ARC Centre of Excellence in Population Ageing Research

Working Paper 2013/16

Multi-State Actuarial Models of Functional Disability.

Joelle H. Fong, Adam W. Shao and Michael Sherris.

School of Risk and Actuarial Studies and ARC Centre of Excellence in Population Ageing Research (CEPAR), University of New South Wales, Sydney, Australia.

Corresponding author Joelle H. Fong: h.fong@unsw.edu.au

We are grateful to the Australian Research Council for generous financial support.

Multi-State Actuarial Models of Functional Disability

Joelle H. Fong[a,*], Adam W. Shao[a], Michael Sherris[a]

[a] *ARC Centre of Excellence in Population Ageing Research, University of New South Wales, Australia*

Abstract

We apply generalized linear models to evaluate disability transitions for individuals in old age based on a large sample of U.S. elderly. We estimate a multi-state model for long-term care insurance applications, and find significant differences in disability rate patterns and levels from the commonly-used Robinson (1996) model. Our results suggest that the elderly face a 10% chance of becoming long-term care disabled only at ages past 90, rather than in their 80s. Furthermore, age patterns of recovery are found to differ significantly between the sexes. These estimates of transition probability are sensitive to the definition of 'long-term care disability', which has implications for the design of benefit triggers for private and public long-term care insurance programs.

Keywords: morbidity, health transitions, long-term care insurance, generalized linear models

JEL Classifications: G22; H51, H75; C24

1. Introduction

Benefit payouts from long-term care (LTC) insurance covers are typically triggered when the insured becomes functionally disabled or chronically ill, and requires support in various facets of living over prolonged periods of time. Despite the large literature that explores the trends in functional status and disability patterns among elderly persons, surprisingly few studies have attempted to characterize the full set of morbidity and mortality transition probabilities across different functional states, and take into account the insurance reimbursement eligibility of those states. Understanding individual transitions into and out of states of disability severe enough to trigger LTC insurance payouts is essential to product pricing and development.

*Corresponding author. Tel.: +61 2 9385 6367; fax: +61 2 9385 6956.

Email addresses: h.fong@unsw.edu.au (Joelle H. Fong), wenqiang.shao@unsw.edu.au (Adam W. Shao), m.sherris@unsw.edu.au (Michael Sherris)

The goal of this paper is to estimate age-specific disability transition probabilities among a sample of U.S. older adults aged 50 to 100 at baseline in 1998. Our analysis adopts a time-continuous inhomogeneous Markov chain setup, and specifically considers only insurance reimbursement-eligible disability state(s). Our results are of interest for several reasons. Firstly, this study provides new evidence regarding how morbidity-mortality transition rates vary by age and sex. We apply a generalized linear modeling (GLM) approach that is suited to graduating both mortality and disability transition intensities while also flexible in allowing for alternative model specifications. Secondly, we seek to determine the effects of different benefit trigger configurations on the sensitivity of the disability incidence rates. While public and private LTC insurance programs often utilize measures of activities of daily living (ADLs) as a trigger for payouts, individual policies could differ in terms of the threshold number of ADLs, specific ADLs used, associated waiting period, and inclusion of an alternative trigger.

Older adults are most at risk of experiencing losses in physical functioning that cumulate in chronic physical impairment. The Congressional Budget Office reports that approximately two-thirds of the expenditures on LTC in the U.S. are directed towards services for elderly people over age 65; in 2004, LTC expenditures for this group totals about US $135 billion (Congressional Budget Office, 2004). At the same time, the number of elderly people will significantly increase over the coming decades. Fueled by the aging of the baby-boomer generation, the proportion of people aged 65 or older in the U.S. population is expected to grow from 13.0% in 2010 to 20.2% in 2050 (Census Bureau, 2008).

Prior research on ADL disability have mainly focused on the prevalence and trends in morbidity over time and across cohorts (e.g. Crimmins, 2004; Cutler, 2001; Freedman et al., 2004); the impact of disability on the risk of nursing home admittance (Fong et al., 2012); and the associations between individual risk factors and disability (Beckett et al., 1996; Mendes de Leon et al., 1997; Smedley and Syme, 2001). There are fewer studies devoted to the construction of nationally representative estimates of functional status transition rates. Among those that did is Manton (1988) which examined two-year disability, institutional, and mortality transitions but the transition probabilities derived were not specific by exact ages. In the 1990s, the Society of Actuaries (SOA) LTC Insurance Valuation Methods Task Force embarked on a large-scale effort to construct a LTC morbidity table for the U.S. noninstitutional population.[1] The Task Force relied on longitudinal data from the National LTC Surveys (NLTCS) and derived age-specific transition rates using parametric gradua-

[1]The Task Force was set up in 1991 at the request of the National Association of Insurance Commissioners to address the valuation needs for LTC insurance.

tion methods based on a carefully-specified function (Society of Actuaries, 1995).[2] James Robinson, who is a member of the Task Force, also expanded the original analysis to include finer health states that are tied to LTC insurance reimbursement eligibility (Robinson, 1996). What became subsequently known as the 'Robinson model' is highly regarded and widely cited. It has been used by academics, economists, regulators, as well as government agencies administering public LTC benefit programs. The American insurance industry also employs the model and its derivatives to develop premiums and reserves for LTC insurance.

One of the main advantages of the SOA/Robinson study is that it draws on large-scale longitudinal data on disabilities which facilitates the analysis of two-period disability transitions. Some other researchers, lacking longitudinal data, have relied on prevalence rate data instead. For example, Olivieri and Pitacco (2001) and Rickayzen and Walsh (2002) demonstrate methods to derive transition rates from cross-sectional prevalence rate data obtained from the U.K. Office of Population Censuses and Surveys. Our present analysis shares some commonalities with Olivieri and Pitacco (2001) in that we consider just a single level of LTC disability tied to insurance benefit payout, and derive transition probabilities through the transition intensity approach.[3] However, unlike the earlier study, we allow for the possibility of recovery from disability. Rickayzen and Walsh (2002) identify functional forms directly for health deterioration and improvement transition probabilities rather than adopt the transition intensity approach. Extending prior work by Nuttall et al. (1994), the authors consider the observed prevalence rates across a total of ten disability categories but do not take into account LTC insurance reimbursement eligibility.

In this paper we apply a GLM approach to graduating the morbidity-mortality transition intensities. Graduation refers to methods by which a set of observed probabilities are fitted and smoothed to provide a suitable basis for inferences (such as calculation of premiums) to be made (Haberman and E., 1996). The GLM methodology provides a flexible approach to parametric graduation in actuarial science because it unifies many of the parametric graduation formulae within the exponential family of distributions (Forfar et al., 1988; Renshaw, 1991). Specifically, it has been implemented in several contexts including survival modeling and mortality forecasting (Alai and Sherris, 2012; Hatzopoulos and Haberman, 2009; Verrall, 1996), motor and marine insurance (McCullagh and Nelder, 1983), non-life insurance (Haberman and E., 1989), and multi-state models for permanent health insurance (Renshaw

[2]The NLTCS is a longitudinal survey of a random sample of Medicare enrollees and aims to study changes in the health and functional status of older Americans (aged 65+). The survey began in 1982, with follow-ups in 1984, 1989, 1994, 1999, and 2004. Society of Actuaries (1995) utilized data from the first two waves only, and Robinson (1996) utilized data from the first three waves.

[3]The 'transition intensity approach' refers to the method of deriving the differential equations for calculating transition probabilities from transition intensities (see Haberman and Pitacco, 1999).

and Haberman, 1995). None of these studies, however, have applied the GLM framework towards enumerating disability transition intensities in a multi-state model for LTC insurance applications. For this analysis, we focus on the Poisson modeling distribution, which is known to be useful for modeling count data and consistent with our continuous-time setup. To identify the most appropriate graduation formulae within the GLM framework, we rely on a combination of statistical tests and diagnostic checks. We believe it is also of interest to appraise the effects of different graduation methods on the sensitivity of the transition rates, and thus pick the Robinson model as a basis of comparison. That is, we separately apply Robinson (1996)'s method to our present dataset. The two sets of transition rate estimates derived from each approach are then compared.

The remainder of the paper is organized as follows. Section 2 describes the GLM methodology and data. We also present the transition rates results using the most suitable graduation formulae within the GLM framework and perform some diagnostic checks. In section 3, we compare these transition rate estimates with population sources, and with rates derived from the SOA/Robinson method. Section 4 evaluates the impact of different benefit trigger configurations on disability incidence estimates, and the last section concludes.

2. A GLM framework for graduating transition intensities

2.1. Markov chain

We consider a three-state Markov process with states "N" (nondisabled), "F" (functionally disabled), and "D" (dead) as the basis for modeling LTC status transitions. This is represented in Fig. 1 in which states N and F are transient and state D is absorbing. Of interest are the four transition intensities given by: σ, the intensity of becoming functionally disabled; μ, the mortality intensity for the healthy; ν, the mortality intensity for a disabled person; and φ, the intensity of recovery from being disabled. LTC insurance benefits are triggered upon entry into state F. Our setup features a single level of disability to resemble product design; most stand-alone LTC insurance policies provide only one level of disability benefit.

