### Welfare Effects of Uniform Variable Annuities for Individuals with Different Educational Levels

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#### Research Question

What is the magnitude of the welfare losses or gains that annuitants with different educational levels face due to mismatches in survival rates, risk preferences, or both?

#### Heterogeneous Mortality Rates for Educational Groups



- We employ education- and gender-specific mortality projections of the Dutch population constructed by Nusselder et al. (2022)
- Cumulative survival rates of different educational groups (black lines) show substantial gaps around the average population (red lines)

#### Heterogeneous Mortality Rates for Educational Groups

Table 1: Remaining	Life	Expectancy	of	Individuals	aged	65	in	2020
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	Edu-Low	Edu-Mid	Edu-High	AG2020					
A. RLE of a person aged 65 in 2020									
Men	21.6	24.2	26.7	23.5					
Women	25.2	28.9	29.2	26.1					
B. RLE gap with AG2020 rates									
Men	-1.9	0.7	3.2	-					
Women	-0.9	2.8	3.1	-					

 The effect of education on mortality is more pronounced among the male population (e.g., Backlund et al. (1999))

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- We model a VA contract using the so-called "annuity bucket"  $W_0(t)$  that is reserved on date t = 0 to finance the annuity payments at t
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  - $W_0(t)$  increases by the investment return over the period [0,t]
- The annuity provider needs to determine (i) the bucket sizes and (ii) the investment strategy of each bucket so as to maximize the discounted expected utility of the corresponding annuity payments

Financial market dynamics follow standard Merton (1971) setting:

$$dS_t = \mu S_t dt + \sigma S_t dZ_t$$
  
=  $(r + \lambda \sigma) S_t dt + \sigma S_t dZ_t$ 

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It then remains to determine the size of the annuity buckets such that the expected discounted utility of the corresponding consumption levels is optimized, given that the Merton strategy is used for each bucket.

Let R<sub>[0,t]</sub> be the gross return over the period [0, t] of a continuously rebalanced portfolio with weight f\* on the risky asset:

$$egin{split} &\mathcal{R}_{[0,t]}\sim \exp\left\{(r+f^*\sigma\lambda-rac{1}{2}(f^*)^2\sigma^2)t+f^*\sigma\sqrt{t}Z
ight\},\ &\sim \exp\left\{(r+rac{\lambda^2}{2\gamma^2}(2\gamma-1))t+rac{\lambda}{\gamma}\sqrt{t}Z
ight\} \end{split}$$

The optimal bucket sizes must satisfy:

$$\max_{\{W_0(t):t=0,1,...,T_{max}\}} \mathbb{E}\bigg[\sum_{t=0}^{T_{max}} e^{-\beta t} \cdot {}_t p_x \cdot W_0(t)^{1-\gamma} \cdot (R_{[0,t]})^{1-\gamma}\bigg]$$
  
s.t.  $\sum_{t=0}^{T_{max}} W_0(t) \cdot {}_t p_x = W_0$ 

A straightforward extension of Balter and Werker (2020) yields that the optimal bucket sizes are given by:

$$W_0^*(t|\gamma, \boldsymbol{p}) = W_0 \cdot \kappa_t^{\frac{1}{\gamma}} \Big/ \Big( \sum_{s=0}^{T_{max}} \kappa_s^{\frac{1}{\gamma}} \cdot {}_s \boldsymbol{p}_x \Big),$$

where  $\kappa_t$  equals:

$$\kappa_t = expigg\{ -eta t + igg(r + rac{\lambda^2}{2\gamma}igg)(1-\gamma)tigg\}$$

Correspondingly, the optimal consumption in period t is:

$$C^*_t(\gamma, \boldsymbol{p}) = R_{[0,t]}(\gamma) \cdot W^*_0(t|\gamma, \boldsymbol{p})$$

- ► We use the following notation:
  - We denote γ<sub>i</sub> > 1 and γ<sub>a</sub> > 1 for the risk aversion level used by the annuity provider (insurers or pension funds) and the annuitant's true risk aversion level, respectively.

