Managing Life Insurer Risk and Profitability: Annuity Market Development using Natural Hedging Strategies

Andy Wong[*], Michael Sherris[†], and Ralph Stevens[‡]

23rd January 2013

Abstract

Changing demographics creates the potential for the expansion of existing and new products to manage longevity risk. Life annuities address this risk, yet these annuity product markets are thin. Insurers are concerned about the long term risks associated with these longevity products and capital requirements. Life insurers also offer life insurance products, whole-of-life and term, that provide an opportunity to offset longevity risks. This can allow capital efficient longevity risk products to be sold as part of a product portfolio. Natural hedging, or the offsetting of risks in life insurance and annuity business, provides a way of managing capital efficiently as well as improving profitability. This paper uses stochastic mortality and interest rate models to assess life and annuity capital requirements and quantify the benefits of natural hedging taking into account relative profit loadings on products. The benefits of offering longevity products, in terms of capital requirements, as well as the importance of the type of life insurance products offered are illustrated using standard life and annuity products. The impact of capital requirements, such as solvency II with a one-year horizon, are considered and compared to multiple period risk measures to confirm the results hold for regulatory capital requirements.

Keywords: Life insurance, annuity, risk based capital, profit loading, natural hedging

JEL Classifications: G22, G23

---

[*] Risk and Actuarial Studies, CEPAR, Australian School of Business, University of New South Wales, andywong89@hotmail.com

[†] Risk and Actuarial Studies, CEPAR, Australian School of Business, University of New South Wales, m.sherris@unsw.edu.au

[‡] Risk and Actuarial Studies, CEPAR, Australian School of Business, University of New South Wales

1 Introduction

Life annuities are an important product that allows individuals to manage longevity risk in retirement. The recent global financial crisis, which resulted in significant falls and high volatility in equity markets, has highlighted the benefits of such products in a retired individual's portfolio. With increasing life expectancies in most developed countries, many retirees face the risk of outliving their retirement funds and savings, even if they have no equity market risk. Blake et al. (2008) highlights how life annuities provide retirees with a stable income that protects against longevity and also protects retirees from investment risk. Despite this, life annuity markets are thin but hold potential as a growth market for life insurers given the changing demographics in most developed countries.

Annuities involve the transfer of longevity risk from annuitants to the life insurer. Longevity risk can potentially have severe financial implications for pension funds and annuity providers as unforeseen increases in life expectancies will lead to more payments being made by annuity providers and pension funds. The impact of longevity risk on life insurers was demonstrated in Bauer and Weber (2008). Since there are significant social costs if a life insurer or pension fund becomes insolvent, regulations require insurers to hold sufficient risk based capital. This makes the provision of life products more expensive. In order to make life annuity products attractive to individuals, annuity providers and pension funds need to manage their exposure to longevity risk as efficiently as possible.

Risk management of insurance products involves both managing the costs of capital required to back the liabilities, as well as charging profit loadings that are low enough to ensure a viable market. In order to manage capital for longevity risk, there are limited options available to a life insurer. The primary option for life insurers to transfer their exposure to longevity risk is through reinsurance. Recently, there have been attempts to transfer longevity risk from annuity providers and pension funds to capital markets through securitization of longevity risk, as covered in Blake et al. (2008). Reinsurance capacity is limited and there has yet to be a well developed securitization market.

Natural hedging, or the offsetting of risks in life insurance and annuity business, provides another way of managing capital efficiently as well as improving profitability. Natural hedging is a risk management strategy to hedge against longevity risk that has the potential to support existing business and product markets and to develop a new product market through more efficient use of capital and more effective pricing. Cox and Lin (2007) used an empirical study to show how the portfolio composition of an insurer had an impact on the pricing of annuities. They found that life insurers issuing both annuities and life insurance had a lower risk premium compared to life insurers that solely held annuities. Cox and Lin (2007) did not consider natural hedging strategies.

The rationale for natural hedging is that values of annuity policies and life insurance policies have opposite exposures to changes in mortality. In general, annuities become more expensive with improving mortality since policy holders will on average live for longer. As a consequence, the life insurer will have to make more payments to the annuity holders. On the other hand, life insurance becomes less expensive with improvements in mortality and the life insurer will make fewer payments or later payments to policy holders. For whole life insurance the life insurer will make payments at a later point in time as mortality improves, whilst for term insurance fewer payments are also made since fewer insured lives will die during the term of the contract.

In countries where there are significant annuity markets, natural hedging can be applied to hedge longevity risk. For example, US life insurers received $124.6 billion in life insurance premiums and $231.6 billion in annuity premiums in 2009[1]. There is significant potential for natural hedging in many countries. Australian life insurers received $6.2 billion in revenue from their life insurance business over the financial year ending June 2012[2]. This amount of life insurance business could hedge up to $24 billion worth of annuity liabilities each year based on natural hedging. Australia's annuity market is currently very small, with revenue from annuities with longevity risk of $17 million in the June 2012 quarter. In contrast, Australia's superannuation industry received total contributions over the 2011 financial year of $104.8 billion.

There have been several proposed strategies for natural hedging, ranging from static hedging approaches in Tsai and Tzeng (2010) and Gatzert and Wesker (2012) to the dynamic hedging approaches in Wang et al. (2010) and Luciano et al. (2012*b*). Static hedging strategies involve managing the risk level of a portfolio or managing the surplus of an insurer through natural hedging and portfolio structuring. On the other hand, dynamic hedging strategies hedge the value of the liabilities through minimizing the sensitivity of a portfolio to changing mortality.

What has not been considered is the effects on natural hedging of the relative profit loadings in contracts and the impact of different risk measures for establishing capital requirements. In this paper we use a stochastic mortality and interest rate model to assess the benefits of natural hedging for a life insurer in terms of risk based capital. The mix of life insurance business is determined and the effect of different product types and measures of capital are analyzed. The impact of profit loadings on the different products is assessed and shown to have important implications for the business strategy of the life insurer aiming to grow its life annuity business in conjunction with its life insurance business.

2 Methodology

The assessment of natural hedging strategies involves the simulation of different product types with stochastic mortality and interest rates.

