# Welfare Effects of Uniform Variable Annuities for Individuals with Different Educational Levels

#### Jun-Hee An, Anja De Wagaenaere, Theo Nijman

Tilburg University

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#### Research Question

What is the magnitude of the welfare losses or gains that annuitants with different educational levels face due to mismatches in survival rates, risk preferences, or both?

# Heterogeneous Mortality Rates for Educational Groups



- ▶ We employ education- and gender-specific mortality projections of the Dutch population constructed by Nusselder et al. (2022)
- ▶ Cumulative survival rates of different educational groups (black lines) show substantial gaps around the average population (red lines)

Heterogeneous Mortality Rates for Educational Groups



Table 1: Remaining Life Expectancy of Individuals aged 65 in 2020

▶ The effect of education on mortality is more pronounced among the male population (e.g., Backlund et al. (1999))

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	- $\triangleright$   $W_0(t)$  increases by the investment return over the period [0,t]
- $\triangleright$  The annuity provider needs to determine (i) the bucket sizes and (ii) the investment strategy of each bucket so as to maximize the discounted expected utility of the corresponding annuity payments

 $\blacktriangleright$  Financial market dynamics follow standard Merton (1971) setting:

$$
dS_t = \mu S_t dt + \sigma S_t dZ_t
$$
  
=  $(r + \lambda \sigma) S_t dt + \sigma S_t dZ_t$ 

where  $\mu$  is the instantaneous expected return of stock,  $\sigma$  the stock volatility,  $r$  the risk-free rate, and  $Z$  the Geometric Brownian motion

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▶ It then remains to determine the size of the annuity buckets such that the expected discounted utility of the corresponding consumption levels is optimized, given that the Merton strategy is used for each bucket.

 $\blacktriangleright$  Let  $R_{[0,t]}$  be the gross return over the period  $[0,t]$  of a continuously rebalanced portfolio with weight  $f^*$  on the risky asset:

$$
R_{[0,t]} \sim \exp\left\{ (r + f^*\sigma\lambda - \frac{1}{2}(f^*)^2\sigma^2)t + f^*\sigma\sqrt{t}Z \right\},
$$
  

$$
\sim \exp\left\{ (r + \frac{\lambda^2}{2\gamma^2}(2\gamma - 1))t + \frac{\lambda}{\gamma}\sqrt{t}Z \right\}
$$

 $\blacktriangleright$  The optimal bucket sizes must satisfy:

$$
\max_{\{W_0(t): t=0,1,\ldots,T_{\text{max}}\}} \mathbb{E}\bigg[\sum_{t=0}^{T_{\text{max}}} e^{-\beta t} \cdot {}_{t}p_{x} \cdot W_0(t)^{1-\gamma} \cdot (R_{[0,t]})^{1-\gamma}\bigg]
$$
\ns.t. 
$$
\sum_{t=0}^{T_{\text{max}}} W_0(t) \cdot {}_{t}p_{x} = W_0
$$

▶ A straightforward extension of Balter and Werker (2020) yields that the optimal bucket sizes are given by:

$$
W_0^*(t|\gamma,\boldsymbol{p})=W_0\cdot\kappa_t^{\frac{1}{\gamma}}\bigg/\bigg(\sum_{s=0}^{T_{max}}\kappa_s^{\frac{1}{\gamma}}\cdot {}_s\boldsymbol{p}_x\bigg),
$$

where  $\kappa_t$  equals:

$$
\kappa_t = \exp\bigg\{-\beta t + \bigg(r + \frac{\lambda^2}{2\gamma}\bigg)(1 - \gamma)t\bigg\}
$$

 $\triangleright$  Correspondingly, the optimal consumption in period t is:

$$
C_t^*(\gamma, \mathbf{p}) = R_{[0,t]}(\gamma) \cdot W_0^*(t|\gamma, \mathbf{p})
$$

 $\blacktriangleright$  We use the following notation:

▶ We denote  $\gamma_i > 1$  and  $\gamma_a > 1$  for the risk aversion level used by the annuity provider (insurers or pension funds) and the annuitant's true risk aversion level, respectively.