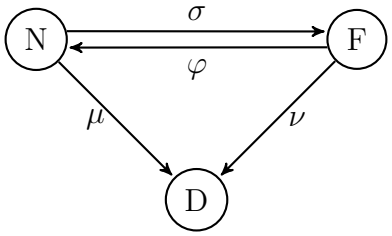


Fig. 1. Three-state Markov process.

We adopt a time-continuous, inhomogeneous Markov process with transition probabilities and transition intensities respectively denoted by:

$$P_{ij}(t,u) = Pr\left\{S(u) = j \mid S(t) = i\right\} \qquad 0 \leq t \leq u, \qquad i,j \in \{\text{N, F, D}\}, \tag{1}$$

$$TI_{ij}(t) = \lim_{u \to t^+} \frac{P_{ij}(t,u)}{u-t}, \qquad t \geq 0, \qquad i \neq j. \tag{2}$$

In the equations, P_{ij} is the transition probability from state i into state j, and t is the time unit in years from the time of policy issue. Let θ denote the age at policy issue, then $\theta + t$ represents the attained age of the insured at any point in time. The instantaneous transition intensities (TI_{ij}) are age-dependent, and are assumed to be integrable on compact intervals. We follow the conventional transition intensity approach by first estimating the intensities from data, and then deriving the transition probabilities via Kolmogorov differential equations (Haberman and Pitacco, 1999).

Notably, Ferri and Olivieri (2000) also specify a three-state Markov chain in examining LTC disability but they disregard the possibility of recovery from disability stating the 'commonly chronic character of LTC disability'. We are agnostic about the exact nature of LTC disability; in particular, existing research on the patterns of change in disability suggests that younger-old adults of age 65+ tend to exhibit a pattern of functional 'decline and recovery' up to about age 85 (Fauth et al., 2007; Fonda et al., 2002). We therefore include the possibility of recovery from disability for a more complete characterization of the process.

State F is specified according to the minimum level of functional disability which triggers LTC insurance benefit payouts. Benefit triggers - which may vary slightly from policy to policy - are used as a means of defining when the policyholder is considered a 'chronically ill individual'. By and large, the majority of private LTC insurance policies do not pay benefits until after the claimant has limitations in two or more activities of daily living for a period of at least 90 days (Wiener et al., 2000).[4] Six types of ADLs are specified under the Health Insurance Portability and Accountability Act of 1996, including dressing, continence, bathing, eating, toileting, and transferring. The Act also requires that at least five of these ADLs be used for benefit evaluation. Public LTC programs reportedly use an even stricter ADL trigger. For example, the State Medicaid programs typically will not process an application unless the person applying has limitations in three or more ADLs (Congressional

[4]Note that the period of 90 days is not a requirement for a 90-day elimination period. Instead, it means that the policyholder have to obtain certification from a licensed health care practitioner that he or she is expected to continue to meet the ADL trigger for the next 90 days from now.

Budget Office, 2004; Stone, 2002). Medicaid also pays benefits when the individual requires substantial supervision due to severe cognitive impairment (CI). By contrast, not all private insurers allow for an alternative CI trigger. Consequently, the ADL trigger is construed as primary across all LTC insurance policies while the CI trigger is secondary.

To accommodate these variations in LTC benefit triggers, we explore three different specifications of state F. For the main analysis presented in the following section, individuals are classified as functionally disabled if they have difficulty with two or more (i.e. 2+) ADLs. This criterion is in line with the minimum threshold level of ADL disabilities used in LTC insurance policies. It is used to derive the transition rates reported in Section 3. The other two formulations of state F are explored later in Section 4: one based on a disability criterion of 3+ ADLs, and the other based on a criterion of 2+ ADLs and/or CI.

2.2. Data

Our analysis draws on the Health and Retirement Study (HRS), an ongoing detailed nationally representative survey of Americans over the age of 50 and their spouses. The survey has been conducted every two years since 1992, and offers a rich source of information on health, retirement, assets, pensions, and family structure in older households both over time and across cohorts. We use panel data for the 12-year observation window beginning wave 1998 to the latest wave 2010. This is because questions on difficulty with ADLs were asked in a consistent manner only starting from survey wave 1998 onwards.[5] Our final analytic sample comprises 19,547 community-based individuals who are alive as at 1998, and give valid yes/no responses to all ADL questions in that baseline interview.

At baseline, all respondents are noninstitutional and are either in an able or functionally disabled state. Vital statuses of respondents are assessed between survey waves through HRS tracking efforts and linkages to the governmental National Death Index. We rely on the exact dates of death from the dataset, when relevant. The six ADL items assessed in the HRS are dressing, walking, bathing, eating, transferring in/out of bed, and toileting. Respondents are asked whether they have any difficulty with each of these everyday activities due to a physical, mental, emotional, or memory problem, whereby the difficulty is expected to last at least three months. Those who responded 'yes', 'can't do', or 'don't do' are coded as having that particular ADL disability. We total the number of self-reported ADL disabilities in each wave to determine health state classification.

As part of the overall functional assessment, the HRS survey also includes questions

[5]Prior to the 1998 wave, the survey question on ADLs was inconsistent across waves in terms of specific question wording, number of activities assessed, and response collection (see Fong et al., 2012). Data is obtained from the RAND HRS file version L.

relating to mobility limitations (e.g. climb stairs, stoop, and carry weights); instrumental activities of daily living (IADLs) (e.g. shop for groceries, manage money, and prepare meal); and cognitive health status. We disregard mobility limitations and IADLs since these do not directly trigger LTC insurance benefits. We do, however, use information on cognitive health in order to assess an individual's eligibility for LTC benefits based on the alternative CI trigger. The HRS measures cognition in terms of episodic memory and mental status among self-respondents using a battery of well-validated cognitive performance tests such as word recall, serial 7s, and backwards count (Health and Retirement Study, HRS, 2012). Respondents are scored on a 35-point scale. As in prior studies (e.g. Langa et al., 2008), we consider a respondent severely cognitively impaired if he or she scored seven or lower on the HRS cognitive scale.

While functional disability can strike at any age, the elderly age 65 and above are at much greater risk than the rest of the population. Accordingly it is of interest not only to examine LTC disability onset at ages past 65, but also at pre-retirement ages leading up to 65. This can provide insights into how functional disability triggering LTC use varies over the life course of older adults, so as to help anticipate the level of care needed for individuals at various time points. Data from the National LTC Survey used in the SOA/Robinson study, while also nationally representative, under-samples younger adults (sample population is restricted to age 65+). Moreover that survey started in the early 1980s and is repeated at protracted five-year intervals between 1984 and 2004. In contrast, our HRS sample of 19,547 individuals spans a wide age range and respondents are assessed at regular two-year intervals. At baseline, 48% are below age 65, 47% are between ages 65 and 85, and 5% are age 85 and above. By the wave 2010 cut-off, about 14% of the surviving sample respondents are past age 85. The richness of this data - combined with a fairly long observation window - allows the derivation of transition rates for each exact age spanning from 50 to 100; an issue we turn to next.

2.3. Methodology

Let each of the four transition intensities of interest ($\sigma,\ \mu,\ \nu,$ or, φ) be a function of the age of onset x. For illustration, we focus on the disability onset intensity σ_x. The class of functions available in the GLM framework takes the form:

$$\eta_x = g(m_x), \tag{3}$$

where m_x is the expected value of the response variable, $g(\cdot)$ is an invertible link function and η is a linear function of regressors. The link function is differentiable over its domain and monotonic such that its inverse $g^{-1}(\cdot)$ exists, and basically transforms the expectation of the

response variable to the linear predictor. The linear predictor η is a pre-specified function of the explanatory variables.[6] Since age x is deemed to be the only covariate affecting the morbidity-mortality transitions in this context, η is given by:

$$\eta_x = \sum_{s=1}^{k} x_s \beta_s = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_k x^k. \tag{4}$$

Equation (3) shows that the polynomial predictor η_x of fixed degree k is linear in the unknown parameters β_s.

The vector of independent response variables in the GLMs is given by the number of transitions at age x (denoted i_x). The distribution of i_x is a member of the exponential family, and has mean $m_x \equiv E(i_x)$. In common with Renshaw and Haberman (1995), we focus on the Poisson modeling distribution and its canonical log link function where:

$$i_x \sim Poi(e_x \sigma_x) \quad \text{independently} \ \ \forall x, \tag{5}$$

$$E(i_x \mid e_x, \sigma_x) = e_x \sigma_x = m_x, \tag{6}$$

$$Var(i_x \mid e_x, \sigma_x) = \phi m_x. \tag{7}$$

It is to be recalled here that σ_x is the intensity of becoming functionally disabled (and is substitutable here with μ, ν, or φ). The dispersion parameter (ϕ) is set to the value of unity by convention in Poisson family. We relax this assumption later in estimating the second moments but find that this dispersion parameter is not statistically significant. The associated log likelihood function with log-link is:

$$\log\left[l(\mathbf{i}; \mathbf{m})\right] = \sum_{x} \left\{-m_x + i_x \log(m_x)\right\} + C, \tag{8}$$

where $\mathbf{i}$ is the vector of response variables, $\mathbf{m}$ is the vector of mean values of the response variables, and C is a constant. AppendixA provides further technical details on the residual deviance statistic and the variance of the fitted transition rates.

