Intergenerational Actuarial Fairness in Pay-as-you-go Pension Schemes

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Outline

Introduction

- Some facts in recent years
  - Declining mortality over time
  - Declining fertility
  - Population aging
- Concerns about sustainability of PAYG pension system
- Parametric reform to maintain financial sustainability
  - Increasing the contribution rate
  - Lowering the benefit ratio
- Concerns about intergenerational actuarial fairness

Literature Review

- How to measure actuarial fairness in pension schemes
  - The ratio between the present value of lifetime benefits and the accumulation at retirement(see,e.g.Queisser & Whitehouse (2006)).
  - The internal rate of return of the pension schemes(see,e.g.Settergren & Mikula (2005),Knell (2010)).
- Designing pension systems with the goal of achieving intergenerational actuarial fairness(see,e.g.Howse (2007),Bravo et al. (2023),Grossmann et al. (2024)).
  - Intergenerational risk sharing(see,e.g.Wang et al. (2018),Cui et al. (2011),Andersen (2014)).

Literature Review

- Assessing actuarial fairness of different cohorts
  - Earlier cohorts vs. younger cohorts (see,e.g.Myers & Schobel (1983) and Leimer (2007)).
- Actuarial fairness under different pension schemes
  - Defined Contribution(DC) vs. Defined Benefits(DB) vs. Notional Defined Contribution(NDC)(see,e.g.Queisser & Whitehouse (2006),Belloni & Maccheroni (2013),Alonso-García et al. (2018)).

Gaps in the Literature

- Lack of a Unified Framework for Assessing Intergenerational Actuarial Fairness
  - Existing studies tend to focus on isolated indicators, without integrating them into a cohesive framework that can be applied universally across different types of pension systems.
- Insufficient Attention to the Key Factors Influencing Actuarial Fairness
  - The interactions between factors such as contribution rates, benefit, and demographic changes are often insufficiently explored.

Main contribution

- We propose a general framework for measuring intergenerational actuarial fairness
  - Identifying the factors influencing intergenerational actuarial fairness and compare their effects in pure DB, pure DC, and hybrid pension schemes
  - Assessing the impact of changing demographic structures on the actuarial fairness of different cohorts.
  - Examining how variations in key parameters affect intergenerational actuarial fairness.

PAYG pension schemes

Let $N(x,t)$ denote the amount of individuals aged x at time t, i.e., born at time $t-x$ and belonging to generation $h(h=t-x)$, which is given by

$$N(x,t) = N(0,t-x)S(x,t-x) = N(0,h)S(x,h) = N(h)S(x,h).$$

where $N(h)$ denotes the amount of newborn cohort h, $S(x,h)$ denotes the survival probability that a member of cohort h survives to age x.

Wages are earned by the active population and are assumed to be paid at the beginning of the calendar year. For $x \geq a$, the wage of an individual aged x at time t is given by:

$$W(x,t) = W(a,t-x+a)\prod_{j=a+1}^{x}(1+g_j),$$

where g_j is the rate of wage variation for a cohort from age a to age x.

PAYG pension schemes

Consider a pay-as-you-go(PAYG) pension system, with benefits to retirees covered by the current contributions. The aggregate contribution of active members at time t is given by

$$C(t) = \pi(t)\overline{W}(t)A(t) = \sum_{x=a}^{r-1} \pi(t)W(x,t)N(x,t), \quad (1)$$

where $\pi(t)$ denotes the contribution rate, $\overline{W}(t)$ denotes the mean wage of all contributors, $A(t)$ denotes the number of active population at time t.

PAYG pension schemes

The pension expenditures paid by the pension system at time t can be expressed as:

$$E(t) = \overline{P}(t)R(t) = \delta(t)\overline{W}(t)R(t) = \sum_{x=r}^{w} P(x,t)N(x,t), \tag{2}$$

where $\overline{P}(t)$ is the mean pension benefit at time t for all retirees, $\delta(t)$ represents benefit ratio which is given by the ratio of the mean pension and the mean wage at time t, $R(t)$ represents the number of retire population at time t.

In a balanced PAYG pension schemes, the contributions $C(t)$ must be sufficient to cover the pension expenditures $E(t)$ at time t, that is,

$$C(t) = E(t) \Rightarrow \pi(t) = \frac{R(t)}{A(t)}\delta(t) = D(t)\delta(t) \tag{3}$$

where $D(t)$ denotes the dependency ratio at time t.