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	- $\triangleright$  Optimal tailor made annuity: the annuity based on the annuitant-specific characteristics (i.e.,  $C^*(\gamma_{\mathsf{a}},\boldsymbol{p}_{\mathsf{a}}))$

## Welfare Evaluation: Certainty Equivalent

- ▶ We use the Certainty Equivalent Consumption Loss (CECL) to quantify the welfare effects due to a mismatch in risk aversion level, survival rates, or both
- $\blacktriangleright$  For any given stochastic consumption stream  $\boldsymbol{C} = \{C_t : t = 0, 1, \ldots, T_{max}\}$ , the CECL is defined as:

$$
CE[\mathbf{C}] = U_{\gamma_a}^{-1} \bigg( \sum_{t=0}^{T_{max}} e^{-\beta t} \cdot {}_{t}p_x^{a} \cdot \mathbb{E} \bigg[ \frac{(C_t)^{1-\gamma_a}}{1-\gamma_a} \bigg] \bigg)
$$

where  $\boldsymbol{C}$  can either be  $C^*(\gamma_i, \boldsymbol{p}_i)$  or  $C^*(\gamma_a, \boldsymbol{p}_a)$ 

▶ The CECL from being offered the optimal uniform annuity instead of the optimal tailor made annuity is given by:

$$
L(\gamma_i, \gamma_a, \boldsymbol{p}_i, \boldsymbol{p}_a) = 1 - \frac{CE[\boldsymbol{C}^*(\gamma_i, \boldsymbol{p}_i)]}{CE[\boldsymbol{C}^*(\gamma_a, \boldsymbol{p}_a)]}
$$

Inadequate mortality asssumption and the MWR

- ▶ The seminal work of Mitchell et al. (1999) have introduced the money's worth ratio (MWR) which is the ratio of monetary value of the expected discounted annuity payments to the annuitant and the price charged by the annuity provider
- ▶ If the annuity priced is normalized to one, the MWR is equal to the value to the annuitant of the optimal uniform annuity:

$$
MWR(\gamma, \mathbf{p}_i, \mathbf{p}_a) = \sum_{t=0}^{T_{max}} W_0^*(t|\gamma, \mathbf{p}_i) \cdot {}_tp_x^a = \frac{\sum_{t=0}^{T_{max}} \kappa_t^{1/\gamma} \cdot {}_tp_x^a}{\sum_{t=0}^{T_{max}} \kappa_t^{1/\gamma} \cdot {}_tp_x^i}
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Inadequate mortality asssumption and the MWR

 $\triangleright$  We show that the MWR is equal to a scaling factor between two consumption streams with the same risk preference parameter:

$$
\pmb{C}^*(\gamma,\pmb{p}_i) = \textit{MWR}(\gamma,\pmb{p}_i,\pmb{p}_a) \cdot \pmb{C}^*(\gamma,\pmb{p}_a)
$$

 $\blacktriangleright$  Then, it holds that:

$$
\frac{\text{CE}[\boldsymbol{C}^*(\boldsymbol{\gamma},\boldsymbol{p}_i)]}{\text{CE}[\boldsymbol{C}^*(\boldsymbol{\gamma},\boldsymbol{p}_a)]} = \text{MWR}(\boldsymbol{\gamma},\boldsymbol{p}_i,\boldsymbol{p}_a),
$$

which in turn implies:

$$
L(\gamma,\gamma,\boldsymbol{p}_i,\boldsymbol{p}_a)=1-MWR(\gamma,\boldsymbol{p}_i,\boldsymbol{p}_a)
$$

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- $▶$  We test  $\gamma_i \in \{2, 3, 5, 8, 20\}$  and  $\gamma_a \in \{2, 3, 5, 8, 20\}$ 
	- ▶ Our financial market parameters:  $\gamma = 2$  (20) indicates  $f^* = 50\%$  (5%)

