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Product Pricing and Solvency Capital Requirements for Long-term Care Insurance

Adam W. Shao[1] Michael Sherris[2] and Joelle H. Fong[3]

[1]Centre of Excellence in Population Ageing Research (CEPAR), UNSW Business School, UNSW Australia, email: wenqiang.shao@unsw.edu.au
[2]CEPAR, School of Risk and Actuarial Studies, UNSW Business School, UNSW Australia, email: m.sherris@unsw.edu.au
[3]CEPAR, UniSIM College, SIM University, Singapore, email: joellefonghy@unisim.edu.sg

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Adam W. Shao*, Michael Sherris, and Joelle H. Fong

School of Risk and Actuarial Studies and ARC Centre of Excellence in Population Ageing Research (CEPAR), UNSW Australia

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Abstract

This paper presents a comprehensive assessment of premiums, reserves and solvency capital requirements for long-term care (LTC) insurance policies using Activities of Daily Living (ADLs) and U.S. data. We compare stand-alone policies, rider benefit policies (LTC insurance combined with whole life insurance), life care annuities (LTC insurance combined with annuities), and shared LTC insurance in terms of premium cost and solvency capital requirements. Premiums and best-estimate reserves for generic LTC insurance policies are determined using Thiele's differential equation. Product features such as the elimination period and the maximum benefit period are compared using a simulation-based model. Solvency capital requirements for longevity risk and disability risk are based on the Solvency II standard formula. We quantify the extent to which rider benefit policies and life care annuities provide lower solvency capital requirements than stand-alone LTC insurance policies. We show how a maximum benefit period can reduce costs and risks for LTC insurance products.

Keywords: Long-term care insurance; Solvency II; solvency capital requirements

JEL Classifications: G22, G32, G32, I11, I13

*[Corresponding author]. Email: wenqiang.shao@unsw.edu.au; Postal address: Australian Research Council Centre of Excellence in Population Ageing Research, UNSW Business School, University of New South Wales, Sydney NSW 2052, Australia; Phone: +61-2-9385 7005; Fax: +61-2-9385 6956.

1 Introduction

Long-term care (LTC) costs have shown a significant increase over recent decades and the increasing trend is projected to continue in future (e.g., Congressional Budget Office, 2004; Productivity Commission of Australia, 2013; Shi and Zhang, 2013). LTC expenses are significantly higher if the insured moves into LTC facilities (see Fong *et al.*, 2012, for a discussion on nursing home admittance). The primary funding for LTC costs in Australia is the lifetime stop-loss mechanism funded through the pay-as-you-go scheme (Productivity Commission of Australia, 2011, 2013). In the U.S., the base funding programme for LTC costs is Medicaid (Meiners, 2008). In particular in the U.S., community-based LTC costs are primarily funded through two public programmes: Medicaid and Medicare; institutionalised LTC expenses are primarily funded through Medicaid and personal co-payments (Kaye *et al.*, 2010). Organisation for Economic Co-operation and Development (2005) and Colombo *et al.* (2011) provide a comprehensive review on the LTC funding systems in OECD countries including Australia and the U.S.

Recent discussions in Australia and many other countries have focused on developing the private LTC insurance market as an important supplement for public funding sources (see e.g., Colombo *et al.*, 2011; Glendinning *et al.*, 2004; Productivity Commission of Australia, 2011, 2013). Though the private insurance is an important source, the share of the private market is small. In the U.S., only 4% of LTC costs are reimbursed from private insurance (Brown and Finkelstein, 2008). Motivated by the small share of private LTC insurance, Brown and Finkelstein (2008) investigate the interaction of the public Medicaid program and private LTC insurance. They find that Medicaid has a very large crowd-out effect due to the implicit tax imposed on the benefits of private LTC insurance. Against this background, a comprehensive analysis of LTC insurance in terms of premium costs, policy designs and solvency capital will allow a more informed consideration of the role and effectiveness of private LTC insurance.

A LTC insurance policy entitles the insured to receive benefits when the insured becomes functionally disabled according to the definition pre-specified in the policy (Haberman and Pitacco, 1999). LTC insurance policies, however, do not have a uniform definition for the benefit eligibility in the market. The most frequently used criteria for defining functional disability in LTC insurance are the number of Activities of Daily Livings (ADLs) that individuals cannot perform independently and cognitive impairment (Haberman and Renshaw, 1996; Murtaugh *et al.*, 2001; Pritchard, 2006). The Australian Bureau of Statistics defines individuals' functional disability based on the Core Activity Restrictions (CARs) that can be linked to scales of ADLs (Leung, 2004, 2006). We focus on ADLs as the basis for a private LTC insurance contract.

LTC insurance policies can be categorised into four different types (Haberman and Pitacco, 1999; Leung, 2006): fixed benefit policies sold to healthy individuals, fixed benefit policies sold to the elderly entering or already staying in LTC facilities, indemnity-based benefit policies, and policies that allow the insured to choose between fixed benefit and LTC service. The fixed benefit policy is the most typical and widely used type in the private LTC insurance market. Fixed benefit LTC insurance can be stand-alone policies, included as a rider benefit in the whole life insurance, or life care annuities (Haberman and Pitacco, 1999).

A stand-alone policy pays out the predetermined benefit when the insured becomes functionally disabled. In practice LTC policies can be combined with other forms of insurance. LTC cover included as a rider benefit in a whole life insurance policy, referred to as the rider benefit policy, is a financial product that allows the insured to draw the death benefit for LTC costs before death (Haberman and Pitacco, 1999). In a rider benefit policy, the insured is eligible for LTC benefits when becoming functionally disabled and also becomes eligible upon death for the death benefit net of drawn LTC benefits. Another version of the rider benefit policy is to directly include LTC benefits in a whole life insurance where the death benefit is a fixed amount Leung (2006). This paper focuses on the rider benefit policy of the type in Leung (2006).

LTC insurance can also be combined with annuities, which is usually referred to as the life care annuity (Brown and Warshawsky, 2013; Murtaugh *et al.*, 2001; Warshawsky, 2007). The life care annuity reduces the adverse selection problem by pooling annuitants who are vulnerable to longevity risk and LTC insurance policyholders who are vulnerable to disability risk (Murtaugh *et al.*, 2001). This risk pooling of the life care annuity provides a natural hedge and therefore reduces insurance premiums (Brazell and Warshawsky, 2008; Murtaugh *et al.*, 2001; Warshawsky, 2007). Such annuities offer a valuable product structure for both the insurer and the insured especially given the increasing need for individuals to fund their own retirement income and the potential role of a private annuity market.

The generic LTC insurance in this paper is a LTC insurance policy with no elimination period or maximum benefit period. In order to minimise adverse selection and to make LTC insurance more affordable, insurers usually include an elimination period and a maximum benefit period in the product. The elimination period is the required minimum number of consecutive payment periods before the insured becomes eligible for benefits. The elimination period can span from three months to two years. Most LTC insurers provide a lifetime elimination period, which means that the insured does not have to go through the elimination period each time before he or she is eligible for receiving benefits. The maximum benefit period is also a useful tool in managing risks and making the product more affordable. Analogous to the upper limit in property and casualty insurance, the maximum benefit period is the maximum periods of payment that the insured can possibly receive. The commonly used maximum benefit periods are 3 years, 4 years and 5 years. These alternative product designs have implications for both costs and capital requirements.

This paper considers product design for LTC and analyses lump sum and regular premiums for a broad range of fixed benefit LTC insurance policies, taking into account product features such as the elimination period and the maximum benefit period. Different combinations of the elimination period and the maximum benefit period are analysed to highlight how more affordable products can be offered. Solvency capital requirements under Solvency II for different types of LTC insurance policies are assessed to determine the extent of capital reductions in stand-alone policies for disabled lives compared to healthy lives. Capital reductions for combined LTC insurance with life insurance and annuities are quantified. For example, under the Solvency II standard formula framework, it is shown that 80% less capital per unit premium at policy issue is required for life care annuities compared to stand-alone policies for policies sold to 65-year-old healthy males.

The paper is arranged as follows. Section 2 describes the Markov model framework for health dynamics. Section 3 presents the methodology on pricing, reserving and deriving capital requirements for LTC insurance policies. The first part of Section 3 outlines Thiele's differential equation approach used for the pricing and reserving of generic LTC insurance policies. The second part focuses on policies with flexible product features which require a simulation-based approach to derive premiums and reserves. The third part discusses the solvency capital requirement in the Solvency II Directive (European Insurance and Occupational Pension Authority, 2011). Section 4 briefly describes the data used to derive health dynamics and presents the transition rates assumed for the analysis. The demographic characteristics of the experience assumed in the analysis is also provided in Section 4. Section 5 presents results for premiums based on the methods described in Section 3 and compares premiums for different types of LTC insurance policies. Section 6 gives results for the best-estimate liabilities and solvency capital requirements for the different types of policies. Section 7 concludes.

2 Markov Model Framework for Health Dynamics

A LTC insurance policy pays benefits to the insured when the insured becomes functionally disabled, i.e. the benefits are dependent on the current health state. We employ a four-state continuous time Markov model to describe the health dynamics of retirees.

The health states are categorised based on the number of difficulties in independently performing Activities of Daily Livings (ADLs). The transition diagram is shown in Figure 1, where "H" denotes the healthy state, "M" denotes the mildly disabled state (defined as having 1 - 2 ADL difficulties)[1], "S" denotes the severely disabled state (defined as having 3 - 6 ADL difficulties), and "D" denotes the dead state. As shown in the diagram, the paper allows for recovery from disability, which is in line with prior studies (e.g., Ameriks *et al.*, 2011; Brown and Warshawsky, 2013; Leung, 2006; Pritchard, 2006; Robinson, 1996). Some prior studies (such as Ferri and Olivieri, 2000; Olivieri and Pitacco, 2001) do not take into account recovery, based on the argument that LTC disability has the chronic characteristic that makes it very hard to recover from the disabled state. Based on Fong *et al.* (2015) the recovery rate is comparatively high and should be taken into account in the analysis.

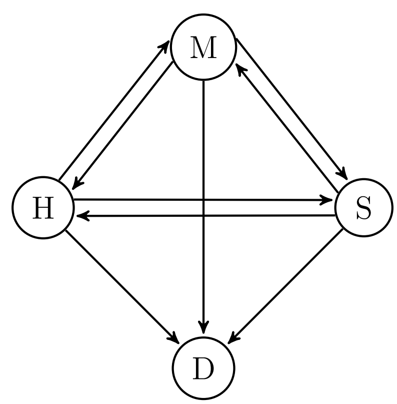

Figure 1. Four-state Markov transition diagram.

The transition rates and probabilities are age- and sex-dependent. Let $\Omega_\chi = \{\text{H, M, S, D}\}$ denote the state space, x the individual's age last birthday, $\chi(x) \in \Omega_\chi$ the health state at age x. For $t \geq 0$ and $i, j \in \Omega_\chi$, the transition probability from state i at age x into state j

[1]Typical private LTC insurance, particularly in North America, pays a benefit when the insured has difficulties with either 2+ or 3+ ADLs. This paper uses 3+ ADLs as a base and shows the effect of 2+ ADLs in a sensitivity analysis in Table 9.

at age $x+t$ is defined as follows:

$$p_{ij}(x, x+t) = \Pr\left\{\chi(x+t) = j \mid \chi(x) = i\right\}. \quad (1)$$

Instantaneous transition intensities are assumed to be integrable on compact intervals. For $i \neq j$, the transition intensity is defined as:

$$\mu_{ij}(x) = \lim_{\Delta x \to 0^+} \frac{p_{ij}(x, x+\Delta x)}{\Delta x}. \quad (2)$$

We use a Generalised Linear Model (GLM) with a log link function and a Poisson distribution to graduate the estimated transition rates. In this model, the number of transitions is assumed to follow a Poisson distribution with mean that depends on age in the following function:

$$m_x = e_x \sum_{s=0}^{k} \beta_s x^s, \quad (3)$$

where e_x denotes the exposure for x-year-old individuals, β_s denotes the coefficients for the s^{th} order polynomial function of age, and k is the maximum polynomial order. We select the optimal k based on AICc, BIC and model deviance (see Fong *et al.*, 2015, for the detailed methodology). The graduated transition rates are then used as inputs to the pricing, reserving and capital requirements of LTC insurance.

3 Methodology

3.1 Thiele's Differential Equation

The generic LTC insurance policy has no elimination period or maximum benefit period. Since we assume a Markov process for our transitions and benefits, we can compute premiums with the generalised Thiele's differential equation (Christiansen *et al.*, 2014; Hoem, 1969;

Leung, 2006). The Thiele's differential equation approach, first published in Gram (1910), provides a set of simultaneous differential equations that are used to calculate premiums and reserves for life insurance policies where only alive and dead states are involved. The generalised Thiele's differential equation is applied to life contingencies that involve multiple health states Linnemann (1993) and Norberg (1992, 1995).

