When Should Intra-Household Risk Sharing Affect Life-Cycle Portfolio Choice?

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32nd Colloquium on Pensions and Retrirement Research
IPRA Sponsored Online Session

3 December 2024

Contribution

- Using a quantitative, collective life-cycle portfolio choice model, we show that couples optimally should invest a larger share of financial wealth in risky assets than singles if partners differ in relative risk aversion.
- This finding holds true at any age and carries significant implications for the design of target date funds, given that most individuals live in couples during their working life and that partners in nearly half of these couples differ in relative risk aversion (Mazzocco, 2004, Gu et al., 2023).
- Our finding is driven by the potential to share idiosyncratic earnings risk within the household. Risk sharing implies a consumption sharing rule that optimally allocates household consumption across the partners.

Background

- Large literature on unitary life-cycle portfolio choice models treats every household as if it were a single person. Recent overviews are provided by Gomes (2020) and Gomes, Haliassos, and Ramadorai (2021).
- Collective models of consumption (and often labor supply) decisions in the tradition of Chiappori (1988) are designed to reflect opportunities for intra-household risk sharing but typically abstract from portfolio choice.
- We extend the dynamic, collective model proposed by Mazzocco (2004) and reviewed in Browning, Chiappori and Weiss (2014) to incorporate the life-cycle portfolio choice decisions of dual-income couples.
- We focus on dual-income households because most couples consist of partners with their own sources of individual income.

Intra-Household Risk Sharing

- Intra-household risk sharing ensures that the ratio of the individual marginal utilities of consumption of both partners remains constant.
- Consequently, each partner experiences consumption volatility that aligns with their individual risk preference. More on this below.
- The efficient allocation of risk within the household enables the couple to take on more financial risk compared to a household that cannot share risk. This is the main insight from our paper.
- It is important to distinguish risk sharing from the potential to diversify earnings risk in dual-income couples. Unlike risk diversification, risk sharing impacts both single-income and dual-income couples, provided that the partners differ in relative risk aversion.

Related Portfolio Choice Literature

- Collective life-cycle portfolio choice models proposed by Love (2010), Hong and Ríos-Rull (2012), and Hubener, Maurer and Mitchell (2015).
  - Equal sharing of consumption, thus no intrahousehold risk sharing.
- Collective mean-variance portfolio choice model proposed by Gu et al. (2023), who estimate a higher bargaining power for male partners.
  - No consumption, thus no role for consumption sharing & risk sharing.
- Collective life-cycle portfolio choice model with limited commitment (due to time-varying bargaining power) proposed by Addoum et al. (2017), in which wives are more risk-averse than husbands (as in Addoum, 2017).
  - However, the authors attribute risk sharing to risk diversification.
- We focus on portfolio choice implications of intra-household risk sharing.

Collective Life-Cycle Portfolio Choice

- Couple maximizes π-weighted average of the partners' individual life-time power utilities in non-durable consumption by choosing a sequence of consumption (C_{At}, C_{Bt}) and the share of wealth in risky assets (α_t):

$$\max_{\{C_{At},C_{Bt},\alpha_t\}_{t=1}^{T}} \left\{ (1-\pi)\, E_1 \left[\sum_{t=1}^{T} \beta_A^{t-1} p_{At} \frac{C_{At}^{1-\gamma_A}}{1-\gamma_A} \right] + \pi E_1 \left[\sum_{t=1}^{T} \beta_B^{t-1} p_{Bt} \frac{C_{Bt}^{1-\gamma_B}}{1-\gamma_B} \right] \right\}$$

- Subject to budget constraint involving stochastic earnings and returns.
- The time-invariant Pareto weight π reflects the bargaining power of partner B at the time the couple is formed ($t = 1$, age 25). We abstract from divorce to focus on the risk sharing mechanism. $T = 85$ (age 109).

Consumption Sharing Rule

- From equating the f.o.c. for individual consumption, we obtain:

$$\frac{\beta_A^{t-1} p_{At} C_{At}^{-\gamma_A}}{\beta_B^{t-1} p_{Bt} C_{Bt}^{-\gamma_B}} = \frac{\pi}{1-\pi}$$

- The ratio of discounted marginal utilities is constant, a standard characterization of efficient risk sharing (Browning et al, 2014).
- The sharing rule determines the optimal allocation of household consumption, $C_t = C_{At} + C_{Bt}$, among the partners.
- Equal sharing of consumption is a special case that is unlikely to hold ($p_{At} \neq p_{Bt}$), even if partners have identical preference parameters.
- We optimize w.r.t. C_t and then find C_{At} and C_{Bt} from the sharing rule.