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- ▶ The annuity provider employs survival probabilities depending on the gender composition of the fund



# Results 1: Effect of Uniform Annuity Pricing



▶ The level of expected annuity payment shifts upward or downward depending on  $p$ 

# Results 1: Welfare Effects of Inadequate Mortality Assumption

Table 2: Welfare Losses of Annuitants with Different Education Levels, for  $\gamma_i = 5$ 



▶ Substantial differences in welfare effects by educational attainments ▶ Even low-educated women can benefit from favorable annuity pricing in most cases

# Results 2: Welfare Effects of Inadequate Risk Preference Parameter

Table 3: Welfare Loss Due to Inadequate Risk Aversion Parameters: Average Population



▶ Welfare losses  $\leq 15\%$ , except for extreme cases

▶ Does this depend on survival probabilities?

# Results 2: Welfare Effects of Inadequate Risk Preference Parameter



▶ Yes, but only marginally - relatively insensitive to the survival rates of the annuitant

### Results 3: Combined Welfare Effects - Are They Additive?

 $\blacktriangleright$  We previously have shown that:

$$
\boldsymbol{C}^*(\gamma_i,\boldsymbol{p}_i) = \textit{MWR}(\gamma_i,\boldsymbol{p}_i,\boldsymbol{p}_a) \cdot \boldsymbol{C}^*(\gamma_i,\boldsymbol{p}_a)
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$$

▶ Taking the certainty equivalent and dividing both sides of the equation by  $\mathit{CE}[\mathit{C}^*(\gamma_a,\bm{\rho}_a)]$  yields:

$$
\underbrace{1 - L(\gamma_i, \gamma_a, \mathbf{p}_i, \mathbf{p}_a)}_{\text{Mismatch } \mathbf{p} \text{ and } \gamma} = \underbrace{MWR(\gamma_i, \mathbf{p}_i, \mathbf{p}_a)}_{\text{Mismatch } \mathbf{p}} \cdot \underbrace{[1 - L(\gamma_i, \gamma_a, \mathbf{p}_a, \mathbf{p}_a)]}_{\text{Mismatch } \gamma}
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 $\blacktriangleright$  Rewriting the equations yields:

$$
L(\gamma_i, \gamma_a, \mathbf{p}_i, \mathbf{p}_a) = L(\gamma_i, \gamma_a, \mathbf{p}_a, \mathbf{p}_a) + L(\gamma_i, \gamma_i, \mathbf{p}_i, \mathbf{p}_a) -L(\gamma_i, \gamma_a, \mathbf{p}_a, \mathbf{p}_a) \cdot L(\gamma_i, \gamma_i, \mathbf{p}_i, \mathbf{p}_a)
$$

## Results 3: Combined Welfare Effects

Table 4: Welfare Gains/Losses Due to Inadequate  $\gamma$  and  $\boldsymbol{p}$  (Men)



- $\blacktriangleright$  The annuity provider assumes  $\gamma_i = 5$
- ▶ When risk preference parameter and survival probs are mismatched, low- and mid-educated men experience substantial welfare losses
- ▶ High-educated men mostly get welfare gains due to favorable annuity pricing unless extremely risk-averse

#### Results 3: Combined Welfare Effects

#### Table 5: Welfare Gains/Losses Due to Inadequate  $\gamma$  and  $\boldsymbol{p}$  (Women)



 $\triangleright$  Even for women with the lowest educational level, the welfare gains from favorable pricing of the uniform annuity can outweigh the welfare losses from a mismatch in risk aversion parameter

# Conclusion

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- $\triangleright$  We find that the welfare effects of mismatch in the survival probabilities can be substantially higher than those from inadequate risk preference parameters
	- ▶ Welfare gains due to the favorable annuity pricing for high-educated men and women of all education levels **easily cancel out** the welfare losses of having inadequate investment strategies
- ▶ The results of our study suggest that having funds with relatively homogeneous groups in terms of survival probabilities can alleviate the welfare losses of individuals, particularly those with low education levels



# Thank you!