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Developing Forecasting Mortality Trends allowing for Cause-of-Death Mortality Dependence.

Severine Gaille 1, Michael Sherris 2

1 Professor of Actuarial Science
Department of Actuarial Science, Faculty of Business and Economics, University of Lausanne,
Switzerland, Tel +41 21 692 33 72, Severine.Gaille@unil.ch

2 Professor of Actuarial Studies and Chief Investigator ARC Centre of Excellence in Population
Ageing Research
MBA, FIAA, FIA, FSA, FFin
Risk and Actuarial Studies, CEPAR, Australian School of Business, University of New South
Wales, Australia, Tel +61-2-9385 2333, Fax +61-2-9385 1883,
M.Sherris@unsw.edu.au

Forecasting Mortality Trends allowing for Cause-of-Death Mortality Dependence

Séverine Gaille[1] Michael Sherris[2]

Abstract

Longevity risk is amongst the most important factors to consider for pricing and risk management of longevity products. Past improvements in mortality over many years, and the uncertainty of these improvements, have attracted the attention of experts, both practitioners and academics. Since aggregate mortality rates reflect underlying trends in causes of death, insurers and demographers are increasingly considering cause-of-death data to better understand risks in their mortality assumptions. The relative importance of causes of death has changed over many years. As one cause reduces, others increase or decrease. The dependence between mortality for different causes of death is important when projecting future mortality. However, for scenario analysis based on causes of death, the assumption usually made is that causes of death are independent. Recent models, in the form of Vector Error Correction Models (VECM), have been developed for multivariate dynamic systems and capture time dependency with common stochastic trends. These models include long-run stationary relations between the variables, and thus allow a better understanding of the nature of this dependence. This paper applies VECM to cause-of-death mortality rates in order to assess the dependence between these competing risks. We analyze the five main causes of death in Switzerland. Our analysis confirms the existence of a long-run stationary relationship between these five causes. This estimated relationship is then used to forecast mortality rates, which are shown to be an improvement over forecasts from more traditional ARIMA processes, that do not allow for cause-of-death dependencies.

Keywords: Mortality forecasts, Causes of death, VECM, Dependence, Common trends
JEL Classifications: C32, C52, J11, G22, I13

[1]Professor of Actuarial Science
Department of Actuarial Science, Faculty of Business and Economics, University of Lausanne, Switzerland, Tel +41 21 692 33 72, *Severine.Gaille@unil.ch*

[2]Professor of Actuarial Studies and Chief Investigator ARC Centre of Excellence in Population Ageing Research
MBA, FIAA, FIA, FSA, FFin
Risk and Actuarial Studies, CEPAR, Australian School of Business, University of New South Wales, Australia, Tel +61-2-9385 2333, Fax +61-2-9385 1883, *M.Sherris@unsw.edu.au*

1 Introduction

Cause-of-death trends have important implications for forecasting aggregate mortality rates. The need for better methods of forecasting mortality trends including the impact of cause-of-death trends has been recognised for some time (Stoto and Durch, 1993). Official projections in some countries are based on cause-specific mortality, with each cause forecasted in isolation and subsequently aggregated to produce total mortality (Wong-Fupuy and Haberman (2004)). An important limitation is allowing for dependence between cause-of-death mortality rates. Dependence between several causes is not observable. For any given death, it is not possible to know what would have been the future cause of death if the person had remained alive. Therefore, the common assumption is that of independence between causes of death, following early work of Chiang (1968).

A better understanding of trends in the underlying causes of death has the potential to improve mortality projections. The use of cause-of-death data is seen as being beneficial (Tuljapurkar (1998), Gutterman and Vanderhoof (1998) and Tabeau *et al.* (2001)) as well as having limitations (Booth and Tickle (2008) and Richards (2009)). Despite this potential, most mortality projections are based on aggregate rates (Pitacco (2004), Booth and Tickle (2008)) or assume independence between causes (Andreev and Vaupel (2006)).