We use $V_i(t,T)$ to denote the time t expected present value of benefits paid to an individual in state i within the period $(t,\ T)$, where $i \in \Omega_\chi$ is the health states and T is the terminal period. As discussed in Fong *et al.* (2015), it is difficult to extrapolate the transition rates past age 100 due to limited exposure at very old ages. Therefore, the maximum attainable age is assumed to be 100. Based on the four-state health transition diagram defined in Figure 1, the expected present value is given by:

$$V_i(t,T) = \int_t^T e^{-\left(\delta+\sum_{j\neq i}\mu_{ij}(x+s)\right)(s-t)}\left[b_i(s) + \sum_{j\neq i}\mu_{ij}(x+s)\Big(B_{ij}(s) + V_j(s,T)\Big)\right]ds, \qquad (4)$$

where $i,\ j \in \Omega_\chi = \{\text{H, M, S, D}\}$ denote health states as defined in Section 2, δ is the continuously compounded interest rate, $b_i(s)$ is the annuity payment to the insured while in state i at time s, $\mu_{ij}(x+s)$ is the transition intensity from state i to state j for individuals aged $x+s$, and $B_{ij}(s)$ is the benefit payment upon transitions from state i to state j at time s. When the insured dies, the reserve becomes zero after the payment of death benefit, if any, i.e. $V_i(t,T) \equiv 0$ when $i = D$. $b_i(s)$ is usually referred to in the literature as the sojourn benefit and $B_{ij}(s)$ as the transition benefit (Christiansen *et al.*, 2014). Since this paper focuses on fixed benefit products, the sojourn and transition benefits are fixed for generic stand-alone policies, rider benefit policies of the type in Leung (2006) and life care annuities.

Differentiating both sides of Equation (4) with respect to t, the generalised Thiele's differential equation is derived and expressed as follows:

$$\frac{dV_i(t,T)}{dt} = \delta V_i(t,T) - b_i(t) - \sum_{j\neq i} \mu_{ij}(x+t)\Big(B_{ij}(t) + V_j(t,T) - V_i(t,T)\Big), \tag{5}$$

where notations are consistent with those in Equation (4). Equation (5) explicitly shows that the change in the reserve during an infinitesimal time can be decomposed into four parts: the accrued interest, the paid-out annuity to individuals staying in certain states if any, the benefit payment upon transitions if any, and the jump in the reserve if the individual transitions into a different state.

When the transition intensities are simple functions with respect to age, the above simultaneous differential equations can be easily solved to derive a closed formula for the reserve function. Based on the assumed values for the interest rate δ and the graduated transition rate $\mu_{ij}(x+t)$, the reserve function evaluated at time 0 is the lump sum premium of a generic LTC insurance policy.

Since graduated transition rates are parametrised as exponential polynomial functions with respect to age, the reserve functions cannot be directly solved for in the simultaneous equations as in Equation (5). We use the Euler's rule to derive a numerical solution of the reserve function $V_i(t,T)$. The Euler's rule is a commonly used approach in discretising differential equations. The dt term in Equation (5) is replaced by a very short period of time h, such as a day. Rearranging the discretised version of Equation (5), the process of calculating the reserve for a previous period is as follows:

$$\begin{aligned} V_i(T,T) &= 0, \quad \forall\, i \in \Omega_{\chi}, \qquad (6)\\ V_i\big(T-(u+1)h,T\big) &= V_i(T-uh,T)\left(1 - h\delta - h\sum_{j\neq i}\mu_{ij}(x+T-uh)\right) + hb_i(T-uh) \\ &\quad + h\sum_{j\neq i}\mu_{ij}(x+T-uh)\Big(B_{ij}(T-uh) + V_j(T-uh,T)\Big), \end{aligned}$$

where $i,\ j \in \Omega_\chi$, $u \in \{0,\ 1,\ 2, \cdots,\ \frac{T}{h} - 1\}$ is a non-negative integer, and other notations are consistent with those in Equation (5). An implicit assumption is that the transition intensity is constant within the short period of time h. Numerical values are then solved for based on backward iterations of Equation (6) starting from the terminal period.

The lump sum premium of a generic LTC insurance sold to an individual in state i is equal to the expected present value of future benefits, based on the principle of equivalence. The expected present value of a unit benefit while the insured is in the severely disabled state can be calculated as the reserve function evaluated at the present time, which can be expressed as follows:

$$v_{\text{S}} = V_i(0, T), \tag{7}$$

where $i \in \Omega_\chi$, $\forall\ t \in (0,\ T)$, $b_{\text{S}}(t) = 1$, $b_k(t) = 0$ for $k \neq \text{S}$, and $B_{ij}(t) \equiv 0$. The lump sum premium of insurance policies sold to individuals in state i, denoted by P_i^L, is then calculated in the following equation:

$$P_i^L = y\ v_{\text{S}}, \tag{8}$$

where y is the predetermined annual amount of LTC insurance benefits paid while the insured is in the severely disabled state.

Continuously paid premiums are also of interest in the actuarial field. In particular, premiums paid on a very frequent basis, such as weekly premiums, are usually approximated as continuous premiums. The expected present value of a unit payment while the individual stays in the healthy or mildly disabled state is first calculated based on the discretised simultaneous differential equations, as shown in Equation (6), with a very small step size such as 0.001. In mathematical expressions, the first step is to calculate the following expected present value:

$$v_{\text{H,M}} = V_i(0, T), \tag{9}$$

where $i \in \Omega_\chi$, $\forall\ t \in (0,\ T)$, $b_{\text{H}}(t) = b_{\text{M}}(t) = 1$, $b_{\text{S}}(t) = b_{\text{D}}(t) = 0$, and $B_{ij}(t) \equiv 0$. The expected present value of a unit payment while the individual stays in the severely disabled state is then calculated using the same specifications as in the case of the lump sum premium, as shown in Equation (7). Based on the principle of equivalence, the continuously paid premium per annum of insurance policies sold to individuals in state i, denoted by $\bar{P}_i$, is solved for in the following equation:

$$\bar{P}_i\ v_{\text{H,M}} = y\ v_{\text{S}}. \tag{10}$$

Instead of paying lump sum premiums at the outset or continuously paying insurance premiums, policyholders usually choose to pay LTC insurance premiums on an annual, quarterly, or monthly basis while the insured is not eligible for receiving LTC insurance benefits[2]. The Thiele's differential equation is a very useful tool in dealing with these regular premiums.

For regular premiums of a generic LTC insurance policy, the unit payment while the insured stays in the healthy or the mildly disabled state are calculated using the discretised version of Thiele's differential equation, as shown in Equation (6), with a step size of the corresponding frequency, for example $h = \frac{1}{12}$ for premiums paid on a monthly basis. Let $v'_{\text{H,M}}$ denote the expected present value of a unit payment while the insured is in the healthy or the mildly disabled state at the beginning of each assessment interval (for example, at the beginning of each month for premiums on a monthly basis). v_{S} is calculated using the same specifications on the benefit payments as in Equation (7). Based on the principle of equivalence, the premium on a regular basis of insurance policies sold to individuals in state i, denoted by $P_i^{(f)}$, can be solved for in the following equation:

$$f P_i^{(f)}\ v'_{\text{H,M}} = y\ v_{\text{S}}, \tag{11}$$

[2]For some types of LTC insurance policies, only lump-sum premiums are taken into account since regular or continuous premiums are not feasible. For example, policies sold to individuals who are already severely disabled and life care annuities.

where f denotes the number of payments in a year. For example, $f = 12$ and 1 for monthly and annual premiums respectively.

3.2 Simulation-Based Approach

The generalised Thiele's differential equation is widely used for pricing and reserving of insurance in a Markov model framework. When flexible product features such as the elimination period and the maximum benefit period are allowed, the LTC benefit depends on durations in one or multiple states and the benefit process is therefore no longer Markov. Consequently, the generalised Thiele's differential equation approach cannot be directly used for LTC insurance policies with more flexible features (Christiansen *et al.*, 2014; Hoem, 1969). An alternative method is to simulate health trajectories of a large number of homogeneous and independent individuals and to calculate the expected present values of benefit payments based on the simulated health trajectories.

Given graduated annual transition rates from Fong *et al.* (2015), transition probabilities for a short period of time can be calculated as the matrix exponential of the annual transition rate matrix multiplied by the short period, assuming that transition rates are constant within integer ages. Based on the calculated transition probabilities for a short period, an individual's health state in the next period given his or her current health state follows a multinomial distribution. The health transition trajectories of a large number of N homogeneous individuals aged x are simulated using the multinomial distribution. Based on the simulated health trajectories, the present value of future benefit payments to the insured currently in state i at time t is:

$$PV_i(l,t) = \sum_{s=t}^{\omega-x} \Delta_i(l,t)\ B(l,0,1,2,\cdots,s)e^{-\delta(s-t)}, \tag{12}$$

where $i \in \Omega_\chi$ is the health state, l denotes the l^{th} individual, ω is the maximum attainable age, $\Delta_i(l,t)$ is an indicator variable that equals 1 if the l^{th} insured is in state i at time t and equals 0 otherwise, $B(l,0,1,2,\cdots,s)$ is the benefit payment to the l^{th} insured based on the simulated historical and current health status up to time s, and δ is the continuously compounded interest rate. The benefit payment function $B(l,0,1,2,\cdots,s)$ takes into account the elimination period and the maximum benefit period. In particular, the value of $B(l,0,1,2,\cdots,s)$ is set to zero for those periods spent in functional disability before the duration surpasses the elimination period; the value of $B(l,0,1,2,\cdots,s)$ is set to zero if the total number of payments exceeds the maximum benefit period.

The lump sum premium of insurance policies sold to individuals in state i, denoted by $\tilde{P}_i^L$, is calculated as the sample mean of the present values of benefits across all the simulated homogeneous individuals:

$$\tilde{P}_i^L = \frac{1}{N}\sum_{l=1}^{N} PV_i(l,0), \tag{13}$$

where N is the number of simulations. In addition, the estimation standard error are calculated as the standard deviation divided by the square root of the number of simulations. The premium paid on a regular basis of insurance policies sold to individuals in state i, denoted by $\tilde{P}_i^{(f)}$, can be calculated in the following equation:

$$\tilde{P}_i^{(f)}\ \tilde{a}_{\text{HM}}^{(f)}(0) = \frac{1}{N}\sum_{l=1}^{N} PV_i(l,0), \tag{14}$$

where $\tilde{a}_{\text{HM}}^{(f)}(0)$ is the time-0 present value of unit payments at the beginning of each $\frac{1}{f}$ year while the insured is healthy or mildly disabled.

The premium of LTC insurance calculated using the simulation approach is only an approximation to the price calculated using the Thiele's differential equation approach. If the number of simulations is very large and the step size h of the Euler's approximation approach is small enough in deriving numerical solutions to the differential equations, the premiums of generic LTC insurance policies calculated using the two approaches should give very close results.

3.3 Best-Estimate Reserves and Solvency Capital Requirements

LTC insurance providers take risks that span a long period of time. Accurate estimations of premiums and reserves are therefore critical in the risk management for product providers. The best-estimate reserve is the expected present value of future liabilities while the solvency reserve ensures that the insurer survives losses resulting from extreme events that occur with low probabilities.

3.3.1 Best-Estimate Reserves Based on Thiele's Differential Equation

Best-estimate reserves for individuals in each alive health state are determined from Equation (6) using the Thiele's differential equation approach. Let $\chi(t) \in \Omega_\chi$ denote the health state that the individual stays at time t. The time t best-estimate reserve for a LTC insurance policy issued to an individual in state k, denoted by $V(t, T \mid \chi(0) = k)$, is the expected value of best-estimate reserves for individuals in different health states. The time t best-estimate reserve is determined as follows:

$$V\big(t, T \mid \chi(0) = k\big) = \sum_i \Pr\big(\chi(t) = i \mid \chi(0) = k\big)\ V_i(t, T), \tag{15}$$

where $i,\ k \in \Omega_\chi$, $\Pr(\chi(t) = i \mid \chi(0) = k)$ is the probability of staying in heath state i at time t given the insured is in state k at the outset, and $V_i(t, T)$ is the best-estimate reserve for an insured in state i at time t which is calculated from Equation (6).

3.3.2 Best-Estimate Reserves Based on the Simulation Approach

Let $\tilde{V}_i(t, T)$ denote best-estimate reserves for an insured in state i at time t calculated using the simulation approach. $\tilde{V}_i(t, T)$ can be estimated as follows:

$$\tilde{V}_i(t, T) = \frac{\sum_{l=1}^{N} PV_i(l, t)}{\sum_{l=1}^{N} \Delta_i(l, t)}, \tag{16}$$

where $i \in \Omega_\chi$, N is the number of individuals at the outset, $PV_i(l, t)$ is calculated using Equation (12), and $\Delta_i(l, t)$ is the indicator variable that equals 1 if the l^{th} insured is in state i at time t and equals 0 otherwise. Based on the simulation approach, the best-estimate reserve for a LTC insurance policy issued to an individual in state k at the outset, denoted by $\tilde{V}\big(t, T \mid \chi(0) = k\big)$, can be calculated as follows:

$$\tilde{V}\big(t, T \mid \chi(0) = k\big) = \frac{\sum_{i \in \Omega_\chi} \sum_{l=1}^{N} PV_i(l, t)}{N} = \sum_{i \in \Omega_\chi} \frac{\Delta_i(l, t)}{N} \tilde{V}_i(t, T), \tag{17}$$

where the simulation is on a cohort of N individuals who are in state k at the outset.