---

[1] United States Census Bureau: http://www.census.gov/

[2] APRA LI Quarterly performance downloadable at www.apra.gov.au

2.1 Mortality model

The mortality model for the natural hedging analysis is that of Schrager (2006). This is an affine mortality model with closed form solutions for some specifications. The existence of closed form solutions for survival probabilities greatly simplify valuations and pricing of insurance policies.

Affine stochastic mortality models have a mortality intensity which has a stochastic differential equation of the following form:

$$d\mu_x(t) = a(t, \mu_x(t))dt + \sigma(t, \mu_x(t))d\widetilde{W_x}(t) + dJ_x(t), \tag{1}$$

where $\mu_x(t)$ is the mortality intensity at time t for a life aged x, $J_x(t)$ is the value of a jump process at time t for a life aged x, $\widetilde{W}_x(t)$ is a d-dimensional Brownian motion at time t for a life aged x, $a(t, \mu_x(t))$ is the drift and $\sigma(t, \mu_x(t))\sigma(t, \mu_x(t))'$ is the covariance matrix.

The probability of survival for a life aged x from time t to T is

$$S_x(t, T) = \mathbb{E}\left[\exp\left(-\int_t^T \mu_x(s)ds\right)\middle| \mathcal{F}_t\right] \tag{2}$$

where $\mathcal{F}_t$ is the filtration generated by the process up until time t as shown in Luciano et al. (2012*a*).

Schrager (2006) proposes that a stochastic Makeham mortality model of the following form can be used to model mortality,

$$\mu_x(t) = Y_1(t) + Y_2(t)c^x, \tag{3}$$

where $\mu_x(t)$ is the mortality rate for a life aged x at time t, c is the shape parameter, $Y_1(t)$ and $Y_2(t)$ are stochastic factors.

The stochastic component of the mortality model arises from Y_i, assumed to follow an Ornstein-Uhlenbeck process with stochastic differential equations of the form

$$dY_i(t) = a_i(\theta_i - Y_i(t))dt + \sigma_i dW_{it}^P, \tag{4}$$

where $Y_i(0) = \bar{Y}_i$ and $dW_{1t}^P dW_{2t}^P = \rho dt$. With the further assumption that $\theta_2 = 0$, there is a closed form solution for survival probabilities as shown by Schrager (2006).

The advantage of the affine mortality model proposed by Schrager (2006) is that the whole mortality curve is modelled instead of that for specific ages and generations. This is critical for natural hedging because policy holders of different contracts may have different ages. Typically annuity holders will have different ages to life insurance policy holders. The Schrager mortality model allows for the dependence between the ages to be modeled in their model set up.

Estimation of the Schrager mortality model was carried out on UK male mortality data over the period 1960-2009 between the ages of 35-100 using the Kalman filter procedure outlined in Koopman and Durbin (2000).The number of ages was 66 and the number of years of data is 50. The estimated parameters are in Table 1.

Table 1: Estimated parameters for the Schrager mortality

Parameters	Estimate
a_1	0.00580
a_2	0.00891
σ_1	0.0000482
σ_2	0.00000116
c	1.103
s	0.0969
ll	14272
ll/(M*N)	4.325

2.2 Interest rate model

The interest rate risk is important in assessing a natural hedging strategy. Interest rate models are necessary for valuation and modelling interest rate risk. Existing natural hedging analysis, such as Wang et al. (2010) and Tsai and Tzeng (2010), assume constant interest rates. Stochastic interest dynamics are incorporated to model interest rate risk in natural hedging.

The interest rate model chosen is the Cox-Ingersoll-Ross (CIR) model proposed in Cox et al. (1985). This is widely used in finance and standard for interest rate term structure models, see Hull (2012) and Wilmott (2006). The CIR model uses the following stochastic differential equation to describe the dynamics of the interest rate process $r(t)$,

$$dr(t) = \kappa(\theta - r(t))dt + \sigma\sqrt{r(t)}dW(t) \tag{5}$$

where $r(t)$ is the short term interest rate at time t, θ is the long term mean of interest rate, κ is the speed of adjustment and σ is the volatility of the interest rate process.

The properties of the CIR process is that it is mean reverting to a level θ. Additionally, this process is strictly positive subject to the technical restriction,

$$2\kappa\theta \geq \sigma^2. \tag{6}$$

since interest rates are always greater than or equal to 0. This condition makes the Cox Ingersoll Ross process preferred to models such as the Vasicek model, which can have negative interest rates.

For the CIR model, the bond price process has a closed form solution which makes discounting future cash flows relatively straight forward and fast. The price of a zero coupon bond can be used as a discount factor and for the Cox Ingersoll Ross interest rate model, has a closed form solution as shown in Cox et al. (1985):

$$B(r,t,T) = C(t,T)\exp(-D(t,T)r)$$

where

$$\begin{aligned} C(t,T) &= \left[\frac{2\gamma \exp \frac{(\kappa+\lambda+\gamma)(T-t)}{2}}{(\gamma+\kappa+\lambda)(e^{\gamma(T-t)}-1)+2\gamma}\right]^{\frac{2\kappa,\theta}{\sigma^2}}, \\ D(t,T) &= \frac{2(e^{\gamma(T-t)}-1)}{(\gamma+\kappa+\lambda)(e^{\gamma(T-t)}-1)+2\gamma}, \end{aligned}$$

and

$$\gamma = ((\kappa+\lambda)^2+2\sigma^2)^{\frac{1}{2}}.$$

The calibration procedure of the Cox Ingersoll Ross interest rate model is maximum likelihood as outlined in Kladivko (2007). The Cox Ingersoll Ross interest rate model is calibrated to Australian 90 day bank accepted bills over the period of 1990-2009. The estimated parameters from the Cox Ingersoll Ross model are given in Table 2.

