific (observable) features, individuals are grouped into risk classes

> *Risk classes are not fully homogeneous, but they should show a reduced heterogeneity*

The several risk classes show different values for the expected lifetime

- Standard risks: Individuals whose specific features are considered to be the normal ones for the product dealt with
- Substandard and preferred risks: Individuals in poorer (substandard) or better (preferred) conditions than the standard ones, and then with a lower or higher life expectancy

Heterogeneity in life insurance – III

In the case of life annuities:

"Substandard" risks ⇒ "Special-rate" annuities

Same technical structure, but different mortality profile than standard annuities

⇒ Higher annuity rates

Heterogeneity in life insurance – IV

Mortality for substandard or preferred risks

In actuarial practice: higher or lower mortality levels obtained by adjusting the population mortality rates

For example (further details available in [Olivieri, 2006], [Haberman and Olivieri, 2014]):

$$q_x^{(A)} = a \cdot q_x + b$$

$$\mu_x^{(A)} = a \cdot \mu_x + b$$

$$\mu_x^{A} = \mu_{x+z}$$

$$\ldots$$

- This choice is not (always) justified from a formal point of view
- Heterogeneity is not explicitly modelled
- Conversely, interesting results can be obtained modelling explicitly the heterogeneity of the population

Modelling heterogeneity – I

Discrete approaches

Heterogeneity is expressed through a (finite) mixture of mortality functions (e.g. forces of mortality), where each function is referred to a homogeneous group inside the heterogeneous population

Contributions provided in particular by [Keyfitz and Littman, 1979], [Levinson, 1959], [Redington, 1969]

For a review: [Olivieri, 2006]

Recent contributions: [Avraam et al., 2014], [Boumezoued et al., 2016]

Continuous approaches

Based on a non-negative real-valued variable, which expresses the individual frailty, i.e. the unobservable risk factors affecting individual mortality

> *Those people with a higher frailty have a lower expected lifetime than others*

Modelling heterogeneity – II

The fixed frailty approach

The individual frailty level is unknown, but keeps constant lifelong

Approach proposed by [Beard, 1959] and [Vaupel et al., 1979], followed by many contributions: [Hougaard, 1984, Hougaard, 1986], [Manton et al., 1986], [Yashin et al., 1985], [Yashin and Iachine, 1997], [Steinsaltz and Wachter, 2006], and many others

For a compact review, see: [Olivieri, 2006], [Haberman and Olivieri, 2014]

The changing frailty approach

The individual frailty stochastically changes with age

Model proposed by [Le Bras, 1976]

Fixed frailty approach and changing frailty approach compared by [Thatcher, 1999] and [Yashin et al., 1994]

Markov ageing models, generalizing Le Bras's assumption, adopted by: [Su and Sherris, 2012], [Lin and Liu, 2007], [Liu and Lin, 2012], [Sherris and Zhou, 2014]

Modelling heterogeneity – III

In the following: Fixed-frailty approach

[Yashin et al., 1994] show that changing frailty models cannot be distinguished from a fixed frailty model

The fixed frailty model, especially under the traditional Gompertz-Gamma assumption, is more convenient and provides a satisfactory fitting to empirical data

The frailty – I

Approach proposed by [Beard, 1959] and [Vaupel et al., 1979]

Basic assumptions

Reference: heterogeneous cohort, defined at age 0 and closed to new entrants

Heterogeneity is expressed by the individual frailty

The individual frailty keeps constant in time, and unknown

The average frailty level in the whole population is expected to decline with age, given that people with lower frailty are expected to live longer

The frailty – II

Individual force of mortality

- Standard force of mortality: μ_x
- Force of mortality for an individual with frailty level z: $\mu_x(z) = z \cdot \mu_x$
- Depending on the value of z (z takes value in $(0, \infty)$): $\mu_x(z) \gtreqless \mu_x$
- If $z = 1 \Rightarrow \mu_x(1) = \mu_x$

Expected frailty and population force of mortality

- Z_x: Random value of the frailty, whose probability distribution is measured on the population at age x
- $g_x(z)$: Probability density function
- Average force of mortality of the population at age x (or: Population force of mortality): $\overline{\mu}_x = \mu_x \cdot \int_0^\infty z \cdot g_x(z) \, \mathrm{d}z = \mu_x \cdot \underbrace{\mathbb{E}[Z_x]}_{\overline{z}_x}$