Distributional risk measures, such as the Value-at-Risk (VaR), are widely used in actuarial practice, in particular in the context of estimating solvency capital requirements and solvency reserves. Based on the simulation approach, these distributional risk measures of the reserve can be easily calculated. The VaR_α of liabilities for the insured in state i at time t is calculated as the $100\alpha\%$ quantile of $PV_i(l, t)$ across all simulated individuals at time t, which is expressed as follows:

$$\begin{aligned} & VaR_\alpha\Big(t, T \mid \chi(0) = k\Big) \\ = \;& \arg\min_x \left\{ \forall\, l \in \{1,\ 2, \cdots,\ N\},\ \Pr\left(\sum_{i \in \Omega_\chi} PV_i(l, t) > x \right) \leq 1 - \alpha \right\}. \end{aligned} \tag{18}$$

3.3.3 Solvency Capital Requirements

The Solvency Capital Requirement (SCR) is defined under Solvency II as the amount of capital required to cover losses that occur with a probability of 99.5% over one year (Kochanski, 2010; Meyricke and Sherris, 2014; Olivieri and Pitacco, 2009). SCRs are required to take into account a broad range of risks that insurers are faced with, including longevity risk, the risk of higher disability rates, the risk of lower recovery rates, interest rate risk, etc. Planchet and Tomas (2014) consider the impact of mortality risk of disabled lives on solvency capital requirements for LTC insurance. Pitacco (2015) uses a sensitivity analysis to assess the impact of mortality risk and disability risk on premiums of LTC insurance, which provides insights into the level of solvency capital for product providers.

Let NAV_t denote the net asset value at time t, which is calculated as the difference between the value of assets and the best estimate of liability at time t. The SCR at time t is then defined as the smallest amount of capital held at time t so that the probability of a positive NAV next year is no less than 0.995. This can be expressed as follows:

$$SCR_t = \arg\min_x \Big\{ \Pr\left(NAV_{t+1} > 0 \mid NAV_t = x\right) \geq 99.5\% \Big\}. \tag{19}$$

An equivalent expression frequently adopted in practice for the SCR is shown as follows (Börger, 2010; Meyricke and Sherris, 2014):

$$SCR_t = \arg\min_x \Big\{ \Pr\left(NAV_t - NAV_{t+1} e^{-\delta_{t+1}} > x\right) \leq 0.5\% \Big\}, \tag{20}$$

where δ_{t+1} is the continuously compounded interest rate per annum from time t to $t+1$.

An alternative approach to the above framework is to use the standard formula in Solvency II. The SCR in the standard formula is calculated as the negative change in NAV_t in the presence of a shock that represents a one-in-two-centuries crisis. The SCR calculated using the standard formula, denoted by SCR_t^S, can be expressed as follows:

$$SCR_t^{Shock} = NAV_t - NAV_t^{Shock}, \tag{21}$$

where NAV_t^{Shock} is the net asset value at time t if the one-off permanent shock occurs.

The paper focuses on the analysis of two major risks that LTC insurance providers are faced with: longevity risk and disability risk. Under Solvency II, longevity risk is assessed as a permanent 20% decrease in mortality rates at all ages; disability risk is assessed as an increase of 35% in disability rates at all ages for the next year, a permanent increase of 25% at all ages for the following years and a permanent decrease of 20% in recovery rates at all ages (European Insurance and Occupational Pension Authority, 2011). The SCR for the aggregate risk is to calculate the SCR for each risk and to aggregate these SCRs via a correlation matrix (Kochanski, 2010). Based on the assumption of zero correlation between longevity risk and disability risk, the SRC for longevity risk and disability risk is calculated as follows:

$$SRC_t^S = \sqrt{\left(SRC_t^{Longevity}\right)^2 + \left(SRC_t^{Disability}\right)^2}, \tag{22}$$

where $SRC_t^{Longevity}$ and $SRC_t^{Disability}$ are the SRCs for longevity risk and disability risk respectively which are calculated using Equation (21).

In the standard formula, insurers are also required to hold a risk margin in addition to the best-estimate reserve, in order to cover residual risks associated with those captured in SCRs (European Insurance and Occupational Pension Authority, 2011; Olivieri and Pitacco, 2009). The risk margin also represents the fair value amount that another insurer would require to take over the liabilities (Meyricke and Sherris, 2014). The risk margin, denoted by RM_t, is linked to current and future SCRs and is determined as follows:

$$RM_t = \sum_{k=0}^{m} c \, \frac{SCR_{t+k}^S}{(1+r_f)^k}, \tag{23}$$

where m is the time to exhaustion of the portfolio of LTC insurance policies, c is the cost of capital and r_f is the risk-free interest rate. The total capital requirement at time t in the Solvency II standard formula, denoted by TCR_t^S, is the sum of the solvency capital requirement and the risk margin at time t, i.e.:

$$TCR_t^S = SCR_t^S + RM_t. \tag{24}$$

4 Health Dynamics

4.1 Data Description

In order to provide realistic estimates we use health transitions data from the University of Michigan Health and Retirement Study (HRS), which is a U.S. nationally representative ongoing survey of people aged 50 and above. Starting from 1992, the survey has been conducted biennially to collect information on physical and mental health functioning, health insurance, health expenditures, retirement plans, and assets. Since there was an inconsistent structure of questions asked before wave 1998, we use data from wave 1998 onward to the latest available wave in 2010.

The data has detailed information on self-reported difficulties in six Activities of Daily Livings (ADLs) and an assessment of mental functioning. The six ADLs are dressing, walking, bathing, eating, transferring, and toileting. There is also information on whether the respondent moves into a nursing home, but information on the respondent thereafter is no longer tracked. Based on the number of ADLs that the individuals cannot perform independently, we categorise alive health states into healthy, mildly disabled, severely disabled and dead (see Section 2 for the detailed definition of each state). Exposure years and the number of transitions are summarized in Table 1.

Table 1. Summary of exposure years and the number of transitions. For reporting purposes, we report these summaries for 5-year age groups. Data for lives aged 101 and above is truncated.

Age Band	Exposure years			Number of transitions								
	H	**M**	**S**	**H-M**	**H-S**	**H-D**	**M-H**	**M-S**	**M-D**	**S-H**	**S-M**	**S-D**
Males												
50-54	1,946.6	165.6	85.7	32	4	8	15	7	3	4	7	3
55-59	6,602.6	589.3	209.8	143	27	60	111	22	10	13	22	18
60-64	12,139.0	1,147.7	408.2	267	42	159	234	69	54	30	59	31
65-69	13,963.7	1,361.0	530.1	347	62	270	301	75	80	27	60	69
70-74	11,766.0	1,444.9	628.1	410	80	349	250	87	105	28	44	87
75-79	8,881.3	1,431.0	717.0	397	87	394	224	97	103	17	37	117
80-84	5,557.1	1,195.4	693.6	337	89	356	182	96	150	11	35	166
85-89	2,564.4	787.2	624.4	213	92	257	103	84	132	11	30	156
90-94	786.2	328.2	310.8	85	56	131	36	39	96	8	8	125
95-100	129.6	92.5	89.5	21	21	39	9	7	30	0	3	47
Total	**64,336.3**	**8,542.7**	**4,297.5**	**2,252**	**560**	**2,023**	**1,465**	**583**	**763**	**149**	**305**	**819**
Females												
50-54	4,539.7	381.6	171.1	67	21	8	52	13	2	10	13	4
55-59	10,855.1	1,202.5	494.3	280	40	55	212	69	27	37	63	16
60-64	15,767.3	1,932.4	887.5	458	74	114	436	129	37	42	112	36
65-69	16,652.7	2,293.9	971.2	553	112	193	474	147	86	41	145	79
70-74	14,029.9	2,170.3	1,096.9	575	107	226	441	178	97	53	95	86
75-79	10,853.4	2,267.5	1,216.2	579	144	257	349	157	116	41	95	171
80-84	7,546.3	2,227.3	1,377.6	570	162	315	338	190	166	37	94	242
85-89	3,905.6	1,907.7	1,558.5	445	172	302	235	211	212	36	82	312
90-94	1,250.9	1,016.1	1,060.3	218	92	160	86	156	172	18	50	296
95-100	240.7	273.8	429.4	52	24	51	18	76	75	3	13	174
Total	**85,641.6**	**15,673.2**	**9,262.8**	**3,797**	**948**	**1,681**	**2,641**	**1,326**	**990**	**318**	**762**	**1,416**

4.2 Graduated Transition Rates

Table 2 shows goodness of fit comparisons for the nested models for the relationship between transition rates and age based on the three selection criteria: AICc, BIC and the difference between model residual deviances. It is found that a quadratic specification is optimal for two and seven health transition rates for males and females respectively. This specification results in lower values of AICc and BIC criteria than other predictor structures. Likelihood-ratio tests of the difference in residual deviance also confirm that the inclusion of the quadratic age term (i.e. moving from $k = 1$ to $k = 2$) is beneficial and improves the fit ($p < 0.01$) for these transition rates, despite the additional parameter involved. In contrast, the subsequent inclusion of age-cubed is found to be over-parameterised and not statistically significant. The model selection criteria supports the use of a linear function of age for the remaining health transition rates.

Transition rates are then graduated based on the optimal specification for each transition. The fitted parameters for each of the transition rates are shown in Table 3.

4.3 Demographic Characteristics of the Simulated Individuals

In order to provide a large enough sample we simulate health trajectories of 40,000 males and 40,000 females starting at various ages and from the healthy, mildly disabled and severely disabled states based on the monthly transition probabilities. Monthly transition probabilities are calculated as the matrix exponential of the graduated annual transition rates divided by 12. For illustrative purposes, simulations for the 65-year old male and female cohorts starting from the healthy state are as follows.

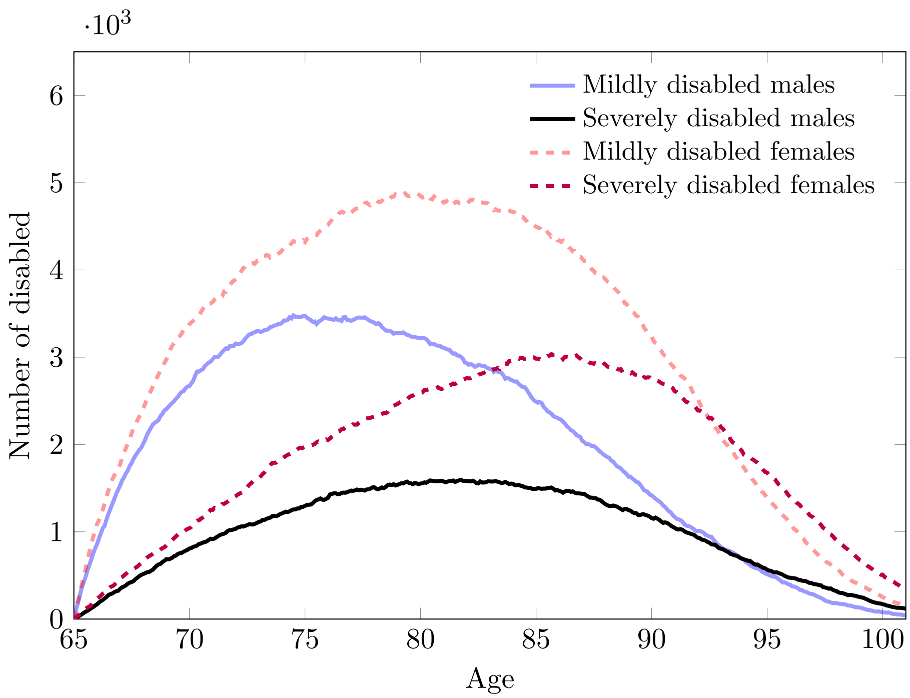

Figure 2. Number of disabled among the simulated cohorts of 65-year-old healthy males and females.