Table 2: Estimated parameters of CIR model

Parameter	Estimate
κ	0.533432
μ	0.051004
σ	0.034160
log-likelihood	4.608138

2.3 Life insurer model

The impact of natural hedging is considered on the balance sheet of a life insurer. The balance sheet of the insurer consists of the assets, liabilities and the surplus or equity. The insurer has assets which are from the premiums received from policy holders, the initial capital contributed from shareholders $E(0)$, and investment returns from invested assets. The money obtained from premiums and capital is assumed to be invested in riskless interest bearing assets. The value of the assets of the insurer at time t is denoted by $A(t)$.

Liabilities of the life insurer arise from the insurer either selling different types of life insurance policies or annuity contracts. The liabilities of the life insurers are the actuarial present value of future payments from both life insurance policies and annuity policies. The value of the liabilities at time t from life insurance policies is denoted by $L_{life}(t)$ whilst the value of liabilities from annuities is denoted by $L_{annuity}(t)$. Surplus of the insurer at time t, $E(t) = A(t) - L(t)$, is the value of the assets less the liabilities of the insurer.

We assume that at the beginning of each year the insurer invests its funds into interest bearing securities, such as a government bonds, yielding a return of $r(t)$ in year t. At the end of the year the insurer receives its premiums for life insurance policies in force next year denoted by $n_L(t)$.

$$A(t) = A(t-1)(1 + r(t)) + n_L(t) \cdot P_L - n_A(t) \cdot a - d_L(t) \cdot DB, \tag{7}$$

where $n_A(t)$ is the proportion of annuity policy holders in force at time t, $d_L(t)$ is the proportion of deaths of life insurance policy holders during the period $t-1$ to t, P_L is the premium for life insurance policies, DB is the death benefit for life insurance claims and a is the annual annuity payment.

Cash outflows for an insurer are the annuity payments and death benefits paid out to their respective policy holders. The proportion of annuity holders alive at time t is denoted by $n_A(t)$, which means that the life insurer pays out a total $n_A(t)$ for its annuity portfolio in year t. For the insurer's life insurance portfolio, the proportion of its initial life insurance policies dying between year $t-1$ and t is $d_L(t)$, resulting in the life insurer paying death benefits of $DBd_L(t)$ in year t.

There are two main drivers which cause the variability of a life insurer's surplus: the mortality of its policy holders and its investment returns. The interest rate will affect the insurer's asset accumulation and the rate which the insurer discounts its cash flows. Mortality experience of the insurer will influence both the premiums it receives, which is dependent on the number of life insurance policy holders alive as well as the payments the life insurer has to make to policy holders. Idiosyncratic mortality risk is not considered, because as shown in Gatzert and Wesker (2012) this can be diversified by an insurer through selling a sufficiently large number of policies.

3 Results

Existing studies identify various factors affecting natural hedging potential of longevity risk and these have been analyzed and quantified. Our analysis considers the effect of an insurer issuing different types of life insurance contracts and the relative pricing of policies. This analysis is useful for an insurer considering issuing new products and the effect pricing has on the overall risk of a portfolio of different policies. Insurers may sell different types of life insurance ranging from fixed term to whole life insurance. Premiums the insurer receives may either be a lump sum in the form of a single premium or paid annually to the insurer in the form of level premiums.

This section considers the impact of the policy type on the effectiveness of natural hedging. The three types of life insurance policies considered are the single premium whole life insurance, level premium whole life insurance and level premium 30 year term insurance. Moreover, we look at the effect profit loadings have on the risk minimizing portfolio. Finally, we investigate the sensitivity of the risk measure on the natural hedging potential of a life insurers capital and solvency requirements.

The risk measure that is being used to compare risks between the different types of policies is the 99% Value at Risk (VaR). The 99% Value at Risk is the amount of money the insurer needs to set aside today for the insurer to have a positive surplus in the final year with a probability of 99%. We express VaR as a percentage of the initial liabilities of the insurer. This allows our results to be generalized to life insurance portfolios of any size and is used in other studies such as Ngai and Sherris (2011).

3.1 The effect of life insurance product types

First, we consider in Table 3 and Figure 1 the effect of insurance products on the risk levels of a life insurer. All three products have a profit loading of 0%. Figure 1 shows the VaR as a percentage of liabilities across different policy types and for a range of portfolio compositions. Five thousand simulations are used, which leads to relatively narrow confidence intervals for estimates of the 99% VaR.

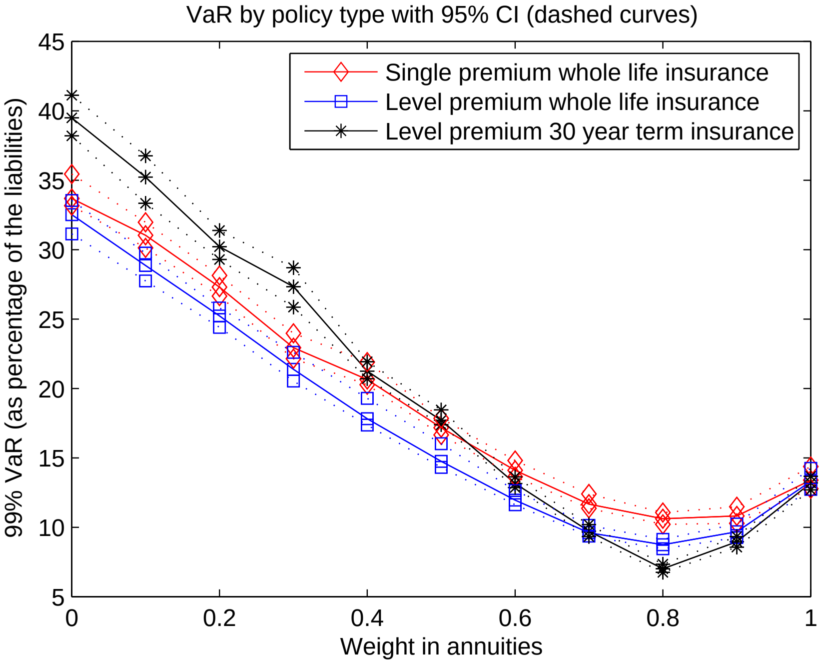


Figure 1: Life insurance product types

Table 3: Risk levels of policy types

Insurance product	99% VaR
Whole life insurance single premium	33.7%
Whole life insurance level premiums	32.5%
Term insurance level premiums	39.5%
Whole life annuity	13.3%

From Table 3 and Figure 1, we observe that life insurance business is significantly more risky than the whole life annuity business. Life insurance products are much more sensitive to mortality risk and interest rate risk than annuities. Moreover, the results indicate that an insurer has a higher risk exposure when the insurer receives a single premium for life insurance products than when it receives level premiums. This is due to the investment smoothing effect for level premiums life insurance. Where premiums are received as an initial lump sum, the insurer is significantly more impacted by investment risk than in the case of premiums as a stream of payments.

