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Abstract

Mortality models used to assess longevity risk and retirement funding have been extended to stochastic models with trends and systematic risk. Systematic risk cannot be readily diversified in an insurance pool or pension fund. It is an important factor in assessing solvency and highlighting the tail risk in longevity insurance and pension products. Idiosyncratic risk can be diversified in typical pool sizes, although less effectively at the older ages. Mortality heterogeneity is not usually taken into account in stochastic mortality models. This is a mortality risk that reduces the effectiveness of idiosyncratic mortality risk pooling. Heterogeneity has been modelled with frailty models and more recently with Markov multiple state ageing models. This paper overviews recent developments in models for mortality heterogeneity and uses a model calibrated to both population mortality and health condition data to consider the impact of model risk and heterogeneity in assessing solvency and tail risk for longevity risk products.

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Mortality improvements have been systematic, in that they have impacted individuals of all ages, although to varying extents by age and across time for many countries. Mortality improvement rates have also shown varying trends (Njenga and Sherris 2011). Pension funds and insurance companies issuing life annuities have been exposed to this systematic risk and this has the potential to impact solvency especially in the tail of the distribution of survivors. Although some of this risk has been transferred to reinsurers using reinsurance and longevity swaps, much of this risk is accumulating with insurers, pension funds and reinsurers and has not been diversified into the broader financial markets (Blake, Courbage, MacMinn, and Sherris 2011).

Systematic risk is modelled using a doubly stochastic survival model where the mortality rate follows a stochastic process and all individuals of the same age and gender experience the same realised mortality rate. Given the mortality rate, individual survival is subject only to idiosyncratic risk, which can be diversified in large pools of lives. Even if there is only idiosyncratic risk, at the older ages the number of lives surviving becomes small and variability in benefit payments and liability values increases in the tails of the survival distribution. This is exacerbated by systematic risk in the form of uncertain but common rates of improvement across individuals.

Many models of systematic mortality risk have been proposed. These vary from models such as the Lee-Carter model (Lee and Carter 1992), and variations, to models that model random changes in a parametric survival curve (Cairns, Blake, and Dowd 2006), to those that model the dynamics of mortality rates in a financial framework similar to that used for interest rate models (Biffis 2005). These models do not include allowance for heterogeneity. Individuals of the same age experience the same aggregate mortality rate.

Increasingly attention is being given to the impact of mortality heterogeneity and its impact on insurers and pension funds (Su and Sherris 2012; Lin and Liu 2007; Liu and Lin 2012). Along with systematic mortality risk, heterogeneity has implications for solvency and tail risk of annuity and pension providers. Even if there were no systematic, or aggregate, mortality risk, heterogeneity generates variability in future experience and volatility in financial results. Heterogeneity requires underwriting of risks to avoid adverse selection. Without full information about the risks that insurers underwrite, the financial impact of adverse selection has its greatest impact for annuities in the tail of the survival distributions long after the annuities have been issued.

Solvency and tail risk for life annuities and pensions have two dimensions. There is an impact on insurer profitability from adverse experience as well as an impact on variability at the older ages. Trends in mortality that arise from uncertain mortality improvements and from the deaths of less healthy lives in a heterogeneous pool have their greatest impact at the older ages. The volatility of financial results arises from both systematic mortality changes, with higher volatility experienced at older ages, and from heterogeneity also producing in higher volatility at older ages (Su and Sherris 2012; Meyricke and Sherris 2013).

There are many different approaches to modelling mortality heterogeneity. Recent advances have seen the calibration and application of more advanced models in the form of Markov ageing models (Su and Sherris 2012; Lin and Liu 2007; Liu and Lin 2012) that are extensions of the Le Bras model (Le Bras 1976). The other, more commonly used, approach is to apply frailty models to capture unobserved heterogeneity (Vaupel, Manton, and Stallard 1979).

In this paper we develop and apply a stochastic Markov ageing model of heterogeneity that is calibrated to population aggregate mortality and health data that also includes systematic

mortality risk. We compare results with a well-known frailty model and the Le Bras Markov multiple state model to assess model risk, neither of which includes systematic mortality risk. The models are used to quantify solvency and tail risk for a portfolio of life annuities using risk measures standard deviation and value-at-risk for fund values at the older ages. Results demonstrate the impact of heterogeneity and model risk on the assessment of longevity risk for these portfolios, as well as the impact of selection and pool size.

Mortality heterogeneity models

The main approaches to modelling mortality heterogeneity we consider are frailty models and Markov multiple state models. Frailty models treat heterogeneity as unobservable. An often used frailty model is that of Vaupel, Manton, and Stallard (1979) where an individual is assumed to have frailty Z at age x with force of mortality: $\mu(x, Z) = Zae^{bx} + c$. The frailty factor Z is gamma distributed $Z \sim \text{Gamma}(1, \sigma^2)$ so that the average frailty at age x is

$$\bar{Z}(x) = \left(1 + \sigma^2 \frac{a}{b} (e^{bx} - 1) \right)^{-1}$$

and the average force of mortality is given by $\bar{\mu}(x) = \bar{Z}(x)ae^{bx} + c$.

The Markov multiple state mortality model was developed by Le Bras (1976). Le Bras (1976) used a continuous time Markov chain with an infinite number of states and a discrete state space to model senescence. The model starts at state 1, and progresses to state 2, 3, etc. In any state, the rate of jump to the next higher state and the rate of death are proportional to the state number. All individuals start in state 0 at time 0. In state i , the transition rate to state $i+1$ is $\lambda_0 + i\lambda$, and the transition to death (an absorbing state) is $\mu_0 + i\mu$. For the Le Bras model the probability of being in state i at time t is (Yashin, Iachine and Begun 2000: 13):

$$P_i(t) = \frac{e^{-(\lambda_0 + \mu_0)t}}{i!} \left(\frac{\lambda - \lambda e^{-(\lambda + \mu)t}}{\lambda + \mu} \right)^i \prod_{k=1}^i \left(\frac{\lambda_0}{\lambda} + (k-1) \right)$$

The probability of survival to time t , given the individual was in state n at time 0 is give by

$$S_n(t) = e^{-(\lambda_0 + \mu_0 + n(\lambda + \mu))t} \left(\frac{\lambda + \mu}{\mu + \lambda e^{-(\lambda + \mu)t}} \right)^{\frac{\lambda_0 + n\lambda}{\lambda}}$$

Yashin, Vaupel and Iachine (1994) show the representation of the average force of mortality in the fixed frailty model to be equivalent to the Le Bras' model. The two are equivalent when:

$$a = \frac{\lambda_0}{\lambda} \times \mu$$

$$b = \lambda + \mu$$

$$c = \mu_0 - \frac{\lambda_0}{\lambda} \times \mu$$

$$\sigma^2 = \frac{\lambda}{\lambda_0}$$

Markov ageing models have the potential to account for observed heterogeneity. Although there have been several applications of the distribution of failure time of a Markov chain to mortality, also known as phase-type distributions, Lin and Liu (2007) developed a deterministic survival rate model based on a Markov ageing process. Each state in the model represents a “physiological age” as opposed to calendar age. The model assumes that there is a maximum physiological age n and they assumed that $n = 200$ is appropriate as an approximation to the potentially infinite ageing process, assumed in the Le Bras's model theory. Su and Sherris (2012) developed the Lin and Liu (2007) to assess population heterogeneity for life annuity portfolios and relate states and mortality rates to aggregate population mortality.