Figure 2 shows the number of mildly disabled and severely disabled individuals among the

Table 2. Poisson GLM: goodness-of-fit of nested models. AICc is the Akaike Information Criterion corrected for sample size, BIC is the Bayesian information criterion, and D_c is the residual deviance statistic. $k = 1$ implies age term only; $k = 2$ implies age and age-squared terms; and $k = 3$ implies age, age-squared, and age-cubed terms. The optimal criteria value is bolded for each set of nested models. * is for statistic that is significant at the 10% level, ** at the 5% level, and *** at the 1% level.

k	Males				Females			
	AICc	BIC	D_c	ΔD_c	AICc	BIC	D_c	ΔD_c
Disability rates								
μ_{HM}: *healthy to mildly disabled*								
1	256.77	**260.38**	**33.91**		334.42	338.03	87.15	
2	**256.37**	261.66	31.26	2.65	305.15	**310.44**	**55.63**	**31.52***
3	258.66	265.52	31.19	0.07	**304.20**	311.05	52.31	3.32*
μ_{HS}: *healthy to severely disabled*								
1	237.77	241.38	62.62		260.51	264.13	64.70	
2	**217.44**	**222.73**	**40.04**	**22.59***	248.16	**253.45**	**50.09**	**14.61***
3	219.55	226.40	39.78	0.25	**246.87**	253.73	46.44	3.65*
μ_{MS}: *mildly disabled to severely disabled*								
1	**214.46**	**218.07**	**43.54**		314.73	318.34	99.05	
2	215.97	221.25	42.79	0.75	278.90	**284.19**	**60.96**	**38.09***
3	215.29	222.15	39.75	3.04*	**278.50**	285.36	58.21	2.76
Recovery rates								
μ_{MH}: *mildly disabled to healthy*								
1	**244.31**	**247.92**	**44.62**		300.61	304.23	72.82	
2	245.44	250.72	43.49	1.13	**290.77**	**296.06**	**60.72**	**12.10***
3	247.71	254.57	43.41	0.08	293.05	299.91	60.64	0.08
μ_{SH}: *severely disabled to healthy*								
1	**147.94**	**151.55**	**42.01**		182.83	186.45	41.31	
2	149.37	154.65	41.18	0.83	**179.44**	**184.73**	**35.66**	**5.65**
3	151.59	158.45	41.04	0.14	181.80	188.66	35.66	0.00
μ_{SM}: *severely disabled to mildly disabled*								
1	**184.84**	**188.45**	**45.95**		**239.36**	**242.97**	**58.64**	
2	185.53	190.82	44.38	1.56	240.57	245.85	57.60	1.05
3	187.88	194.74	44.37	0.01	242.76	249.62	57.43	0.17
Mortality rates								
μ_{HD}: *healthy to dead*								
1	248.70	**252.31**	**21.86**		271.48	275.09	50.02	
2	**247.81**	253.09	18.70	3.15*	**264.39**	**269.67**	**40.67**	**9.35***
3	248.28	255.14	16.82	1.89	266.24	273.10	40.17	0.51
μ_{MD}: *mildly disabled to dead*								
1	228.91	232.53	36.62		245.42	249.04	43.72	
2	**223.39**	**228.68**	**28.84**	**7.78**	**242.61**	**247.90**	**38.65**	**5.07**
3	225.11	231.96	28.19	0.65	242.68	249.54	36.36	2.29
μ_{SD}: *severely disabled to dead*								
1	**239.71**	**243.32**	**47.29**		**245.43**	**249.04**	**30.06**	
2	241.70	246.98	47.02	0.27	247.67	252.95	30.04	0.02
3	243.49	250.35	46.46	0.56	247.28	254.13	27.29	2.75

Table 3. Parameter estimates of the Poisson GLM with log link.

Intensity	**Males**			**Females**		
	β_0	β_1	β_2 ($\times 10^{-2}$)	β_0	β_1	β_2 ($\times 10^{-2}$)
Disability Rates						
μ_{HM}	-7.12***	0.05***	-	-2.73***	-0.06***	0.08***
μ_{HS}	1.32	-0.25***	0.23***	-1.87	-0.16***	0.16***
μ_{MS}	-4.59***	0.03***	-	1.22	-0.13***	0.11***
Recovery Rates						
μ_{MH}	-0.75***	-0.01***	-	-7.64***	0.18***	-0.14***
μ_{SH}	0.06	-0.05***	-	-4.82**	0.08	-0.08*
μ_{SM}	0.18	-0.04***	-	-0.05	-0.03***	-
Mortality Rates						
μ_{HD}	-9.71***	0.09***	-	-7.67***	0.01	0.06***
μ_{MD}	-4.36**	-0.01	0.05	-4.20***	-0.03	0.06**
μ_{SD}	-5.47***	0.05***	-	-6.62***	0.06***	-

* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$

simulated independent and homogeneous 65-year-old healthy males and females respectively. It can be seen from Figure 2 that there are fewer mildly disabled and severely disabled individuals for males than females at all ages in the simulated cohorts. In addition, the ages at which the numbers of mildly disabled and severely disabled individuals respectively reach the peak are smaller for males than females. For both males and females, the number of mildly disabled individuals is larger than that of the severely disabled at all ages except after around age 93 from which severely disabled individuals start to dominate. This is also confirmed in Table 4 that shows the proportion of survivors in each health state on a 5-year basis for the simulated 65-year-old healthy individuals.

Based on the simulated health trajectories for the 80,000 individuals (40,000 males and 40,000 females) who are healthy at age 65, relevant demographic characteristics can be computed (Brown and Warshawsky, 2013). The results are shown in Table 5. These demographic characteristics include the life expectancy, time spent in each level of disability, share of people ever becoming disabled, and the average age of first disability conditional on becoming disabled. Life expectancy is the expected survival time of the cohort plus 65, expected time

Table 4. Proportion of survivors in each health state for the simulated 65-year-old healthy male and female cohorts.

Age	Survivors	Healthy	Mildly Disabled	Severely Disabled
			Males	
65	40,000	100.00%	0.00%	0.00%
70	35,834	90.30%	7.45%	2.25%
75	29,735	83.96%	11.68%	4.36%
80	22,129	78.44%	14.54%	7.02%
85	13,912	71.37%	17.91%	10.72%
90	6,612	61.04%	21.46%	17.50%
95	2,064	47.77%	24.90%	27.33%
100	340	27.35%	22.94%	49.71%
			Females	
65	40,000	100.00%	0.00%	0.00%
70	37,597	88.25%	8.98%	2.77%
75	33,587	81.26%	12.86%	5.88%
80	27,735	73.17%	17.48%	9.35%
85	20,001	62.62%	22.47%	14.91%
90	11,375	47.41%	28.44%	24.15%
95	4,273	28.32%	32.44%	39.25%
100	855	11.46%	29.12%	59.42%

in each health state is calculated as the mean of total time spent in each state across all simulated individuals. Share with disability is the proportion of people who are ever disabled. Share with severe disability is the proportion of people who are ever severely disabled. It can be seen from Table 5 that 65-year-old healthy females are expected to live three years longer than their male counterparts. The expected time of the remaining life spent in the severely disabled state for a healthy female aged 65 is nearly twice that for a 65-year-old male. More than half of males and nearly three quarters of females in the simulated 65-year-old healthy cohorts are expected to experience mild or severe disability. Among those males who are ever disabled, nearly half of them are expected to experience severe disability. For females, nearly 60% of those who are ever disabled are expected to ever become severely disabled. Given that a 65-year-old healthy individual ever becomes disabled in the remaining life, disability after age 65 is expected to first strike at age 76 for both males and females. Conditional on ever becoming severely disabled, the expected age of first severe disability after age 65 for females is about 1 year older than that for males.

Table 5. Demographic characteristics of the simulated 65-year-old healthy individuals (40,000 males and 40,000 females).

Demographic Characteristics	**Males**	**Females**
Mean years of life after age 65	16.33	19.43
Mean years with mild disability	1.78	2.80
Mean years with severe disability	0.89	1.68
Share with disability	56.43%	72.70%
Share with mild disability	47.89%	63.37%
Share with severe disability	26.82%	42.39%
Average age of first disability, conditional on becoming disabled	76.23	76.52
Average age of first mild disability, conditional on becoming mildly disabled	75.83	76.38
Average age of first severe disability, conditional on becoming severely disabled	80.51	81.70

5 LTC Insurance Premiums

This section presents the estimated premiums based on the approaches covered in Section 3. The results for the base case analysis for generic LTC insurance policies, i.e. products without the elimination period or the maximum benefit period are presented. These products include stand-alone policies sold to the healthy, the mildly disabled and the severely disabled, rider benefit policies, and life care annuities. Different combinations of the elimination period and the maximum benefit period are taken into account. Thiele's differential equation and the simulation approach are also compared.

5.1 Base Case Results: Generic Policies

The premiums of generic stand-alone long-term insurance policies sold to the healthy, the mildly disabled and severely disabled are calculated using the Thiele's differential equation approach as described in Section 3. The generic stand-alone policy pays \$100 per day when the insured is severely disabled and no elimination period is included in the generic policy. The benefit is unlimited as long as the insured stays in the severely disabled state since there is no maximum benefit period in the generic policy. The generic policies are assumed sold to healthy individuals aged 55, 60, 65, 70, 75, and 80 respectively in exchange for lump sum premiums or premiums paid on a regular basis. The continuously compounded interest rate is assumed to be a constant 4% per annum.

Lump sum, continuous, annual and monthly premiums of the above generic stand-alone policy sold to individuals in different initial health states are shown in Table 6. It can be seen from the table that LTC insurance premiums for females are considerably higher than those for their male counterparts. This is due to the dual effects of females' higher disability rates and lower mortality rates compared to those of males. Higher disability rates increase the probability of claiming LTC insurance benefits and lower mortality rates decrease the probability of ceasing benefit payments due to deaths.

The premium generally increases as the age at policy issue goes up, except for very old females who pay lump sum premiums. The impact of policy purchase age on the premium amount results from the combined effects of disability, recovery and mortality. On the one hand, older individuals have higher probabilities of getting disabled and lower probabilities of recovery that increase the amount of premiums. On the other hand, older individuals also have higher death probabilities that result in higher probabilities of ceasing benefit payments and therefore lower the amount of premiums. For females ageing from 75 to 80, the effects of the increasing mortality rate slightly outweigh the joint effects of the higher disability rate and the lower recovery rate. Subsequently the lump sum premiums charged to 80-year-old females are slightly higher than 75-year-old females, as shown in Table 6. Since the probability of staying in or coming back to the healthy state decreases as the age goes up, the present value of unit payments while the insured is in the healthy state, such as υ_{H} in Equation (10) and υ'_{H} in Equation (11), is lower for older ages. Consequently the continuous, annual and monthly premiums charged to 80-year-old females are higher than the corresponding premiums charged to 75-year-old females.

Premiums of generic stand-alone policies sold to mildly disabled and severely disabled individuals are shown in the second and third panels of Table 6. The generic stand-alone LTC insurance sold to disabled individuals also pays $100 per day while the insured is severely disabled. Benefit payment amount, eligibility for receiving benefits, interest rate and other parameters are the same as in the analysis of stand-alone policies sold to the healthy. The only difference is the starting health state in which the insured stays. It can be seen from the table that policies sold to the mildly disabled and the severely disabled are considerably more expensive than those sold to the healthy, since individuals already in the disabled state have

Table 6. Premiums ($) of generic stand-alone LTC insurance policies sold to individuals in different health states and at different ages. The generic stand-alone LTC insurance pays $100 per day while the insured is severely disabled.

Age	**Males**				**Females**			
	Lump sum	Continuous	Annual	Monthly	Lump sum	Continuous	Annual	Monthly
Stand-alone policies sold to the healthy								
55	15,923	1,138	1,126	95	27,526	1,825	1,806	152
60	16,766	1,350	1,333	112	28,913	2,127	2,101	177
65	17,448	1,619	1,596	135	30,313	2,535	2,501	211
70	17,915	1,964	1,933	163	31,469	3,084	3,036	257
75	18,193	2,428	2,383	202	32,099	3,824	3,753	318
80	18,403	3,094	3,025	257	31,924	4,828	4,719	402
Stand-alone policies sold to the mildly disabled								
55	28,694	2,326	2,295	194	48,865	3,647	3,607	304
60	31,230	2,935	2,892	244	47,727	3,977	3,926	331
65	32,622	3,639	3,581	303	47,391	4,550	4,482	379
70	32,590	4,417	4,340	368	47,163	5,412	5,318	450
75	31,096	5,242	5,139	436	46,333	6,615	6,483	550
80	28,328	6,075	5,942	505	44,260	8,188	8,001	681
Stand-alone policies sold to the severely disabled								
55	130,655	-	-	-	157,337	-	-	-
60	136,521	-	-	-	159,954	-	-	-
65	136,771	-	-	-	159,412	-	-	-
70	131,552	-	-	-	154,487	-	-	-
75	121,918	-	-	-	144,742	-	-	-
80	109,382	-	-	-	130,743	-	-	-

higher probabilities of staying longer in the severely disabled state than healthy individuals.

Table 7. Premiums ($) of generic rider benefit policies and life care annuities. The generic rider benefit policy pays $100 per day while the insured is severely disabled and pays a death benefit of $500,000 when the insured dies. The generic life care annuity pays $50 per day while the insured is alive and additional $50 per day while the insured is severely disabled.