PAYG pension schemes

The initial pension benefit $P(r,t)$ for retirees retiring at age r is determined based on the replacement rate, which is given by:

$$P(r,t) = \gamma(t) W(r,t)$$

where $\gamma(t)$ represents the replacement rate, defined as the ratio of the first pension of an individual and the last wage of the individual.
For those who retired in the previous year, the pension in payment for an individual aged $x (x > r)$ at time t is given by

$$P(x,t) = P(x-1, t-1)(1 + \lambda_{t-1})$$

where λ_t denotes the indexation of pensions in payment at time t.

Intergenerational Actuarial Fairness

Following the definition of actuarial fairness by Queisser & Whitehouse (2006), the intergenerational actuarial fairness ratio $AF(h)$ is calculated as the ratio of the present value of the benefits received by an individual until the death to the present value of the contributions to the system, which is given by:

$$AF(h) = \frac{\sum_{i=r}^{w} P(i, h+i) S(i, h) \prod_{m=a+1}^{i} (1 + k_{h+m})^{-1}}{\sum_{i=a}^{r-1} \pi(h+i) W(i, h+i) S(i, h) \prod_{m=a+1}^{i} (1 + k_{h+m})^{-1}}$$

where k_t denotes the interest rate at time t.

Intergenerational Actuarial Fairness

By rewriting the benefit $P(i, h+i)$ and the wage $W(i, h+i)$, in the case where the wage growth rates g_t and the discount factor coincide each year, i.e., $1 + g_t = \frac{1}{1+k_t}$, it can be derived that $AF(h)$ is given by

$$AF(h) = \frac{\sum_{i=r}^{w} \gamma(h+r)S(i,h) \ \prod_{m=r}^{i-1}(\frac{1+\lambda_{h+m}}{1+g_{h+m}})}{\sum_{i=a}^{r-1} \pi(h+i)S(i,h)} \tag{4}$$

During the retirement period, we assume that pensions are indexed following the evolution of average wages adjusted by a sustainability factor as

$$\frac{1+\lambda_{h+m}}{1+g_{h+m}} = \beta_{h+m}$$

Intergenerational Actuarial Fairness

In this section, based on the analysis of intergenerational risk sharing between contributors and retirees in classical pension schemes by Morsomme et al. (2024), we discuss how the intergenerational actuarial fairness indicator varies across three classic pension schemes:

- Pure defined contribution(Pure DC)
- Pure defined benefit(Pure DB)
- Hybrid pension scheme that adapts either the contribution rate $\pi(t)$ and/or the benefit $\delta(t)$ ratio.

Pure DB

In pure DB pension schemes, the level of the real pension benefit is constant. This has the natural implication that pensions for new retirees will be equivalent to those of older generations.

- Indexation cannot divert from the wage-increase which implies that the sustainability factor should be equal to $1 (\beta_t = 1)$.
- The replacement rate used to calculate the first pension needs to be constant, i.e., $\gamma(t) = \gamma(t - x + r) = \gamma$, from which it follows that the benefit ratio $\delta(t)$ is constant and simplifies to $\delta(t) = \gamma = \delta$.

Pure DB Without Buffer Fund

According to (3), the contribution rate $\pi(t)$ is adjusted in response to changes in the dependency ratio $D(t)$:

$$\pi(t) = D(t)\delta$$

The intergenerational actuarial fairness indicator $AF(h)$ in (4) without buffer fund under pure DB can expressed as follows:

$$AF^{B-DB}(h) = \frac{\sum_{i=r}^{w} \gamma(h+r)S(i,h)\ \prod_{m=r}^{i-1}(\frac{1+\lambda_{h+m}}{1+g_{h+m}})}{\sum_{i=a}^{r-1} \pi(h+i)S(i,h)} = \frac{\sum_{i=r}^{w} S(i,h)}{\sum_{i=a}^{r-1} D(h+i)S(i,h)} \tag{5}$$

Pure DB With Buffer Fund

Assumed that there exists buffer fund, which indicates the balance of PAYG pension system are not guaranteed each year. The contribution rate can be expressed as:

$$\pi(t) = \pi(t_0)(1+\alpha)^{t-t_0}$$

where α is growth rate of contribution rate, t_0 is the time point when the demographic change begin. The intergenerational actuarial fairness indicator $AF(h)$ with buffer fund under pure DB pension system is given by

$$AF^{I-DB}(h) = \frac{\sum_{i=r}^{w} \gamma(h+r)S(i,h) \ \prod_{m=r}^{i-1}(\frac{1+\lambda_{h+m}}{1+g_{h+m}})}{\sum_{i=a}^{r-1} \pi(h+i)S(i,h)} = \frac{\sum_{i=r}^{w} S(i,h)}{D(t_0)\sum_{i=a}^{r-1}(1+\alpha)^{h+i-t_0} S(i,h)} \tag{6}$$