The frailty – III

Survival function

- Survival function for an individual (newborn) with frailty level z: $S(x|z) = \mathrm{e}^{-\int_0^x \mu_t(z)\,\mathrm{d}t} = \mathrm{e}^{-z \cdot H(x)}$ (where: $H(x) = \int_0^x \mu_t\,\mathrm{d}t$)
- Survival function of the population, *or* expected share of individuals alive at age x out of the initial newborns $\overline{S}(x) = \int_0^\infty S(x|z) \cdot g_0(z)\,\mathrm{d}z$

Probability density function of Z_x

Thanks to the multiplicative assumption for $\mu_x(z)$: $g_x(z) = \frac{g_0(z) \cdot S(x|z)}{\overline{S}(x)}$

Thus, the frailty model is defined once we assign:

- The probability density function of Z_0, $g_0(z)$
- The standard force of mortality μ_x

The traditional setting – I

See [Beard, 1959] and [Vaupel et al., 1979]

The probability distribution of the frailty

- Let $Z_0 \sim \text{Gamma}(\delta, \theta)$
- Then (thanks to the multiplicative model), we have: $Z_x \sim \text{Gamma}(\delta, \theta + H(x))$
- Shortly: $\theta(x) = \theta + H(x)$. Then: $Z_x \sim \text{Gamma}(\delta, \theta(x))$

The traditional setting – II

Summary statistics

- Average frailty level in the population at age x: $\overline{z}_x = \frac{\delta}{\theta(x)}$
- Variance: $\mathbb{V}\text{ar}[Z_x] = \frac{\delta}{(\theta(x))^2}$
- Coefficient of variation: $\mathbb{CV}[Z_x] = \frac{\sqrt{\mathbb{V}\text{ar}[Z_x]}}{\mathbb{E}[Z_x]} = \frac{1}{\sqrt{\delta}}$ (constant)

  δ measures, in relative terms, the level of heterogeneity of the population
  - *Small values for $\delta \Rightarrow$ High degree of heterogeneity*
  - *$\delta \to \infty \Rightarrow$ (Almost) homogeneous population*

- Note: decreasing value of the expected frailty, but constant relative variability

Setting the parameters

- Usually, parameters are chosen so that $\overline{z}_0 = 1 \Rightarrow \theta = \delta$
- δ is chosen to reflect the degree of heterogeneity of the population

The traditional setting – III

Population force of mortality

- Assume the Gompertz law for the standard force of mortality: $\mu_x = \alpha \cdot \mathrm{e}^{\beta \cdot x}$
- Then: average force of mortality of the population:

$$\overline{\mu}_x = \frac{\alpha' \cdot \mathrm{e}^{\beta \cdot x}}{1 + \delta' \cdot \mathrm{e}^{\beta \cdot x}}$$

where: $\alpha' = \frac{\alpha \cdot \delta}{\theta - \frac{\alpha}{\beta}}$ and $\delta' = \frac{\alpha}{\beta \cdot \theta - \alpha}$

$\Rightarrow$ Logistic age-pattern (First Perks law)

The individual frailty in a cohort implies a deceleration in the population mortality

Actuarial applications of the frailty model

Frailty models are very well-known in demography, less in actuarial science

Actuarial applications:

- Fitting to insurance data: [Butt and Haberman, 2002, Butt and Haberman, 2004], [Avanzi et al., 2015]
- Impact of heterogeneity and frailty on the actuarial values of life annuities: [Meyricke and Sherris, 2013]
- Impact on tail risk and solvency capital: [Olivieri, 2006], [Sherris and Zhou, 2014]

Identifying the risk classes through a frailty model

Identifying risk classes through a frailty model

We assume to hold the probability distribution of the frailty for the general population

A risk class is identified by a given range of values for the frailty

- The frailty of an individual in the general population takes value in $(0, \infty)$
- The frailty of an individual in risk class G_j takes value in $(z_{j-1}, z_j]$

- G_1: Class of standard risks
- G_j, $j > 1$: Special-rate annuities

The set $\{G_j; j = 1, \ldots, J\}$ constitutes a partition of the sample space of the frailty

Note: a standard risk is not assigned the so-called standard force of mortality

Indeed, standard risks are those with a frailty value in $(0, z_1]$, where z_1 can be lower than 1