Age	**Males**				**Females**			
	Lump sum	Continuous	Annual	Monthly	Lump sum	Continuous	Annual	Monthly
				Rider benefit policies sold to the healthy				
55	226,927	16,219	16,042	1,350	209,708	13,906	13,759	1,158
60	258,649	20,826	20,570	1,734	239,785	17,637	17,426	1,468
65	291,614	27,053	26,675	2,252	272,847	22,820	22,509	1,900
70	324,797	35,615	35,044	2,964	307,940	30,183	29,708	2,512
75	357,067	47,658	46,767	3,965	343,570	40,930	40,171	3,406
80	387,212	65,096	63,649	5,415	377,597	57,100	55,821	4,750
				Life care annuities sold to the healthy				
55	267,773	-	-	-	298,983	-	-	-
60	240,319	-	-	-	273,634	-	-	-
65	211,479	-	-	-	245,530	-	-	-
70	182,067	-	-	-	215,110	-	-	-
75	153,053	-	-	-	183,191	-	-	-
80	125,472	-	-	-	150,957	-	-	-
				Life care annuities sold to the mildly disabled				
55	250,787	-	-	-	290,061	-	-	-
60	222,786	-	-	-	263,741	-	-	-
65	194,002	-	-	-	234,859	-	-	-
70	165,388	-	-	-	204,025	-	-	-
75	137,878	-	-	-	172,404	-	-	-
80	112,263	-	-	-	141,551	-	-	-
				Life care annuities sold to the severely disabled				
55	270,261	-	-	-	323,363	-	-	-
60	239,606	-	-	-	292,932	-	-	-
65	208,682	-	-	-	259,514	-	-	-
70	178,927	-	-	-	224,654	-	-	-
75	151,417	-	-	-	190,228	-	-	-
80	126,759	-	-	-	157,930	-	-	-

Premiums of rider benefit policies and life care annuities calculated using the Thiele's differential equation approach are shown in Table 7. The rider benefit policy has a fixed death benefit of $500,000 and pays the LTC benefit of $100 per day while the insured is severely disabled. Since the death benefit is a large component in the rider benefit policy and males have higher mortality rates than their female counterparts, the calculated premium of the rider benefit policy charged to the male insured is larger than the premium charged to their female counterparts[3]. Results show that rider benefit products become very expensive for old individuals. When the insured is disabled, expected values of LTC and death benefits both increase. Therefore, rider benefit policies sold to disabled individuals are expected to be very expensive and are not feasible in the market.

The life care annuity pays $50 per day while the insured is healthy or mildly disabled and the benefit is upgraded to $100 per day if the insured is in the severely disabled state. It can be seen from Table 7 that life care annuities are more affordable as the insured becomes older. It is also interesting to note that premiums for life care annuities sold to individuals who are mildly disabled are lower than those charged to the healthy and the severely disabled. The results provide evidence for the insurability of LTC costs for old and impaired individuals and also provide insights into the design of more affordable LTC insurance policies.

Inflation protection is a typical feature included in most LTC insurance policies. LTC insurance policies with inflation protection provide the insured with benefits that increase with inflation. We assume a continuously compounded inflation rate of 3% per annum. Lump sum premiums for generic LTC insurance policies with inflation protection are shown in Table 8. It can be seen that including inflation protection leads to a large increase in the premium of all types of generic insurance policies. This increase in the premium reduces as the purchasing age is older or the insured is in a worse health state. In addition, including inflation protection makes the insurance policies more expensive for females than males of the same age and health condition.

We also show lump sum premiums for generic LTC insurance policies where the insured become eligible for LTC benefits when they have difficulties with 2 or more ADLs. The results are also compared with prior results where the definition for receiving LTC benefits is to have difficulties with 3 or more ADLs. The results are shown in Table 9. It can be seen

[3]The discrepancy between premiums of rider benefit policies that are charged to males and females becomes larger when the death benefit is increased due to higher mortality rates of males.

Table 8. Lump sum premiums for generic LTC insurance policies with inflation protection.

Age	**Males**			**Females**		
	Without Inflation	With Inflation	Increase	Without Inflation	With Inflation	Increase
			Stand-alone policies sold to the healthy			
55	15,923	29,708	86.57%	27,526	54,264	97.14%
60	16,766	28,945	72.64%	28,913	52,697	82.26%
65	17,448	27,930	60.08%	30,313	50,946	68.07%
70	17,915	26,681	48.94%	31,469	48,790	55.04%
75	18,193	25,308	39.11%	32,099	46,042	43.44%
80	18,403	24,008	30.46%	31,924	42,560	33.32%
			Stand-alone policies sold to the mildly disabled			
55	28,694	44,193	54.02%	48,865	78,748	61.16%
60	31,230	45,339	45.18%	47,727	73,980	55.01%
65	32,622	44,992	37.92%	47,391	69,990	47.69%
70	32,590	42,958	31.82%	47,163	66,041	40.03%
75	31,096	39,360	26.57%	46,333	61,471	32.67%
80	28,328	34,556	21.98%	44,260	55,767	26.00%
			Stand-alone policies sold to the severely disabled			
55	130,655	154,702	18.40%	157,337	197,103	25.27%
60	136,521	159,375	16.74%	159,954	196,134	22.62%
65	136,771	157,321	15.03%	159,412	191,237	19.96%
70	131,552	149,008	13.27%	154,487	181,256	17.33%
75	121,918	135,968	11.52%	144,742	166,109	14.76%
80	109,382	120,155	9.85%	130,743	146,862	12.33%
			Rider benefit policies sold to the healthy			
55	226,927	240,711	6.07%	209,708	236,446	12.75%
60	258,649	270,828	4.71%	239,785	263,569	9.92%
65	291,614	302,097	3.59%	272,847	293,480	7.56%
70	324,797	333,564	2.70%	307,940	325,262	5.63%
75	357,067	364,183	1.99%	343,570	357,513	4.06%
80	387,212	392,818	1.45%	377,597	388,233	2.82%
			Life care annuities sold to the healthy			
55	267,773	386,968	44.51%	298,983	450,833	50.79%
60	240,319	332,297	38.27%	273,634	394,043	44.00%
65	211,479	280,065	32.43%	245,530	337,567	37.48%
70	182,067	231,296	27.04%	215,110	282,469	31.31%
75	153,053	186,933	22.14%	183,191	230,037	25.57%
80	125,472	147,737	17.74%	150,957	181,654	20.33%
			Life care annuities sold to the mildly disabled			
55	250,787	357,529	42.56%	290,061	430,790	48.52%
60	222,786	303,319	36.15%	263,741	374,509	42.00%
65	194,002	252,675	30.24%	234,859	318,576	35.65%
70	165,388	206,604	24.92%	204,025	264,409	29.60%
75	137,878	165,758	20.22%	172,404	213,778	24.00%
80	112,263	130,384	16.14%	141,551	168,436	18.99%
			Life care annuities sold to the severely disabled			
55	270,261	363,643	34.55%	323,363	455,549	40.88%
60	239,606	307,057	28.15%	292,932	393,576	34.36%
65	208,682	255,889	22.62%	259,514	332,848	28.26%
70	178,927	211,169	18.02%	224,654	275,812	22.77%
75	151,417	173,076	14.30%	190,228	224,554	18.04%
80	126,759	141,166	11.37%	157,930	180,252	14.13%

that relaxing the LTC disability to 2+ ADLs makes generic stand-alone LTC insurance much more expensive but the impact on rider benefit policies and life care annuities is minimal. The increase in premium of stand-alone policies reduces for older ages and where the insured is in a worse health state. This is slightly larger for males than females in the healthy state and the mildly disabled state and is larger for females than their male counterparts in the severely disabled state.

5.2 Policies with Typical Product Features

Based on the simulated health trajectories of the different individuals, premiums and reserves for policies with typical product features are calculated using the simulation approach described in Section 3.2. Different combinations of the elimination period and the maximum benefit period are allowed for in calculating premiums based on the simulation approach. For illustrative purposes, lump sum premiums of stand-alone policies, rider benefit policies and life care annuities issued to 65-year-old healthy individuals are shown in Table 10 and Table 11. When the elimination period is zero and the maximum benefit period is unlimited, the products are generic policies, i.e. the first two rows in the first panels of Tables 10 and 11.

It can be seen from Table 10 that the elimination period and the maximum benefit period are effective tools in making the premium more affordable. For example, premiums of generic stand-alone policies sold to 65-year-old females would be 46% cheaper if a 90-day elimination period and a maximum benefit period of 3 years are included. The elimination period and the maximum benefit period are not effective in making rider benefit policies and life care annuities more affordable as shown in Table 11, because LTC benefits account for only a small proportion in rider benefit policies and life care annuities.

We also investigate the premiums for shared LTC insurance issued to couples. The shared LTC insurance allows each spouse to access his or her partner's unused LTC funds. As a result, a shared LTC insurance policy is cheaper than two stand-alone policies purchased

Table 9. Lump sum premiums for generic LTC insurance policies with different LTC disability definitions: 2+ ADLs v.s. 3+ ADLs.

Age	**Males**			**Females**		
	3+ ADLs	2+ ADLs	Difference from 3+ ADLs	3+ ADLs	2+ ADLs	Difference from 3+ ADLs
			Stand-alone policies sold to the healthy			
55	15,923	26,657	67.41%	27,526	45,000	63.48%
60	16,766	27,487	63.95%	28,913	46,157	59.64%
65	17,448	28,059	60.82%	30,313	47,473	56.61%
70	17,915	28,296	57.95%	31,469	48,459	53.99%
75	18,193	28,202	55.02%	32,099	48,615	51.45%
80	18,403	27,852	51.35%	31,924	47,493	48.77%
			Stand-alone policies sold to the mildly disabled			
55	28,694	45,261	57.74%	48,865	72,121	47.59%
60	31,230	47,794	53.04%	47,727	69,362	45.33%
65	32,622	48,756	49.46%	47,391	68,184	43.88%
70	32,590	47,836	46.78%	47,163	67,330	42.76%
75	31,096	45,006	44.73%	46,333	65,596	41.57%
80	28,328	40,503	42.98%	44,260	61,956	39.98%
			Stand-alone policies sold to the severely disabled			
55	130,655	146,985	12.50%	157,337	187,343	19.07%
60	136,521	150,330	10.11%	159,954	181,076	13.21%
65	136,771	148,445	8.54%	159,412	175,380	10.02%
70	131,552	141,413	7.50%	154,487	168,242	8.90%
75	121,918	130,106	6.72%	144,742	157,759	8.99%
80	109,382	115,896	5.96%	130,743	142,987	9.36%
			Rider benefit policies sold to the healthy			
55	226,927	237,845	4.81%	209,708	227,297	8.39%
60	258,649	269,470	4.18%	239,785	257,161	7.25%
65	291,614	302,290	3.66%	272,847	290,170	6.35%
70	324,797	335,271	3.22%	307,940	325,128	5.58%
75	357,067	367,248	2.85%	343,570	360,311	4.87%
80	387,212	396,926	2.51%	377,597	393,395	4.18%
			Life care annuities sold to the healthy			
55	267,773	272,975	1.94%	298,983	307,601	2.88%
60	240,319	245,590	2.19%	273,634	282,118	3.10%
65	211,479	216,727	2.48%	245,530	253,939	3.42%
70	182,067	187,176	2.81%	215,110	223,396	3.85%
75	153,053	157,906	3.17%	183,191	191,206	4.37%
80	125,472	129,966	3.58%	150,957	158,477	4.98%
			Life care annuities sold to the mildly disabled			
55	250,787	261,300	4.19%	290,061	303,750	4.72%
60	222,786	233,782	4.94%	263,741	277,020	5.03%
65	194,002	205,040	5.69%	234,859	247,994	5.59%
70	165,388	176,002	6.42%	204,025	216,992	6.36%
75	137,878	147,647	7.09%	172,404	184,884	7.24%
80	112,263	120,854	7.65%	141,551	152,969	8.07%
			Life care annuities sold to the severely disabled			
55	270,261	291,262	7.77%	323,363	346,964	7.30%
60	239,606	260,166	8.58%	292,932	314,357	7.31%
65	208,682	227,793	9.16%	259,514	279,606	7.74%
70	178,927	195,657	9.35%	224,654	243,408	8.35%
75	151,417	165,134	9.06%	190,228	207,043	8.84%
80	126,759	137,230	8.26%	157,930	172,111	8.98%

Table 10. Premiums ($) of stand-alone policies with different combinations of the elimination period and the maximum benefit period issued to 65-year-old healthy individuals. The stand-alone LTC insurance pays $100 per day while the insured is severely disabled. Standard errors are shown in brackets under the corresponding premium estimates.