The GLMs are fitted to the data using maximum likelihood methods to obtain the estimates of the regression coefficients (β_s) and their associated asymptotic standard errors. Specifically to graduate a transition intensity, we organize the data into a set of ordered pairs (i_x, e_x) defined over integer values of x. i_x is the number of transitions as earlier

[6] As in Renshaw and Haberman (1995), we consider only linear predictors for graduation purposes here. See Forfar et al. (1988) for a discussion of non-linear predictors.

described, and e_x is the corresponding central exposed to risk recorded for the duration of the 12-year observation window approximately from 1998 to 2010. The transition intensities are assumed to be constant in each age cell (i.e. the one-year age intervals), and the amount of time until transition follows an exponential distribution. We also assume that morbidity transitions from state N to F (and vice versa) occur, on average, halfway between two known survey dates.[7] Mortality transitions utilize exact dates of death. Analysis time is measured in months.

To create several alternative models for possible consideration, we vary the k term in Equation (4) fitting first a straight-line predictor structure, and thereafter adding higher order age terms.[8] Several model selection criteria are applied to evaluate the goodness of fit of the nested models. These include the Akaike Information Corrected Criterion (AICc) and Bayesian Information Criterion (BIC). The AICc, a corrected version of the more familiar AIC criterion, is appropriate for small samples as it avoids selecting models with excessive parameters and ensures parsimony (Burnham and Anderson, 2004). We also calculate the residual deviance (denoted D_c), which is defined as two times the difference between the maximized likelihood under the saturated model and that of the model in question.[9] Likelihood-ratio tests based on the differences in the deviances are assessed in order to determine whether there is a significant improvement in fit moving from $k = 1$ to $k = 2$, then from $k = 2$ to $k = 3$.[10] Finally, we perform graphical diagnostic checks for the adequacy of the chosen model in representing the raw data.

3. Results

3.1. Descriptive statistics

The in-sample tabulations of i_x and e_x by sex are summarized in AppendixB for each transition intensity ($\sigma,\ \mu,\ \nu,$ or, φ) in the three-state Markov process. The crude data to be smoothed by graduation is defined for individual integer ages spanning 50 to 100, but for reporting purposes we group by five-year intervals. Note that the final age category is [100, 101) i.e. data pertaining to age 101 and above is truncated. About 57% of the sampled respondents (or 11,051) are females. Their aggregate exposed to risk in the nondisabled state 'N' is 93,633 years and that in the disabled state 'F' is 16,029 years over the observation

[7] This is akin to the uniform distribution of death assumption conventionally adopted in life table computations.

[8] Note $k + 1$ is the total number of unknown parameters.

[9] See AppendixA for mathematical formulae of the residual deviance statistic.

[10] In the likelihood-ratio test, the test statistic (e.g. differences in residual deviances between Model 2 and Model 1) is asymptotically distributed as a chi-square with $k_2 - k_1$ degrees of freedom.

period.[11] There are 3,217 disability transitions and 3,767 deaths observed among females.

For the male sample of $N = 8{,}496$, the aggregate exposure years are 71,756 and 7,650 in states N and F respectively. There are much fewer disability transitions for males (1,743) as compared to females, but the raw number of deaths observed (3,435) is comparable. The crude mortality rate for the sample under consideration is 37% (or 7,202 deaths). The majority of deaths are observed to have occurred over ages 70-84 for males and at slightly older ages of 75-89 for females. Note that the number of observed recoveries is nontrivial for both sexes: 812 and 1,708 instances for males and females respectively. The bulk of these recovery transitions also appear to take place well before age 85.

Older adults may vary considerably in their ability to function in their everyday life. To better understand the nature of functional disability , we categorize the individuals by their number of reported morbidity transitions starting from their initial/baseline health state (either nondisabled state N or disabled state F). Table 1 shows the proportion of respondents having no transitions, one transition (either N$\mapsto$F or F$\mapsto$N), two transitions, or multiple (three or more) transitions. We see that higher proportions of initially disabled individuals experience health state transitions than initially nondisabled individuals. For example, more than 40% of initially disabled males reported recovery from disability prior to death or cut-off. Furthermore about half of those males who recovered experience a subsequent transition into disability. The female sample exhibit similar patterns.

Table 1. Proportion of respondents by number of morbidity transitions.

	Initial Age	All	All	< 65	< 65	65+	65+	85+	85+
	Initial State	A	F	A	F	A	F	A	F
Male	None	82%	57%	88%	33%	77%	66%	65%	88%
	One only	12%	20%	6%	33%	17%	15%	30%	4%
	Two	4%	15%	4%	20%	3%	14%	3%	6%
	Multiple	2%	8%	2%	14%	3%	5%	2%	3%
Female	None	75%	51%	85%	31%	67%	54%	49%	77%
	One only	16%	20%	7%	28%	23%	19%	43%	8%
	Two	6%	18%	5%	20%	6%	18%	3%	12%
	Multiple	3%	12%	3%	21%	4%	10%	4%	2%

Whether a disabled individual remains disabled over time also depends critically on his/her age. Segregating the individuals by age group (initial age at baseline), we find that LTC disability is largely persistent for the oldest-old age 85+. By contrast, the health state

[11] The numbers of exposure years in the 50-54 and 95-100 age groups are relatively smaller than those for other age groups so care is taken in interpreting any results pertaining to these ages.

of younger disabled individuals (< age 65) tends to be less stable: many of these individuals experience a 'recovery and decline' pattern, some even report multiple transitions into and out of disability. We conclude from these statistics that functional disability is not necessarily chronic in nature for our sample of older adults, which points to the importance of allowing for recovery transitions in the Markov process. Another striking observation in Table 1 is that - among the initially nondisabled respondents - there is a significant proportion who first transit into LTC disability only past age 85. One reason is because our oldest-old subsample (from the AHEAD and CODA cohorts) is likely to be a select group of elderly adults with above-average health; these individuals were first interviewed at ages 70 or older whilst in the community.

3.2. Model selection

Within the exponential family of distributions, binomial GLMs and Poisson GLMs are standard for modeling count data. In particular, the Poisson distribution is consistent with models of the hazard rate (mortality/morbidity force) function in our continuous-time setup. Thereafter, the probabilities of death (and morbidity) are derived on the basis of modeled values of the hazard rate. A visual inspection of the natural log of the crude transition intensities by age in Fig. 2 reveals three key observations.[12] First, the generally linear patterns support the use of the proposed log transformation based on Poisson GLMs. Second, curvatures in some of the plots - especially that for disability inception - indicate the need to include higher-order age terms in the predictor structure. Third, we note substantial variations in the log crude mortality rates particularly between ages 50-60. After trimming the data at ages below 60, subsequent fitting is found to be more reasonable and reflective of historical patterns of morbidity-mortality.

The Poisson GLMs based on a hierarchical series of model predictor structures are fitted using maximum likelihood estimation. Table 2 shows the model comparisons using various model selection criteria; k denotes the the number of age-related parameters.[13] We find that, for σ and μ, the use of the quadratic predictor is optimal for both sexes. This specification results in better (lower) AICC and BIC criteria values than other predictor structures. Likelihood-ratio tests of the difference in residual deviance also confirm that the inclusion of

[12]For a given age x, the crude (or raw) transition intensity is the raw transition count divided by the number of central exposure years. Since the waiting time to transition follows an exponential distribution, the crude transition intensity is the maximum likelihood estimator of the true transition intensity.

[13]We compared the AICC and BIC criteria values of four GLMs (the Poisson GLM with log-link, the binomial GLM with complementary log-log link, binomial GLM with logit-link, and binomial GLM with probit-link) for each of the four transition intensity of interest and find no substantive differences (results not reported in detail here).