Lifetime and frailty for the risk classes – I

The probability distribution of the frailty of risk group G_j can be assessed as a conditional distribution of the frailty for the whole population

Relative size of group G_j at age x

$$\rho_{j;x} = \mathbb{P}[z_{j-1} < Z_x \leq z_j] = F(z_j; \delta, \theta(x)) - F(z_{j-1}; \delta, \theta(x))$$

where $F(z; \delta, \theta(x))$ is the probability distribution function of a Gamma($\delta, \theta(x)$)-distributed random variable

Probability distribution function of the frailty in risk group G_j, age x

$$F(z; \delta, \theta(x)|G_j) = \begin{cases} 0 & \text{if } z \leq z_{j-1} \\ \frac{F(z;\delta,\theta(x)) - F(z_{j-1};\delta,\theta(x))}{\rho_{j;x}} & \text{if } z_{j-1} < z \leq z_j \\ 1 & \text{if } z > z_j \end{cases}$$

Lifetime and frailty for the risk classes – II

Expected value of the frailty in group G_j, age x

$$\mathbb{E}[Z_x|G_j] = \mathbb{E}[Z_x] \cdot \frac{F(z_j; \delta+1, \theta(x)) - F(z_{j-1}; \delta+1, \theta(x))}{\rho_{j;x}}$$

Variance of the frailty in group G_j, age x

$$\mathbb{V}\mathrm{ar}[Z_x|G_j] = \mathbb{V}\mathrm{ar}[Z_x] \cdot \Big((\delta+1) \cdot \frac{F(z_j;\delta+2,\theta(x)) - F(z_{j-1};\delta+2,\theta(x))}{\rho_{j;x}} - \delta \cdot \frac{F(z_j;\delta+1,\theta(x)) - F(z_{j-1};\delta+1,\theta(x))}{\rho_{j;x}}\Big)$$

Average survival function in group G_j, age x

$$\overline{S}(x|G_j) = \overline{S}(x) \cdot \frac{\rho_{j;x}}{\rho_{j;0}}$$

Next step: setting the frailty limits z_{j-1}, z_j

Model calibration – I

Assumptions

Immediate life annuities, paid in arrears

A cohort of males, initial age $x_0 = 65$

G_1: standard risks

G_2, G_3: preferred risks

Reference life tables

The Gompertz-Gamma model is calibrated on two Italian projected life tables

- TG62, describing mortality for the general population (source: ISTAT)
- A62I, describing mortality for voluntary immediate life annuities (source: ANIA)

Model calibration – II

Steps for the calibration

1. Gompertz-Gamma model for the general population referring to life table TG62 (assuming $\overline{z}_0 = 1$, i.e. $\delta = \theta$)
2. Setting the parameter z_1 for standard risks, referring to life table A62I
3. Setting the parameter z_2 assuming appropriate benchmarks for the reduced values of the expected lifetime

Risk classes

Group	**Frailty interval** $(z_{j-1}, z_j]$	**Relative size at age 65 of group G_j in the general population** $\rho_{j;65}$	**Expected value of the frailty** $\mathbb{E}[Z_{65} \| G_j]$	**Coefficient of variation** $\mathbb{CV}[Z_{65} \| G_j]$	**Expected lifetime** $\mathbb{E}[T_{65} \| G_j]$
G_1	$(0, 1.038741]$	60.121%	0.845593	15.243%	22.81
G_2	$(1.038741, 1.307144]$	30.111%	1.152338	6.479%	20.36
G_3	$(1.307144, \infty)$	9.769%	1.445866	8.736%	18.71
Population	$(0, \infty)$	100%	0.996594	23.308%	21.67

The present value of future benefits – I

Annuity rates

Based on the traditional equivalence principle

For each class, a different mortality assumption (i.e., different conditional values for the Gompertz-Gamma model)