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  - Optimal uniform annuity: the annuity based on the assumptions imposed by the annuity provider (i.e., C\*(\(\gamma\_i, \mathbf{p}\_i)\))

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  - Optimal uniform annuity: the annuity based on the assumptions imposed by the annuity provider (i.e., C\*(\(\gamma\_i, \mathbf{p}\_i)\))
  - Optimal tailor made annuity: the annuity based on the annuitant-specific characteristics (i.e., C<sup>\*</sup>(γ<sub>a</sub>, p<sub>a</sub>))

#### Welfare Evaluation: Certainty Equivalent

- We use the Certainty Equivalent Consumption Loss (CECL) to quantify the welfare effects due to a mismatch in risk aversion level, survival rates, or both
- For any given stochastic consumption stream  $C = \{C_t : t = 0, 1, ..., T_{max}\}$ , the CECL is defined as:

$$CE[\boldsymbol{\mathcal{C}}] = U_{\gamma_{\boldsymbol{\partial}}}^{-1} igg( \sum_{t=0}^{T_{max}} e^{-eta t} \cdot {}_t p_{\scriptscriptstyle X}^{\boldsymbol{\partial}} \cdot \mathbb{E}igg[ rac{(C_t)^{1-\gamma_{\boldsymbol{\partial}}}}{1-\gamma_{\boldsymbol{\partial}}} igg] igg)$$

where **C** can either be  $C^*(\gamma_i, \boldsymbol{p}_i)$  or  $C^*(\gamma_a, \boldsymbol{p}_a)$ 

The CECL from being offered the optimal uniform annuity instead of the optimal tailor made annuity is given by:

$$L(\gamma_i, \gamma_a, \boldsymbol{p}_i, \boldsymbol{p}_a) = 1 - \frac{CE[\boldsymbol{C}^*(\gamma_i, \boldsymbol{p}_i)]}{CE[\boldsymbol{C}^*(\gamma_a, \boldsymbol{p}_a)]}$$

Inadequate mortality asssumption and the MWR

- The seminal work of Mitchell et al. (1999) have introduced the money's worth ratio (MWR) which is the ratio of monetary value of the expected discounted annuity payments to the annuitant and the price charged by the annuity provider
- If the annuity priced is normalized to one, the MWR is equal to the value to the annuitant of the optimal uniform annuity:

$$MWR(\gamma, \boldsymbol{p}_i, \boldsymbol{p}_a) = \sum_{t=0}^{T_{max}} W_0^*(t|\gamma, \boldsymbol{p}_i) \cdot {}_t\boldsymbol{p}_x^a = \frac{\sum_{t=0}^{T_{max}} \kappa_t^{1/\gamma} \cdot {}_t\boldsymbol{p}_x^a}{\sum_{t=0}^{T_{max}} \kappa_t^{1/\gamma} \cdot {}_t\boldsymbol{p}_x^i}$$

Inadequate mortality asssumption and the MWR

We show that the MWR is equal to a scaling factor between two consumption streams with the same risk preference parameter:

$$oldsymbol{C}^*(\gamma,oldsymbol{p}_i)=MWR(\gamma,oldsymbol{p}_i,oldsymbol{p}_a)\cdotoldsymbol{C}^*(\gamma,oldsymbol{p}_a)$$

► Then, it holds that:

$$\frac{CE[\boldsymbol{C}^*(\gamma, \boldsymbol{p}_i)]}{CE[\boldsymbol{C}^*(\gamma, \boldsymbol{p}_a)]} = MWR(\gamma, \boldsymbol{p}_i, \boldsymbol{p}_a),$$

which in turn implies:

$$L(\gamma,\gamma,oldsymbol{p}_i,oldsymbol{p}_a)=1-MWR(\gamma,oldsymbol{p}_i,oldsymbol{p}_a)$$

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- **b** Discount parameter:  $\beta = 0.02$
- We test  $\gamma_i \in \{2, 3, 5, 8, 20\}$  and  $\gamma_a \in \{2, 3, 5, 8, 20\}$ 
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- The annuity provider employs survival probabilities depending on the gender composition of the fund

Gender Composition	% Men	% Women
All Men	100	0
All Women	0	100
Balanced	50	50
Men-dominant	70	30
Women-dominant	30	70