The natural hedging effect is well illustrated in Figure 1, for example there exists portfolios with a lower Value at Risk than a portfolio consisting of only annuities or life insurance. The optimal portfolio in terms of lowest Value at Risk comprises of approximately 80% annuities and 20% life insurance in terms of premium values. Our findings are consistent with the results documented in existing literature. Gatzert and Wesker (2012) also finds that a portfolio consisting of around 80% annuities leads to the lowest risk. The paper suggests that by utilizing such a portfolio, the insurer's sensitivity to mortality risk and overall risk will be minimized through the natural hedging effect.

The type of life insurance policies the life insurer sells significantly affects the natural hedge potential of a portfolio. As we can see from Figure 1, the annuity portfolio weight that minimizes the Value at Risk is similar for all three portfolios, namely approximately 80%. Although the portfolio compositions are similar, the degree of risk reduction relative to an annuity portfolio is different. Therefore, in Table 4 we illustrate the relative risk reduction for those natural hedged portfolios compared to a portfolio consisting of annuities only.

Table 4: Relative risk reduction relative to annuity portfolio

Insurance product	Relative reduction in VaR
Whole life insurance single premium	20.7%
Whole life insurance level premiums	34.3%
Term insurance level premiums	47.4%

First, from Table 4 we observe that life insurance with level premiums has a larger hedge effectiveness than single premium life insurance. This is due to a cash flow matching effect in the case the insurer issues life insurance policies with level premiums. The stream of premiums the life insurer receives from life insurance policy holders has the same effect as the insurer buying an annuity. The longevity risk the insurer is exposed to through level premium life insurance is partly of the form of going short in an annuity due to the stream of premiums it receives. With improving mortality, the life insured will pay premiums for longer whereas the insurer has to pay out more annuity payments as annuitants live for longer. Therefore, the hedge potential for level premium life insurance product is larger than for single premium life insurance products.

Second, from Table 3, Table 4 and Figure 1 we observe that term insurance can be used to construct a portfolio that has the lowest risk, although term insurance has the highest VaR of the investigated products. This is because the value of term insurance has a relatively high sensitivity to mortality risk. More importantly, the duration of term insurance matches more closely to the duration of the whole life annuity portfolio. Due to the different ages of insured for life insurance and annuities, the annuity business will be run off in approximately 35 years whilst the whole life insurance business will be run off over approximately 65 years. Term insurance on the other hand is run off over 30 years, which means the cash flows from the term insurance more closely match the cash flows of the annuity business. The two products are exposed to the same shocks in mortality, whereas for the whole life insurance the life insurance product is exposed to longevity risk beyond 30 years, which has a limited effect on longevity risk for annuities. Therefore, when using whole life insurance business for hedging longevity risk in annuities, there is still approximately 30 years worth of mismatched cash flows. Further payments can still hedge current exposure to mortality risk, however the effectiveness diminishes as the mismatch in the common factors becomes larger.

3.2 The effect of annual renewal of life insurance

In the previous subsection we investigated the hedge potential of different life insurance products. In this subsection we investigate the effect of dynamic pricing of life insurance products. In this case, each year the premiums are set by the life insurer depending on the observed mortality experience and the interest rate. In this dynamic pricing strategy we assume that the insurer issues a series of 1-year term life insurance at an actuarial fair value for 30 years instead of issuing a 30-year term insurance at an actuarial fair value. We assume that the insured renew annually, if they are alive, for 30 years and that there are no lapses in life insurance renewals.

For comparability, the death benefits of the 1 year term insurance are set equal to those of the 30 year term insurance. Figure 2 illustrates the effect of having dynamic premiums in case of actuarial fair premiums.

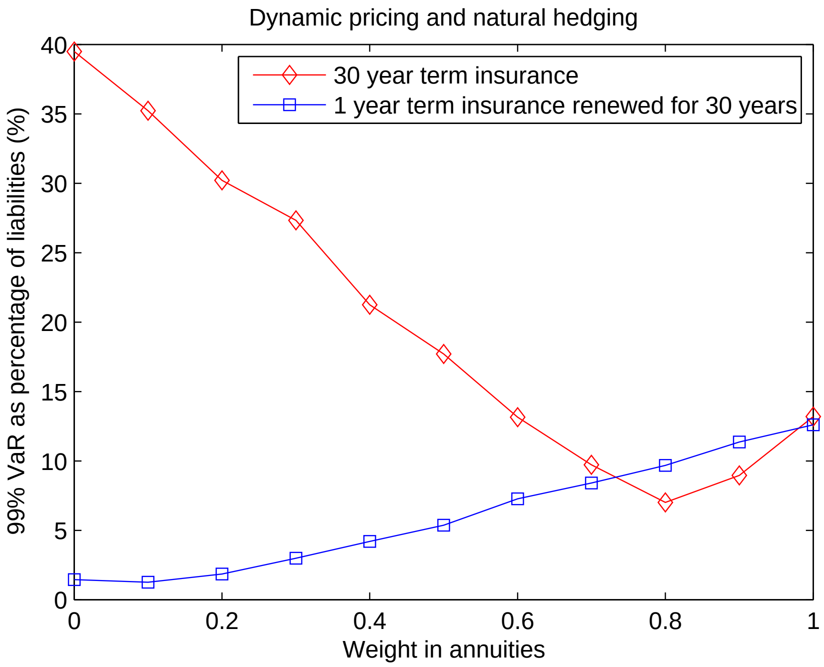


Figure 2: Dynamic pricing

The first observation from Figure 2 is that annual renewed life insurance products are much less riskier than 30 year term insurance, because the risk is shifted from the insurer to the insured. Longevity risk is largely borne by the insured through the yearly adjustment in the actuarial fair price to reflect its mortality experience. The life insurer is only exposed to the sequence of one year uncertainty in mortality and interest rate risk given the updated information of the underlaying processes. The risk exposure for the insurer is substantially reduced, because changes in mortality and interest rates are priced into contracts after one year. Thus, for annual renewed life insurance products most of the systematic mortality risk is transferred to the insured.

