Reference-dependent Preferences, Time Inconsistency and Pay-as-you-go Pensions

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Motivation

- Social security has been around for a century and nearly ubiquitous.
- Old-age support v.s. redistribution.
- Consumption-smoothing: (pay-as-you-go) tax the young to finance the payment for the current elderly.

Motivation

- Why pensions if the capital markets is complete and even offers a higher return than pensions (dynamically efficient).
- Standard preferences cannot generate a role for pay-as-you-go pensions in a dynamically efficient economy (pension is return dominated) (Aaron, 1966; Samuelson, 1975).
    - Most developed economies are most likely dynamically efficient (Abel et al., 1989; Barbie et al., 2004).

Motivation

- Time-inconsistent preferences $\Rightarrow$ undersave for retirement (Imrohoroglu et al., 2003, QJE; Andersen and Bhattacharya, 2011, ET).
  - True v.s. choice utility, where the paternalistic government maximizes the true utility.
  - Time-inconsistent agents can offset the forced saving by reducing their own saving.
  - $+$ Borrowing constraints $\Rightarrow$ all private retirement savings are zero to generate pensions in a small open economy.

Motivation

- Reference-dependent preferences $\Rightarrow$ undersaving for retirement (Koszegi and Rabin, 2009, AER)
  - The young sets the reference consumption for her future.
  - Undersave or overconsume by the middle-aged.
- This paper: a welfare role for pensions under KR framework?
  - Yes and the under-saving problem is addressed to some extent.
  - Private retirement saving could be positive.
  - No paternalistic government.

The Model: Environment

- Three-period OLG model: the young, the middle-aged, the old.
- Dynamic Efficiency: the return on saving is greater than the population growth rate, $R > 1$.
- The government levies taxes at rate τ, and pays pensions $b = \tau w$.
- The lifetime utility of the agent is

$$\Omega_m = u(c_m) + \Phi[u(c_m) - u(c_m^r)] + \beta\gamma\Phi[u(c_o) - u(c_o^r)] + \beta u(c_o).$$

The Model: Environment

- The Timeline

t = 0

The young, with beliefs about her preference, makes consumption plans c_m^r, c_o^r.

t = 1

The middle-aged, endowed with income w, consumes c_m, saves s and pays taxes at tax rate τ, $c_m = (1-\tau)w - s$.

Updates belief $c_o^{r'} = c_o$.

t = 2

The old consumes what he saves and pensions, i.e. $c_o = Rs + b$.

The Model: Budgets

- The middle-aged consumption $c_m = (1 - \tau)w - s$.
- The old's consumption $c_o = Rs + b$.
- The budget constraints can be combined in equilibrium to

$$c_m + \frac{c_o}{R} = w - b(1 - \frac{1}{R}) \equiv Y.$$

The Model: The young

- Naive young: no gain-loss (Kramer, 2016, ET)
- Projection bias in predicting future utility (Loewenstein, ODonoghue, Rabin, 2003; DellaVigna, 2009)
- The young's problem

$$\max_{s^r} \Omega_y = u(c_m) + \beta u(c_o)$$
$$= u((1-\tau) w - s^r) + \beta u(R s^r + b).$$

The optimal consumption plan (c_m^r, c_o^r) satisfies

$$\frac{d\Omega_y}{ds^r} = -u'(c_m^r) + \beta R u'(c_o^r) = 0.$$

The Model: The old

- The old consumes what the middle-aged saved and the pension received, $c_o = R\left[(1-\tau)\,w - c_m\right] + b$.
- The old's actual consumption c_o coincides with the middle-aged updated belief of c_o.
- The instantaneous utility of the old is $u\left(c_o\right)$, where the gain-loss utility is 0.

The Model: The middle-aged

- The middle-aged consumes c_m and get $u(c_m)$.
- Contemporaneous gain-loss utility $G\left[u(c_m) - u(c_m^r)\right]$.
- Prospective gain-loss utility $\lambda G\left[u(c_o^r) - u(c_o)\right]$
- For $c_m \geq c_m^r$, given c_m^r, c_o^r the middle-aged chooses s, c_m, c_o to maximize

$$\Omega_m = u(c_m) + G[u(c_m) - u(c_m^r)] - \beta\gamma\lambda G[u(c_o^r) - u(c_o)] + \beta u(c_o).$$

$$s.t. \quad c_m = (1-\tau)w - s,$$

$$c_o = Rs + b.$$

The Model: The middle-aged

- The first-order condition is

$$u'(c_m) = \beta \kappa R u'(c_o),$$

where

$$\kappa = \frac{1 + \gamma\lambda G'(u(c_o^r) - u(c_o))}{1 + G'(u(c_m) - u(c_m^r))}.$$