These two Markov ageing models have parameters that naturally capture the changes in the observed period life tables. Liu and Lin (2012) make the model stochastic by adding a time change component. The small number of states and the transition matrix facilitate the incorporation of health information. The time change allows a probabilistic statement of mortality uncertainty. The initial distribution is estimated from health condition data and closed forms for the expected value and variance of the survival probability exist if the stochastic time change process has a closed form moment generating function.

These Markov ageing models are the basis of the model used in this paper. We extend the Su and Sherris (2011) model to include health states calibrated to health conditions data as well as population aggregate mortality data. We also subordinate this underlying model to a Gamma time change so that survival distributions are stochastic. The underlying model allows an assessment of model risk by comparison of results for solvency and tail risk with the other models of heterogeneity. The subordinated model shows the significance of heterogeneity if mortality is stochastic.

The Markov ageing model used has 3 sets of transition matrices, each with 5 transient states and 1 absorbing state, fitted separately to ages 40-70, 70-90 and 90-110. Transition occurs as a Markov process from one transient state to its next state, or to the absorbing state, and the model takes into account both health status and mortality data. Aggregate survival rates are determined by a deterministic underlying multiple states survival model $S_0(\cdot)$ and a time change process γ_t . The underlying model assumes the individual mortality process moves through a series of deteriorating health statuses. Health and mortality is made stochastic by a random time change. The aggregate survival rate at time t is $S(t) = S_0(\gamma_t)$. The time until absorption or death, in this system has a phase type representation (π, T) , where π is the initial distribution

on the transient states, and T is the states' transition rates matrix. The probability of survival up to time x is $S_0 = \pi \exp(Tx) e$ where e is a column of 1's. Under the assumption that deterioration in health is more likely than improvement, transition is assumed to be acyclic. Since all acyclic phase type distributions have a Coxian representation, T can be written as:

$$\begin{pmatrix} -(\lambda_1 + q_1) & \lambda_1 & 0 & 0 & 0 \\ 0 & -(\lambda_2 + q_2) & \lambda_2 & 0 & 0 \\ 0 & 0 & -(\lambda_3 + q_3) & \lambda_3 & 0 \\ 0 & 0 & 0 & -(\lambda_4 + q_4) & \lambda_4 \\ 0 & 0 & 0 & 0 & -q_5 \end{pmatrix}$$

where λ_i is the rate of transition from state i to state $i + 1$, and q_i is the rate of transition from state i to the absorbing (death) state. A restriction was placed on the values of q such that the five transient states must have increasing q . This ensures that individuals in better health states do not have higher death rates.

The 5 states are 5 divisions of the full range of physiological ages. The model is made non-homogeneous using splines. $S_0(x)$ with splines at s_i 's can be represented as:

$$S_0(x) = \begin{cases} \pi \exp(T_1 x) e & \text{for } 0 \leq x < s_1 \\ \pi \exp(T_1 s_1) \exp(T_2(x - s_1)) e & \text{for } s_1 \leq x < s_2 \\ \pi \exp(T_1 s_1) \exp(T_2(s_2 - s_1)) \exp(T_3(x - s_2)) e & \text{for } s_2 \leq x < s_3 \\ \text{etc} & \end{cases}$$

The position of the splines were determined through trial and error, taking into account of goodness of fit and the number of parameters, since each additional spline requires another transition matrix of 5 q 's and 4 λ 's.

The time change is modelled as a Gamma process which is non-decreasing, additive, and has a closed form moment generating function. It is defined as starting at $\gamma_0 = 0$ with independent increments $(\gamma_{t+s} - \gamma_t)$, which are Gamma distributed with mean s and variance vs .

The Markov ageing model is used in two ways. Its deterministic component (i.e. the underlying Markov process) is used for comparisons with the other deterministic heterogeneity

models. The subordinated model, which introduces the stochastic component, is used to assess the impact of systematic mortality risk since it introduces a risk factor that is common across the risk groups.

Data

Modelling mortality heterogeneity requires a basis to divide the population into groups of individuals expected to experience similar rates of mortality, distinct from the other groups. Calibration of these models requires information about the health status distribution and survival probability. This can be done using socioeconomic status, health conditions or health risk factors. Socioeconomic status and income level are related to mortality. However, the correlation is not definitive and mortality is driven by more specific factors than socioeconomic status. Health risk factors based on individual panel data can be used to relate failure time to health characteristics of individuals. Characteristics include various factors such as diastolic and systolic blood pressure, body mass index, cholesterol, blood sugar, vital capacity, cigarettes per day. This approach has significant data availability imitations at a population level.

Health risk factors such as obesity or smoking habits are less effective in capturing heterogeneity than existing health conditions such as heart disease or lung cancer. In addition, health condition data is more readily available than health risk factor information, which requires both the risk factor and its duration. The ideal form of data is that which records a cohort's experience through time. However, health data is generally only available for the population alive in a particular year so that period mortality data has to be used to match period health data.

For calibration of the models, the data used for estimating health status distribution, severity of the health conditions and population survival probabilities came from a variety of

sources. The National Health Survey (NHS) data used were for the prevalence of long term conditions, at 10 year intervals from age 15 to 75, across years 2007-2008. Self-assessed health for age intervals: 15-17, 18-24, 25-65, 65+, for years 2007-2008 and estimated average dementia prevalence at 5 year age intervals from 60 to 85 were also used.

The Australian Cancer Incidence and Mortality Books (ACIMB) were used for cancers incidence and mortality, for 5 year age intervals up to 85, to year 2008 and the WHO mortality database for Australia gave the number of deaths from a health condition, for 5 year age intervals until 95, to year 2006. The Australian Bureau of Statistics Causes of Death (ABSCD) provided the number of deaths from each condition, the aggregate of all ages, to year 2010. Australian life tables (from Human Mortality Database) up to age 110, to year 2010 along with Australian cohort mortality rates (from Human Mortality Database) up to year 2008 were used for mortality rate data.

Population health status distributions were estimated from prevalence of health conditions. Health conditions were ranked according to their severity and divided into 5 groups (or health states) and the distribution of the population for these 5 health states was determined from the data. The model does not take into account infectious diseases or accidents. All individuals are assumed to have the same exposure to these baseline risks. Health conditions were ranked by the probability of death from cause of death data given the prevalence of a condition. Since deaths by cause from WHO is only available up to 2006, and prevalence available for 2007-2008 data, the 2006 WHO data was scaled by the ratio of 2008 to 2006 numbers of deaths in the ABSCD.

To estimate the proportion of the population in each health state, health data is available at 10 year intervals, but the model requires distributions across ages. It was assumed that the prevalence of a condition for individuals for a 10 year age range could be used to represent the

expected prevalence at the midpoint age. It was also assumed that long term conditions are independent and that for a person affected by more than one condition, the highest death rate among all of the conditions was assumed to be the death rate. The proportion of individuals with a specific condition as their most severe condition was assumed equal to the proportion of individuals not affected by any worse condition multiplied by the proportion of the total population affected by the specific condition. The remaining proportion of individuals was assumed to have the best health status.

Calibration of Mortality Heterogeneity Models

Figure 1 shows the survival curve for the fitted Le Bras model and the Australian 2008 life table used for calibration. The model provides a better fit to the survival curve when fitted for ages above 20. The parameter values estimated for the Le Bras model 20+ are given in Table 1. The model is equivalent to the frailty model.