Bellman Equation

- The couple's value function of the Pareto problem is defined as:

$$V_{Ct}\left(X_t, Y_{At}^p, Y_{Bt}^p\right) =$$

$$\max_{C_{At}, C_{Bt}, \alpha_t} \left\{ (1-\pi) \left[\frac{C_{At}^{1-\gamma_A}}{1-\gamma_A} + \beta_A p_{At+1} E_t \left[V_{At+1}\left(X_{t+1}, Y_{At+1}^p, Y_{Bt+1}^p\right)\right]\right] \right.$$
$$\left. + \pi \left[\frac{C_{Bt}^{1-\gamma_B}}{1-\gamma_B} + \beta_B p_{Bt+1} E_t \left[V_{Bt+1}\left(X_{t+1},, Y_{At+1}^p, Y_{Bt+1}^p\right)\right]\right] \right\},$$

- where the state variables are cash-on-hand (X_t) and the persistent components (Y_{At}^p, Y_{Bt}^p) of the joint earnings process of the partners.
- We will now simplify the problem to focus on risk sharing implied by intra-household heterogeneity in relative risk aversion. We assume that the partners have identical discount factor β, and that survival is certain.

Simplified Problem

- In this case, the Bellman equation is of the usual recursive form:

$$V_{Ct}\left(X_t, Y^p_{At}, Y^p_{Bt}\right) = \max_{C_{At}, C_{Bt}, \alpha_t} \left\{ (1-\pi)\frac{C_{At}^{1-\gamma_A}}{1-\gamma_A} + \pi \frac{C_{Bt}^{1-\gamma_B}}{1-\gamma_B} + \beta E_t\left[V_{Ct+1}\left(X_{t+1}, Y^p_{At+1}, Y^p_{Bt+1}\right)\right] \right\}$$

- The consumption sharing rule becomes (as in Apps et al., 2014):

$$\frac{C_{At}^{-\gamma_A}}{C_{Bt}^{-\gamma_B}} = \frac{\pi}{1-\pi}$$

- For ease of interpretation, define the individual coefficients of absolute risk aversion: $\rho_A = \frac{\gamma_A}{C_{At}}$, $\rho_B = \frac{\gamma_B}{C_{Bt}}$
- Also define the couple's relative (absolute) risk aversion: γ_u $\left(\rho_u = \frac{\gamma_u}{C_t}\right)$.

Risk Sharing Explained

- Ortigueira and Siassi (2013) show that:

$$\frac{dC_{At}}{dC_t} = \frac{\rho_u}{\rho_A} \qquad \frac{dC_{Bt}}{dC_t} = \frac{\rho_u}{\rho_B}$$

- Assume partner A is more risk tolerant than partner B. Then, if C_t decreases, C_{At} decreases by more than C_{Bt}. Thus, partner A's consumption share, C_{At}/C_t, decreases while C_{Bt}/C_t increases.
- Vice versa, if C_t increases, C_{At} increases by more than C_{Bt}. The more risk tolerant partner bears most of the variation in C_t.
- Intuition: Partner B buys partial consumption insurance from A by giving up upward potential. We investigate the portfolio choice implications.

Household Relative Risk Aversion

- Ortigueira and Siassi (2013) also derive γ_u:

$$\gamma_u = \frac{\gamma_A \gamma_B}{\gamma_B \frac{C_{At}}{C_t} + \gamma_A \frac{C_{Bt}}{C_t}}$$

- Risk aversion is time-varying unless consumption shares are constant.
- In a seminal paper, Mazzocco (2004) shows that the consumption shares are constant (and determined by π) if $\gamma_A = \gamma_B = \gamma$. In this case, $\gamma_u = \gamma$ and the collective model can be represented by a unitary model:

$$V_{Ut}\left(X_t, Y^p_{At}, Y^p_{Bt}\right) = \max_{C_t, \alpha_t} \left\{ \frac{C_t^{1-\gamma}}{1-\gamma} + \beta E_t \left[V_{Ut+1}\left(X_{t+1}, Y^p_{At+1}, Y^p_{Bt+1}\right) \right] \right\}$$