- Overconsumption when evaluated at the young's consumption plan

$$J|_{s=s^r} = -G'(0)\, u'(c_m^r)\,(1 - \gamma\lambda) \leq 0 \text{ if and only if } \gamma < 1/\lambda.$$

The Model: The government

- Without borrowing constraints, the government is choosing pension b to maximize

$$max_b \Omega(Y) = max_b \, argmax_{\{c_m, c_m^r, c_o, c_o^r\}} \Omega_m(c_m, c_o, c_m^r, c_o^r).$$

- **Proposition** Under any preferences, if there exists an interior pension b^* alongside interior voluntary savings, then the optimal pension b^* is

$$b^* = b_0 + \frac{R}{R-1} w.$$

The Model: The government

- Assume that $\beta = 1$ since it does not affect the overconsumption.
- Choose b to maximize the ex-ante expected utility of the agent

$$\Omega_m = u(c_m) + G[u(c_m) - u(c_m^r)] - \gamma\lambda G[u(c_o^r) - u(c_o)] + u(c_o)$$

Taking derivative w.r.t. b, we can easily get

$$\begin{aligned}\frac{d\Omega_m}{db} &= \left(1 + G'(u(c_m) - u(c_m^r))\right) u'(c_m)(-1 - s'(b)) \\ &+ \left(1 + \gamma\lambda G'(u(c_o^r) - u(c_o))\right) u'(c_o)(1 + Rs'(b)) \\ &- G'(u(c_m) - u(c_m^r)) u'(c_m^r)\left(-1 - s^{r\prime}(b)\right) \\ &- \gamma\lambda G'(u(c_o^r) - u(c_o)) u'(c_o^r)\left(1 + Rs^{r\prime}(b)\right).\end{aligned}$$

The Model: The government

- First-order

$$\begin{aligned}\frac{d\Omega_m}{db} = & \left(1 + G'(u(c_m) - u(c_m^r))\right) u'(c_m)(-1 - s'(b)) \\ & + \left(1 + \gamma\lambda G'(u(c_o^r) - u(c_o))\right) u'(c_o)(1 + Rs'(b)) \\ & - G'(u(c_m) - u(c_m^r)) u'(c_m^r)\left(-1 - s^{r\prime}(b)\right) \\ & - \gamma\lambda G'(u(c_o^r) - u(c_o)) u'(c_o^r)\left(1 + Rs^{r\prime}(b)\right) .\end{aligned}$$

- Summing the first-two terms, we have

$$-(1 - \frac{1}{R})\left(1 + G'(u(c_m) - u(c_m^r))\right) u'(c_m) < 0.$$

  - The reason is the decline in lifetime income as a result of pension

$$Y = w - b(1 - \frac{1}{R}).$$

- First-order

$$\frac{d\Omega_m}{db} = \left(1 + G'(u(c_m) - u(c_m^r))\right) u'(c_m)(-1 - s'(b))$$
$$+ \left(1 + \gamma\lambda G'(u(c_o^r) - u(c_o))\right) u'(c_o)(1 + Rs'(b))$$
$$- G'(u(c_m) - u(c_m^r)) u'(c_m^r) \left(-1 - s^{r\prime}(b)\right)$$
$$- \gamma\lambda G'(u(c_o^r) - u(c_o)) u'(c_o^r) \left(1 + Rs^{r\prime}(b)\right) .$$

- The last two terms are positive, that is,

$$- G'(u(c_m) - u(c_m^r)) u'(c_m^r) \left(-1 - s^{r\prime}(b)\right) > 0.$$

$$- \gamma\lambda G'(u(c_o^r) - u(c_o)) u'(c_o^r) \left(1 + Rs^{r\prime}(b)\right) > 0.$$

Existence

- **Lemma** For CES utility with $\sigma > 1$ and constant elasticity gain-loss function G, the old-age consumption relative to middle-aged consumption is increasing in pension b, that is, $d\kappa/db > 0$, where

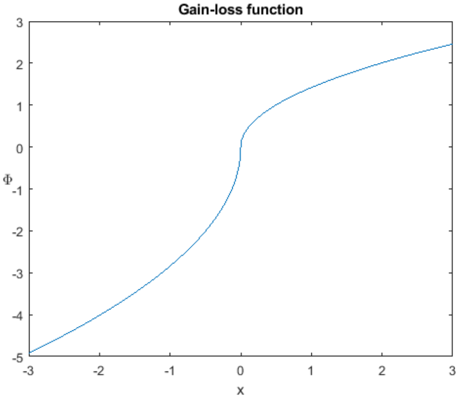
$$u'(c_m) = \kappa R u'(c_o).$$

- **Proposition** For CES utility $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. Suppose $\gamma\lambda < 1$ and the gain-loss function satisfies $G_x(0) > \underline{G}_x(0)$, positive pensions and positive savings exists for income $w \in (\underline{w}, \bar{w})$ provided $\sigma > 1$.