Figure 1 here

Table 1 here

Least squares was used to fit the models based on estimated health distributions and expected survival rates. The data used to calibrate the model are actual observations, so the estimation uses observed health and survival distributions as expected values. Fitting by least squares involves two steps. Firstly, a life table is used for expected values of survival rates. The sum of squared difference with model's estimation of $E(S(t))$ is minimized. A lower limit of

0.001 was imposed for ν to prevent a near zero denominator in the numerical estimation procedure. Other parameters were assumed to have a lower limit of 0. The estimated transition matrix was applied to the survival data to calculate the time difference between data points. Finally a Gamma distribution with mean 1 was fitted to the time differences to estimate ν .

The model does not assume birth cohorts differ and was fitted to period data.

The three matrices fitted to age intervals 40-70, 70-90 and 90-110 are shown below. This was found to be the optimal placement of splines.

Matrix 1:

$$\begin{pmatrix} -0.040674 & 0.040674 & 0 & 0 & 0 \\ 0 & -0.038392 & 0.038390 & 0 & 0 \\ 0 & 0 & -0.077902 & 0.077895 & 0 \\ 0 & 0 & 0 & -0.041452 & 0.036872 \\ 0 & 0 & 0 & 0 & -0.324648 \end{pmatrix}$$

Matrix 2:

$$\begin{pmatrix} -0.538303 & 0.538173 & 0 & 0 & 0 \\ 0 & -0.286794 & 0.286664 & 0 & 0 \\ 0 & 0 & -0.197219 & 0.197089 & 0 \\ 0 & 0 & 0 & -0.142874 & 0.142744 \\ 0 & 0 & 0 & 0 & -0.163605 \end{pmatrix}$$

Matrix 3:

$$\begin{pmatrix} -0.942212 & 0.942212 & 0 & 0 & 0 \\ 0 & -0.922036 & 0.922036 & 0 & 0 \\ 0 & 0 & -0.594132 & 0.594132 & 0 \\ 0 & 0 & 0 & -0.383907 & 0.383907 \\ 0 & 0 & 0 & 0 & -0.386949 \end{pmatrix}$$

The best estimate of the time change variance ν was 0.095. The estimation of variance from an expected value (i.e. $E(S(t))$) is possible because the Gamma distribution is skewed. However, the accuracy of the estimate is low. The goodness of fit is not significantly different for values of ν less than 0.1.

Figure 2 shows the fitted survival curve and Figure 3 the fitted versus observed data for the health states. The model provides a good fit to the survival distribution used for calibration.

Figure 2 here

Figure 3 here

Solvency and Tail Risk

In order to assess solvency and tail risk arising from heterogeneity a portfolio of life annuities is projected using simulation. Annuity contracts are assumed written at age 65 under differing assumptions about the health status of the lives purchasing the annuity. All annuities are for \$1 p.a. There are no expenses or other costs assumed. The distribution of health status is based on the distribution estimated by each model. For comparison purposes ranges of health status were aggregated into groups for the purpose of calculating premiums and simulating annual balances.

Premiums are determined to equal the actuarial expected present value of all payments. Survival rates conditional on health states are used to allow for selection and population average survival rates are used for the case of no anti-selection. A fixed interest rate of 3% p.a. was assumed as well as an assumption of random investment returns.

Random returns were simulated using a model (including calibration) directly taken from Nirmalendran et al (2012). Assets were assumed allocated according to APRA's 2010 statistics of 5.5% in cash, 86.8% in bonds, and 7.7% in stocks (rebalanced every year). Cash rates and stock prices were modeled with geometric Brownian motion. The short rates generated by the Vasicek model were used for single period bond returns. For the random returns case, premiums were calculated with discount factor based on bonds yields. However, unlike Nirmalendran et al (2012), the market price of investment risk was not considered.

The distributions of healthy states for the Markov ageing model are given in Table 2. These percentages were calibrated to the health data and show the shift from the healthier states to the less healthy states and eventually to the death states with age. Figure 4 shows the distribution of heterogeneity at age 65 given by the three models based on the distribution of expected future lifetimes for the models. Both the Vaupel frailty model and the Le Bras Markov model have similar distributions, although not identical. The Markov ageing model has a markedly different distribution and this reflects a more accurate calibration to health status.

Table 2 here

Figure 4 here

Impact of heterogeneity and adverse self-selection.

The impact of heterogeneity is demonstrated in Figure 6 through the comparison of a 'best health' case and a 'mixed' case. The best health case assumes only individuals in the best health class of the Markov ageing model, and equivalent states in the Le Bras and Vaupel models (equivalent

top x% of the population in terms of mortality rate), purchase annuities. The mixed cases assume a portfolio of annuitants with similar health proportions to that of the population and no selection based on the annuity premium.

The standard deviation of the pool amount increases with older ages for all models. Even though frailty models imply reduced relative heterogeneity at older ages, there is an increase in variability of pool fund amounts. The Le Bras and Vaupel models produce similar results although the Vaupel model gives higher standard deviations.

The most interesting aspect shown here is the Markov ageing model, whose measure of heterogeneity is specifically calibrated to population health data. The cases where only the best health states lives purchase annuities differ significantly from the mixed population health states pool. These differences do not arise in the other two models, where heterogeneity in health is derived from aggregate survival rates only.

Figure 5 here

Figure 7 shows the Markov ageing model results for the best health state compared with the mixed health case. In the population case the distribution of fund sizes is much wider with higher probabilities of adverse fund sizes. (The other two models show a smaller magnitude, see later in Table 3.) The selection strategy of writing annuities for a select group of individuals reduces the volatility arising from heterogeneity and is a lower risk strategy.

Figure 6 here

The self-selection case assumes that the premium is charged based on the mixed population distribution of health states but individuals select to purchase the annuities based on an assumption they know their health state. As shown in Figure 8, the effect of this anti-selection is that the average fund size drops significantly, as expected, and the chance of major losses increases. Although adverse selection results in lower standard deviations of pool fund balances, this is primarily because the mean level of the fund falls rapidly.

Table 3 compares the premiums and risk measures for the cases of best health, mixed health and adverse selection. The three models agree on the impact of self-selection, although they differ on the amount of reduction in volatility when the best health group is priced separately.

Figure 7 here

Table 3 here

Impact of random investment returns.

Table 4 shows the annuity premiums and risk measures for pool sizes of 1000 assuming random investment returns. Risk is substantially increased with the addition of investment return risk. The Le Bras and Vaupel models show similar risk measures for the different cases of selection. For the Markov ageing model the better health states contribute significantly to overall portfolio risk due to high level of fund.

Table 4 here

Impact of stochastic mortality.

As discussed earlier, the subordinated Markov ageing model incorporates stochastic mortality through a Gamma time change. The degree of uncertainty is expressed through its variance ν . Survival data in 2008 supports values of ν being less than 0.1 with a best estimate of 0.095. Table 5 shows the diminishing difference between the standard deviation of funds with different health status composition (i.e. best health only and mixed health) as variance ν increases. Heterogeneity's relative effect on fund fluctuation diminishes as systematic uncertainty increases.

Table 5 here

Impact of pool size.

Table 6 compares the standard deviation at age 110 for pool sizes 10^2 to 10^5 given by a deterministic Markov model and its stochastic equivalent. With deterministic mortality rates, standard deviation increases disproportionately to pool size, showing a diversification of idiosyncratic risk. In contrast, with the inclusion of systematic risk, the effect of diversification from is almost cancelled out by increases in volatility from the increased exposure to systematic mortality risk due to the larger pool size.

Table 6 here

Figure 5 shows the standard deviation of the pool amount for ages above 90 for the deterministic and subordinated Markov ageing models, for pool sizes 500 and 1000. At the older

ages a larger pool size increases the standard deviation more significantly because of the effect of systematic risk.