### Results 1: Effect of Uniform Annuity Pricing



The level of expected annuity payment shifts upward or downward depending on p

#### Results 1: Welfare Effects of Inadequate Mortality Assumption

Table 2: Welfare Losses of Annuitants with Different Education Levels, for  $\gamma_i = 5$ 

Gender		Men				Women	
Composition	Edu-Low	Edu-Mid	Edu-High	_	Edu-Low	Edu-Mid	Edu-High
All Men	6.8%	-1.5%	-9.6%		-7.7%	-17.3%	-18.9%
All Women	16.2%	8.7%	1.4%		3.1%	-5.5%	-7.0%
Men-dominant	9.6%	1.5%	-6.3%		-4.5%	-13.8%	-15.4%
Balanced	11.5%	<mark>3.6%</mark>	-4.1%		-2.4%	-11.5%	-13.0%
Women-dominant	13.3%	5.6%	-1.9%		-0.2%	-9.1%	-10.6%

Substantial differences in welfare effects by educational attainments
 Even low-educated women can benefit from favorable annuity pricing in most cases

# Results 2: Welfare Effects of Inadequate Risk Preference Parameter

 Table 3: Welfare Loss Due to Inadequate Risk Aversion Parameters: Average

 Population

γ <sub>i</sub> (f*) Men								Women		
	$\gamma_{a}=2$	$\gamma_{a}=3$	$\gamma_{a} = 5$	$\gamma_{a}=8$	$\gamma_{a}=20$	$\gamma_{a}=2$	$\gamma_{a}=3$	$\gamma_{a}=5$	$\gamma_{a}=8$	$\gamma_{a}=20$
2 (50.0%)	0.0%	1.7%	10.0%	31.9%	96.9%	0.0%	1.8%	11.0%	34.4%	97.0%
3 (33.0%)	1.1%	0.0%	1.9%	8.6%	62.6%	1.2%	0.0%	2.1%	9.4%	64.0%
5 (20.0%)	3.5%	1.1%	0.0%	1.0%	16.1%	3.9%	1.2%	0.0%	1.1%	17.5%
8 (12.5%)	5.5%	2.6%	0.6%	0.0%	3.0%	6.0%	2.8%	0.6%	0.0%	3.3%
20 (5.0%)	7.9%	4.8%	2.3%	0.9%	0.0%	8.6%	5.2%	2.5%	1.0%	0.0%

▶ Welfare losses  $\leq$  15%, except for extreme cases

Does this depend on survival probabilities?

# Results 2: Welfare Effects of Inadequate Risk Preference Parameter



Yes, but only marginally - relatively insensitive to the survival rates of the annuitant

#### Results 3: Combined Welfare Effects - Are They Additive?

► We previously have shown that:

$$\boldsymbol{C}^*(\gamma_i, \boldsymbol{p}_i) = MWR(\gamma_i, \boldsymbol{p}_i, \boldsymbol{p}_a) \cdot \boldsymbol{C}^*(\gamma_i, \boldsymbol{p}_a)$$

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Taking the certainty equivalent and dividing both sides of the equation by CE[C\*(\(\gamma\_a, \mathbf{p}\_a)\)] yields:

$$\underbrace{1 - L(\gamma_i, \gamma_a, \boldsymbol{p}_i, \boldsymbol{p}_a)}_{\text{Mismatch } \boldsymbol{p} \text{ and } \gamma} = \underbrace{MWR(\gamma_i, \boldsymbol{p}_i, \boldsymbol{p}_a)}_{\text{Mismatch } \boldsymbol{p}} \cdot \underbrace{[1 - L(\gamma_i, \gamma_a, \boldsymbol{p}_a, \boldsymbol{p}_a)]}_{\text{Mismatch } \gamma}$$

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Rewriting the equations yields:

$$L(\gamma_i, \gamma_a, \boldsymbol{p}_i, \boldsymbol{p}_a) = L(\gamma_i, \gamma_a, \boldsymbol{p}_a, \boldsymbol{p}_a) + L(\gamma_i, \gamma_i, \boldsymbol{p}_i, \boldsymbol{p}_a) \\ -L(\gamma_i, \gamma_a, \boldsymbol{p}_a, \boldsymbol{p}_a) \cdot L(\gamma_i, \gamma_i, \boldsymbol{p}_i, \boldsymbol{p}_a)$$