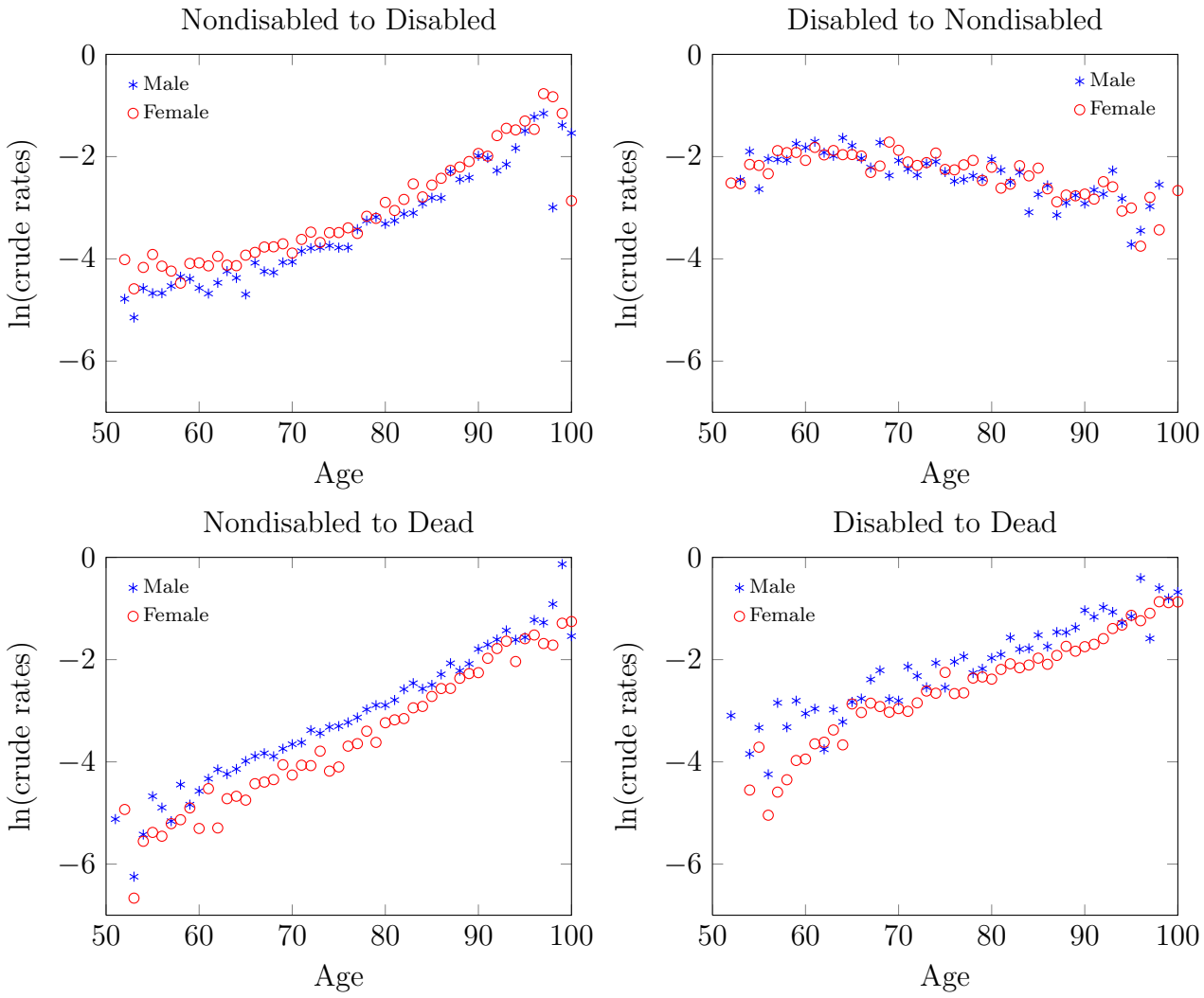


Fig. 2. Log transformation of crude transition intensities.

the quadratic age term (i.e. moving from $k = 1$ to $k = 2$) is beneficial and improves the fit ($p < .01$), despite the additional parameter involved. In contrast, the subsequent inclusion of age-cubed is found to be too complex and not statistically significant. The model selection criteria also support the use of a linear predictor for φ and ν for males, and separately for females, the use of the quadratic predictor for φ and the cubic predictor for ν. An intuition for the cubic fit for ν (F$\mapsto$D; females) as follows: unlike disabled males who experience a linear decrease in the log rates of recovery over their life course, disabled females actually enjoy an increasing rate of recovery up to about age 65 before the rates start to decline. Consequently, their log rate of mortality increases at a decreasing rate up to about age 65 before continuing to increase at an increasing rate, thus forming the cubic shape. These results support the observation that age patterns in mortality differ between the disabled

and nondisabled groups within the age 50-100 bracket. We are unable to extrapolate these results after age 100 due to sparse data.

Table 2. Poisson GLM: goodness-of-fit of nested models.

k	Males				Females			
	AICc	**BIC**	D_c	ΔD_c	**AICc**	**BIC**	D_c	ΔD_c
$\sigma : \text{A} \mapsto \text{F}$								
1	272.6	276.2	52.6		372.6	376.2	124.2	
2	**260.1**	**265.4**	**37.8**	**14.76*****	**299.4**	**304.7**	**48.7**	**-75.46*****
3	262.4	269.3	37.8	-0.04	300.5	307.4	47.5	-1.23
$\varphi : \text{F} \mapsto \text{A}$								
1	**209.1**	**212.7**	**31.0**		272.1	275.7	57.7	
2	211.3	216.6	30.9	-0.03	**265.2**	**270.4**	**48.5**	**-9.24*****
3	213.6	220.4	30.9	-0.07	267.5	274.3	48.5	-0.02
$\mu : \text{A} \mapsto \text{D}$								
1	260.7	264.3	25.0		290.5	294.1	57.2	
2	**255.9**	**261.2**	**18.0**	**-7.00*****	**281.8**	**287.1**	**46.2**	**-10.93*****
3	257.7	264.5	17.4	-0.62	282.4	289.3	44.5	-1.74
$\nu : \text{F} \mapsto \text{D}$								
1	**241.8**	**245.4**	**41.2**		250	253.6	28.0	
2	243.8	249.1	40.9	-0.28	250.4	255.7	26.1	-1.91
3	245.9	252.8	40.7	-0.22	**246.6**	**253.5**	**20.0**	**-6.10****

* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$

Note: AICc is the Akaike Information Criterion corrected for sample size, BIC is the Bayesian information criterion, and D_c is the residual deviance statistic. $k = 1$ implies age term only; $= 2$ implies age and age-squared terms; and $= 3$ implies age, age-squared, and age-cubed terms. The optimal criteria value is bolded for each set of nested models.

The fitted parameters for each of the four transition intensities are shown in Table 3. We also investigate here the possibility of over-dispersion by relaxing the assumption that $\phi = 1$. Over-dispersion occurs when the variability of the data is greater than that anticipated by the Poisson distribution assumed, in which case the quasi-Poisson model should be used instead.[14] We find that the estimated values of ϕ are close to unity (range from 0.8 to 1.1). Furthermore, the Pearson chi-squared test results confirm that the dispersion parameter is not statistically significant in models for each outcome of interest. Under the null hypothesis that there is no over- or under-dispersion, the calculated test statistic follows a chi-squared

[14]Over-dispersion is given by $\phi > 1$, implying that the conditional variance of the response variable increases more rapidly than its mean. Much less common, although also possible for count data, is the case of under-dispersion. See AppendixA for a mathematical representation of the dispersion parameter.

distribution. Consequently we set ϕ to unity in estimating the second moments.

Table 3. Poisson GLM: parameter estimates (MLE).

Parameter	**Transition Intensity**			
	σ	φ	μ	ν
Males				
β_0	-2.563**	-0.275	-7.916***	-6.852***
β_1	-0.105***	-0.027***	0.038	0.062***
β_2 ($\times 10^{-2}$)	0.121***	–	0.032*	–
Females				
β_0	0.085	-6.333***	-6.679***	-41.577***
β_1	-0.165***	0.137***	-0.016	1.384***
β_2 ($\times 10^{-2}$)	0.158***	-0.108***	0.073***	-1.680***
β_3 ($\times 10^{-2}$)	–	–	–	0.010***

* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$

3.3. Transition rates from GLM approach

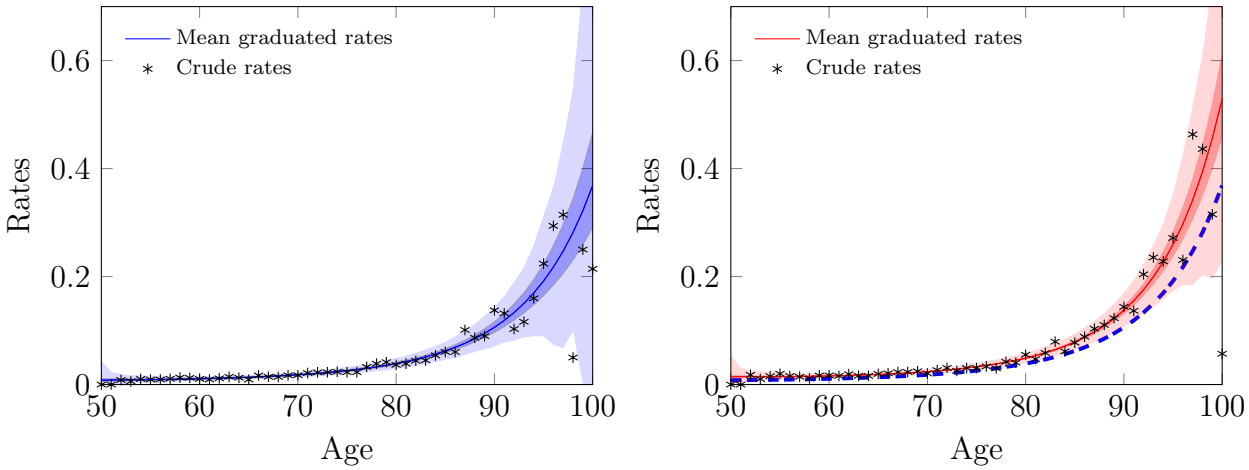
Fig. 3 shows the graduated (smoothed) disability and recovery transition intensities for ages 50 to 100. To enable comparison by sex, we superimpose the rates for males as a dotted line within the females' plots. Not surprisingly, we see that the risk of developing LTC disability increases more rapidly for each additional year of age among females than males after age 80. Results from a paired sample t-test indicate that the age-adjusted disability incidence rates are significantly higher for women. At the advanced ages, there is also more variability in incident disability for males than females. The shaded portions in each plot show the 95% confidence intervals of the fitted estimates. This confidence interval comprises two portions: the darkly-shaded portions capture parameter uncertainty while the lightly-shaded portions capture variations from the residuals.[15] We see that the higher variance for males largely stems from larger variations in the crude disability rates (lightly-shaded portion) which are primarily a result of data sparseness i.e. fewer males are observed to become disabled at the advanced ages compared to females.