Second, from Figure 2 we observe that the natural hedging benefit are limited for portfolios consisting of annuities and term life insurance products which are renewed each year. This is a consequence of the low remaining longevity risk in the life insurance product. The natural hedging benefit from 30 year term life insurance is so substantial that, although the risk in 1 year term life insurance is small, there are portfolios of term life insurance and annuities which have a lower risk than portfolios with 1 year term insurance and annuities. Hence, an insurer which is exposed to longevity risk through annuities, can reduce longevity risk by offering products insuring more risk i.e., offering 30 year term insurance instead of 1 year term insurance renewed for 30 years. In this case both the insurer and the insured are better off, because the risk of the life insurance product away from the insured and hedges the risk for the insurer.

3.3 The effect of profit loading

In the previous subsections we have investigated the effect of combining different products has on the required capital an insurer should hold in addition to the premiums received from selling the product. Life insurance products and annuities are both exposed to systematic longevity risk. We illustrate the effect profit loadings, for example as compensation for systematic longevity risk, could have on the risk minimizing portfolio.

First, we investigate the effect of parallel shifts in the profit loadings for both insurance products. Figure 3 illustrates the effect of the profit loading for the cases where both the profit loading for 30 year term insurance and annuities are 0%, 10% and 20%.

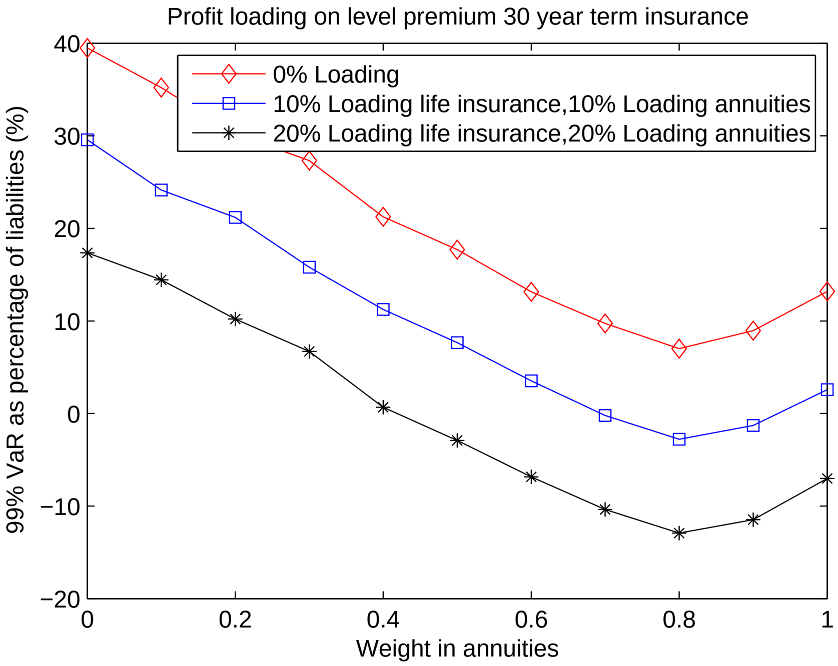


Figure 3: Level premium 30 year term insurance with equal profit loading

From Figure 3 we observe that the Value at Risk minimizing portfolio in all three cases consists of 80% annuities and is insensitive to profit loading when both products have equal profit loadings. Moreover, we observe that shifts in the 99% Value at Risk appear to be parallel when profit loadings are equal. Hence, the optimal portfolio weights in this case do not depend on the size of the profit loading, but it might depend on the profit loading, when the profit loading of the products is different.

Next, we investigate the effect of the different profit loadings for the products on the hedge effectiveness of combining different products. Using traditional actuarial pricing methods, such as VaR or standard deviation principle, the profit loading for life insurance products would be larger than for annuities as they are more risky. However, in case of a complete and liquid market, longevity risk might have its own market beta, and given that the exposure to longevity risk for life insurance products and annuities are opposites, one product would have a positive loading and the other product a negative loading. Hence, it is likely that the the loading for the products would be different. To illustrate the impact of profit loading on the risk minimizing portfolio, the following cases are considered:

i) 0% profit loading life insurance 0% profit loading annuity

ii) 20% profit loading life insurance 0% profit loading annuity

iii) 0% profit loading life insurance 20% profit loading annuity

The first case, illustrated in Figure 4, is a single premium life insurance and annuity portfolio.