There are a number of studies that consider cause-of-death forecasts using the independence assumption including Rogers and Gard (1991), Wilmoth (1996), Tabeau *et al.* (1999), Caselli *et al.* (2006). In these studies, total future mortality rate results from the summation of cause-specific mortality forecasts, each cause being projected without taking into account trends in other causes of death. For example, in McNown and Rogers (1992), univariate ARIMA models are used to forecast parameters of a function fitted to the age pattern of mortality. They forecast the four main causes of death (heart diseases, cancer, vascular diseases, accident and violence) and other causes to 1985 using data from 1960 to 1975. A similar approach is used in Knudsen and McNown (1993). Caselli (1996) forecasts mortality by cause for ages 60 and over for 25 European countries over the period 1988-2020, using data from 1950 to 1990 and an Age-Period-Cohort model. She adds cause-specific rates to estimate future aggregate mortality rates.

In this paper, we propose an approach for capturing the dependence between causes of death using Vector Autoregression (VAR) and Vector Error Correction Models (VECM). They have been developed in econometrics to model multivariate dynamic systems including time dependency between economic variables and allowing for common stochastic trends. VECM also include long-run equilibrium relationships using cointegrating relations. We apply VECM to cause-of-death mortality rates. The analysis supports the existence of long-run stationary relationships between the five main causes of death and provides insights into the form of dependence between these competing risks over recent years. These long-run stationary relationships are then used to forecast mortality rates. The benefits gained from modeling the underlying cause trends with these methods are quantified by comparing the resulting mortality forecasts with those from more traditional ARIMA processes, that do not allow for cause-of-death dependencies.

This paper presents a new modeling approach for researchers interested in understanding the dependence between cause-of-death mortality rates. The model-

ing approach provides the potential to assist practitioners in setting dependence assumptions for scenario analysis based on cause-of-death mortality. The benefits of incorporating cause dependence in longevity and mortality risk models are demonstrated and can be used in official projections for countries interested in cause-specific mortality as well as by insurers issuing longevity and mortality risk products.

The paper starts with introducing the data in Section 2. After a brief explanation of the features of VAR and VECM modeling, these models are applied to cause-specific mortality for females in Switzerland (Section 3). In Section 4, the estimated relationships between the causes of death are used to forecast mortality rates and the results are compared with forecasts from more traditional ARIMA processes. Finally, Section 5 concludes.

2 Data

The World Health Organization (WHO) provides a comprehensive database for causes of death (World Health Organization (2012)). Mid-year population and number of deaths according to the underlying cause of death are maintained for various countries over the last 50 to 60 years. The data are typically divided into five-year age groups. We use data for Switzerland from 1951 to 2007 to illustrate the methods. Similar benefits would apply to other countries experiencing changes in causes of death.

The WHO database classifies the causes of death according to the International Classification of Diseases (ICD), thus ensuring consistencies between countries (Table (1)). Under the ICD, the underlying cause of death is specified as *the disease or injury which initiated the train of morbid events leading directly to death, or the circumstances of the accident or violence which produced the fatal injury.* In this study, only the five main ICD primary causes of death are considered, which are the diseases of the circulatory system, cancer, diseases of the respiratory system, external causes, and infectious and parasitic diseases. These major causes of death account for about 80% of the deaths in recent years, while they made up approximately 60% – 70% 50 years ago.

In order to work on data consistent over time and across countries[1], a few adjustments are made to the dataset. First, as recommended by the Human Mortality Database (Human Mortality Database (2012)), the number of deaths of unknown age is divided up across the age range.

Second, ages 85 and over are grouped together as well as ages one to four. Thus, our database is composed of nineteen age groups, the first for infants less than one year old, a second for children aged one to four, thereafter in groups of five years, ending with the group aged 85 and above.

Third, an adaptation is due to the changes of the classification of the diseases over time. Indeed, to improve the classification and to adapt it to changes in science and technology, the ICD evolved from ICD 7 in the 1950's to ICD 10 still used today. Therefore, the data are not directly comparable for different periods and comparability ratios are necessary to allow comparisons, as in Gaille and Sherris (2011). Since Switzerland did not adopt ICD 9, two sets of ratios are developed,

[1]Future researches might be interested in comparisons across countries.

one for 1969 when ICD 8 was adopted and one for 1995 when ICD 10 was adopted. By dividing the death number in a new classification with the comparability ratio linking this classification with the previous one (and previous comparability ratios where appropriate), we remove discontinuities in the death rates in 1969 and 1995. Indeed, the comparability ratios are determined so that the average of the death rates over the last two years of a classification coincides with the average of the death rates over the first two years of the next classification (for details, see Gaille and Sherris (2011)).