Pure DC

In pure DC pension schemes, the level of the contribution is constant, that is, $\pi(t) = \pi$. Furthermore, we assume that the same level of adjustment applied to both new and old retirees by fixing the replacement rate to the benefit ratio :

$$\gamma(t) = \delta(t)$$

From which we can derive that the indexation of the pension in payment for an individual aged $i(i > r)$ is given by

$$\prod_{m=r}^{i-1}\left(\frac{1+\lambda_{t-i+m}}{1+g_{t-i+m}}\right) = \frac{\delta(t-i+r)}{\delta(t)}$$

Pure DC Without Buffer Fund

According to (3), the benefit ratio δ is adjusted in response to changes in the dependency ratio $D(t)$

$$\delta(t) = \frac{\pi}{D(t)}$$

The intergenerational actuarial fairness indicator $AF(h)$ in (4) under pure DB without buffer fund can be expressed as follows:

$$AF^{B-DC}(h) = \frac{\sum_{i=r}^{w} \gamma(h+r)S(i,h)\ \prod_{m=r}^{i-1}(\frac{1+\lambda_{h+m}}{1+g_{h+m}})}{\sum_{i=a}^{r-1} \pi(h+i)S(i,h)} = \frac{\sum_{i=r}^{w} \frac{1}{D(h+i-1)}S(i,h)}{\sum_{i=a}^{r-1} S(i,h)}$$

Pure DC With Buffer Fund

Assumed that there exists buffer fund, which indicates the balance of PAYG pension system are not guaranteed each year. The benefit ratio can be represented as follows:

$$\delta(t) = \delta(t_0)(1+\beta)^{t-t_0}$$

where β is the growth rate of the benefit ratio. Thus, the intergenerational actuarial fairness indicator $AF(h)$ with buffer fund under pure DC pension system is given by

$$AF^{I-DC}(h) = \frac{\sum_{i=r}^{w} \gamma(h+r)S(i,h) \prod_{m=r}^{i-1}(\frac{1+\lambda_{h+m}}{1+g_{h+m}})}{\sum_{i=a}^{r-1} \pi(h+i)S(i,h)} = \frac{\sum_{i=r}^{w}(1+\beta)^{h+i-1-t_0} S(i,h)}{D(t_0)\sum_{i=a}^{r-1} S(i,h)}$$

Hybrid Pension Schemes

The hybrid pension plan adjusts both contributions and benefits simultaneously. A possible intermediate between the DB and DC systems is the Musgrave rule(see e.g. Musgrave (1981)and Morsomme et al. (2024)). The idea is that the net replacement rate denoted as M - the ratio between the pension and the salary net of the contributions-is constant:

$$M = \frac{\overline{P}(t)}{\overline{S}_t(1 - \pi(t))} = \frac{\delta(t)}{1 - \pi(t)} \tag{7}$$

We can obtain $\delta(t)$ and $\pi(t)$ under the Musgrave rule by exploiting the PAYG equilibrium.

$$\pi(t) = \frac{M\ D(t)}{1 + MD(t)} \qquad \delta(t) = \frac{M}{1 + MD(t)}$$

Hybrid Pension Schemes Without Buffer Fund

Along with the Pure DC pension system, we assumed that adjustments for both new and old retirees remain constant, ensuring that longevity risk is shared equally between the two groups. Thus, The intergenerational actuarial fairness indicator $AF(h)$ in (4) under hybrid pension plan without buffer fund can be expressed as:

$$AF^{B-HY}(h) = \frac{\sum_{i=r}^{w} \gamma(h+r)S(i,h) \ \prod_{m=r}^{i-1}(\frac{1+\lambda_{h+m}}{1+g_{h+m}})}{\sum_{i=a}^{r-1} \pi(h+i)S(i,h)} = \frac{\sum_{i=r}^{w} \frac{M}{1+MD(h+i-1)} S(i,h)}{\sum_{i=a}^{r-1} \frac{M\ D(h+i)}{1+MD(h+i)} S(i,h)}$$