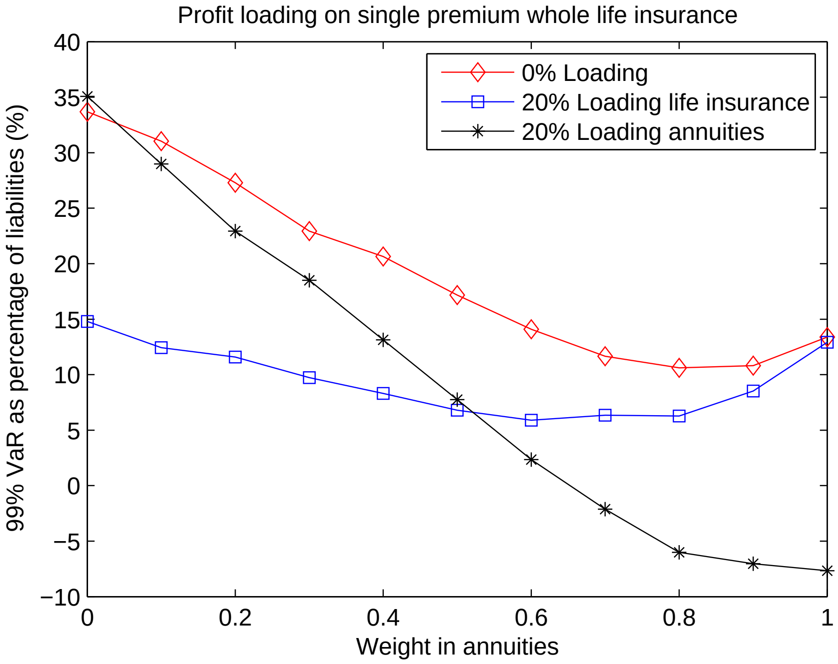
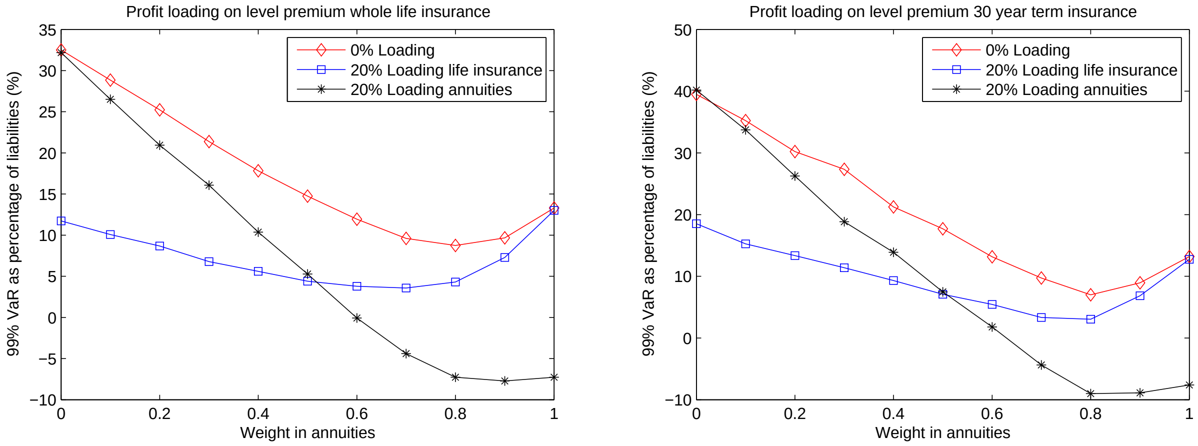


Figure 4: Single premium whole life insurance with 20% profit loading

From Figure 4, we observe that the required additional reserve depends substantially on the profit loadings. When the profit loadings for life insurance and annuities are equal, the risk minimizing portfolio consists of approximately 80% annuities. However, as discussed in the introduction, in most countries the exposure to life insurance is much larger than 20%. That would imply that the risk minimizing portfolio would be obtained by having a substantial higher profit loading for life insurance than annuities. Note that VaR is negative for some portfolios, which implies that the insurer does not have to hold capital in addition to the premium he receives, but could pay out dividends whilst still maintaining a solvency level of 99%.

The second case we consider is the effect of profit loading on the VaR for portfolios with annuities and level premium whole life insurance or level premium 30 year term insurance, which are illustrated in Figure 5.

Figure 5: Level premium whole life insurance and level premium 30 year term insurance with 20% profit loading


(a) Level premium whole life insurance with 20% profit loading

(b) Level premium 30 year term insurance with 20% profit loading

From Figure 5a, we note that the findings observed for single premium whole life insurance and level premium whole life insurance are comparable. However, the sensitivity to profit loading for annuities becomes smaller. Note that the portfolio weights for the risk minimizing portfolio for single premium whole life insurance is different to the portfolio constructed with single premium whole life insurance. The VaR minimizing portfolio for level premium life whole insurance has a higher proportion of assets in life insurance compared to single premium whole life insurance with the same profit loading factors. As can also be observed in Figure 5b, term insurance has the greatest natural hedge potential even with profit loadings. Compared to the other types of life insurance, the diversification benefit from selling term insurance is much higher than that of the other two policy types, even with the addition of profit loading. In contrast to whole life insurance policies, the optimal natural hedging portfolio has an annuity weighting of 80% for all investigated profit loadings.

Based on the relative risk levels of insurance and annuity policies, profit loadings for life insurance should be higher than profit loadings used for annuities to compensate an insurer for accepting higher risk. However, an insurer can reduce risk through selling life insurance policies due to the natural hedging effect, which should result in selling life insurance at a lower price compared to other life insurers. This is similar to the observations of Cox and Lin (2007), who find that life insurers who sell both life insurance and annuities have lower annuity prices compared to those who don't sell life insurance.

3.4 The effect of the risk measure

An insurer will typically not be able to fully diversify all their risks. The hedge effectiveness of combining different products may depend on which risk measure is used. In previous calculations the 99% VaR, i.e., the amount of money the insurer needs to set aside today for the insurer to have a positive surplus in the final year with a probability of 99%, risk measure has been used. This risk measure has been widely used, see e.g., Friedberg and Webb (2007) and Tsai and Tzeng (2010). However, there are different ways to measure risk, which might affect the decision of the portfolio of products for an insurer. Firstly, in this section we will look at the effect of using the risk measure where the surplus of the insurer should be positive at each point in time, instead of only after the last payment. Secondly, we will look at the risk measure where only has to be a positive surplus after one year, which is similar to Solvency II.

3.4.1 Multi period VaR

Traditionally, literature has used a run-off approach to quantify the risk in life insurance products. The corresponding risk measure was the amount of capital the insurer needs to hold in order to be able to pay the liabilities with a certain percentage, i.e., the 99% VaR in the previous subsections. In case of a single premium and no investment risk this is equivalent to having non-negative wealth at each point in time. However, in our analysis we also investigate portfolios with level premium products, where the insurer would be able to recover from a negative surplus because of the level premiums. More importantly, an insurer might not be able to survive with positive wealth, but a negative surplus. To account for that the multi period VaR risk measure, which is the amount of reserves the insurer should hold in order to have nonnegative surplus at *each* point in time. The multi period VaR is the amount of capital required at time 0, $E(0)$ in addition to the premium. Denote the reserve required by $E(0)$, then the optimization problem is:

$$U = \underset{E(0)}{\operatorname{argmin}} \left\{ \Pr\left(E(t) > 0, \forall t \right) = \alpha \right\}, \tag{8}$$

where α is the confidence level of the insurer not defaulting at any point in time. Note that the surplus is also affected by the liability value, due to observed mortality experience and the number of insured still are alive.