Unlike the age patterns of disability, recovery patterns are much more distinct between the sexes. Prior studies (e.g. Beckett et al., 1996; Mendes de Leon et al., 1997; Oman et al., 1999) have reported that men are more likely to recover from ADL disability than women.[16]

[15]Total variance of the responses is thus given by the sum of these two components. The covariance matrix is employed to compute the variance of the fitted transition rates using the delta method. Simulations are used to generate the unconditional distribution of the responses.

[16]Most of these earlier studies employed a definition of disability which is not specific to LTC ADL triggers.

(a) Nondisabled to Disabled, σ



(b) Disabled to Nondisabled, φ

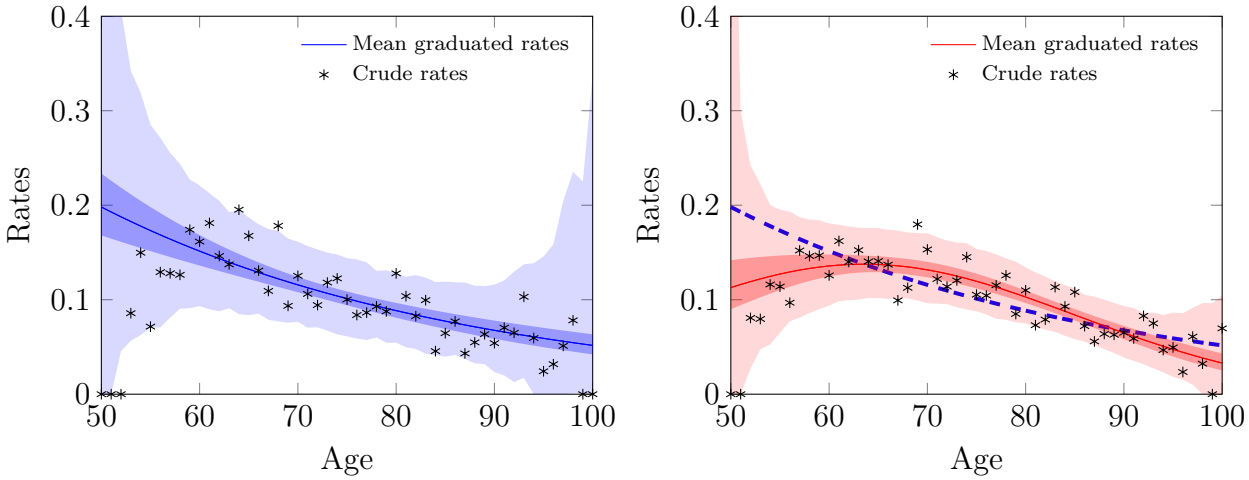


Fig. 3. Graduated morbidity transition intensities for males (left) and females (right). The mean rate for males is superimposed (dotted) in the female plot for comparison.

We find evidence of this only for limited age ranges; the recovery rates for males in our sample exceed those of females only in the 50-65 and 90-100 brackets. Between ages 65 and 90, the recovery rate of females is on average 12% higher than that of males. Additionally, the recovery pattern from LTC disability for females is quite unusual: the rate of recovery actually increases between ages 50 and 65 before gradually declining. On the positive side,

For example, some studies measure only lower-body disability while others define disability as having just one or more ADLs.

this indicates that younger women become increasingly successful in their rehabilitation efforts post-disability in ages leading up to retirement age. On the negative side, the hump-shaped pattern implies that a disabled female in her 50s faces about the chance of recovery as a much older female in her 70s. This suggests that there may be other factors influencing females' response to disability apart from age.

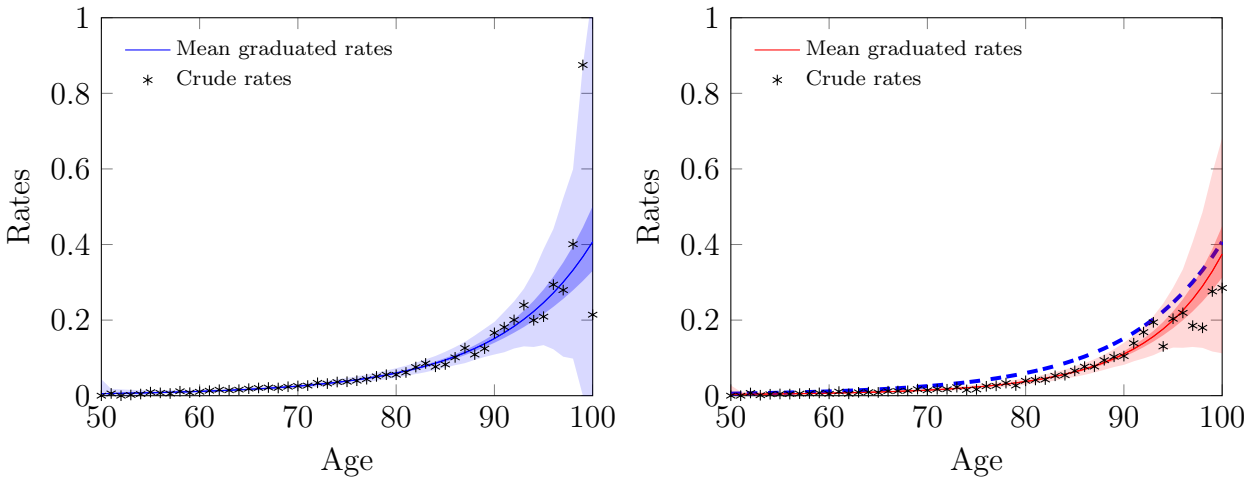
Fig. 4 shows the graduated age-specific mortality transition rates for nondisabled and disabled lives. At each integer age, the rate of death for the disabled individuals is higher than that for the nondisabled presumably because the former are already frail and more prone to accidents such as falls which can prove fatal. Also as expected, the mortality of men is higher than that of women at all ages. This is consistent with the longer life expectancy of females in general. Because the bulk of deaths for males occur between ages 70 and 84, fewer men are observed to die at the advanced ages compared to women. As a result, we see larger variations in the crude mortality rates in the males' plots.

The estimated transition rates are also informative regarding the competing risks of mortality vis-à-vis that of disability, shown in Fig. 5. For nondisabled males, the estimated hazards are fairly even at younger ages. Past age 60, however, their force of mortality exceeds that of LTC disability. Conversely, the rate of incident disability is higher than the death hazard for women of all ages. For a 65-year old woman, for example, the risk of getting 2+ ADL disabilities is about twice that of dying. We conclude that these sex differences in incident disability is an important determinant of the higher prevalence of disability generally observed among women (e.g. Manton, 1988; Fong et al., 2012).[17] In other words, females not only have a higher probability of being LTC disabled, but also have a higher probability of becoming disabled. Our results are consistent with the notion that an average older male can likely spend his retirement years without much worry of experiencing ADL disability levels that trigger LTC use, as noted in Fong et al. (2012). Additionally, we observe that past age 80, the disability incidence as a proportion of death probabilities lies within a narrow range of 0.54-0.55 (males) and 0.78-1.11 (females). This has implications for the pricing of reverse mortgage products, which typically assumes that the mobility rate is a constant proportion of mortality rate for valuation purposes (see Rodda et al., 2004).

The model provides a good fit to the data, with the deviance residuals exhibiting no age trends, as shown in Fig. 6. This indicates that our underlying modeling assumptions are reasonable. Following Renshaw (1991), we also visually check the plots of deviance residuals against fitted values (not reported in detail here). We find the residuals to be generally

[17]For example, also using recent waves of HRS data, Fong et al. (2012) report that the prevalence rate of 2+ ADL disabilities is 30% for females and 22% for males in the 85-89 age group.

(a) Nondisabled to Dead, μ



(b) Disabled to Dead, ν

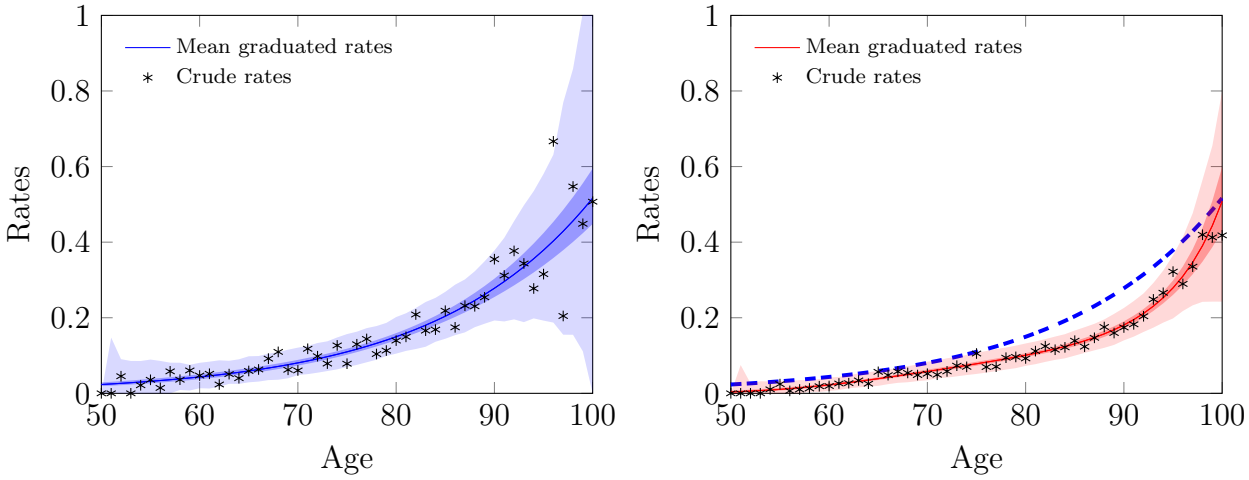


Fig. 4. Graduated mortality transition intensities for males (left) and females (right). The mean rate for males is superimposed (dotted) in the female plot for comparison.

well-behaved and supportive of the assumed variance function.