#### Results 3: Combined Welfare Effects

Table 4: Welfare Gains/Losses Due to Inadequate  $\gamma$  and  $\boldsymbol{p}$  (Men)

Men													
Gender	Edu-Low					Edu-Mid				Edu-High			
Composition	$\gamma_a = 2$	$\gamma_a = 5$	$\gamma_a = 8$	$\gamma_a = 20$	$\gamma_a = 2$	$\gamma_a = 5$	$\gamma_a = 8$	$\gamma_a = 20$	$\gamma_a = 2$	$\gamma_a = 5$	$\gamma_a = 8$	$\gamma_a = 20$	
All Men All Women	10.0% 19.0%	6.8% 16.2%	7.7% 17.0%	21.4% 29.3%	2.2% 12.0%	-1.5% 8.7%	-0.5% 9.7%	15.5% 24.0%	-5.4% 5.2%	-9.6% 1.4%	-8.4% 2.5%	10.3% 19.3%	
Men-dominant Balanced Women-dominant	12.6% 14.4% 16.3%	9.6% 11.5% 13.3%	10.4% 12.3% 14.2%	23.7% 25.3% 26.9%	5.1% 7.0% 9.0%	1.5% 3.6% 5.6%	2.5% 4.5% 6.6%	18.0% 19.7% 21.4%	-2.2% -0.1% 2.0%	-6.3% -4.1% -1.9%	-5.2% -3.0% -0.8%	12.9% 14.7% 16.6%	

- The annuity provider assumes  $\gamma_i = 5$
- When risk preference parameter and survival probs are mismatched, low- and mid-educated men experience substantial welfare losses
- High-educated men mostly get welfare gains due to favorable annuity pricing unless extremely risk-averse

#### Results 3: Combined Welfare Effects

#### Table 5: Welfare Gains/Losses Due to Inadequate $\gamma$ and $\boldsymbol{p}$ (Women)

Women													
Gender	Edu-Low					Edu-Mid				Edu-High			
Composition	$\gamma_{a} = 2$	$\gamma_a = 5$	$\gamma_a = 8$	$\gamma_a = 20$	$\gamma_a = 2$	$\gamma_a = 5$	$\gamma_a = 8$	$\gamma_a = 20$	$\gamma_a = 2$	$\gamma_a = 5$	$\gamma_{a} = 8$	$\gamma_{a} = 20$	
All Men All Women	-3.6% 6.8%	-7.7% 3.1%	-6.6% 4.2%	11.5% 20.4%	-12.4% -1.1%	-17.3% -5.5%	-15.9% -4.3%	5.5% 15.0%	-14.0% -2.5%	-18.9% -7.0%	-17.5% -5.7%	4.1% 13.8%	
Men-dominant Balanced Women-dominant	-0.5% 1.6% 3.7%	-4.5% -2.4% -0.2%	-3.4% -1.3% 0.9%	14.1% 15.9% 17.7%	-9.1% -6.8% -4.6%	-13.8% -11.5% -9.1%	-12.5% -10.2% -7.8%	8.3% 10.2% 12.1%	-10.6% -8.3% -6.0%	-15.4% -13.0% -10.6%	-14.0% -11.7% -9.3%	7.0% 8.9% 10.8%	

Even for women with the lowest educational level, the welfare gains from favorable pricing of the uniform annuity can outweigh the welfare losses from a mismatch in risk aversion parameter

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- We find that the welfare effects of mismatch in the survival probabilities can be substantially higher than those from inadequate risk preference parameters
  - Welfare gains due to the favorable annuity pricing for high-educated men and women of all education levels easily cancel out the welfare losses of having inadequate investment strategies
- The results of our study suggest that having funds with relatively homogeneous groups in terms of survival probabilities can alleviate the welfare losses of individuals, particularly those with low education levels



## Thank you!