Finally, trends by cause of death are examined using an age-standardized central death rate, the standard population being equal to the population of the last year under observation, here 2007. We denote by $m^*_{t,d,s}$ the death rate in year t for cause d and gender s, assuming that the age-structure of the population is constant over the complete period under observation and fixed at the level of 2007.

3 Model Fit

Vector AutoRegressive (VAR) models are used to model vectors of variables that are assumed stationary. A pth-order vector autoregression, denoted as VAR(p), is based on p lags of the variables in the model. Thus, expected changes are modeled by allowing for lagged relationships between the variables and also for the correlations between the variables. For mortality modeling, a vector of age-standardized cause-specific death rates transformed to stationary variables can be effectively modeled with a VAR.

More efficient processes exist to model non-stationary or integrated variables. These models are called Vector Error Correction Models (VECM). The intuition behind these models is that the variables may move together with common stochastic trends, even though they are non-stationary. Therefore, even if each variable is non-stationary, a linear combination of these variables may exist such that the relation is stationary. This linear combination represents a long-run equilibrium relationship called cointegration.[2] The system may have more than one cointegrating relation, if each combination is linearly independent from the others. These relations may be included in a VAR model, which is then called a VECM. Comprehensive references on these models are e.g. Hamilton (1994) and Lütkepohl (2005).

Johansen's approach is used to estimate the number of cointegrating relations in the process as well as the parameters of the VECM. The steps to follow to estimate a VECM are summarized in Gaille and Sherris (2011). First, the lag order of the VAR is selected through Akaike's Information Criteria (AIC), Hannan-Quinn Criterion (HQ), Schwarz Criterion (SC), Final Prediction Error (FPE). Second, the stationarity of the variables is considered through several unit root tests: the Kwiatkowski-Phillips-Schmidt-Shin test (KPSS), the Augmented Dickey-Fuller test (ADF), the Phillips-Perron test (PP) or the Elliot-Rothenberg-Stock test (ERS). Third, the Johansen's procedure is applied if some of the variables are integrated in order to find the number of cointegrating relations. For that purpose, the trace test and the maximum-eigenvalue test are used. Besides, the Johansen's procedure allows to include a vector of constants and/or a vector of trends in the model while

[2]In this paper, we consider variables that are integrated of order one. In that special case, cointegrating relations are necessarily stationary. For a more general framework, see Hamilton (1994) and Lütkepohl (2005).

testing for cointegration. Therefore, depending on the specification of the model, the cointegrating relations may be stationary around a constant level or a trend. Forth, a VAR($p-1$) on the first difference is estimated when the variables are integrated and not cointegrated. Otherwise, the appropriate VECM is developed. Finally, model validation tests should be performed, such as tests for residual autocorrelation and non-normality.

This procedure is applied to cause-specific death rates for females in Switzerland, determined as the logarithm of $m^*_{t,d,female}$. The variables of the VECM analysis are the age-standardized death rates for: 1) diseases of the circulatory system; 2) cancer; 3) diseases of the respiratory system; 4) external causes; 5) infectious & parasitic diseases. Since at least 50 years of observation are usually necessary to reliably estimate a VECM, the model is fitted over the period 1951 to 2000 and forecasts are performed until 2007. Some details of tests performed are omitted for ease of presentation and are available from the authors upon request.

Lag order: The four tests show some contradictory results. FPE indicates a lag order of two as optimal (a trend and a constant being included in the VAR or not), while SC indicates a lag order of one, AIC a lag order of five and HQ a lag varying between one and two, the result depending on the deterministic part included in the VAR (a trend, a constant or none). Since this study focuses on forecasting performances and such performances may be improved with a smaller number of parameters, a lag of one is adopted.

Unit root tests: The four tests clearly indicate at a 5% significance level that the diseases of the circulatory system, the external causes of death and the infectious and parasitic diseases are non-stationary. The results are not so evident for the other two causes of death. Indeed, KPSS accepts the null hypothesis of stationarity around a trend for cancer, while ADF, PP and ERS tests accept the null hypothesis of non-stationarity at a 5% significance level. With respect to the diseases of the respiratory system, KPSS, ADF and ERS indicate non-stationarity at a 5% significance level, while PP test rejects the non-stationarity. Since three tests out of four point out non-stationarity for these last two causes of death, the five causes of death are assumed non-stationary for the following analysis.