Using our transition rate estimates, we compute a summary measure given by the lifetime probability of a nondisabled retiree ever becoming LTC disabled in his/her remaining years. Based on the assumed terminal age of 100, this probability works out to be about 37% and 54% for a 65-year old male and female respectively. Nondisabled females face a higher probability of ever becoming LTC disabled over their remaining lifetime because of the dual effect of their higher survival probabilities to older ages and their higher probabilities of

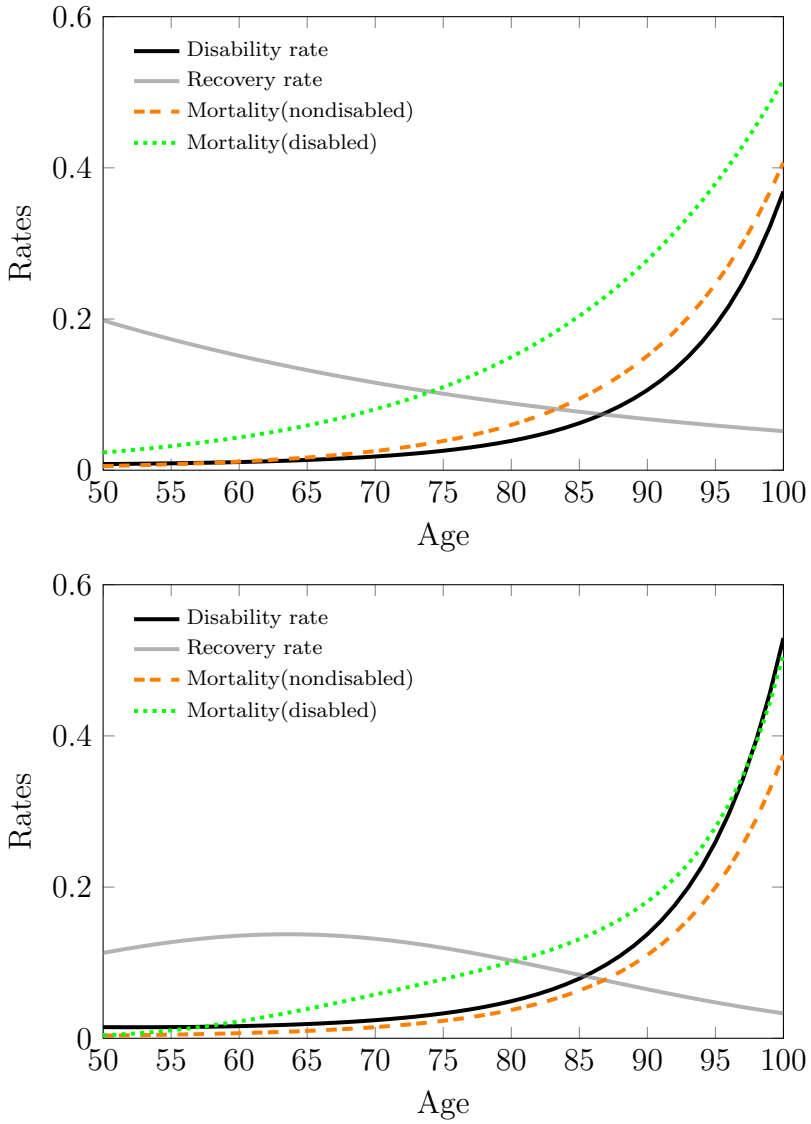


Fig. 5. Competing risks of disability and death for males (top) and females (bottom).

getting disabled at each given age.

To validate our estimates of mortality rates, we convert the mortality hazards to one-year death probabilities (q_x) and compare them with published population sources. A suitable benchmark for comparison is the U.S. Social Security Administration (SSA) cohort life table for the general population. These cohort tables are chosen in lieu of period population tables because our estimates essentially correspond to cohort mortality rates weighted by sample composition (four HRS birth cohorts observed over 12 years). Accordingly, the sex-specific SSA life tables are then weighted to account for the sample composition.[18] Fig. 7 shows the

[18]The SSA cohort tables 1900 - 1940 are sourced from Bell and Miller (2005). For males, the weights

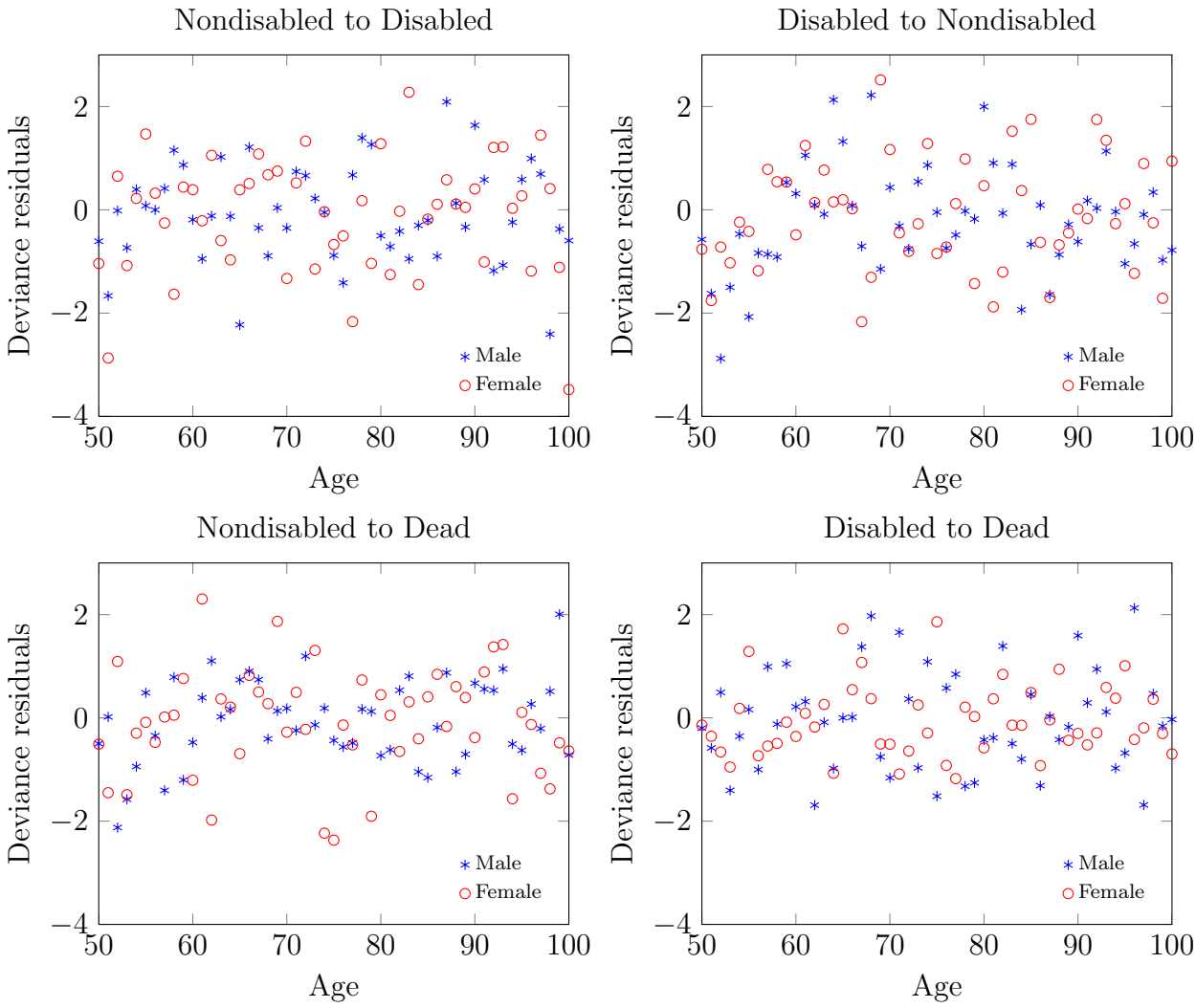


Fig. 6. Deviance residuals of transition rates.

comparisons of death probabilities by sex. By-and-large, our estimated death probabilities are comparable with the SSA mortality probabilities. The SSA probabilities, in fact, lie in the region between our estimated death probabilities for the disabled (dashed black line) and the nondisabled (solid black line), which is not surprising since the general population would consist of a mix of both subgroups. In addition, our mortality estimates for the disabled are also consistent with the rates published in the RP-2000 disabled life tables (which pertain to disabled individuals receiving social security disability benefit payments (Society of Actuaries, 2000)). Note that at the advanced ages this holds more so for disabled

applied based on sample composition are 0.3% (1900 table), 6% (1910), 19% (1920), 32% (1930), and 43% (1940). For females, the weights are 1%, 8%, 20%, 28%, and 42% respectively.

females than disabled males.

