Reference-dependent Preferences, Time Inconsistency and Pay-as-you-go Pensions

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December 6, 2020
Motivation

- Social security has been around for a century and nearly ubiquitous.
- Old-age support v.s. redistribution.
- Consumption-smoothing: (pay-as-you-go) tax the young to finance the payment for the current elderly.
Motivation

- Why pensions if the capital markets is complete and even offers a higher return than pensions (dynamically efficient).
- Standard preferences cannot generate a role for pay-as-you-go pensions in a dynamically efficient economy (pension is return dominated) (Aaron, 1966; Samuelson, 1975).
  - Most developed economies are most likely dynamically efficient (Abel et al., 1989; Barbie et al., 2004).
Motivation

- Time-inconsistent preferences $\Rightarrow$ undersave for retirement (Imrohoroglu et al., 2003, QJE; Andersen and Bhattacharya, 2011, ET).

  - True v.s. choice utility, where the paternalistic government maximizes the true utility.
  - Time-inconsistent agents can offset the forced saving by reducing their own saving.
  - Borrowing constraints $\Rightarrow$ all private retirement savings are zero to generate pensions in a small open economy.
Motivation

- Reference-dependent preferences $\Rightarrow$ undersaving for retirement (Koszegi and Rabin, 2009, AER)
  - The young sets the reference consumption for her future.
  - Undersave or overconsume by the middle-aged.
- This paper: a welfare role for pensions under KR framework?
  - Yes and the under-saving problem is addressed to some extent.
  - Private retirement saving could be positive.
  - No paternalistic government.
Three-period OLG model: the young, the middle-aged, the old.

Dynamic Efficiency: the return on saving is greater than the population growth rate, $R > 1$.

The government levies taxes at rate $\tau$, and pays pensions $b = \tau w$.

The lifetime utility of the agent is

$$\Omega_m = u(c_m) + \Phi[u(c_m) - u(c_m^r)] + \beta \gamma \Phi[u(c_o) - u(c_o^r)] + \beta u(c_o).$$
The Model: Environment

- The Timeline

\[ t = 0 \]

The young, with beliefs about her preference, makes consumption plans \( c^r_m, c^r_o \).

\[ t = 1 \]

The middle-aged, endowed with income \( w \), consumes \( c_m \), saves \( s \) and pays taxes at tax rate \( \tau \),

\[ c_m = (1 - \tau)w - s. \]

Updates belief \( c^r_o = c_o \).

\[ t = 2 \]

The old consumes what he saves and pensions, i.e.

\[ c_o = Rs + b. \]
The Model: Budgets

- The middle-aged consumption $c_m = (1 - \tau)w - s$.
- The old’s consumption $c_o = Rs + b$.
- The budget constraints can be combined in equilibrium to

$$c_m + \frac{c_o}{R} = w - b(1 - \frac{1}{R}) \equiv Y.$$
The Model: The young

- Naive young: no gain-loss (Kramer, 2016, ET)
- Projection bias in predicting future utility (Loewenstein, ODonoghue, Rabin, 2003; DellaVigna, 2009)
- The young’s problem

\[
\max_{s^r} \Omega_y = u(c_m) + \beta u(c_o) \\
= u((1 - \tau)w - s^r) + \beta u(Rs^r + b).
\]

The optimal consumption plan \((c_m^r, c_o^r)\) satisfies

\[
\frac{d\Omega_y}{ds^r} = -u'(c_m^r) + \beta Ru'(c_o^r) = 0.
\]
The Model: The old

- The old consumes what the middle-aged saved and the pension received, $c_o = R \left[(1 - \tau) w - c_m\right] + b$.
- The old’s actual consumption $c_o$ coincides with the middle-aged updated belief of $c_o$.
- The instantaneous utility of the old is $u(c_o)$, where the gain-loss utility is 0.
The Model: The middle-aged