Numerics: S-shaped gain-loss utility

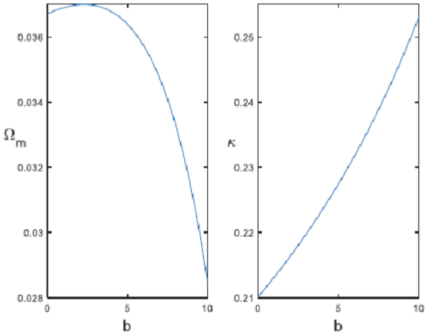
- Suppose $u(\cdot) = (\cdot)^{1-\sigma} / (1-\sigma)$, where $0 < \sigma \neq 1$.
- Gain-Loss Utility

$$\Phi(x) = \begin{cases} Ax^{\alpha}, & \text{if } x \geq 0, \\ -\lambda A(-x)^{\alpha}, & \text{if } x < 0. \end{cases}$$



Numerics: S-shaped gain-loss utility

- The objective function



- Let $R = 1.02^{25}$, $\gamma = 0.1$, $\lambda = 2$, $w = 10$, $\alpha = 1/2$, $A = 1.42$, $\sigma = 3$.
- The optimal pension and saving are both positive, that is,

$$\begin{aligned} b &= 2.3, \\ s &= 1.34, s^r = 2.40, \\ c_m^* &= 6.35, c_m^r = 5.30, \\ c_o^* &= 4.51, c_o^r = 6.25. \end{aligned}$$

Neoclassical Production Technology

- Assume the production function is constant returns to scale $F(K_t, L_t)$ where K_t and L_t denote the capital and labor input.
- Factor markets are perfectly competitive such that factor inputs are paid their marginal products in each period. Therefore,

$$R_t \equiv R(k_t) = f_k(k_t),$$

and

$$w_t \equiv w(k_t) = f(k_t) - k_t f_k(k_t),$$

Neoclassical Production Technology

- The optimal τ solves

$$\max_{\tau} \ \Omega_m(\tau) = u(c_m) + G(u(c_m) - u(c_m^r)) - \gamma\lambda G(u(c_o^r) - u(c_o)) + u(c_o)$$

- To have a welfare role for pensions

$$\begin{aligned} & [1 + G_x(g_m)] \Lambda^{ns} \\ = & \underbrace{-\eta u_c(c_o^r) \left[\left(\gamma\lambda - R(\tilde{k})\right) w\left(\tilde{k}\right) + (\gamma\lambda - 1) R\left(\tilde{k}\right) \frac{\partial \tilde{k}}{\partial \tau} \right]}_{\textit{exog fp}} \\ + & \underbrace{-\eta u_c(c_o^r) \left(\gamma\lambda - R(\tilde{k})\right) (1-\tau) \widetilde{k} f_{kk}\left(\tilde{k}\right) \tilde{k}_\tau(\tau)}_{\textit{endog fp}}. \end{aligned}$$

Neoclassical Production Technology

- To see the intuition, recall $-\widetilde{k} f_{kk}\left(\tilde{k}\right) = w_k(\tilde{k})$ and notice

$$-\eta u_c\left(c_o^r\right)\left[\gamma\lambda - R(\tilde{k})\right](1-\tau)\,\widetilde{k} f_{kk}\left(\tilde{k}\right)\tilde{k}_\tau(\tau)$$

$$=\eta u_c\left(c_o^r\right)\underbrace{\left[R(\tilde{k})-\gamma\lambda\right]}_{>0}(1-\tau)\underbrace{(-)\,w_k(\tilde{k})\,\frac{\partial\tilde{k}}{\partial\tau}}_{>0}.$$

- The pension, via τ, reduces $\tilde{k}$ which via $w\left(\tilde{k}\right)$ reduces the imagined wage income, and therefore, reduces reference consumption, which will increase utility given actual consumption.

Conclusion

- Reference-dependent preference under naive agents can generate a role for PAYG pension, and private savings could be positive.
- Sophisticated agents can work only if the borrowing constraint binds and private savings are zero.
- Extension: what is the optimal pension for partially naive agents or heterogeneous agents with different naivete levels?