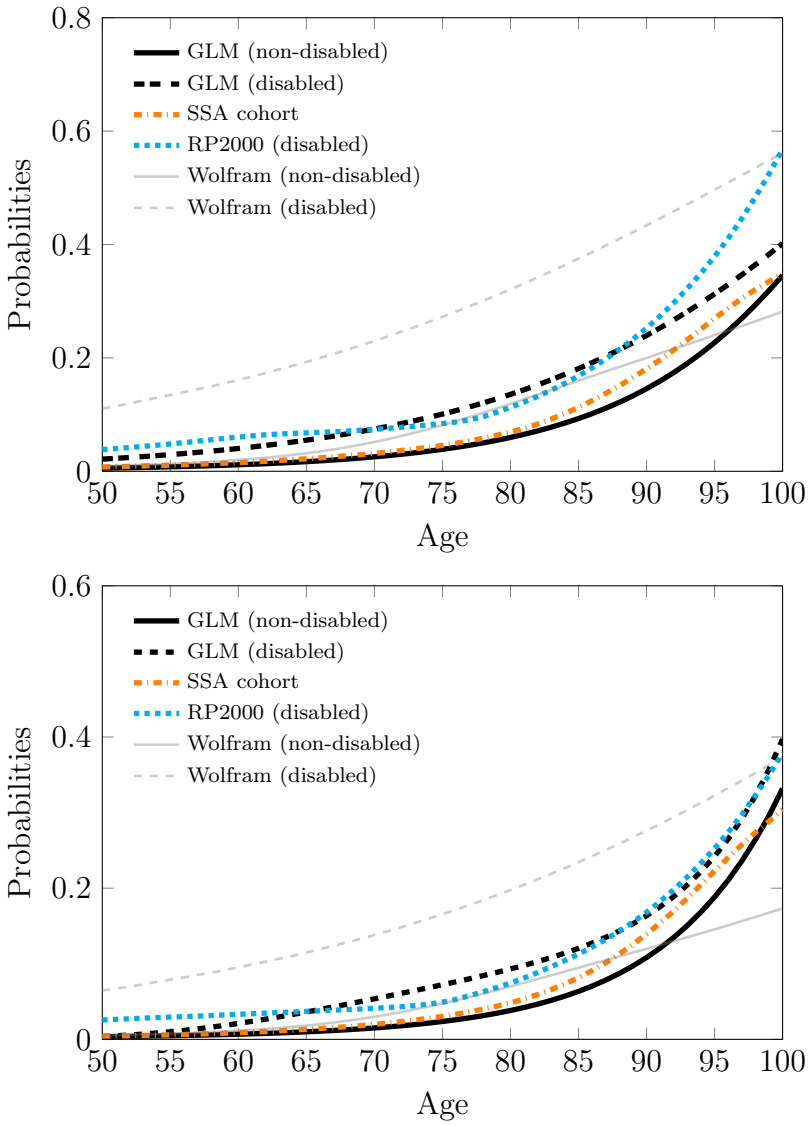


Fig. 7. Comparison of death probabilities for males (top) and females (bottom).

We also compare our estimates against those from the earlier Robinson (1996) study, which relied on a slightly older sample (65+) drawn from NLTCS data in the 1980s. These mortality rates are sourced from Wolfram (nd), and are given by the pair of thinner gray lines in Fig. 7.[19] There are some inconsistencies. First, the Wolfram transition probabilities exhibit a less curvilinear shape than our estimates. Consequently, we see that the mortality

[19]The solid and dashed gray lines reflect mortality rates for the nondisabled and the disabled (2+ ADL disabilities) respectively. To convert the eight-state transition probabilities in Wolfram (nd) to three-state probabilities, we use long-run stationary weights.

(nondisabled) probabilities from Wolfram are higher than our corresponding estimates up to around age 90 after which the reverse then holds. Wolfram's mortality probabilities for solely the nondisabled are even higher than the SSA general population mortality rates. Second, the mortality (disabled) probabilities from Wolfram are significantly higher than those in the present study. These inconsistencies show that the publically-available outputs from the Robinson model may not be directly comparable with our results especially given known differences between the NLTCS and HRS datasets, age of sampled respondents, time of observation, and graduation methodologies. To better understand these differences, we will directly adapt the Robinson model to our HRS sample and compare the outputs.

3.4. Transition rates from Robinson's approach

In this subsection, we seek to apply the graduation methods described in Robinson (1996) to our dataset. The Robinson study uses a time-homogeneous, time-continuous Markov model to derive morbidity transitions with health states that map to LTC insurance eligibility. The key aspects of the published methodology are summarized as follows. Firstly, respondents are grouped by sex, health status, and age categories at the start of each observation period. Observed transition matrices for each sex/age-category/observation-period are tabulated. Robinson (1996) uses, for example, a total of seven initial health states, three age categories (65-74, 75-84, and 85+), and two observation spans (1982-84 and 1984-89). Given a starting status i, the ending status j in each of the observed transition matrix is modeled as an independent trial from a multinomial process.

Secondly, maximum likelihood estimation is used to fit a parametric model of the transition rates. The parametric formulae deemed to be most suitable in that earlier study is:

$$r_{ij}(s,x) = \exp\left[\alpha_{ij} + \beta_{ij}(s-0.5) + \frac{\gamma_{ij}(x-80)}{100}\right], \qquad i \leq 4,\ i \neq j, \tag{9}$$

where r_{ij} is the transition rate from state i to state j, s is the sex variable dummy ($s = 0$ for males and $s = 1$ for females), and x is the average age of the age category. α_{ij}, β_{ij}, and γ_{ij} are parameters to be estimated, whereby constraints are placed on the values of the latter two parameters during estimation. Note that this method yields transition rates only for persons corresponding to the midpoint of each age category (e.g. a 70-year old for the age 65-74 category); exponential interpolation is used to derive the rest of the transition rates for other age values. Finally, matrix exponentiation is applied to convert the transition rates into transition probabilities.

To replicate these methods, we start by creating 48 observed transition matrices based on differences in sex/age-category/observation-spans. We use four age categories (<65, 65-74,

75-84, and 85+) to accommodate the age range of the sampled HRS respondents, and six observation spans (1998-2000, 2000-2002, $\cdots$, 2008-2010). To adhere closely to the health state classifications defined in the earlier study, we define four initial health states: zero, one, two, and 3+ ADLs.[20] We exclude the health states incorporating cognitive impairment since our focus here is primarily on disability defined by the ADL measures.

Next, we implement the parametric formulae referenced above, using maximum likelihood estimation (MLE) to obtain the parameter estimates. Following Robinson (1996), no constraints are imposed on the 16 values of α_{ij}; values for sex-adjustment parameter β_{ij} are constrained to three (one for recovery, one for disability, and one for mortality); and values for the age-slope parameter γ_{ij} are constrained to five (the same three for β_{ij} plus two distinct values when the initial state equals zero ADL). We also set the initial guesses of the α parameters to 0.1 and those of the β and γ parameters to zero as in the earlier study. Corresponding transition probabilities are derived from the transition rates by matrix exponentiation. Table 4 shows the final parameter estimates for our HRS sample.

Table 4. Robinson model: parameter estimates (MLE).

Parameter		**No ADL**	**1 ADL**	**2 ADLs**	**3+ ADLs**	**Dead**
α_{ij}	**No ADL**	–	-2.241	-3.905	-4.433	-3.354
	1 ADL	-0.89	–	-0.818	-2.197	-2.227
	2 ADLs	-13.657	-0.37	–	-0.537	-2.452
	3+ ADLs	-4.465	-3.38	-1.96	–	-1.228
β_{ij}	**No ADL**	–	0.169	0.169	0.169	-0.368
	1 ADL	0.132	–	0.169	0.169	-0.368
	2 ADLs	0.132	0.132	–	0.169	-0.368
	3+ ADLs	0.132	0.132	0.132	–	-0.368
γ_{ij}	**No ADL**	–	6.042	6.042	6.042	7.667
	1 ADL	-1.553	–	2.67	2.67	5.756
	2 ADLs	-1.553	-1.395	–	2.67	5.756
	3+ ADLs	-1.553	-1.395	-1.395	–	5.756

Fig. 8 compares the two sets of results, namely transition probabilities calculated from the GLM approach (solid lines) versus those calculated from Robinson's approach (dotted lines) using our sample of 19,547 individuals drawn from HRS data. Estimation is performed

[20]This is done to adhere as closely as possible to the seven initial health states given in Robinson (1996). Including the death state, there is a total of five states in our replica of the Robinson model. To enable comparability with the GLM estimates, we then apply long-run weights to convert the Robinson 5-state transition probabilities into 3-state transition probabilities. In a separate analysis, we directly derive the Robinson 3-state transition probabilities instead based on a 3-state setup. We find no significant differences between the two sets of Robinson transition probabilities and thus present only results from the former.

for the entire age range of 50-100. For disability onset and recovery transition probabilities, the SOA/Robinson method produces systematically higher probabilities than those from the GLM method. This has important implications in terms of when individuals are likely get inflicted with disability levels severe enough to trigger LTC use and potentially institutionalization. For example, one would conclude from the SOA/Robinson estimates that individuals face a 10% chance of becoming LTC disabled at just age 80. By comparison, our GLM estimates suggest that this does not occur well until about age 90. In terms of recovery from functional disabilities, the GLM probabilities better showcase the complex age patterns of recovery among disabled females than the SOA/Robinson estimates, and thus potentially more reflective of reality.