Figure 8 here

Conclusions

Model risk arises from a misspecification of the underlying process being modelled. Systematic mortality risk models have been developed and applied. Markov ageing models for heterogeneity have also been developed. Using a model that captures only one of these aspects of mortality risk has limitations because of model risk.

This paper has used a recently developed model for mortality heterogeneity, along with more commonly used frailty models to show the impact of this risk on annuity fund values at the older ages, the tail of the mortality distribution. Standard models of heterogeneity do not capture observed health differentials or the effect of systematic mortality risk. They do allow the risk of adverse selection to be quantified.

We show how increasing pool sizes increase tail risk when a mortality model includes systematic risk. This effect is not captured by standard models of heterogeneity. We show how selection of lives in better health states by insurers when writing life annuities is a more profitable and less risky strategy than writing annuities on all health states in the population, even if there is no adverse selection.

Adverse selection has a significant negative impact on mean profitability that outweighs the lower risk in the fund shown in the lower standard deviations of fund values.

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Table 1: Parameter estimates for Le Bras model fitted to ages above 20.

λ_0	0.489972
μ_0	0.000608
λ	0.117869
μ	0.00001

Notes: The table shows the parameters estimates for the Le Bras model based on the Yashin et al (1994) parameterization. Source: authors calculations.

Table 2: Markov ageing model: percentage distribution of health states for ages 40 to 70

Age\State	1	2	3	4	5	deceased
40	47.60%	42.50%	7.50%	0.20%	0.20%	2.20%
50	4.09%	37.90%	12.60%	4.70%	0.30%	3.70%
60	21.10%	41.00%	18.20%	11.80%	0.90%	7.10%
70	13.00%	31.10%	16.90%	21.80%	2.20%	14.90%

Notes: The table shows the distribution of health states for varying ages based on the Markov ageing model. Health state 1 is the best health state with then lowest mortality rate and 5 is the worst health state with the highest mortality rate. Source: authors calculations.

Table 3: Annuity premiums and tail risk measures assuming a fixed investment return for different models of heterogeneity.

Mortality model	Heterogeneity	Annuity premium	Risk measures at age 110		
			Mean	Stdev	95% VaR
Markov	best health only	16.32	-0.07	386.09	631.73
	mixed	14.29	-15.86	710.31	1176.89
	mixed w self selection	14.29	-5872.49	428.07	6566.69
Le Bras	best health only	15.84	4.24	607.33	986.31
	mixed	14.16	11.56	635.70	1022.46
	mixed w self selection	14.16	-3105.13	613.12	4109.81
Vaupel	best health only	16.29	-0.88	658.73	1072.07
	mixed	14.72	-1.61	673.32	1109.78
	mixed w self selection	14.72	-2610.51	666.36	3694.48

Notes: The table shows the premium for a life annuity of 1 p.a. and tail risk measures for a pool of 1000 individuals aged 65 assuming different pool compositions for health statuses for a fixed investment return of 3% p.a.. Results are show for the different deterministic models of heterogeneity. Source: authors calculations.

Table 4: Annuity premiums and tail risk measures assuming random investment returns for different models of heterogeneity.

Mortality model	Heterogeneity	Annuity premium	Risk measures at age 110		
			Mean	Stdev	95% VaR
Markov	best health only	13.48	-199.80	4912.11	7843.93
	state 2	12.54	-198.90	4387.30	7117.17
	state 3	10.04	-111.25	3192.87	5144.76
	state 4	6.74	-54.63	1917.96	3131.44
	state 5	5.00	-35.88	1478.46	2441.54
	mixed	11.99	-132.34	4420.42	7051.55
	mixed w self selection	11.99	-14675.61	4112.85	21204.18
Le Bras	best health only	12.95	-109.05	4901.30	7811.46
	mixed	11.84	-59.61	4283.44	6883.19
	mixed w self selection	11.84	-7006.90	4244.59	13922.83
Vaupel	best health only	13.14	-141.61	5040.23	8067.82
	mixed	12.13	-112.90	4476.47	7234.56
	mixed w self selection	12.13	-5777.86	4397.70	12874.70

Notes: The table shows the premium for a life annuity of 1 p.a. and tail risk measures for a pool of 1000 individuals aged 65 assuming different pool compositions of health statuses for a random investment return. Results are shown for the different deterministic models of heterogeneity. Source: authors calculations.

Table 5: Standard deviation of annuity fund for different assumptions of stochastic mortality risk.

v	Stdev at age 110	
	best health	mixed
0.001	470.09	756.53
0.005	702.46	908.32
0.01	910.17	1052.44
0.05	1862.03	1877.39
0.095	2588.74	2509.45
0.1	2626.27	2563.43

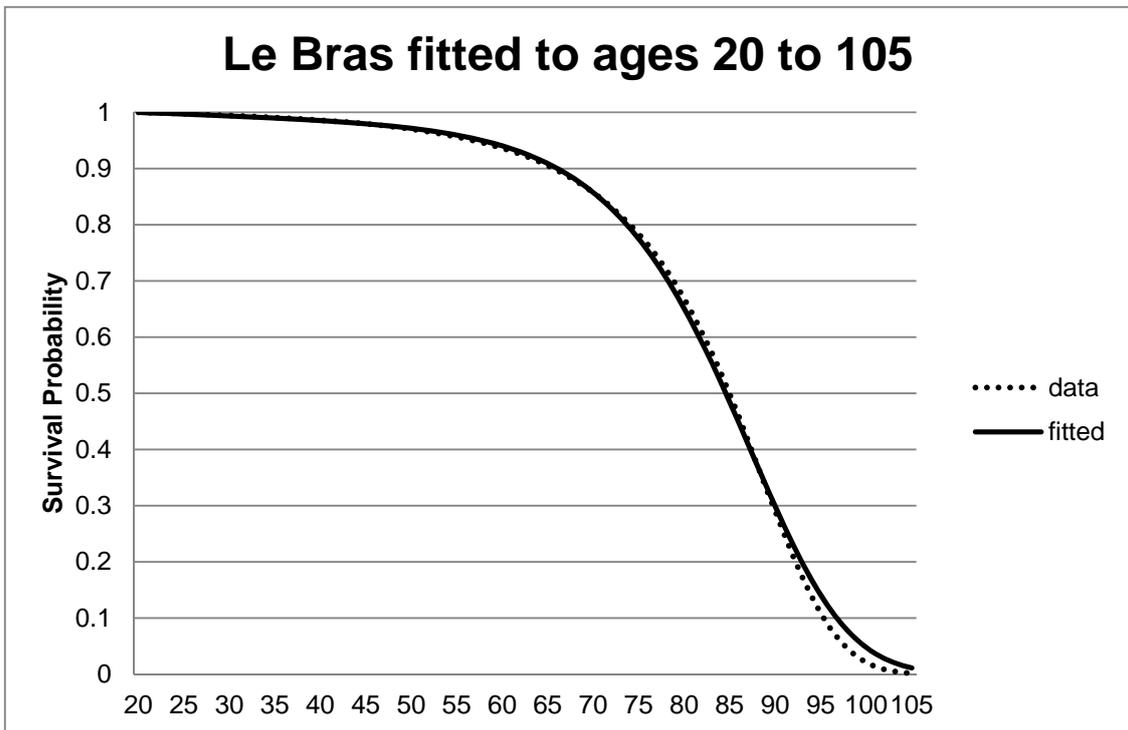
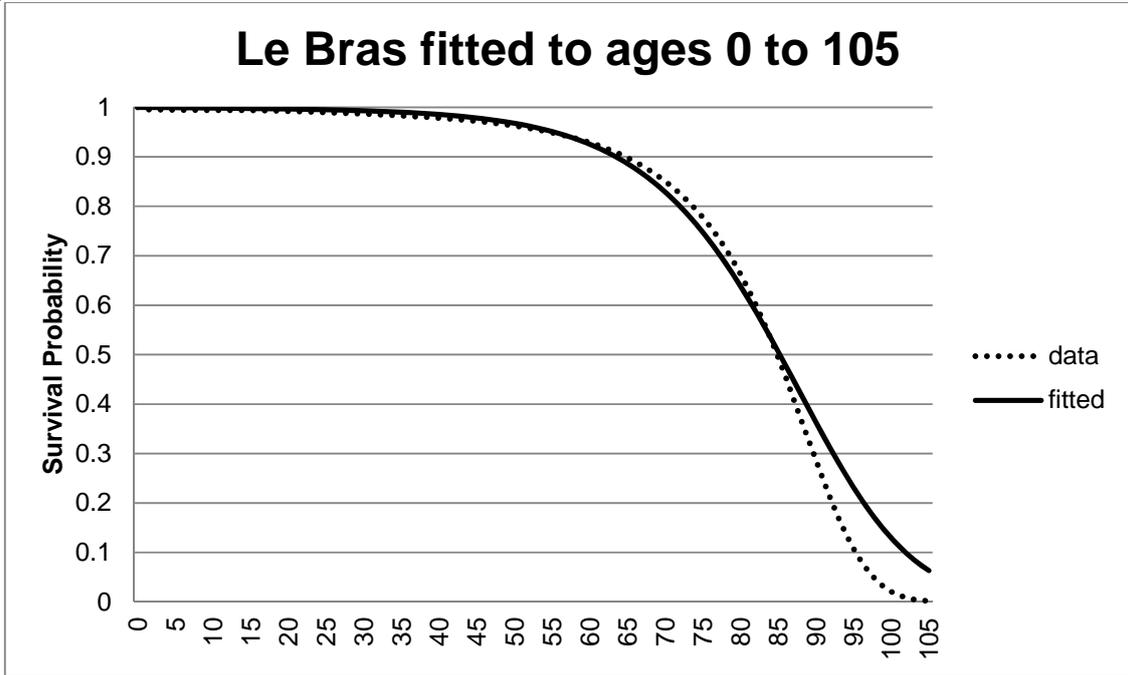
Notes: The table shows the standard deviation for a life annuity fund of a pool of 1000 individuals aged 65 assuming different pool compositions for health statuses for annual payments of \$1 and a fixed investment return of 3% p.a.. Source: authors calculations.