- Actuarial value of the annuity for group G_j:

$$a_{x_0;j} = \sum_{s=1}^{\infty} (1+r)^{-s} \cdot \frac{\overline{S}(x_0 + s|G_j)}{\overline{S}(x_0|G_j)}$$

  r: discount rate, assumed to be deterministic and constant

- Benefit amount for group G_j:

$$b_j = S \cdot \frac{1}{a_{x_0;j}}$$

The present value of future benefits – II

Present value at time t of future benefits

- For group G_j

$$PV_{t;j} = \sum_{s=t+1}^{\infty} b_j \cdot N_{s;j} \cdot v(t,s)$$

$N_{s;j}$: (random) number of individuals in class G_j, time s

$v(t,s)$: discount factor, assumed to be deterministic

$\Rightarrow$ To simplify: $v(t,s) = (1+r)^{-(s-t)}$

- For the whole portfolio

$$PV_t = \sum_j PV_{t;j}$$

Assessment through stochastic simulation

Numerical findings: Portfolios – I

Alternative portfolios

Groups	Portfolios					
	A	B	C	D	E	F
G_1	1 000	1 000	1 000	1 000	1 000	500
G_2	0	200	250	200	501	500
G_3	0	0	0	50	162	0
All	1 000	1 200	1 250	1 250	1 663	1 000

- Portfolio A: base case
- Portfolio E: largest possible size (according to the Gompertz-Gamma model)
- Portfolio B: more heterogeneous than A, but larger
- Portfolio C vs B: same risk classes, but larger size
- Portfolio C vs D: same size, different degree of heterogeneity
- Portfolio F: same size as A, but adverse-selection (thus: more heterogeneous)

Numerical findings: Portfolios – II

Underlying question

When insurers offer special-rate annuities, they can increase the portfolio size

$\Rightarrow$ The pooling effect improves $\Rightarrow$ Risk reduces

However: The heterogeneity increases $\Rightarrow$ Risk increases

What is the result of this trade-off?

In the following:

- Discount rate: $r = 0\%$
- Initial amount (single premium): $S = 100$ for each annuitant

Numerical findings: Benefit amounts

Individual benefit amounts

	Group G_1	Group G_2	Group G_3
Benefit amount b_j	4.483	5.034	5.492
$\frac{b_j}{b_1} - 1$	0%	12.302%	22.515%

Average benefit amount for the portfolio

Additional amount in respect of the base case

Time t	Portfolio A	Portfolio B	Portfolio C	Portfolio D	Portfolio E	Portfolio F
0	0%	2.050%	2.460%	2.869%	5.899%	6.151%
5	0%	2.022%	2.434%	2.826%	5.820%	6.112%
10	0%	1.981%	2.381%	2.740%	5.694%	6.032%
15	0%	1.913%	2.296%	2.633%	5.481%	5.893%
20	0%	1.786%	2.150%	2.402%	5.116%	5.654%
25	0%	1.592%	1.918%	2.073%	4.503%	5.227%
30	0%	1.268%	1.555%	1.587%	3.574%	4.522%
35	0%	0.868%	1.001%	1.120%	2.287%	3.189%
40	0%	0.000%	0.000%	0.000%	0.879%	1.757%
45	0%	0.000%	0.000%	0.000%	0.000%	

Numerical findings: Expected portfolio sizes and compositions

To give an idea: Portfolios C and D

Time t	Portfolio C					Portfolio D				
	Total size		Relative group size			Total size		Relative group size		
	n_t	$\frac{n_t}{n_0}$	Group G_1	Group G_2	Group G_3	n_t	$\frac{n_t}{n_0}$	Group G_1	Group G_2	Group G_3
0	1 250	100.00%	80.000%	20.000%	0.000%	1 250	100.00%	80.000%	16.000%	4.000%
5	1 198	95.84%	80.217%	19.783%	0.000%	1 197	95.76%	80.284%	15.789%	3.926%
10	1 111	88.88%	80.648%	19.352%	0.000%	1 109	88.72%	80.794%	15.509%	3.697%
15	975	78.00%	81.333%	18.667%	0.000%	973	77.84%	81.501%	15.005%	3.494%
20	778	62.24%	82.519%	17.481%	0.000%	774	61.92%	82.946%	14.083%	2.972%
25	526	42.08%	84.411%	15.589%	0.000%	522	41.76%	85.057%	12.644%	2.299%
30	269	21.52%	87.361%	12.639%	0.000%	266	21.28%	88.346%	10.150%	1.504%
35	86	6.88%	91.860%	8.140%	0.000%	86	6.88%	91.860%	6.977%	1.163%
40	13	1.04%	100.000%	0.000%	0.000%	13	1.04%	100.000%	0.000%	0.000%
45	1	0.08%	100.000%	0.000%	0.000%	1	0.08%	100.000%	0.000%	0.000%
50	0	0.00%				0	0.00%			

Note: The relative size of standard risks (i.e., those with the lowest individual benefit amount) increases in time, due to their lower frailty