Cointegrating relations: The trace test and the maximum-eigenvalue test of the Johansen's procedure are performed and presented in Table 2. According to these tests, one cointegrating relation exists between the stochastic death rates of the five main causes of death for females in Switzerland. Indeed, the trace statistic tests the null hypothesis of r cointegrating relations against the alternative of n cointegarting relations, n being the number of variables in the VECM, here the five causes of death, and $r < n$. The test indicates that four, three, two and one cointegrating relations are not rejected against the alternative of five cointegrating relations at a 5% significance level.

The maximum-eigenvalue statistic tests the null hypothesis of r cointegrating relations against the alternative of $r + 1$. Therefore, the null hypothesis of one cointegrating relation is accepted against the alternative of two cointegrating relations, while the null hypothesis of zero cointegrating relation is rejected. Thus, one

long-run equilibrium relationship ties the causes of death together and reveals how the death rates were changing relative to each others over the past 50 years.

The Johansen's approach also allows to test for a potential trend in cointegration. The null hypothesis of no linear trend in cointegration is rejected with a p-value of 0.5%, which indicates that a deterministic trend should be also included in the cointegrating relation and not only in the variables. In other words, the deterministic trend is not eliminated by the cointegrating relation. The cointegrating relation represents a stationary process added to a deterministic linear trend. Finally, the null hypothesis of no quadratic trend is accepted at a 2.5% significance level, with a p-value close to 4%.

Fitted VECM: The Vector Error Correction Model fitted to the five cause-specific death rates is presented in Equation 1.

$$\begin{aligned}
\begin{bmatrix} \nabla \log m^*_{t,I\&P,s} \\ \nabla \log m^*_{t,cancer,s} \\ \nabla \log m^*_{t,circ,s} \\ \nabla \log m^*_{t,resp,s} \\ \nabla \log m^*_{t,external,s} \end{bmatrix} &= \begin{bmatrix} 1.29 \\ -0.91 \\ -3.96 \\ -34.72 \\ 0.48 \end{bmatrix} + \begin{bmatrix} 0.0089 \\ -0.0061 \\ -0.0265 \\ -0.2332 \\ 0.0033 \end{bmatrix} \\
&\times \begin{bmatrix} 1.49 & -19.56 & -6.32 & -4.52 & 1.86 & -0.24 \end{bmatrix} \\
&\times \begin{bmatrix} \log m^*_{t-1,I\&P,s} \\ \log m^*_{t-1,cancer,s} \\ \log m^*_{t-1,circ,s} \\ \log m^*_{t-1,resp,s} \\ \log m^*_{t-1,external,s} \\ t-1 \end{bmatrix}. \qquad (1)
\end{aligned}$$

The cointegrating relation between the causes of death is given by the second term on the right-hand side of this equation. It can be written as

$$\begin{aligned}
z_t = \quad & 1.49 \cdot \log m^*_{t-1,I\&P,s} - 19.56 \cdot \log m^*_{t-1,cancer,s} - 6.32 \cdot \log m^*_{t-1,circ,s} \\
- \quad & 4.52 \cdot \log m^*_{t-1,resp,s} + 1.86 \cdot \log m^*_{t-1,external,s} - 0.24 \cdot (t-1), \qquad (2)
\end{aligned}$$

where z_t is a stochastic and stationary variable representing the deviation from the equilibrium. Thus, the model allows for stochastic trends in mortality rates, while maintaining long-run relationship between the causes of death through Equation 2. Relative changes in mortality between causes are reflected in this relationship, which represents historical evolutions. Mortality was evolving stochastically, but death rates were also driven by this long-run equilibrium relationship between the causes which was maintained stationary over the past 50 years. Thus, by using such a long-run stationary relation in a VECM for forecasting, we assume that this relation will continue in the future.

Model validation: Diagnostic tests are performed on the residuals of the fitted model. As indicated in Table 3, the normality of the residuals is accepted by the three tests. However, some autocorrelations between the residuals remain according to the Portmanteau test. Such a result is not surprising since we fitted a model with a small number of parameters with a lag value of one. A VECM of higher

order may provide better results with respect to this test. Since we are interested in the forecasting performance of the model, this is expected to be better with fewer parameters.