Figure 6 displays the effect of the portfolio weight in annuities for different product types using the 99% VaR (dashed curves) and the multi period 99% VaR.

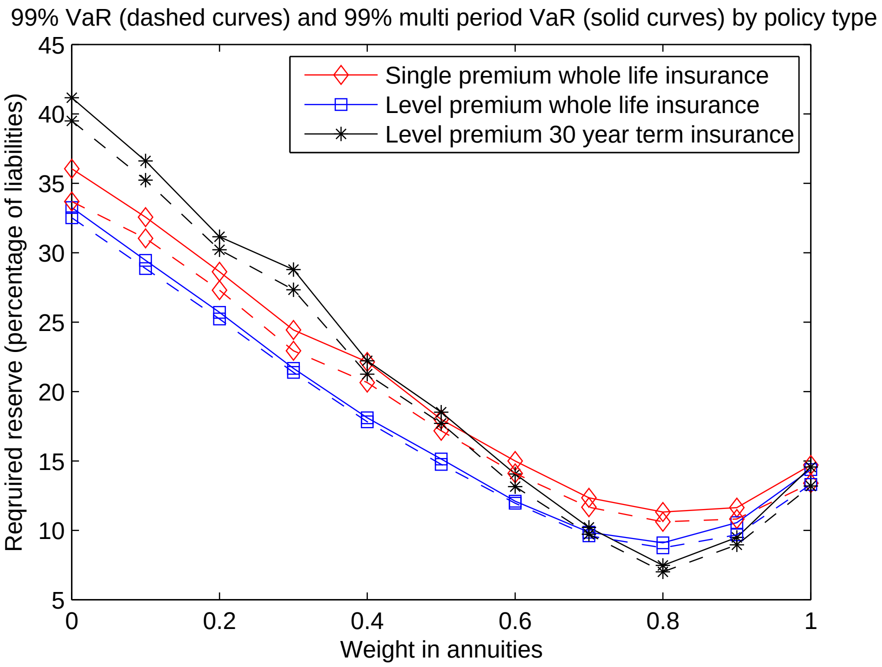


Figure 6: Multi period VaR by product type

From Figure 6 we observe that the 99% multi period VaR is higher than the 99% VaR for the entire range of portfolio compositions. This is because the multi period VaR requires the surplus to be positive at all points in time whilst VaR only accounts for the last time point.

From Figure 6 we also observe that the multi period VaR is similar to the VaR. This indicates that the optimal portfolio for an insurer might not be substantially affected by using either a VaR or a multi period VaR risk measure. In our selected portfolios the hedge potential of the different products using multi period VaR and VaR risk measure is similar. Therefore, insights from existing literature on the hedge potential of life insurance products using the VaR risk measure are expected to hold for the multi period risk measure.

3.4.2 One year VaR

Another risk measure, which has gained more popularity recently due to Solvency II regulation, is the one year VaR. Solvency II stipulates that insurers must hold sufficient capital to have a positive surplus next year with probability 99.5%. Figure 7 displays the effect the portfolio composition has on the capital requirements using the 99% VaR (dashed curves) and the one year 99.5% VaR (solid curves).

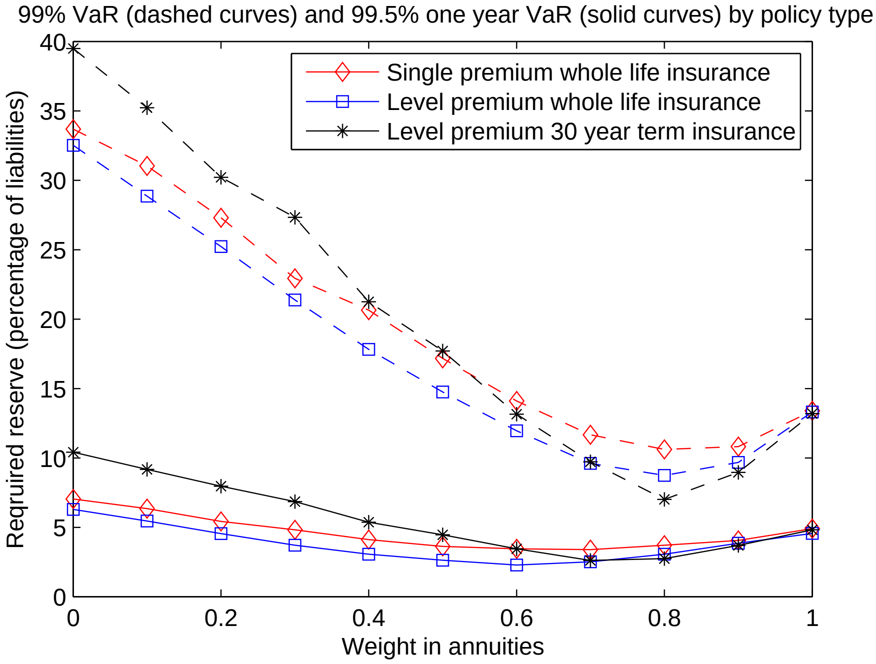


Figure 7: Solvency II capital by product type

From Figure 7 we observe that the required capital savings from holding the optimal natural hedging portfolio is approximately 31%, which is similar to the required capital savings in the VaR risk measure. As expected, Solvency II capital requirements are significantly lower than the VaR, because the Solvency II only considers a 1 year time horizon. Moreover, we observe that whole life term insurance requires relatively much more capital than single premium (48% in case of one year VaR v.s. 17% in case of VaR) or level premium 30 year term insurance (66% in case of one year VaR v.s. 21% in case of VaR). This is because term insurance is more sensitive to mortality risk as shown in Section 3.1. For the Solvency II capital requirements for whole life insurance is approximately 40% higher than that of annuities whilst over a 65 year horizon, whereas it is 150% case of the VaR capital requirements. This indicates that relative riskiness between the whole life insurance and annuities change over time.