Budget Constraint and Earnings Process

- The state variables are X_t and Y_{At}^p, Y_{Bt}^p. Using pooled earnings, $Y_t = Y_{At} + Y_{Bt}$, cash-on-hand evolves according to:

$$X_t = (X_{t-1} - C_{t-1})\left(R^f + \alpha_{t-1}\left(R_t - R^f\right)\right) + Y_t$$

- Log earnings for each partner are affected by deterministic and random components (e_{At}, e_{Bt}). The latter are decomposed into transitory $(\varepsilon_{At}, \varepsilon_{Bt})$ and persistent $(Y_{At}^p = \exp(\eta_{At}), Y_{Bt}^p = \exp(\eta_{Bt}))$ shocks and time-invariant random effects (ω_A, ω_B):

$$e_{At} = \omega_A + \eta_{At} + \varepsilon_{At} \qquad e_{Bt} = \omega_B + \eta_{Bt} + \varepsilon_{Bt}$$

$$\eta_{At} = \phi_A \eta_{At-1} + \zeta_{At} \qquad \eta_{Bt} = \phi_B \eta_{Bt-1} + \zeta_{Bt}.$$

Variance Decomposition

- The process extends Kaplan (2012) to a dual-income household. Unlike Shore (2015) and Blundell et al. (2016), we allow for persistent shocks.
- Random effects and transitory shocks are i.i.d. mean zero normal with variances: $(\sigma^2_{A\omega}, \sigma^2_{B\omega})$, $(\sigma^2_{A\varepsilon}, \sigma^2_{B\varepsilon})$. Persistent earnings shocks and innovations to log stock excess returns (ν_t) are distributed according to:

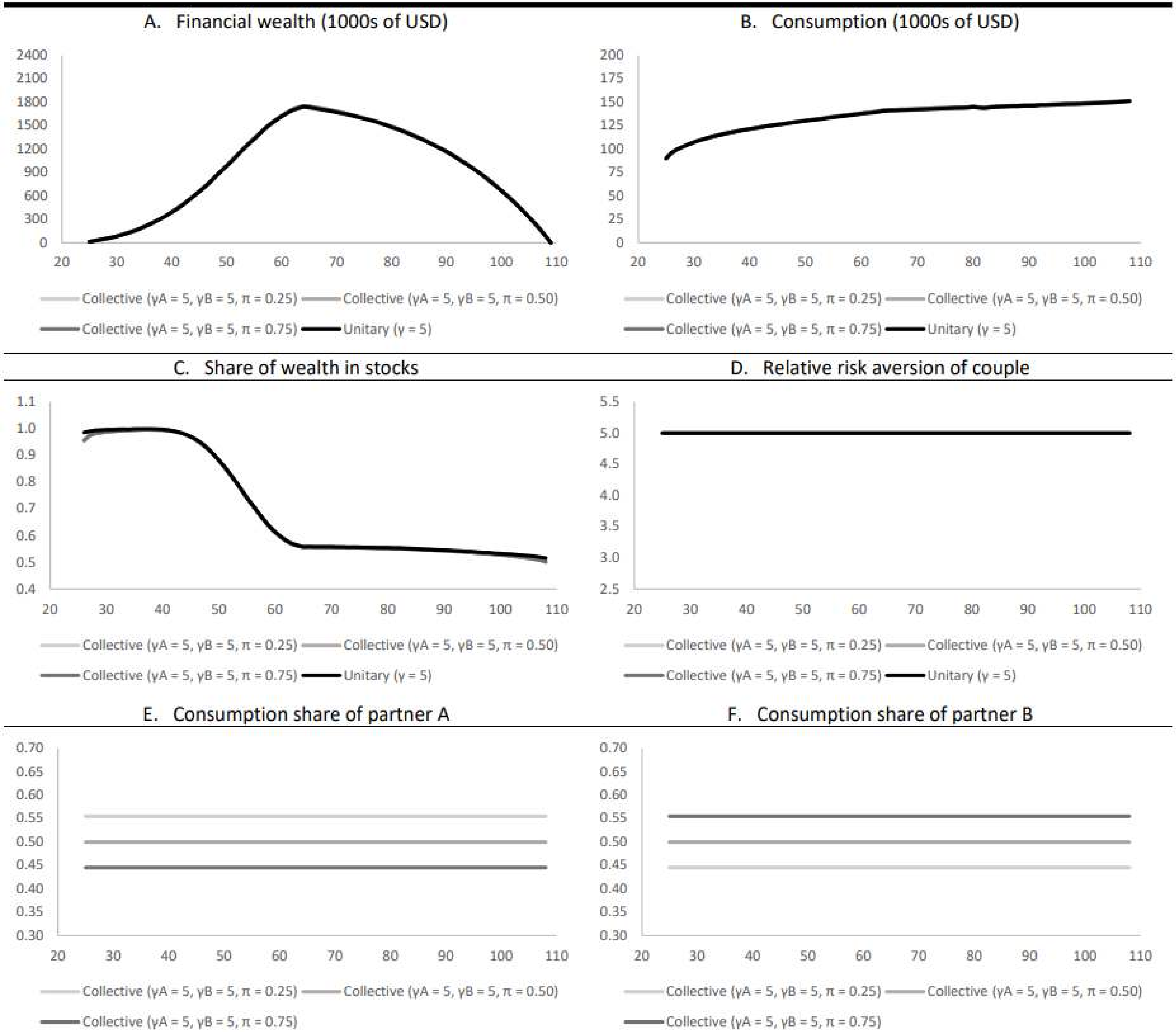
$$\begin{pmatrix} \zeta_{At} \\ \zeta_{Bt} \\ \nu_t \end{pmatrix} \sim i.i.d.N \left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{A\zeta} & \rho_{AB}\sigma_{A\zeta}\sigma_{B\zeta} & \rho_{A\nu}\sigma_{A\zeta}\sigma_{\nu} \\ \rho_{AB}\sigma_{A\zeta}\sigma_{B\zeta} & \sigma^2_{B\zeta} & \rho_{B\nu}\sigma_{B\zeta}\sigma_{\nu} \\ \rho_{A\nu}\sigma_{A\zeta}\sigma_{\nu} & \rho_{B\nu}\sigma_{B\zeta}\sigma_{\nu} & \sigma^2_{\nu} \end{pmatrix} \right]$$