4 Forecasting

The fitted model is used to forecast cause-specific mortality rates. Since data are available until 2007, the forecasts are compared to actual mortality, which gives us some indications on the model forecasting performance. Figure 1 shows the forecasted mortality rates from the fitted VECM compared with the actual data (dots). The curve represents the fitted model until 2000 and the resulting forecasts from 2001 to 2007. The future trend is well captured by the model for the five causes. However, to better evaluate the model performance, it is necessary to compare the results with the outcomes of a more traditional approach, the AutoRegressive Integrated Moving Average (ARIMA) process.

ARIMA processes are very common to model univariate time series. The logarithm of each cause-specific death rate is modeled through an ARIMA process and therefore without taking into account the relations and the dependence that exist between the causes. Each cause is modeled independently from the other causes, as it has been previously done in the literature.

As with the VECM, ARIMA processes are fitted over the period 1951-2000 and used to forecast mortality until 2007. Since the approach developed in Pandit and Wu (2001) is followed, the non-stationarity in the variables is first removed by differencing the variables. To assure stationarity, first differencing on each cause-specific death rate is required and sufficient according to KPSS, ADF, PP and ERS.

ARIMA$(k, 1, k-1)$ models are then successively fitted to each age-standardized cause-specific log-death rate, increasing k by one. Pandit and Wu (2001) suggest the use of the F-criterion to decide which model is the most suitable between an ARIMA$(k, 1, (k-1))$ and an ARIMA$((k+1), 1, k)$, as this criterion tests the assumption that some of the coefficients in a model are restricted to zero. This procedure leads to an optimal ARIMA$(m, 1, (m-1))$ process. If some of the coefficients of that optimal process are not significantly different from zero, the F-criterion is again applied to determine the adequacy of a model without the corresponding coefficients.

Finally, non-correlation among the residuals of the fitted model is checked. The best fitting ARIMA models resulting from this procedure are introduced in Table 4. The ARIMA process for the external causes of death is not introduced since the best model is a random walk without any drift. The Portmanteau test indicates no significant residual autocorrelation with lags of 5, 10, 15, 20 and 25. The only exception is for the infectious and parasitic diseases for which the null hypothesis of no-autocorrelation is rejected at a 1% significance level with a lag of 25.

Figure 1 clearly indicates that taking into account the dependencies and relations existing between the causes of death improves the forecasting performance of the model. Indeed, the VECM captures much better the trends that exist in the data than the ARIMA processes, and this is particularly obvious for the diseases of the respiratory system and the infectious and parasitic diseases.

The forecasting performance of the two models is further evaluated through

two summary statistics. The first one is the well-known mean absolute percentage error statistic (MAPE), the average of the absolute percentage gap between the forecasted and observed death rates. The average is made for a specific year over the five causes.

The second statistic compares the forecasted death rates with the *no-change forecast*. The *no-change forecast* assumes that the future mortality is constant over time and fixed at the level of the last observation, here the death rates of 2000. The mean square error (MSE) between the forecasted and the observed death rates is divided by the MSE between the *no-change forecast* and the observed death rates. The square root of the result represents our second statistic called the *no-change forecast* statistic. The MSE are computed over the five causes of death. A model performs better than the standard assumption of no change in mortality if the ratio is smaller than unity. Table 5 compares the results for the VECM and ARIMA models.

The forecasts of the VECM are much closer to the actual death rates than the forecasts of the ARIMA processes. Indeed, both the MAPE and the *no-change forecast* statistics are smaller for the VECM. Besides, since the value of the *no-change forecast* statistic is below unity, the two models perform better than the standard assumption of no change in mortality over the seven years.

Since the forecasted trend of the ARIMA process for the diseases of the respiratory system and the infectious and parasitic diseases is far from what is actually observed (see Figure 1), these two causes significantly affect the value of the MAPE and the *no-change forecast* statistics. For these two causes of death, the best assumption with an ARIMA process would be that there is no trend and thus, that mortality is constant over time. Under such an assumption, while keeping the ARIMA models described in Table 4 for the three other causes of death, the MAPE and the *no-change forecast* statistics are reduced, as presented in Table 5 under the *adapted ARIMA*, but still higher than the VECM results. Indeed, the VECM performance comes from its ability to capture relationships between the causes of death and to use them in the forecasting process, while the ARIMA models simply ignore these.