Table 6: Standard deviation at age 110 for different pool sizes using Markov model without and with stochastic mortality risk.

Pool size	deterministic Markov	subordinated Markov
100	122.66	286.21
1000	388.23	2588.74
10000	1216.31	25649.07
100000	3914.59	254307.38

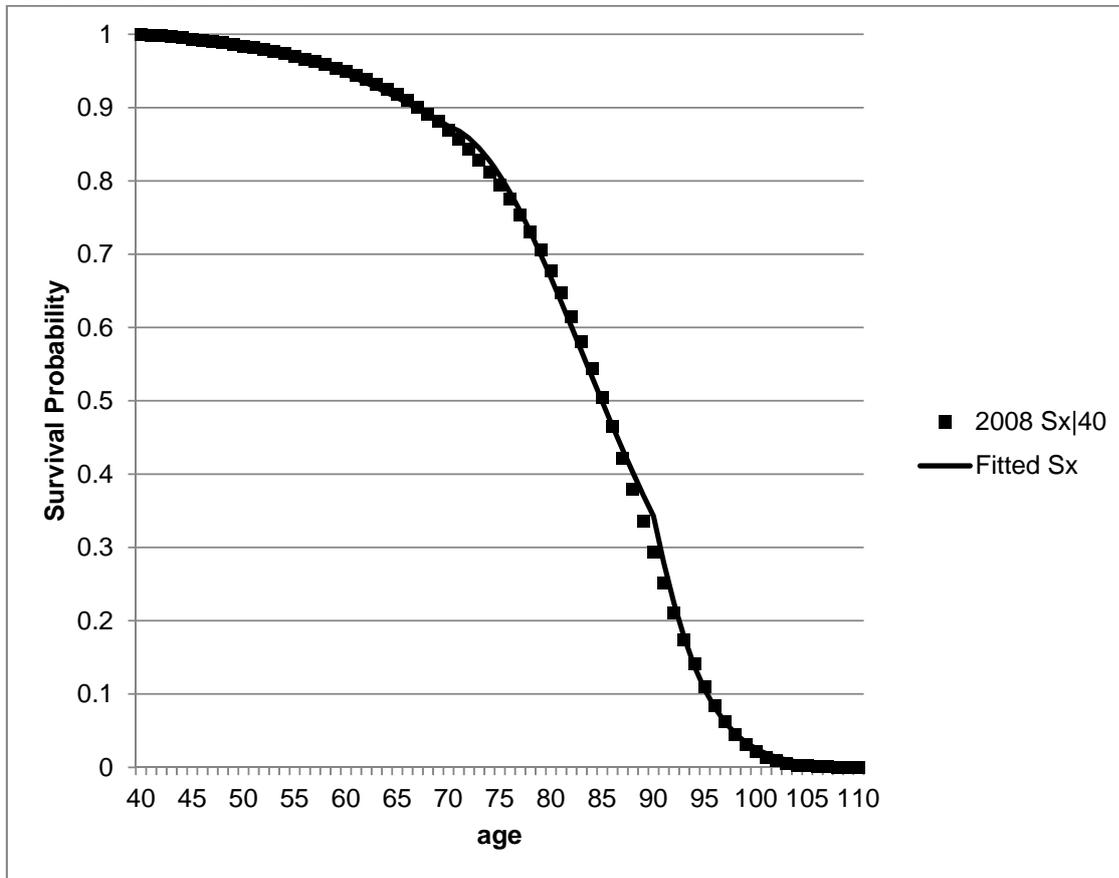
Notes: The table shows the standard deviation of the fund at age 110 for life annuity of 1 p.a. for best health individuals aged 65, assuming a fixed investment return of 3% p.a.. The stochastic model assumes variance of Gamma time change $v = 0.095$. Source: authors calculations.