Whereas there is only a small difference in the hedge effectiveness of different portfolios when comparing the VaR risk measure with the multi period VaR risk measure, there is a substantial difference in the hedge effectiveness when comparing the VaR risk measure with the one year VaR risk measure. Both the product types and the optimal portfolio weight in annuities is substantially different for the two risk measures.

For the one year VaR risk measure the lowest reserve requirement portfolio consists of annuities and level premiums life insurance. This is a different risk minimizing portfolio than using the VaR risk measure, consisting of annuities and single premium whole life insurance. This is because single premium whole life insurance has a higher exposure to interest rate risk. The higher exposure is due to the fact that for the single premium life insurance product the insurer receives its premium at the moment of the transaction and the premium is invested. On the other hand, for level premium whole life insurance the cash flows are spread out which leaves the insurer less exposed to interest rate risk, resulting in a lower overall risk level for the insurer.

Finally, another difference between the hedge potential in the VaR risk measure and the one year VaR risk measure is that the portfolio with the lowest one year VaR reserve requirement has a lower weight in annuities. This is because life insurance products are relatively less risky initially and whole life insurance becomes more risky relative to annuities over time.

4 Conclusion

This paper quantifies the natural hedging effects of portfolios of life insurance and annuity products, taking into account relative profit loadings on products. It illustrates the importance of the type of life insurance products offered using standard life and annuity products. The paper uses stochastic mortality and interest rate models to assess life and annuity capital requirements.

For the product types our main findings are that the capital requirements are higher when offering single premium life insurance than level premium life insurance policies and 30 year term life insurance products best hedge longevity risk for 65 year old annuitants. Both results are due to cashflow hedging effects. The premium for the level premium life insurance has the opposite cashflow for the insurer compared to that of an annuity. Term insurance provides a better hedge because the duration of life insurance products, for which the insured are typically younger, is not much longer than the duration of the annuities. Otherwise life insurance product will be exposed to longevity risk beyond the term of the annuity which reduces the hedge effectiveness of life insurance products.

Another finding for the product types offered is that annual renewal of life insurance products leads to a shift of the risk from insurer to insured. However, this reduces the hedge effect that life insurance provides, resulting in higher capital requirements than in the case of level premiums.

When profit loadings are included we find that the VaR minimizing portfolio is not influenced by the profit loading in the case of equal profit loadings for life insurance and annuities and in the case of 30 year term insurance and annuities. However, in the other cases the VaR minimizing portfolio is substantially affected by the profit loadings. As expected, the profit loading is important when an insurer is deciding which product to offer or the portfolio share in life insurance products when they differ across products.

We show that our results hold for VaR and multi period VaR risk measures. However, the one year VaR risk measure, as in Solvency II, has a higher VaR minimizing portfolio weights in life insurance, because whole life insurance becomes more risky relative to annuities over time. This also implies that in the case of dynamic hedging, instead of static hedging as considered in this paper, the optimal portfolio share for life insurance required to hedge longevity risk in annuities would decrease with the duration of the annuity.

Acknowledgements

Andy Wong acknowledges support from The UNSW Honours Scholarship and the 2012 ARC Linkage Project Managing Risk with Insurance and Superannuation as Individuals Age Honours Scholarship. The authors would like to also acknowledge the financial support of the Australian Research Council Centre of Excellence in Population Ageing Research (project number CE110001029).

References

Bauer, D. and Weber, F. (2008), 'Assessing investment and longevity risks within immediate annuities', *Asia-Pacific Journal of Risk and Insurance* **3**.

Blake, D., Cairns, A. and Dowd, K. (2008), 'The birth of the life market', *Asia-Pacific Journal of Risk and Insurance* **3**.

Cox, J., Ingersoll, J. and Ross, S. (1985), 'A theory of the term structure of interest rates', *Econometrica* **53**, 385–407.

Cox, S. and Lin, Y. (2007), 'Natural hedging of life and annuity mortality risks', *North American Actuarial Journal* **11**, 1–15.

Friedberg, L. and Webb, A. (2007), 'Life is cheap: Using mortality bonds to hedge aggregate mortality risk', *The B.E. Journal of Economic Analysis and Policy* **7**.

Gatzert, N. and Wesker, H. (2012), 'The impact of natural hedging on a life insurer's risk situation'.

Hull, J. (2012), *Options, Futures, And Other Derivatives*, Pearson Education.

Kladivko, K. (2007), 'Maximum likelihood estimation of the cox-ingersoll-ross process: the matlab implementation'.

Koopman, S. J. and Durbin, J. (2000), 'Fast filtering and smoothing for multivariate state space models', *Journal of Time Series Analysis* **21**(3), 281–296.

Luciano, E., Regis, L. and Vigna, E. (2012*a*), 'Delta-gamma hedging of mortality and interest rate risk', *Insurance: Mathematics and Economics* **50**, 402–412.

Luciano, E., Regis, L. and Vigna, E. (2012*b*), 'Natural delta gamma hedging of longevity and interest rate risk'.

Ngai, A. and Sherris, M. (2011), 'Longevity risk management for life and variable annuities: The effectiveness of static hedging using longevity bonds and derivatives', *Insurance: Mathematics and Economics* **49**, 100–114.

Schrager, D. (2006), 'Affine stochastic mortality', *Insurance: Mathematics and Economics* **38**, 81–97.

Tsai, J.T., W. J. and Tzeng, L. (2010), 'On the optimal product mix in life insurance companies using conditional value at risk', *Insurance: Mathematics and Economics* **46**, 235–241.

Wang, J., Huang, H., Yang, S. and Tsai, J. (2010), ‘An optimal product mix for hedging longevity risk in life insurance companies: The immunization theory approach’, *The Journal of Risk and Insurance* **77**, 473–497.

Wilmott, P. (2006), *Paul Wilmott on Quantitative Finance*, Wiley.