Elimination	**Males**			**Females**		
Period	*Lump sum*	*Annual*	*Monthly*	*Lump sum*	*Annual*	*Monthly*
Unlimited benefit period						
0-day	17,018	1,510	131	29,843	2,392	207
	(219)	*(31)*	*(5)*	*(287)*	*(35)*	*(6)*
30-day	16,561	1,470	128	29,155	2,337	202
	(216)	*(30)*	*(5)*	*(284)*	*(35)*	*(6)*
60-day	16,116	1,430	124	28,479	2,283	198
	(213)	*(30)*	*(5)*	*(281)*	*(34)*	*(6)*
90-day	15,680	1,391	121	27,817	2,230	193
	(210)	*(29)*	*(5)*	*(278)*	*(34)*	*(6)*
5-year maximum benefit period						
0-day	13,837	1,228	107	22,907	1,836	159
	(154)	*(22)*	*(4)*	*(184)*	*(24)*	*(4)*
30-day	13,473	1,196	104	22,391	1,795	155
	(153)	*(22)*	*(4)*	*(183)*	*(24)*	*(4)*
60-day	13,117	1,164	101	21,884	1,754	152
	(151)	*(22)*	*(4)*	*(181)*	*(23)*	*(4)*
90-day	12,770	1,133	99	21,386	1,714	148
	(149)	*(21)*	*(4)*	*(179)*	*(23)*	*(4)*
4-year maximum benefit period						
0-day	12,512	1,110	97	20,470	1,641	142
	(135)	*(20)*	*(4)*	*(159)*	*(21)*	*(4)*
30-day	12,183	1,081	94	20,013	1,604	139
	(133)	*(19)*	*(4)*	*(157)*	*(21)*	*(4)*
60-day	11,861	1,053	92	19,564	1,568	136
	(132)	*(19)*	*(4)*	*(156)*	*(20)*	*(4)*
90-day	11,548	1,025	89	19,122	1,533	133
	(130)	*(19)*	*(3)*	*(155)*	*(20)*	*(4)*
3-year maximum benefit period						
0-day	10,700	950	83	17,237	1,382	120
	(111)	*(16)*	*(3)*	*(128)*	*(17)*	*(3)*
30-day	10,418	924	80	16,854	1,351	117
	(109)	*(16)*	*(3)*	*(127)*	*(17)*	*(3)*
60-day	10,142	900	78	16,476	1,321	114
	(108)	*(16)*	*(3)*	*(126)*	*(17)*	*(3)*
90-day	9,873	876	76	16,106	1,291	112
	(107)	*(16)*	*(3)*	*(125)*	*(16)*	*(3)*

Table 11. Lump sum premiums ($) of rider benefit policies and life care annuities with different combinations of the elimination period and the maximum benefit period issued to 65-year-old healthy individuals. The rider benefit policy pays $100 per day while the insured is severely disabled and pays a death benefit of $500,000 when the insured dies. The life care annuity pays $50 per day while the insured is alive and additional $50 per day while the insured is severely disabled. Standard errors are shown in brackets under the corresponding premium estimates.

Elimination	**Rider Benefit Policies**		**Life Care Annuities**	
Period	*Males*	*Females*	*Males*	*Females*
	Unlimited benefit period			
0-day	291,875 *(469)*	273,536 *(465)*	211,475 *(441)*	245,982 *(440)*
30-day	291,419 *(468)*	272,848 *(463)*	211,247 *(441)*	245,638 *(439)*
60-day	290,973 *(468)*	272,173 *(462)*	211,024 *(440)*	245,301 *(438)*
90-day	290,538 *(467)*	271,510 *(461)*	210,806 *(439)*	244,969 *(438)*
	5-year maximum benefit period			
0-day	288,695 *(454)*	266,600 *(431)*	209,885 *(429)*	242,514 *(416)*
30-day	288,331 *(453)*	266,084 *(431)*	209,703 *(429)*	242,256 *(416)*
60-day	287,975 *(453)*	265,577 *(430)*	209,525 *(429)*	242,003 *(416)*
90-day	287,627 *(453)*	265,079 *(430)*	209,351 *(428)*	241,754 *(415)*
	4-year maximum benefit period			
0-day	287,370 *(451)*	264,163 *(426)*	209,222 *(426)*	241,296 *(411)*
30-day	287,041 *(451)*	263,706 *(426)*	209,058 *(426)*	241,067 *(411)*
60-day	286,719 *(451)*	263,257 *(426)*	208,897 *(426)*	240,843 *(411)*
90-day	286,406 *(450)*	262,815 *(426)*	208,740 *(425)*	240,622 *(410)*
	3-year maximum benefit period			
0-day	285,558 *(448)*	260,930 *(423)*	208,316 *(422)*	239,679 *(405)*
30-day	285,276 *(448)*	260,547 *(423)*	208,175 *(422)*	239,488 *(405)*
60-day	284,999 *(448)*	260,169 *(423)*	208,037 *(422)*	239,299 *(405)*
90-day	284,730 *(448)*	259,799 *(422)*	207,902 *(422)*	239,114 *(405)*

by the couple separately. For example, a three-year shared LTC insurance policy gives each spouse the potential to use six year's LTC benefits as long as the total benefits do not exceed six years. For a couple of a male and female who are both 65 years old, the lump sum premiums for shared LTC insurance policies are compared with premiums if they purchase LTC insurance separately. The results are shown in Table 12. We can see that the premium for a 2-year (3-year) shared LTC insurance policy is about 7% (5%) cheaper than the total premium of two 4-year (6-year) stand-alone LTC insurance policies separately purchased by a couple.

Table 12. Lump sum premiums for shared and separately purchased LTC insurance, a couple of a male and a female both aged 65.

Product	**3-year, separate**	**3-year, shared**	**6-year, separate**
Premium	27,937	37,450	39,507
s.e.	*(239)*	*(240)*	*(375)*
Product	**2-year, separate**	**2-year, shared**	**4-year, separate**
Premium	21,171	30,569	32,982
s.e.	*(173)*	*(182)*	*(294)*

6 Reserves and Capital Requirements

6.1 Best-Estimate Reserves

Best-estimate reserves for individuals in each alive state, i.e. $V_i(t, T)$ and $\tilde{V}_i(t, T)$ for any $i \in \{\text{H, M, S}\}$ calculated in Equations (6) and (16) respectively, in generic stand-alone LTC insurance policies purchased by 65-year-old individuals with lump sum premiums are shown in Figure 3. We focus on policies paid with lump sum premiums.

As shown in Figure 3 the best-estimate reserve for the healthy initially increases from accrued interest and then decreases as expected large benefit payments are made. When the individual becomes disabled, the reserve has a sharp increase. Reflecting the impact of dis-

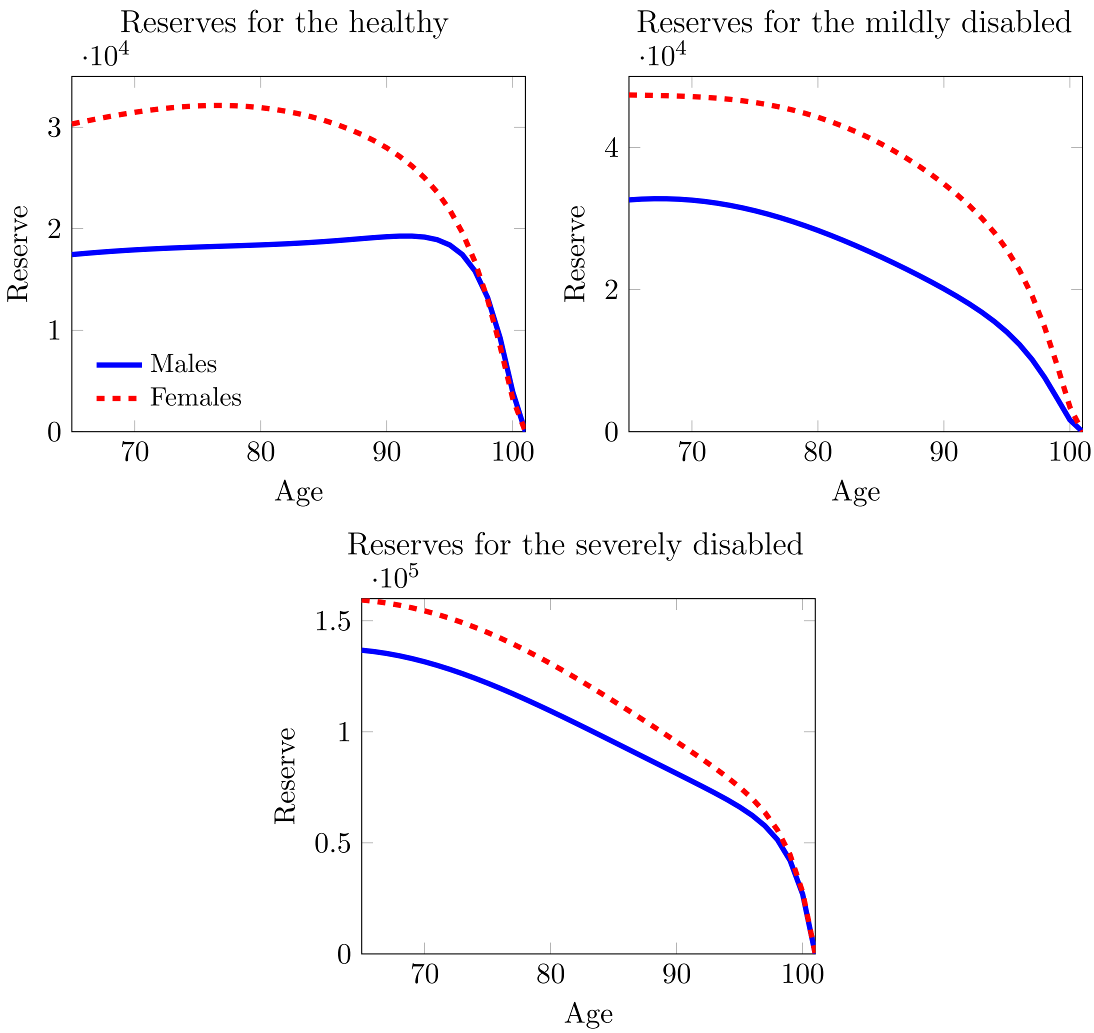

Figure 3. Best-estimate reserves for individuals in each alive state in a generic stand-alone policy sold to the 65-year-old with lump sum premiums.

ability, reserves for females are larger than those for their male counterparts regardless of the health state they are in.

To calculate the best-estimate reserves for policies that are sold to disabled individuals, we also simulate 80,000 65-year-old individuals (including 40,000 males and 40,000 females) who are in the mildly disabled state at the outset and in the severely disabled state at the outset, respectively. Figure 4 shows best-estimate reserves for generic stand-alone policies paid with lump sum premiums. These generic stand-alone policies are sold to 65-year-old individuals in different health states at the outset. The best-estimate reserves, i.e. $V\big(t,T \mid \chi(0)=k\big)$

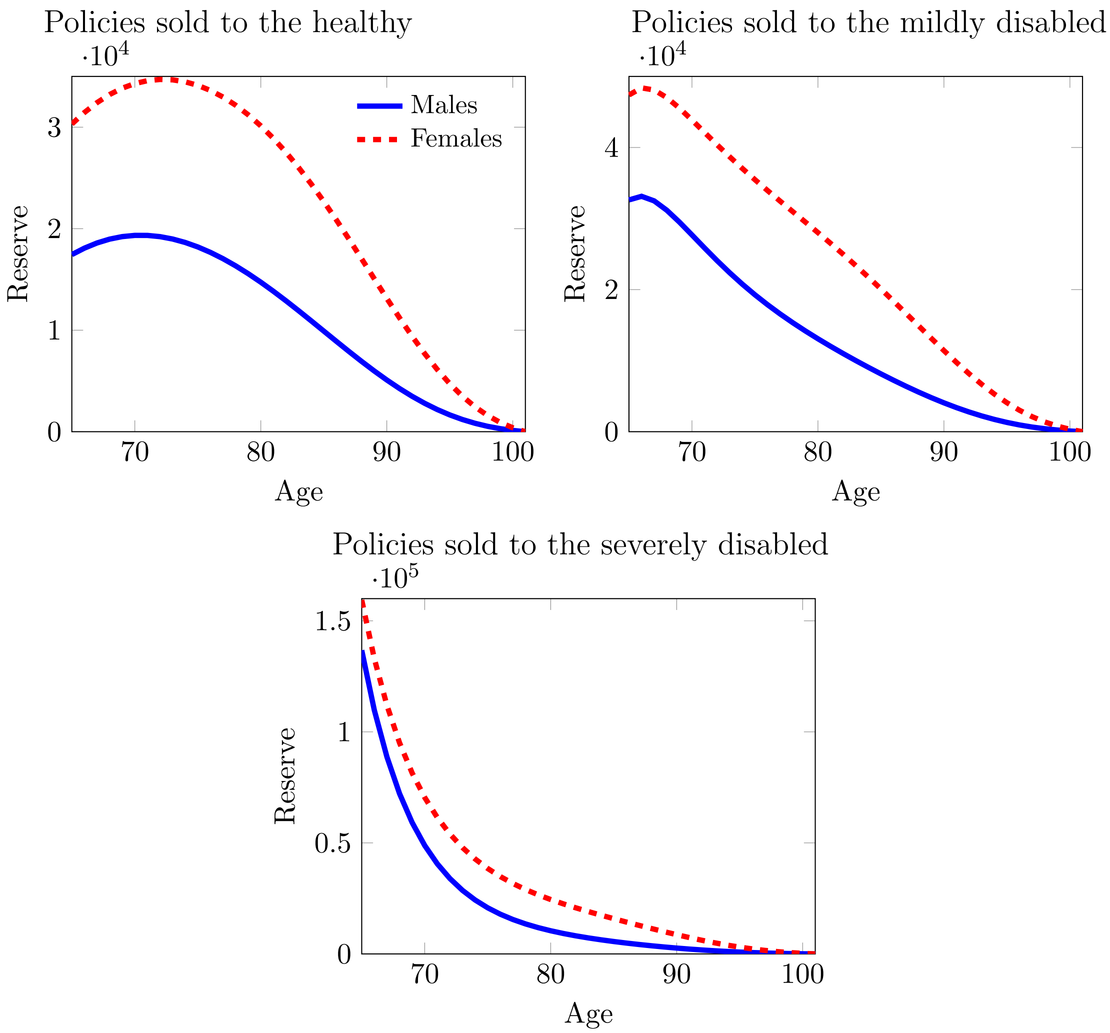

Figure 4. Best-estimate reserves for generic stand-alone policies issued to a 65-year-old in different health states with lump sum premiums.

and $\tilde{V}\big(t, T \mid \chi(0) = k\big)$, are calculated using Equations (6) and (15) respectively. Figure 4 shows that the although reserves for policies sold to more disabled individuals are larger at the outset they decline much faster.