5 Conclusion

This paper presents a new application of VECM to cause-of-death mortality and introduces a new modeling approach for cause-specific mortality that takes into account dependencies between causes. The model is able to capture long-run trends and the stationary relationships between the variables. A long-run equilibrium relationship is shown to exist between the five main causes of death for Swiss females, providing an approach to model the cause-of-death dependence. By including this equilibrium relationship, that is the cointegrating relation, in the modeling framework, forecasting is shown to be improved. If past trends are expected to continue in the future, including them in the model instead of modeling each cause in isolation, such as with traditional ARIMA processes, assists in forecasting future mortality rates.

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Table 1: International Classification of Diseases - Coding system

Causes of death	ICD 7	ICD 8	ICD 9	ICD 10
Circulatory system	A079-A086	A080-A088	B25-B30	1064
Cancer	A044-A060	A045-A061	B08-B17	1026
Respiratory system	A087-A097	A089-A096	B31-B32	1072
External causes	A138-A150	A138-A150	B47-B56	1095
Infectious and parasitic diseases	A001-A043	A001-A044	B01-B07	1001

Notes: The International Classification of Diseases changed three times between 1951 and 2007. The aim of these changes was to account for progresses in science and technology and to achieve more refined descriptions.

Table 2: Tests for the number of cointegrating relations

(a) Trace test

r	Trace stat	Critical values			
		10%	5%	2.50%	1%
4	6.80	10.59	12.49	14.06	16.42
3	18.13	22.95	25.43	27.82	30.55
2	37.64	39.01	42.35	45.23	48.99
1	60.98	58.98	62.71	66.36	70.63
0	118.29	82.29	86.71	90.70	95.19

(b) Maximum-eigenvalue test

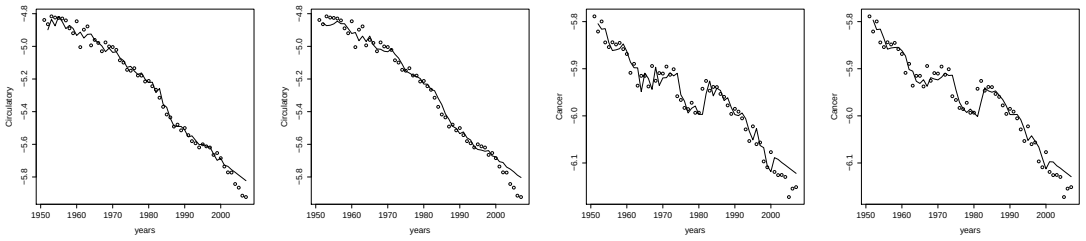
r	Eigen stat	Critical values			
		10%	5%	2.50%	1%
4	6.80	10.59	12.49	14.06	16.42
3	11.33	16.93	19.16	20.87	23.66
2	19.52	23.11	25.44	27.67	30.38
1	23.34	29.04	31.53	34.24	37.15
0	57.31	34.82	37.75	40.05	42.78

Notes: A statistic lower than the corresponding critical value indicates that the null hypothesis of r cointegrating relations is accepted against the alternative of n (trace test) or the alternative of $r+1$ (maximum-eigenvalue test) at a α% significance level. Thus, these tables indicate that one cointegrating relation is accepted at a 5% significance level. These two tests assess the number of long-run equilibrium relationships among the age-standardized log-death rates of the five main causes of death for females in Switzerland over the period 1951–2000.

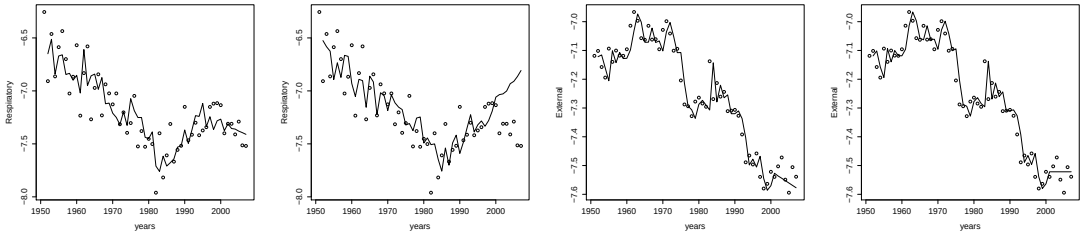
Table 3: Tests on residuals of the fitted VECM, 1951–2000, females in Switzerland