Figure 1: Survival curve fit of the Le Bras model



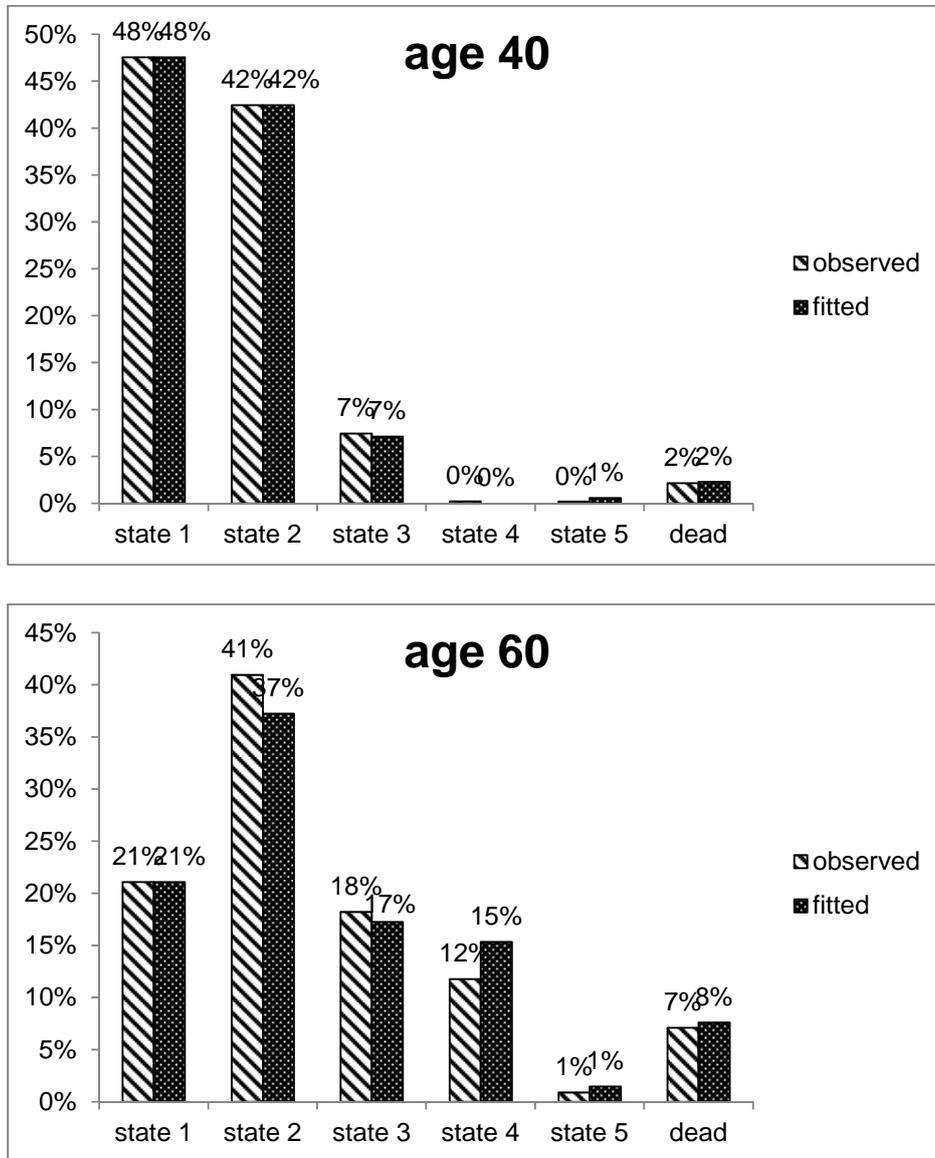
Note: Figures show the fit of the Le Bras model to the 2008 Australian life table survival curve (male and female combined). The model provides a better fit to survival data starting from age 20 than starting from birth. Source: authors calculations.

Figure 2: Survival curve fit of the Markov ageing model of heterogeneity based on both health and survival data



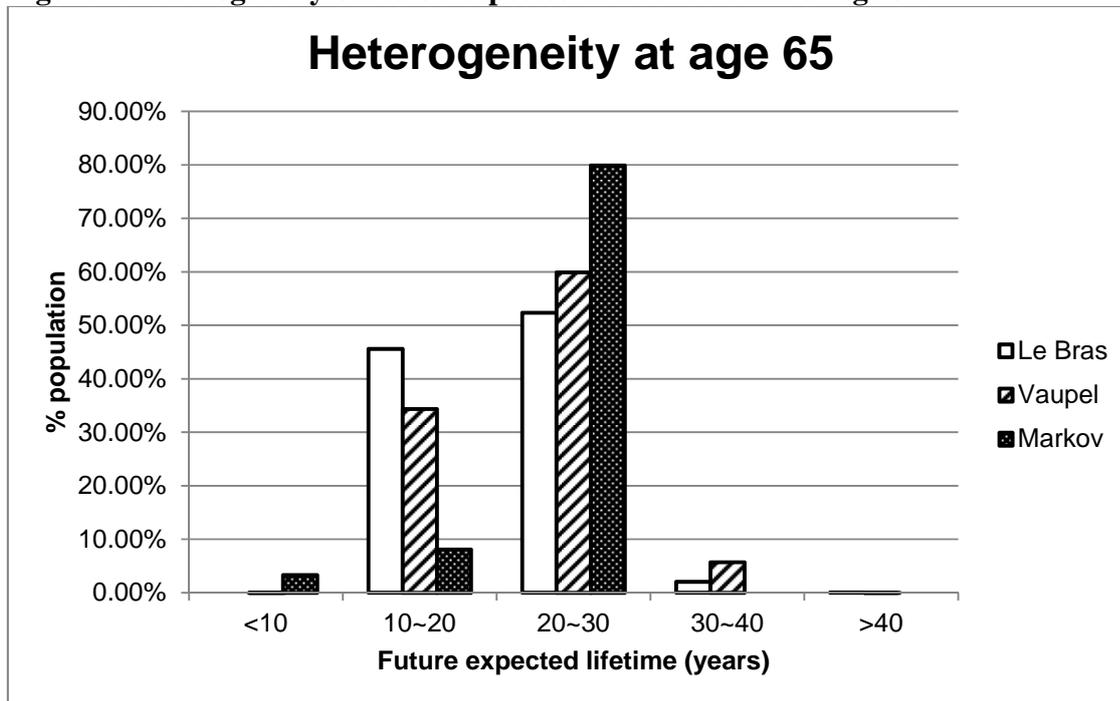
Note: Figure shows the fit of the Markov ageing model used in the paper compared to the 2008 Australian life table survival curve. The model fit is shown for ages 40 and above. Source: authors calculations.