- The middle-aged consumes $c_m$ and get $u(c_m)$.
- Contemporaneous gain-loss utility $G[u(c_m) - u(c'_m)]$.
- Prospective gain-loss utility $\lambda G[u(c'_o) - u(c_o)]$
- For $c_m \geq c'_m$, given $c'_m, c'_o$ the middle-aged chooses $s, c_m, c_o$ to maximize

$$\Omega_m = u(c_m) + G[u(c_m) - u(c'_m)] - \beta \gamma \lambda G[u(c'_o) - u(c_o)] + \beta u(c_o).$$

$$s.t. \quad c_m = (1 - \tau)w - s,$$
$$c_o = Rs + b.$$
The first-order condition is

\[ u'(c_m) = \beta \kappa Ru'(c_o), \]

where

\[ \kappa = \frac{1 + \gamma \lambda G'(u(c'_o) - u(c_o))}{1 + G'(u(c_m) - u(c'_m))}. \]

Overconsumption when evaluated at the young’s consumption plan

\[ J|_{s=s^r} = -G'(0) u'(c'_m) (1 - \gamma \lambda) \leq 0 \text{ if and only if } \gamma < 1/\lambda. \]
Without borrowing constraints, the government is choosing pension $b$ to maximize

$$\max_b \Omega(Y) = \max_b \arg\max\{c_m, c_m^r, c_o, c_o^r\} \Omega_m(c_m, c_o, c_m^r, c_o^r).$$

**Proposition** Under any preferences, if there exists an interior pension $b^*$ alongside interior voluntary savings, then the optimal pension $b^*$ is

$$b^* = b_0 + \frac{R}{R - 1} w.$$
Assume that $\beta = 1$ since it does not affect the overconsumption.

Choose $b$ to maximize the ex-ante expected utility of the agent

$$\Omega_m = u(c_m) + G[u(c_m) - u(c'_m)] - \gamma \lambda G[u(c'_o) - u(c_o)] + u(c_o)$$

Taking derivative w.r.t. $b$, we can easily get

$$\frac{d\Omega_m}{db} = (1 + G'(u(c_m) - u(c'_m))) u'(c_m) (-1 - s'(b))$$
$$+ (1 + \gamma \lambda G'(u(c'_o) - u(c_o))) u'(c_o) (1 + R s'(b))$$
$$- G'(u(c_m) - u(c'_m)) u'(c'_m) (-1 - s''(b))$$
$$- \gamma \lambda G'(u(c'_o) - u(c_o)) u'(c'_o) (1 + R s''(b)) .$$
The Model: The government

- First-order

\[
\frac{d\Omega_m}{db} = (1 + G'(u(c_m) - u(c'_m))) u'(c_m) (-1 - s'(b)) \\
+ (1 + \gamma \lambda G'(u(c'_o) - u(c_o))) u'(c_o) (1 + R s'(b)) \\
- G'(u(c_m) - u(c'_m)) u'(c'_m) (-1 - s''(b)) \\
- \gamma \lambda G'(u(c'_o) - u(c_o)) u'(c'_o) (1 + R s''(b)) .
\]

- Summing the first-two terms, we have

\[-(1 - \frac{1}{R}) (1 + G'(u(c_m) - u(c'_m))) u'(c_m) < 0.\]

- The reason is the decline in lifetime income as a result of pension

\[Y = w - b(1 - \frac{1}{R}).\]
The Model: The government

- First-order

\[
\frac{d\Omega_m}{db} = (1 + G'(u(c_m) - u(c'_m))) \ u'(c_m) (-1 - s'(b)) \\
+ (1 + \gamma \lambda G'(u(c'_o) - u(c_o))) \ u'(c_o) (1 + Rs'(b)) \\
-G'(u(c_m) - u(c'_m))u'(c'_m) (-1 - s'^{(r)}(b)) \\
-\gamma \lambda G' (u(c'_o) - u(c_o)) u'(c'_o) (1 + Rs'^{(r)}(b)) .
\]