- Process estimated by GMM from 1999 – 2017 PSID waves for couples.

Simulation Results

- We obtain consumption and portfolio choice policy functions from the model using numerical methods. We then simulate the optimal policies using repeated draws from the distributions of earnings/return shocks.
- Below, I present average consumption and portfolio choice decisions for the following three intra-household parameter constellations:
  1. Identical risk aversion, different Pareto weights (Mazzocco, 2004)
  2. Differences in relative risk aversion, identical Pareto weights
  3. Differences in relative risk aversion and Pareto weights.
- Discount factors 0.96, risk-free rate 2%, equity premium 4%. The volatility of stock returns is 18%. We use half of the estimated earnings shock volatilities to allow for measurement error (as in Deaton, 1991). We later use the estimated volas to evaluate a mean-preserving spread.

Figure 3: Varying the Pareto weight when both partners have identical risk aversion



Identical RRA.

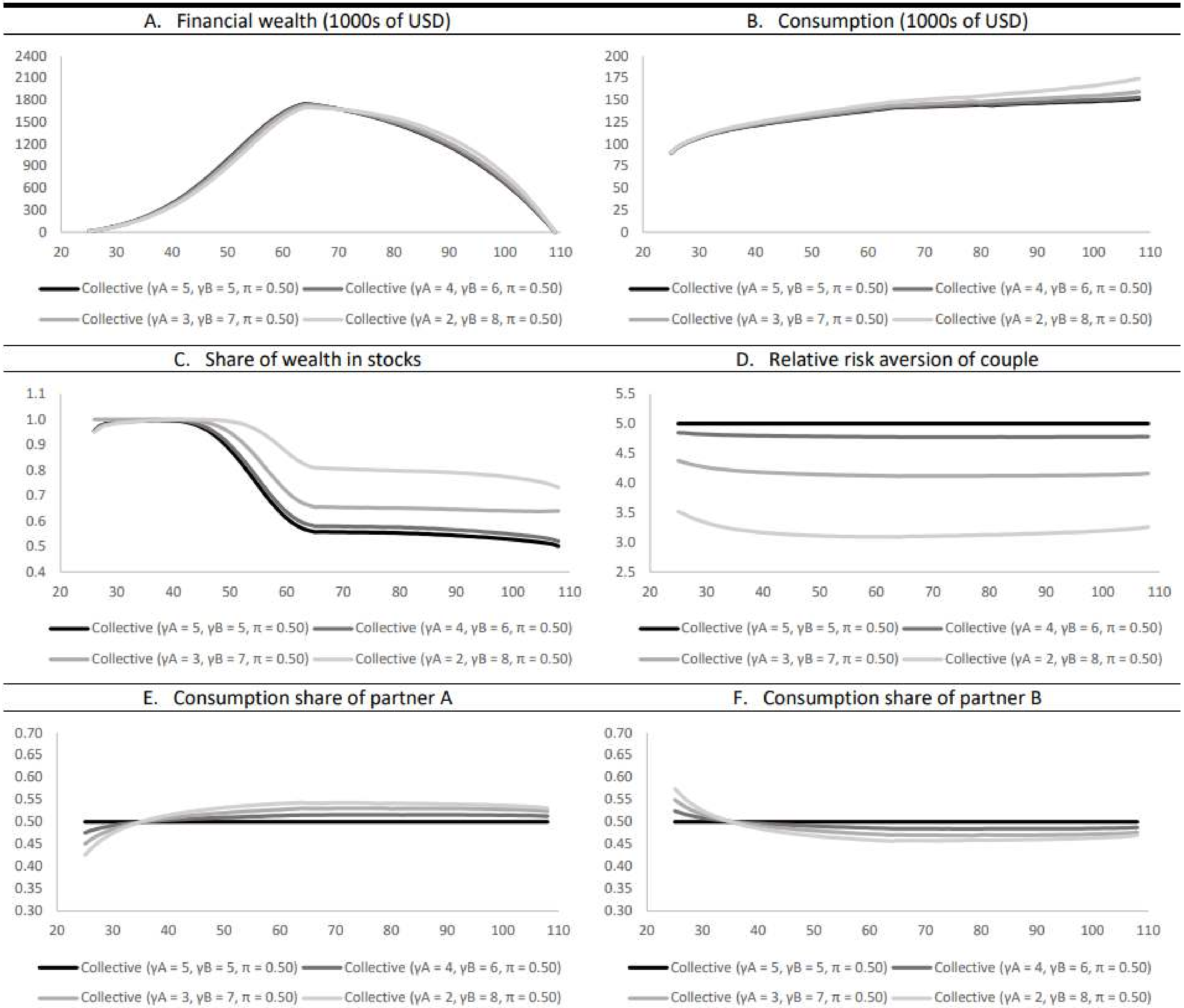
Savings are unaffected by Pareto weight.

Portfolio choice is in line with the unitary model.

No potential to share risk: $\gamma_u = \gamma_A = \gamma_B$.

Partner with higher Pareto weight receives more consumption (age-invariant).

Figure 4: Varying the spread in risk aversion when both partners have identical bargaining power