Type of test	**Name of the test**	**Statistic value**	**p-value**
Autocorrelation	Portmanteau (15 lags)	439.73	0.01
	Portmanteau (25 lags)	716.10	0.00
Normality	Skewness	0.91	0.97
	Kurtosis	5.56	0.35
	Both	6.48	0.77

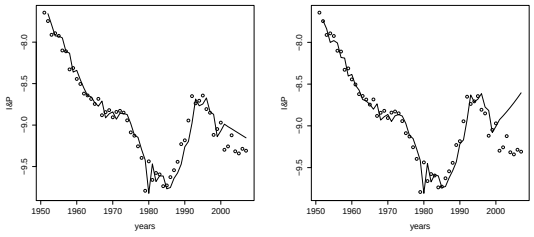
Notes: The null hypothesis of no-autocorrelation among the residuals is tested through the Portmanteau statistic, with a lag of 15 and 25. The skewness statistic, the kurtosis statistic and a combination of these are used to test the normality of the residuals.



(a) Circulatory system, VECM (b) Circulatory system, ARIMA (c) Cancer, VECM (d) Cancer, ARIMA



(e) Respiratory system, VECM (f) Respiratory system, ARIMA (g) External causes, VECM (h) External causes, ARIMA



(i) Infectious & parasitic diseases, VECM (j) Infectious & parasitic diseases, ARIMA

Figure 1: Observed, fitted and forecasted cause-specific log-death rates, females in Switzerland

Notes: The observed age-standardized log-death rates are depicted by the dots. The curve represents the fitted model until 2000 and the forecasted rates since then.

Table 4: Fitted ARIMA$(p, 1, q)$ processes to the logarithm of $m^*_{t,d,female}$, Switzerland

	Circ: ARIMA(2,1,0)			Cancer: ARIMA(1,1,0)		
	Values	CI 95%		Values	CI 95%	
constant	-0.017	-0.023	-0.012	-0.006	-0.010	-0.002
trend	-	-	-	-	-	-
ϕ_1	-1.073	-1.477	-0.668	-0.380	-0.647	-0.113
ϕ_2	-0.537	-0.786	-0.287	-	-	-
θ_1	0.517	0.081	0.954	-	-	-
$\sum \epsilon_t^2$	0.057			0.023		

	Resp: ARIMA(2,1,0)			I&P: ARIMA(0,1,0)		
	Values	CI 95%		Values	CI 95%	
constant	-0.059	-0.104	-0.014	-0.098	-0.174	-0.022
trend	0.002	0.000	0.003	0.003	0.000	0.005
ϕ_1	-0.942	-1.166	-0.719	-	-	-
ϕ_2	-0.609	-0.833	-0.385	-	-	-
θ_1	-	-	-	-	-	-
$\sum \epsilon_t^2$	1.804			0.871		

Notes: All models are identified and estimated over the period 1951 - 2000. First differencing transformation is performed on every variable.

Table 5: Mean absolute percentage error and *no-change forecast* statistics

Year	MAPE			No-change forecast		
	VECM	Original ARIMA	Adapted ARIMA	VECM	Original ARIMA	Adapted ARIMA
2001	8.78%	19.21%	15.62%	0.30	1.08	0.84
2002	7.47%	17.87%	12.96%	0.43	0.91	0.75
2003	4.77%	16.10%	9.35%	0.28	0.81	0.54
2004	8.62%	29.38%	17.82%	0.41	0.86	0.64
2005	10.75%	31.03%	17.26%	0.44	0.74	0.55
2006	10.32%	39.13%	20.55%	0.43	0.82	0.59
2007	9.13%	44.55%	21.14%	0.38	0.85	0.56

Notes: The mean absolute percentage error is written as MAPE. The *no-change forecast* represents the square root of the ratio of the mean square error of one of the two considered models with the mean square error of the *no-change forecast*. The *no-change forecast* assumes the mortality does not change anymore and so is fixed at the level of 2000 for the following seven years. All summary statistics are averages over the five causes of death for females in Switzerland.

Original ARIMA: the statistics result from the ARIMA processes described in Table 4.

Adapted ARIMA: the statistics result from the ARIMA processes described in Table 4, except for the diseases of the respiratory system and the infectious and parasitic diseases for which the trend was removed.