Numerical findings: Summary statistics of the present value of future benefits – I

Expected present value of future benefits, per policy in-force: $\frac{\mathbb{E}[PV_t]}{n_t}$

Time t	**Portfolio A**	**Portfolio B**	**Portfolio C**	**Portfolio D**	**Portfolio E**	**Portfolio F**
	Abs. value	**% of the value obtained for Portfolio A**				
0	100.00	100.00%	100.00%	100.00%	100.01%	100.01%
5	81.26	99.71%	99.65%	99.60%	99.18%	99.13%
10	64.00	99.37%	99.24%	99.15%	98.24%	98.10%
15	48.62	99.00%	98.80%	98.66%	97.24%	96.94%
20	35.44	98.63%	98.35%	98.22%	96.25%	95.67%
25	24.66	98.32%	97.98%	97.89%	95.45%	94.47%
30	16.35	98.18%	97.77%	97.82%	95.13%	93.55%
35	10.34	98.31%	98.04%	97.94%	95.72%	93.71%
40	6.32	100.00%	100.00%	100.00%	97.63%	95.54%
45	3.93	100.00%	100.00%	100.00%	100.00%	0.00%

Numerical findings: Summary statistics of the present value of future benefits – II

Coefficient of variation of the present value of future benefits: $\mathbb{CV}[PV_t]$

Time t	Portfolio A	Portfolio B	Portfolio C	Portfolio D	Portfolio E	Portfolio F
0	1.30%	1.20%	1.17%	1.18%	1.04%	1.87%
5	1.48%	1.37%	1.34%	1.35%	1.19%	1.55%
10	1.75%	1.62%	1.60%	1.60%	1.39%	1.80%
15	2.10%	1.96%	1.91%	1.93%	1.70%	2.19%
20	2.64%	2.45%	2.41%	2.43%	2.17%	2.80%
25	3.55%	3.34%	3.31%	3.31%	3.04%	3.97%
30	5.62%	5.38%	5.32%	5.35%	4.96%	6.54%
35	11.10%	10.78%	10.78%	10.73%	10.28%	13.82%
40	32.19%	32.19%	32.19%	32.19%	31.40%	44.42%
45	136.25%	136.25%	136.25%	136.25%	136.25%	

Numerical findings: Summary statistics of the present value of future benefits – III

99th percentile of PV_t, as a % of $\mathbb{E}[PV_t]$

Time t	Portfolio A	Portfolio B	Portfolio C	Portfolio D	Portfolio E	Portfolio F
0	103.07%	102.81%	102.70%	102.76%	102.44%	104.35%
5	103.46%	103.18%	103.06%	103.13%	102.73%	103.63%
10	104.12%	103.70%	103.77%	103.73%	103.22%	104.17%
15	104.98%	104.58%	104.48%	104.58%	104.00%	105.08%
20	106.36%	105.90%	105.73%	105.79%	105.15%	106.59%
25	108.28%	107.88%	107.78%	107.83%	106.98%	109.49%
30	113.66%	112.77%	112.77%	112.75%	111.91%	115.31%
35	126.74%	125.91%	125.69%	125.67%	124.79%	133.88%
40	185.49%	185.49%	185.49%	185.49%	181.62%	222.70%
45	570.00%	570.00%	570.00%	570.00%	570.00%	

Summarizing the main findings – I

Overall

Most of the convenience for the insurer seems to come from the reduction in time of the average benefit amount and then of the expected present value of future benefits

Risk profile

- Higher degrees of heterogeneity $\Rightarrow$ Higher risk profile
- If matched by a larger total portfolio size, the risk profile can benefit from portfolio diversification

Summarizing the main findings – II

Dependence on the choosen rating structure

We have tested alternative rating structures (in particular: a different definition of the risk group or a different degree of heterogeneity of the general population)

The main impact is on the magnitude of insurer's liabilities, rather than on its dispersion

Details in: [Olivieri and Pitacco, 2016]

Further issues to investigate

- Adverse-selection
- Impact on insurer's liabilities of an incorrect allocation of risks
- Impact of aggregate longevity risk (possibly different mortality dynamics for the several risk classes)
- . . . *?*

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Summary

- Model for risk classification in life insurance, within a frailty setting
- Application to life annuities
- Portfolio size vs heterogeneity
  - Reduction in time of the average benefit amount and then of the expected present value of future benefits