The Value-at-Risk (VaR) of liabilities can be used to assess the idiosyncratic risk, in particular for small insurance providers. The VaR of liabilities for lump sum premium generic stand-alone policies sold to the 65-year-old healthy individuals is calculated and shown in Figure 5. The 99.5% VaR is much higher than the best-estimate reserve, and this reflects large idiosyncratic risk as the lives reduce in number at the older ages.

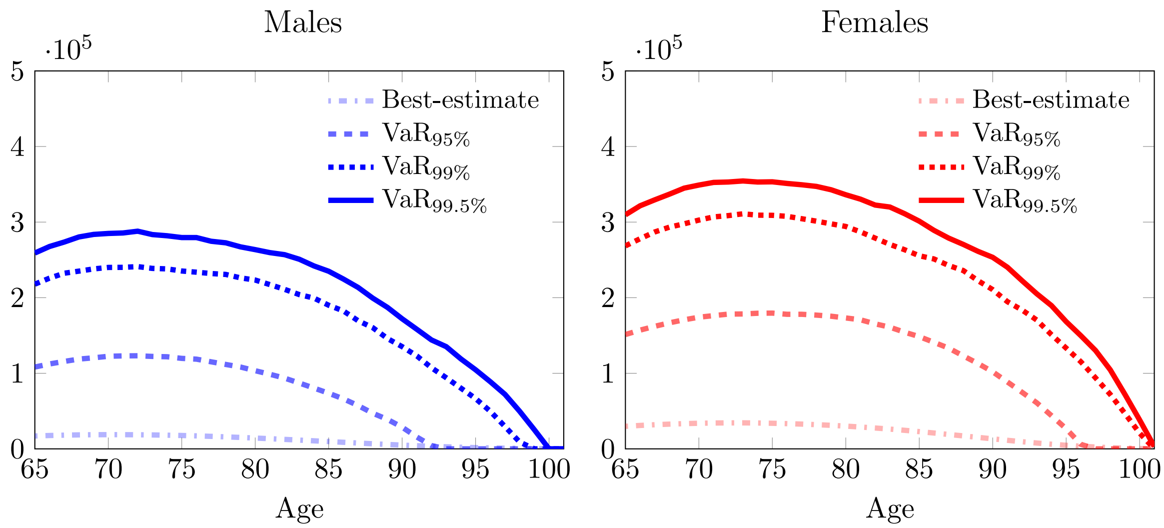

Figure 5. VaR of liabilities of generic lump sum premium stand-alone policies issued to 65-year-old healthy individuals.

The difference between the 99.5% VaR of future liabilities and the best-estimate reserve as a proportion of the best-estimate reserve, which can be expressed as $VaR_{99.5\%}\big(t, T \mid \chi(0) = k\big) \;/\; \tilde{V}\big(t, T \mid \chi(0) = k\big) - 1$, is calculated for stand-alone policies with different elimination periods and maximum benefit periods. The results are shown in Figure 6. The top two panels show results for policies with different elimination periods. The bottom two panels show results for policies with different maximum benefit periods.

Figure 6 shows that the maximum benefit period is effective in reducing extremely large losses, but the elimination period is not effective in reducing idiosyncratic risk.

6.2 Solvency Capital Requirements under Solvency II

This section assesses the impact of longevity risk and disability risk on solvency capital requirements under the Solvency II standard formula framework. In the Solvency II standard formula, the SCR for longevity risk and disability risk is calculated using Equation (22). In addition to SCRs, insurers are required to hold a risk margin that is calculated according to Equation (23). In the following results shown in this section, it is assumed that the cost of capital (denoted by c) is 6% per annum and the risk-free interest rate (denoted by r_f) is the

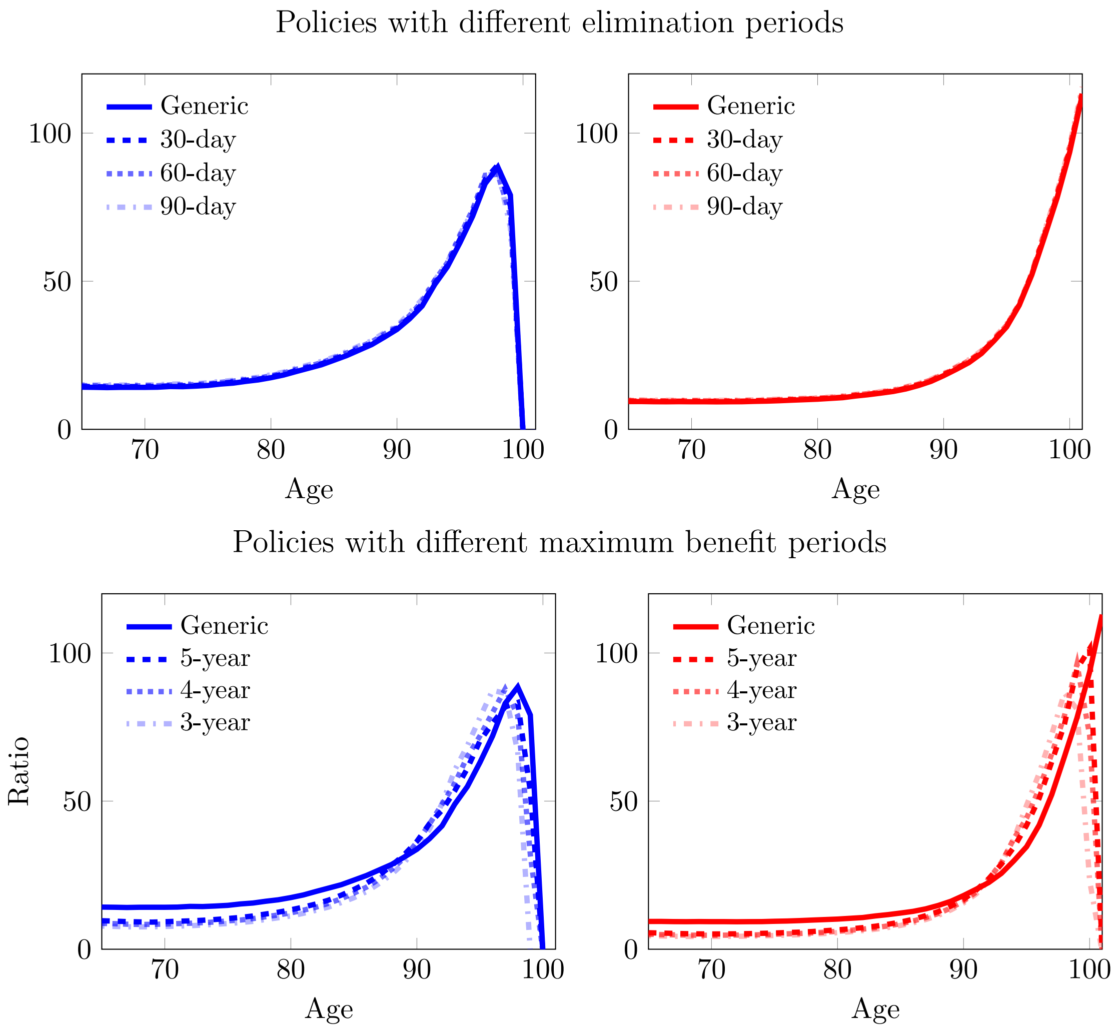

Figure 6. VaR minus best-estimate reserves as a proportion of best-estimate reserves for lump sum premium stand-alone policies with different elimination periods and maximum benefit periods. The policies issued to 65-year-old healthy males (left) and females (right).

annual effective rate equivalent to a continuous compounding rate of 4% per annum.

The ratio of the total capital requirement to the best-estimate reserve is used to assess the total capital required for a unit premium under the Solvency II standard formula framework. The ratio, denoted by $\eta(t)$, is calculated in Equation (25):

$$\eta(t) = \frac{TCR_t^S}{V\big(t, T \mid \chi(0) = k\big)}, \tag{25}$$

where $k \in \Omega_\chi$ is the insured's health state at policy issue, $V\big(t, T \mid \chi(0) = k\big)$ is the best-estimate reserve calculated in Equation (15), and TCR_t^S is the total capital requirement calculated in Equation (24). The ratio, $\eta(t)$, for a generic stand-alone LTC insurance policy is shown in Figure 7.

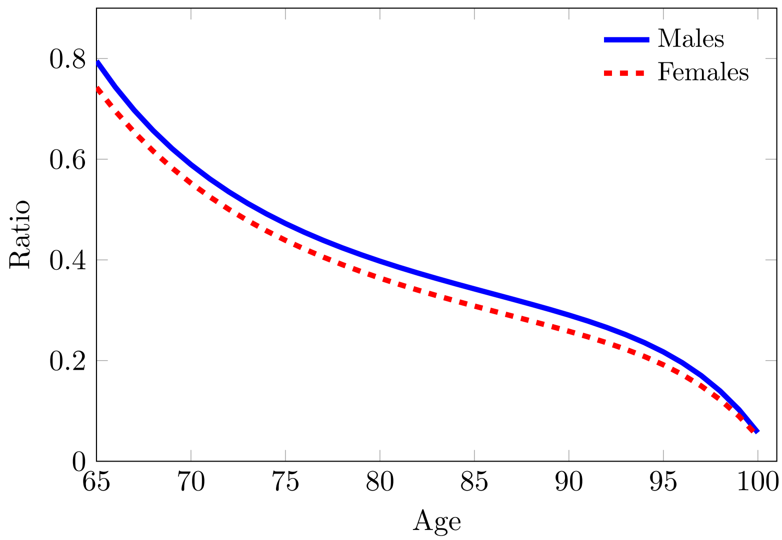

Figure 7. The ratio of total capital requirement to the best-estimate reserve, $\eta(t)$, for generic stand-alone policies sold to the healthy.

Under Solvency II, the total capital requirement as a proportion of the best-estimate reserve, $\eta(t)$, for a lump sum premium generic stand-alone policy sold to 65-year-old healthy individuals decreases significantly as the insured becomes older. At age 80, the total capital requirement is 40% of the best-estimate reserve for males and 36% for females. Figure 7 shows that generic stand-alone policies sold to healthy males require slightly more capital than those sold to healthy females for a unit premium. The difference between capital requirements for generic stand-alone policies sold to the two genders diminishes as the insured reach very old ages.

The ratio of the total capital requirement to the best-estimate reserve, $\eta(t)$, is compared across generic stand-alone policies issued to individuals in different health states. The results are shown in Figure 8. Generic stand-alone policies sold to healthy individuals require high levels of capital per unit premium. Generic stand-alone policies sold to severely disabled individuals are very expensive and require high levels of reserves (as shown in Table 6), they require lower amounts of capital per unit premium compared to those sold to the healthy and mildly disabled. Differences in $\eta(t)$ for policies issued to individuals in different health states diminishes after around 25 years since policy inception.

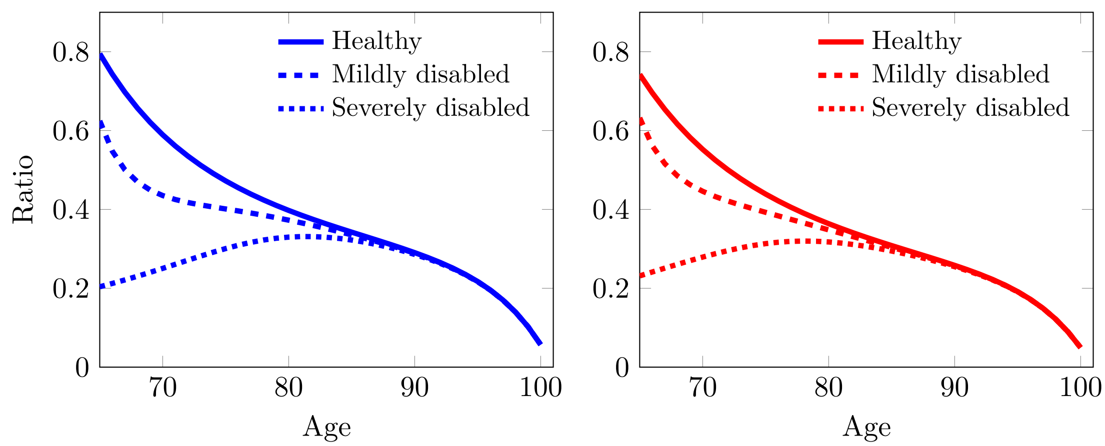

Figure 8. The ratio of the total capital requirement to the best-estimate reserve, $\eta(t)$, for generic stand-alone policies sold to 65-year-old healthy, mildly disabled and severely disabled males (left) and females(right).