Figure 3: Fitted versus observed data for Markov ageing model



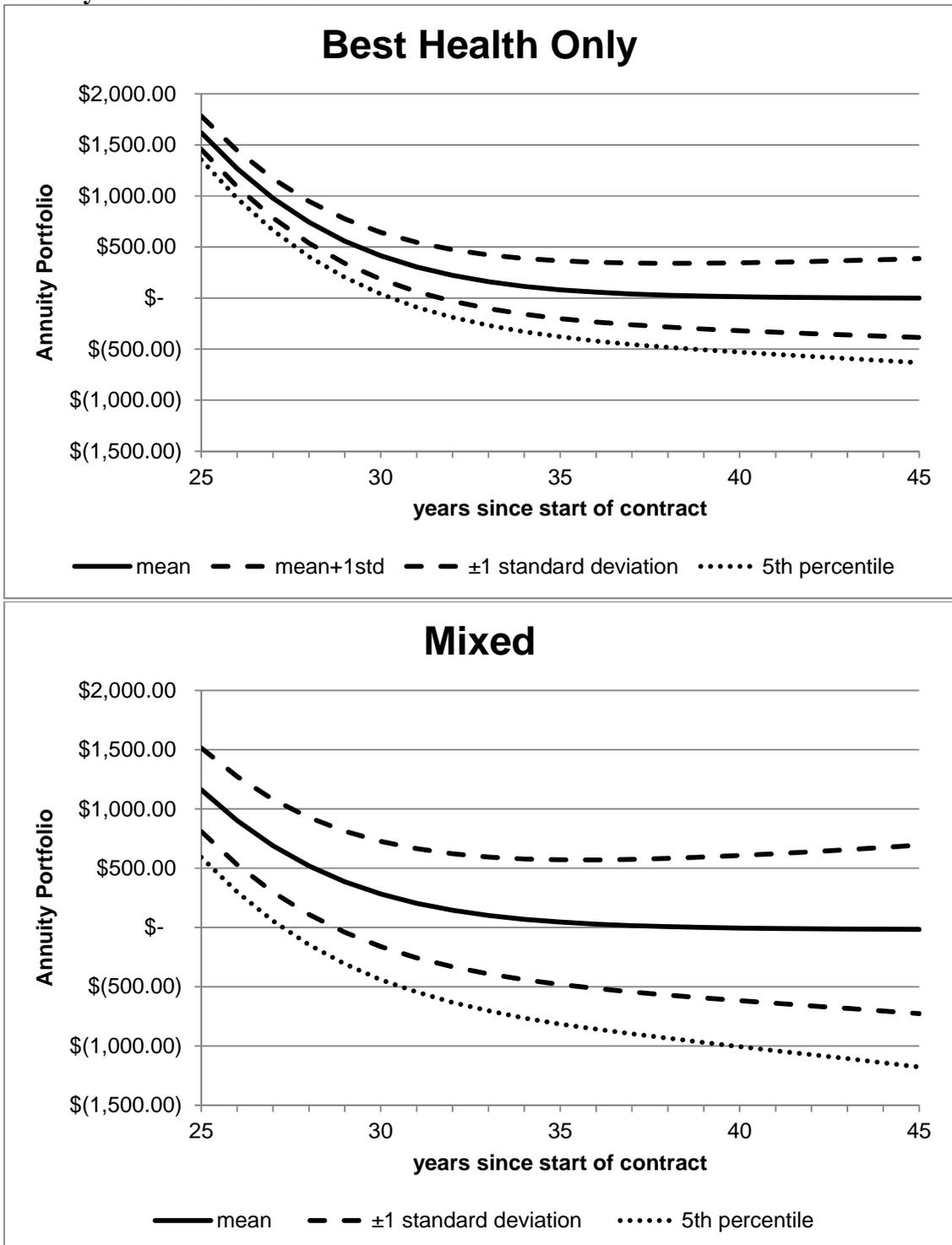
Note: Figure shows distribution of health states for the Markov ageing model used in the paper compared to the actual data used to fit the model. The model fit is shown for ages 40 and 60. Source: authors calculations.

Figure 4: Heterogeneity based on expected future life times at age 65.



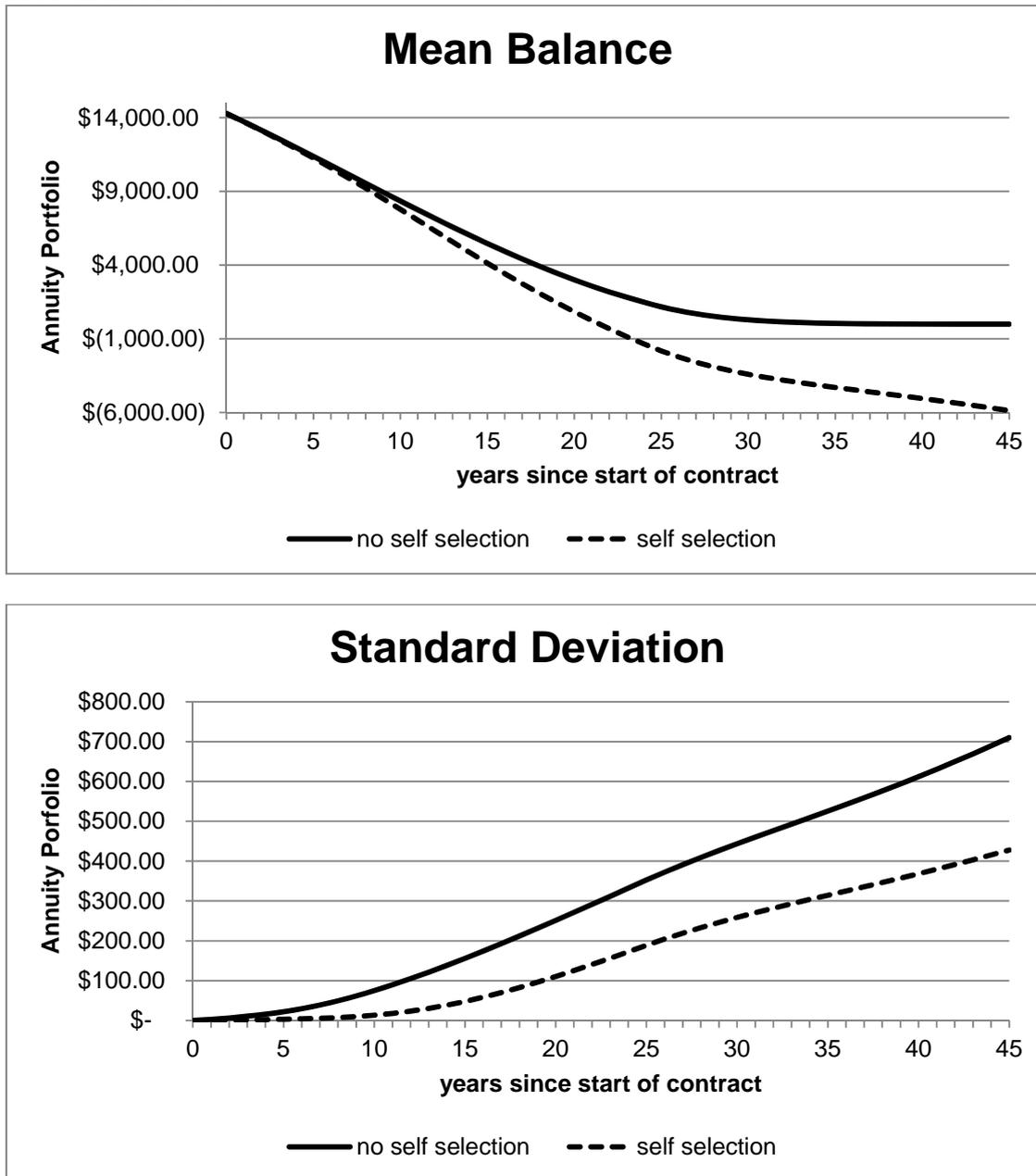
Note: The figure shows the distribution of future expected lifetime according to the three modes used in the paper to quantify heterogeneity of mortality. The Markov model has a noticeably different distribution to the other models reflecting its calibration to both health and survival data. Source: authors calculations.

Figure 5: Balance of annuity fund for the best health state and the population mix showing uncertainty and downside risk.