Hybrid Pension Schemes With Buffer Fund

Assumed that there exists buffer fund, which indicates the balance of PAYG pension system are not guaranteed each year. The contribution rate is given by:

$$\pi(t) = \pi(t_0)(1+\alpha)^{t-t_0}$$

Thus, the intergenerational actuarial fairness indicator AF with buffer fund under hybrid pension system is given by

$$\begin{aligned} AF^{I-HY}(h) &= \frac{\sum_{i=r}^{w} \gamma(h+r)S(i,h)\ \prod_{m=r}^{i-1}(\frac{1+\lambda_{h+m}}{1+g_{h+m}})}{\sum_{i=a}^{r-1} \pi(h+i)S(i,h)} \\ &= \frac{\sum_{i=r}^{w} M(1-\pi(t_0)(1+\alpha)^{h+i-1-t_0})S(i,h)}{\sum_{i=a}^{r-1} \pi(t_0)(1+\alpha)^{h+i-t_0} S(i,h)} \end{aligned}$$

Numerical illustration

- Assumption 1: We assume that the number of people born each year evolves in an exponential manner, that is, for $h \geq t_0$:

$$N(h) = N \exp(-a(h - t_0)),$$

where a is the decreasing rate, t_0 is the time point when the fertility decline begins, and N is the number of initial newborn population.

- Assumption 2: Based on Milevsky (2020), we assume that the mortality rate $q(x, h)$ is given by:

$$q(x,h) = \begin{cases} \rho + \frac{1}{m(h)} e^{\frac{x-\mu(h)}{m(h)}} & x \in [0, x^\star], \\ \rho + e^L & x \in (x^\star, w], \end{cases}$$

where ρ is a nonage-dependent accidental death rate (Makeham constant), and $x^\star, L, \mu$, and m are Gompertz parameters.we assume that for $h \geq t_0$,

$$m(h) = m_0 - d(h - t_0),$$

where d is the parameter that controls the speed of longevity trend.

AF(h) in PAYG Pension Schemes Without Buffer Fund

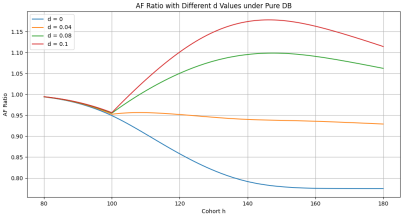

Figure 1: Pure DB

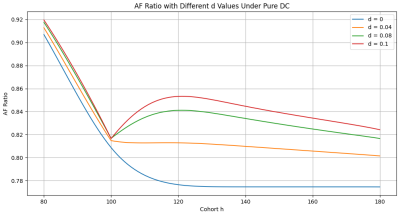

Figure 2: Pure DC

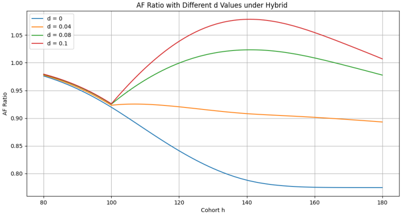

Figure 3: Hybrid

- Commons
  - Decline Period
  - Rise Period
  - Further decline phase
- Difference

AF(h) in PAYG Pension Schemes With Buffer Fund

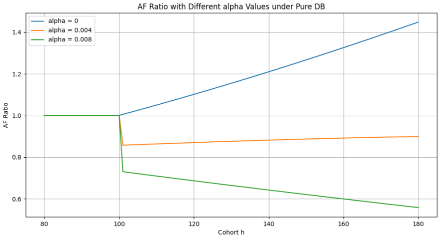

Figure 4: Pure DB

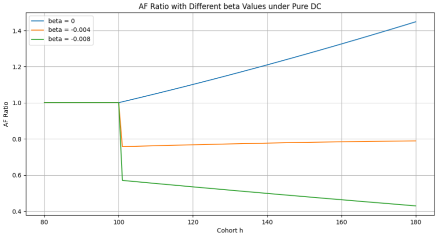

Figure 5: Pure DC

- When the growth rate is low or constant, the value of AF(h) still increases under under demographic aging.
- For equivalent adjustments to the contribution rate and benefit ratio, changes in the benefit ratio have a greater impact on AF(h) than changes in the contribution rate.

Conclusion

- In this study, we develops a general framework to analyze actuarial fairness across different cohorts in PAYG pension schemes.
- Balanced vs. Imbalanced PAYG pension schemes
- Pure DB vs. Pure DC vs. Hybrid pension schemes

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Thank you for your attention!