The impact of longevity risk and disability risk on capital requirements based on the ratios of risk-specific SCRs to the best-estimate reserve is shown in Figure 9. For policies issued to healthy and mildly disabled individuals, disability risk capital requirements are higher than for longevity risk for males (females) in the first eight (twelve) years. After that longevity risk dominates capital requirements. In general, disability risk has more impact on capital requirements for policies issued to disabled females than to disabled males, but the effects of disability risk are very similar for policies sold to healthy males and healthy females. Longevity risk has more impact on the capital requirements for policies sold to males than to females.

The $\eta(t)$ ratios for rider benefit policies and for life care annuities are shown in the left and right panels of Figure 10, respectively. The existing natural hedge in rider benefit policies and life care annuities, results in the total capital requirements per unit premium for these two types of policies being lower than those for stand-alone policies. There are considerable

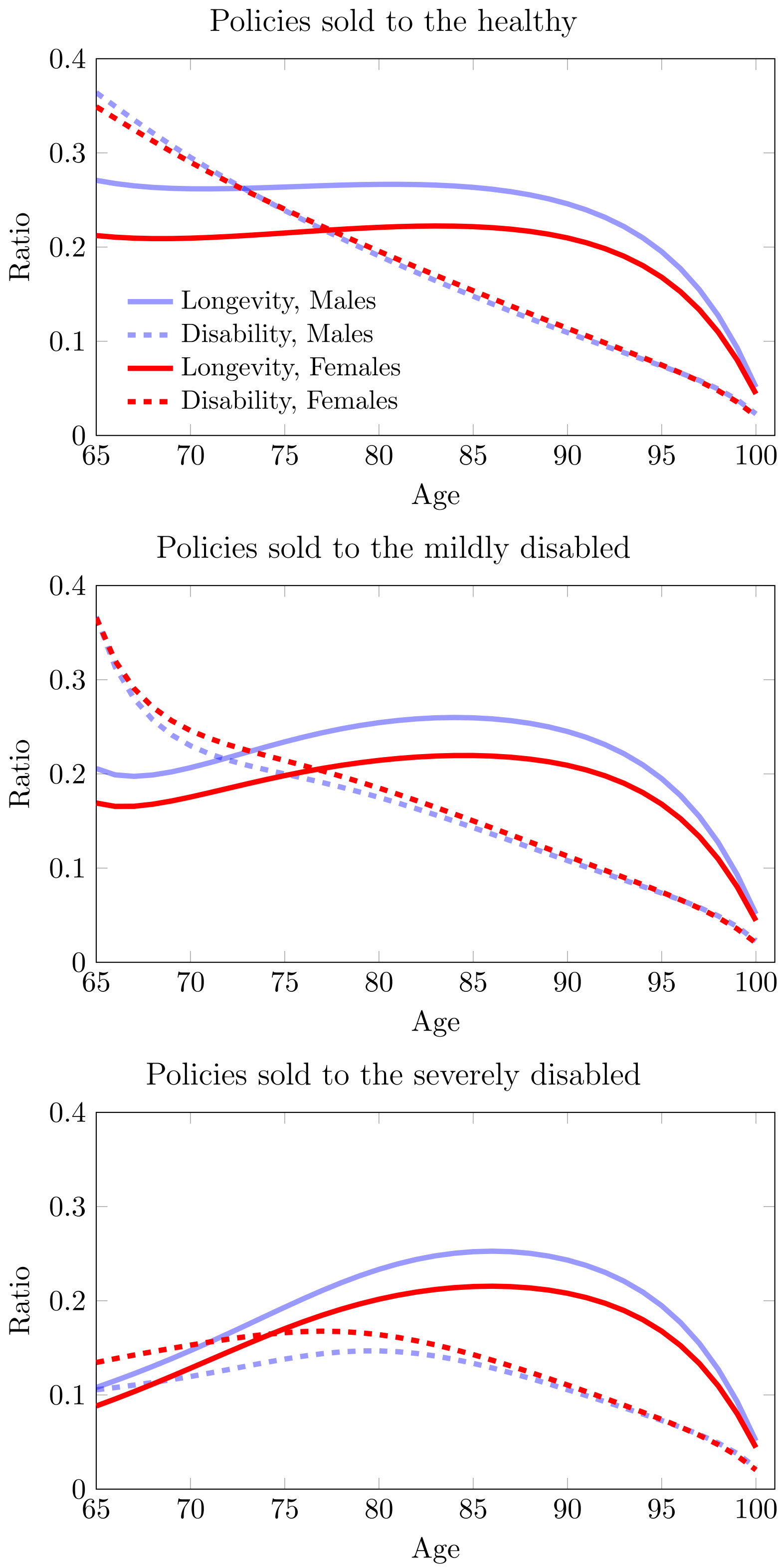

Figure 9. The ratio of SCR for longevity risk and for disability risk respectively to the best-estimate reserve, for generic stand-alone policies sold to the healthy, the mildly disabled and the severely disabled.

capital reductions in policies that combine LTC insurance with life insurance or annuities as noted in Zhou-Richter and Gründl (2011). We show the extent to which life care annuities reduce the required capital level for LTC insurance providers.

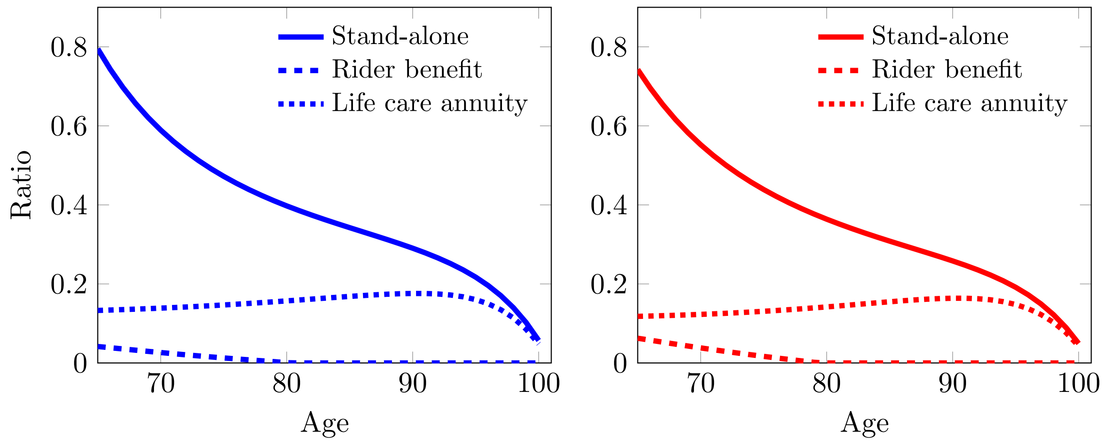

Figure 10. The ratio of the total capital requirement to the best-estimate reserve, $\eta(t)$, for generic LTC insurance policies sold to 65-year-old healthy males (left) and females (right).

The ratios of risk-specific SCRs to the best-estimate reserve for rider benefit policies and for life care annuities are shown in Figure 11. Longevity shocks impact the components of rider benefit policies in opposite directions. Mortality improvements reduce the reserve for the whole life insurance benefit, while longevity risk increases the reserve for the LTC insurance as shown in Figure 9. Overall longevity risk, as in the Solvency II standard formula framework, results in a significant reduction in the liability for rider benefit policies[4].

Disability risk impacts the components of life care annuities in the opposite directions as well. Higher disability rates and lower recovery rates increase the expected value of liabilities for LTC benefits but also increase the average mortality rates, which results in a lower expected value of liabilities for annuity payments. The effect of disability shock on the liabilities of life care annuities is dominated by that of longevity shock.

[4]In calculating the aggregate SCRs for rider benefit policies and for life care annuities, negative values of the risk-specific SCRs are set to zero.

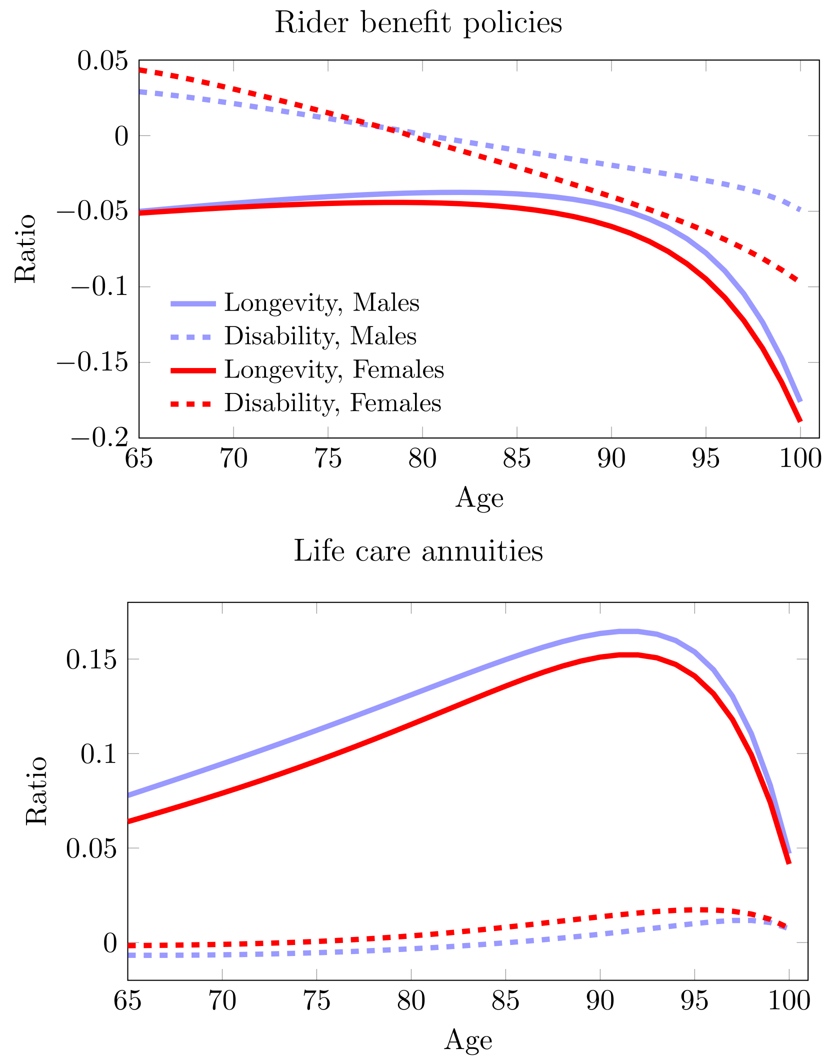

Figure 11. The ratio of the SCR for longevity risk and disability risk respectively to the best-estimate reserve, for generic rider benefit policies and life care annuities sold to 65-year-old healthy individuals.

7 Conclusions

This paper has assessed premiums, best-estimate reserves and solvency capital requirements for a broad range of LTC insurance policies sold to individuals in different health states. Thiele's differential equation approach and a simulation-based method are applied to a range of policy designs. LTC insurance policies considered are stand-alone policies, rider benefit policies (LTC insurance combined with whole life insurance), life care annuities (LTC insurance combined with annuities), and shared LTC insurance.

Policies providing reasonable levels of fixed benefits are relatively affordable for healthy lives. Whereas premiums of stand-alone policies are very high for disabled and older individuals, in particular for those who are severely disabled, life care annuities that combine LTC insurance and annuities are more affordable for disabled and older individuals as well as for healthy lives. Policy design can be used to enhance the insurability of LTC expenses for individuals with impaired health.

The simulation-based approach is required to assess premiums and reserves for policies with different combinations of elimination periods and maximum benefit periods. This also allows the distributional measures of future liabilities, such as the VaR, to be estimated. The elimination period and maximum benefit period are shown to be effective in making a LTC product more affordable. The maximum benefit period is effective in reducing idiosyncratic risk arising from reduced numbers of policies at the older ages.

The Solvency II standard formula framework shows that solvency capital requirements are high for LTC insurance taking into account both longevity risk and disability risk. Stand-alone policies issued to the more disabled require less capital per unit premium compared to healthy lives. Interestingly rider benefit policies and life care annuities show substantial reductions in the required capital per premium compared to stand-alone LTC insurance reinforcing the potential benefits of these combined products. This is in line with the results in Pitacco (2015).

We have provided a thorough analysis of premiums, reserves and capital of LTC polices. We have used U.S. individual data to provide mortality and disability assumptions to allow a realistic assessment. The analysis presented provides valuable insights into the product design of more affordable products and an analysis of the solvency risks faced by LTC insurance providers.

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