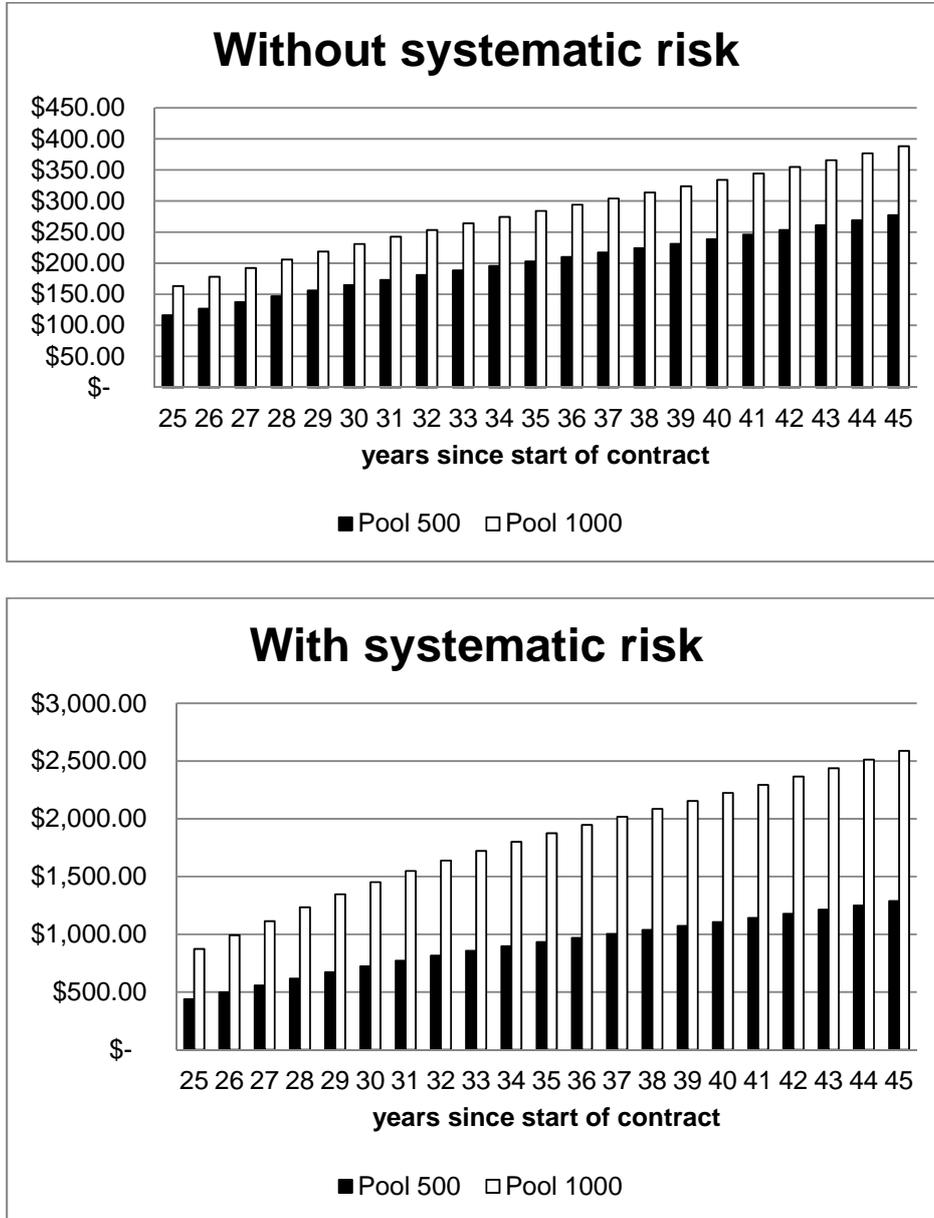
Note: The figures show the annuity fund for annuities commencing at age 65 at the older ages for a pool size of 50 individuals. The top figure is for annuity portfolio with only the best health state and the bottom figure annuities assuming a mixture of health states representative of the population purchase annuities (mixed). Source: authors calculations.

Figure 6: Mean and standard deviation of balance of annuity fund showing the impact of adverse selection.



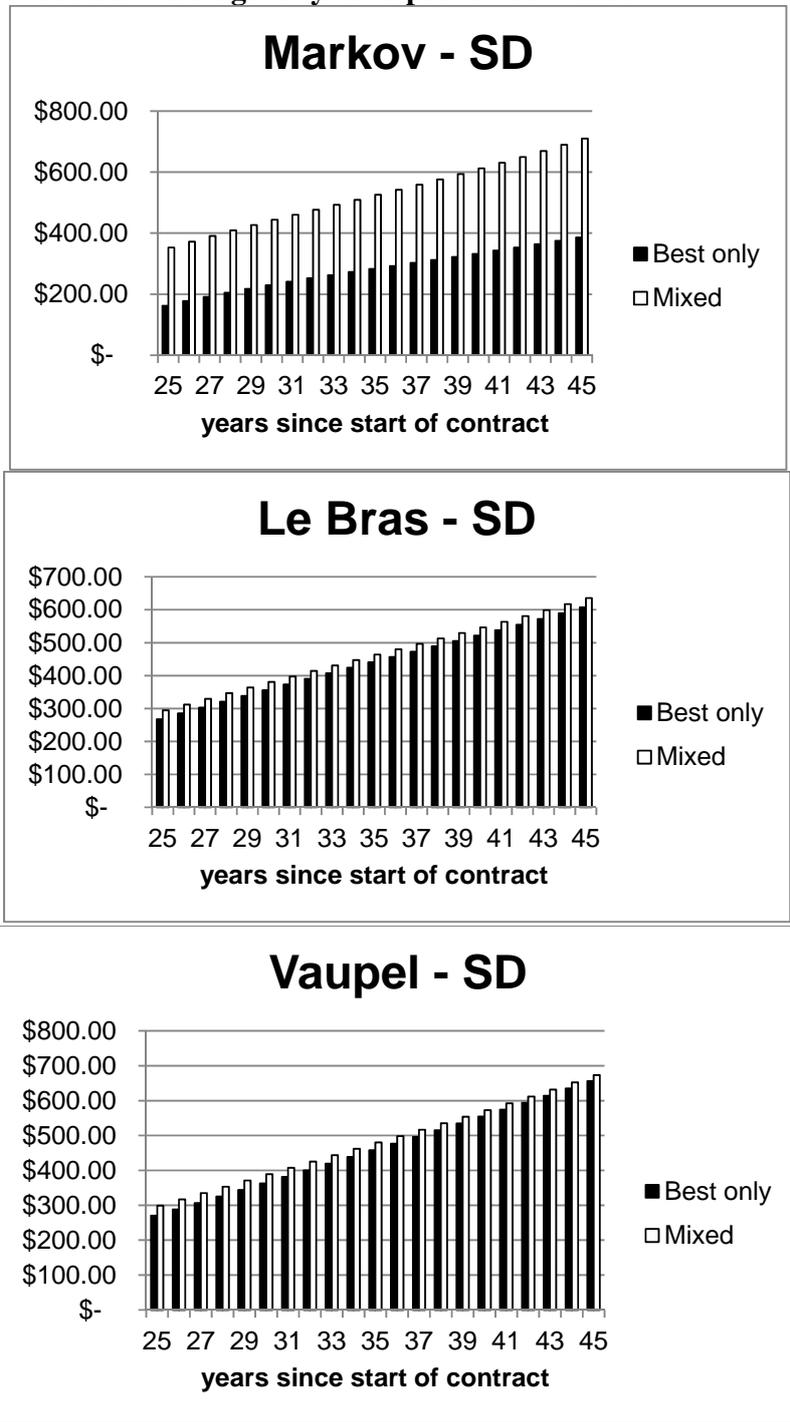
Note: The figures show the annuity fund for annuities commencing at age 65 for a pool size of 50 individuals assuming that a population annuity rate is charged. The top figure shows the mean balance and the bottom figure the standard deviation. Two cases are shown. One where there is no self (adverse) selection and the other where only the healthy lives purchase annuities. Source: authors calculations.

Figure 7: Average and standard deviation of annuity pool amount at older ages for the Markov ageing model



Note: The figures show the standard deviation of the annuity fund for annuities of \$1 p.a. for best health individuals aged 65, assuming a fixed investment return of 3% p.a.. Source: authors calculations.

Figure 8: Standard deviation (SD) risk measure for the annuity pool amount at older ages for the different models of heterogeneity for a pool size of 1000.



Note: The figures show standard deviation of the annuity fund for annuities commencing at age 65 at the older ages for a pool size of 1000 individuals. The standard deviations are shown for the three different models and for the assumption that only the best health individuals purchase annuities (best only) and also assuming a mixture of health states representative of the population purchase annuities (mixed). Source: authors calculations.