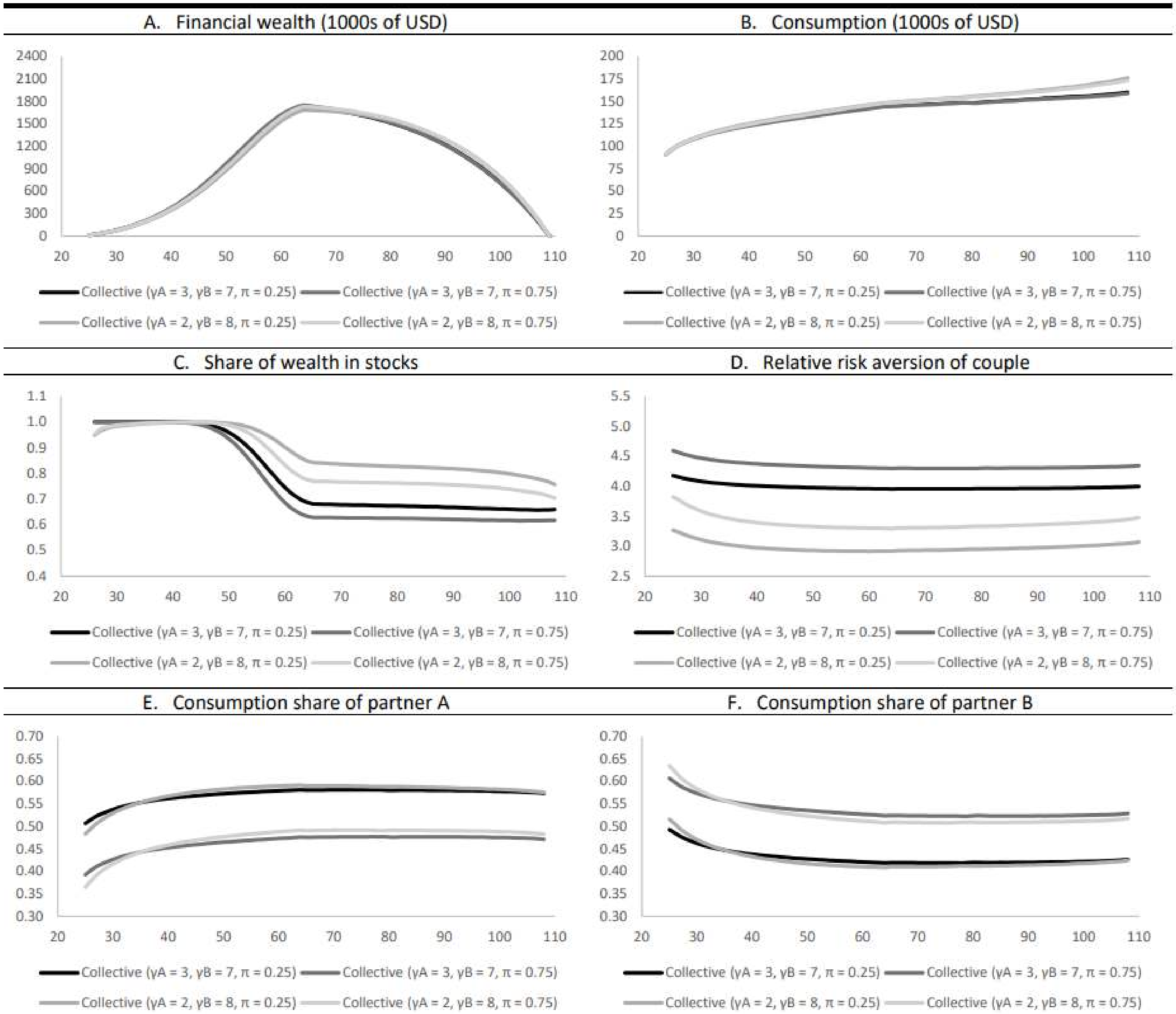
Consumption increases with spread in individual risk aversion.

So does financial risk taking.

Couple's rel. risk aversion < avg. of individual rel. risk aversions (= 5).

The share of consumption of more risk tolerant partner increases.

Figure 5: Varying the Pareto weight when both partners have different risk aversion



Similar results as before, but financial risk taking to a larger extent reflects preferences of the partner with higher bargaining power.

Unitary model cannot replicate collective model. See couple's rel. risk aversion, which varies over the life cycle.

Mean-Preserving Spead in Idiosyncratic Earnings Risk

- We repeat these simulations using the full estimated earnings shock volatility estimates to evaluate the saving and portfolio choice implications of a mean-preserving spread in idiosyncratic earnings risk.
- For the unitary model, Kimball (1990) shows that precautionary savings increase in response to a mean-preserving spread in background risk if the utility function exhibits prudence (positive third derivative of utility).
- According to Apps et al. (2014), this result extends to the collective model if both individual utility functions exhibit prudence.
- We show that saving increases and the share of wealth allocated to stocks decreases. Moreover, the more partners are able to share risk, the less saving and portfolio choice are affected by the increase in risk.

Summary of Findings

- With identical risk preferences, the unitary model describes the couple's optimal portfolio choice. Bargaining only affects consumption shares.
- The more partners differ in risk preferences, the more they benefit from risk sharing, which increases consumption and financial risk taking. Compared to a couple with $\gamma_A = \gamma_B = 5$, a couple with $\gamma_A = 2$ and $\gamma_B = 8$ allocates on avg. 17 percentage points more wealth to stocks ($\pi = 0.5$).
- When partners differ in risk aversion and bargaining power, the portfolio mostly reflects the preferences of the partner with higher Pareto weight.
- Thus, even modest intra-household heterogeneity in risk aversion leads to substantial increases in optimal risk taking if the (Gu et al., 2023: male) partner with higher bargaining power is also the more risk tolerant.