- The last two terms are positive, that is,

\[-G'(u(c_m) - u(c'_m))u'(c'_m) (-1 - s'^{(r)}(b)) > 0.\]

\[-\gamma \lambda G' (u(c'_o) - u(c_o)) u'(c'_o) (1 + Rs'^{(r)}(b)) > 0.\]
Lemma For CES utility with $\sigma > 1$ and constant elasticity gain-loss function $G$, the old-age consumption relative to middle-aged consumption is increasing in pension $b$, that is, $d\kappa/db > 0$, where

$$u'(c_m) = \kappa Ru'(c_o).$$

Proposition For CES utility $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. Suppose $\gamma \lambda < 1$ and the gain-loss function satisfies $G_x(0) > \bar{G}_x(0)$, positive pensions and positive savings exists for income $w \in (\bar{w}, \tilde{w})$ provided $\sigma > 1$. 
Suppose $u(\cdot) = (\cdot)^{1-\sigma} / (1 - \sigma)$, where $0 < \sigma \neq 1$.

Gain-Loss Utility

$$
\Phi(x) = \begin{cases} 
Ax^\alpha, & \text{if } x \geq 0, \\
-\lambda A(-x)^\alpha, & \text{if } x < 0.
\end{cases}
$$
The objective function

Let $R = 1.02^{25}$, $\gamma = 0.1$, $\lambda = 2$, $w = 10$, $\alpha = 1/2$, $A = 1.42$, $\sigma = 3$.

The optimal pension and saving are both positive, that is,

$$b = 2.3,$$
$$s = 1.34, s^r = 2.40,$$
$$c^*_m = 6.35, c^r_m = 5.30,$$
$$c^*_o = 4.51, c^r_o = 6.25.$$
Assume the production function is constant returns to scale $F(K_t, L_t)$ where $K_t$ and $L_t$ denote the capital and labor input.

Factor markets are perfectly competitive such that factor inputs are paid their marginal products in each period. Therefore,

$$R_t \equiv R(k_t) = f_k(k_t),$$

and

$$w_t \equiv w(k_t) = f(k_t) - k_t f_k(k_t),$$
The optimal $\tau$ solves

$$\max_{\tau} \Omega_m(\tau) = u(c_m) + G(u(c_m) - u(c^r_m)) - \gamma \lambda G(u(c^r_o) - u(c_o)) + u(c_o)$$

To have a welfare role for pensions

$$[1 + G_x(g_m)] \Lambda^{ns} = -\eta u_c(c^r_o) \left[ (\gamma \lambda - R(\tilde{k})) w(\tilde{k}) + (\gamma \lambda - 1) R(\tilde{k}) \frac{\partial \tilde{k}}{\partial \tau} \right]$$

\[ \text{exog fp} \]

$$-\eta u_c(c^r_o) \left[ (\gamma \lambda - R(\tilde{k})) (1 - \tau) \tilde{k} f_{kk}(\tilde{k}) \tilde{k}_\tau(\tau) \right]$$

\[ \text{endog fp} \]
Neoclassical Production Technology

- To see the intuition, recall $-\tilde{k}f_{kk} (\tilde{k}) = w_k (\tilde{k})$ and notice

$$-\eta u_c (c_o^r) \left[ \gamma \lambda - R(\tilde{k}) \right] (1 - \tau) \tilde{k}f_{kk} (\tilde{k}) \tilde{k}_\tau (\tau)$$

$$= \eta u_c (c_o^r) \left[ R(\tilde{k}) - \gamma \lambda \right] (1 - \tau) (-) w_k (\tilde{k}) \frac{\partial \tilde{k}}{\partial \tau}.$$  

- The pension, via $\tau$, reduces $\tilde{k}$ which via $w (\tilde{k})$ reduces the imagined wage income, and therefore, reduces reference consumption, which will increase utility given actual consumption.
Reference-dependent preference under naive agents can generate a role for PAYG pension, and private savings could be positive.

Sophisticated agents can work only if the borrowing constraint binds and private savings are zero.

Extension: what is the optimal pension for partially naive agents or heterogeneous agents with different naivete levels?