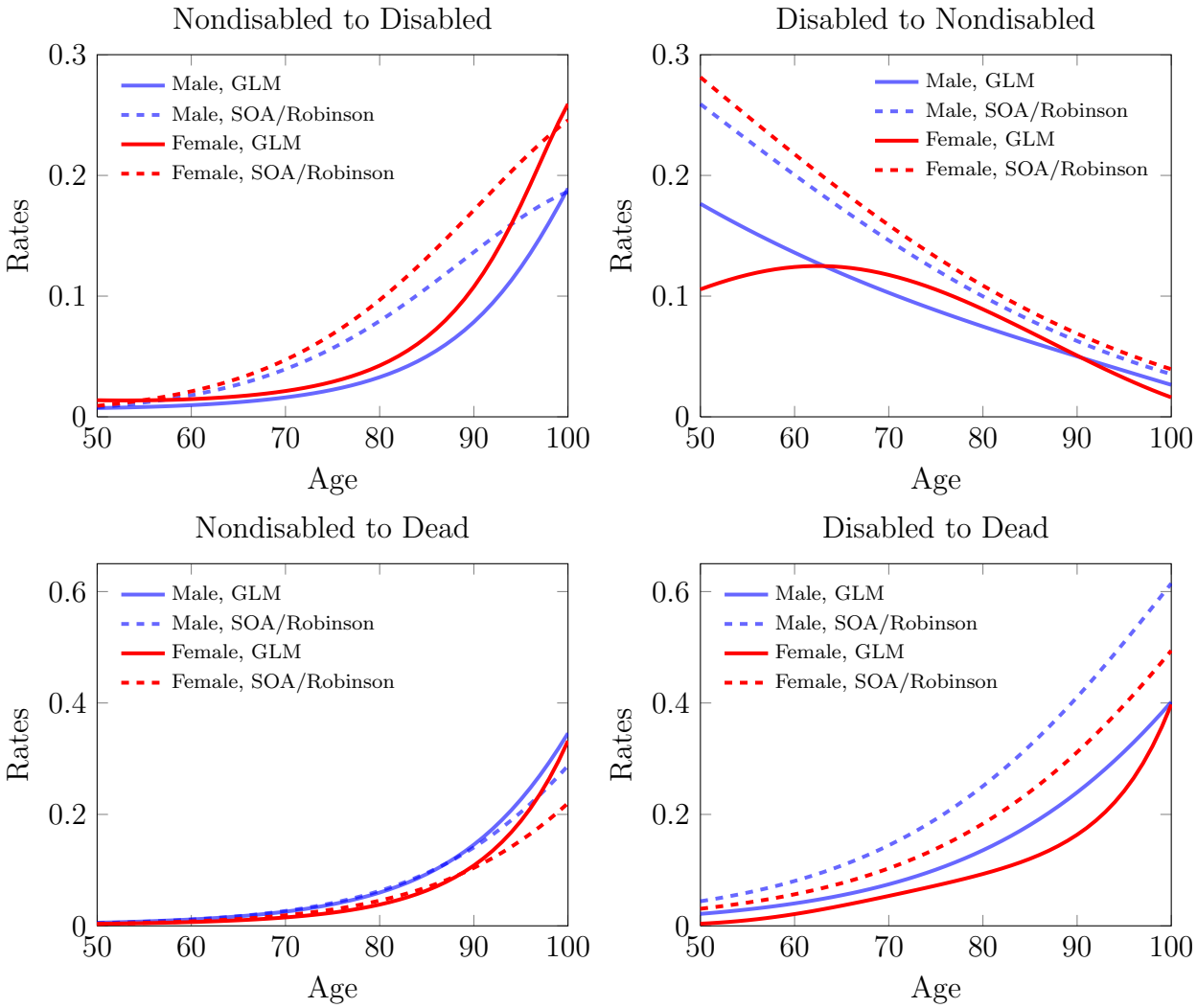


Fig. 8. Comparison of two different graduation approaches.

The main reason for these differences in estimates arises from the parametric form as-

sumed in the SOA/Robinson study. First, the simple log linear specification of the age term does not sufficiently capture the steeper increases in morbidity-mortality probabilities that are expected at older ages. On the other hand, where optimal, we allow for higher-order age terms in the GLM specifications. Consequently most of the probabilities obtained using the Robinson method are visibly less 'curved' and higher when compared alongside the GLM probabilities. This lack of curvature also causes the SOA/Robinson male and female death probabilities not to converge even at very advanced ages. Second, the Robinson (1996) parametric form assumes a constant sex effect across ages. Using this method produces male and female transition rates that are always proportional regardless of age, which explains why the SOA/Robinson transition probabilities for males and females appear parallel in the plots.

4. Alternative benefit triggers

Public programs and private insurance commonly use ADL measures to trigger minimum eligibility criterion. Nonetheless, individual insurers may vary in the way they define the ADL trigger. Arguably the most important difference relates to the threshold number of ADLs used. While most private LTC insurance policies use a 2+ ADL trigger, state Medicaid programs typically use a more stringent 3+ ADL trigger (Congressional Budget Office, 2004; Stone, 2002; Wiener et al., 2000). Some insurers also regulate the ADL types that can be used. Another key difference pertains to the existence of an alternative benefit trigger. Medicaid - as well as some private LTC insurers - allows for benefits to be paid if the individual suffers from severe cognitive impairment and requires substantial supervision. Medical necessity is another form of alternative trigger specified in some LTC insurance policies (Society of Actuaries, 1995).

Thus far, we have defined 'LTC disability' as having 2+ ADL disabilities, as in most private insurance policies. However, since benefit triggers vary in practice, it is useful to extend the transition rate analysis to alternative definitions of LTC disability. This will allow us to determine the sensitivity of different trigger configurations on the transition probabilities. We explore two alternative specifications of 'LTC disability' whereby one specification captures a relatively more stringent criterion (3+ ADLs) and the other specification captures a relatively less stringent criterion (2+ ADLs and/or CI). The latter is the least stringent among the three specifications because individuals satisfy the trigger if they either become moderately functionally disabled or cognitively impaired, whichever occurs first. For each specification, we separately apply the GLM approach to deriving the smoothed transition rates and obtain the transition probabilities.

Results for the female sample appear in Fig. 9; similar patterns are noted for the male

sample. Two observations are worth highlighting. First, we see that different trigger configurations have an evident impact on disability and recovery transition probabilities but little impact on mortality probabilities. Second, assuming a base case criterion of 2+ ADL disabilities, switching to a more stringent 3+ ADLs criterion creates a more pronounced effect on transition probabilities as compared to switching to the less stringent 2+ ADLs and/or CI criterion. Under the stricter criterion, the disability transition probabilities are considerably lower especially at advanced ages and the recovery transition probabilities at the younger ages are considerably higher. Overall this indicates that LTC insurers using a more stringent 3+ ADLs trigger can more competitively price their products since the probability of payout may be substantively lower than insurers using the typical 2+ ADLs trigger. One reason why there is little difference in switching from the 2+ ADL criterion to the 2+ ADLs and/or CI criterion is due to the low proportions of sampled respondents who are severely cognitively impaired. In each wave of the HRS survey, only about 5.5-9.9% of individuals have severe CI. About 1% have mild CI and the majority have normal cognition.

5. Discussion and conclusions

This paper assesses disability and mortality transitions for elderly individuals within a multi-state model for long-term care insurance applications. Using a comprehensive yet flexible GLM graduation approach, we derive four sets of sex- and age-specific transition probabilities pertaining to disability inception, recovery, mortality (nondisabled), and mortality (disabled), for older adults between age 50 and 100. Unlike earlier studies using smaller datasets, we have sufficient observed transition counts across all ages.

Focusing first on 2+ ADL disability incidence and recovery, we confirm results from other studies that the rates of becoming LTC disabled are significantly higher for women than men. The force of disability exceeds the competing mortality hazard for females of all ages, whereas the opposite holds true for males above age 60. Men and women also have distinct age patterns of recovery: while males face a decreasing rate of recovery with age, females actually experience increasing chances of recovery up to about age 65 before it declines. At least 20% of the sampled respondents who were initially LTC disabled reported recovery, showing that the process of functional change among elderly persons - especially those below age 65 - is complex and dynamic. Turning attention to mortality transitions, we find, as expected, that death rates are higher among the disabled than for the nondisabled. Our mortality estimates are also comparable and consistent with the published life table death rates for the U.S. general population, which serve as a source of external validation.

We also explore how transition probability estimates are sensitive to the graduation method adopted. Given its prominence in the existing literature on the LTC status transi-

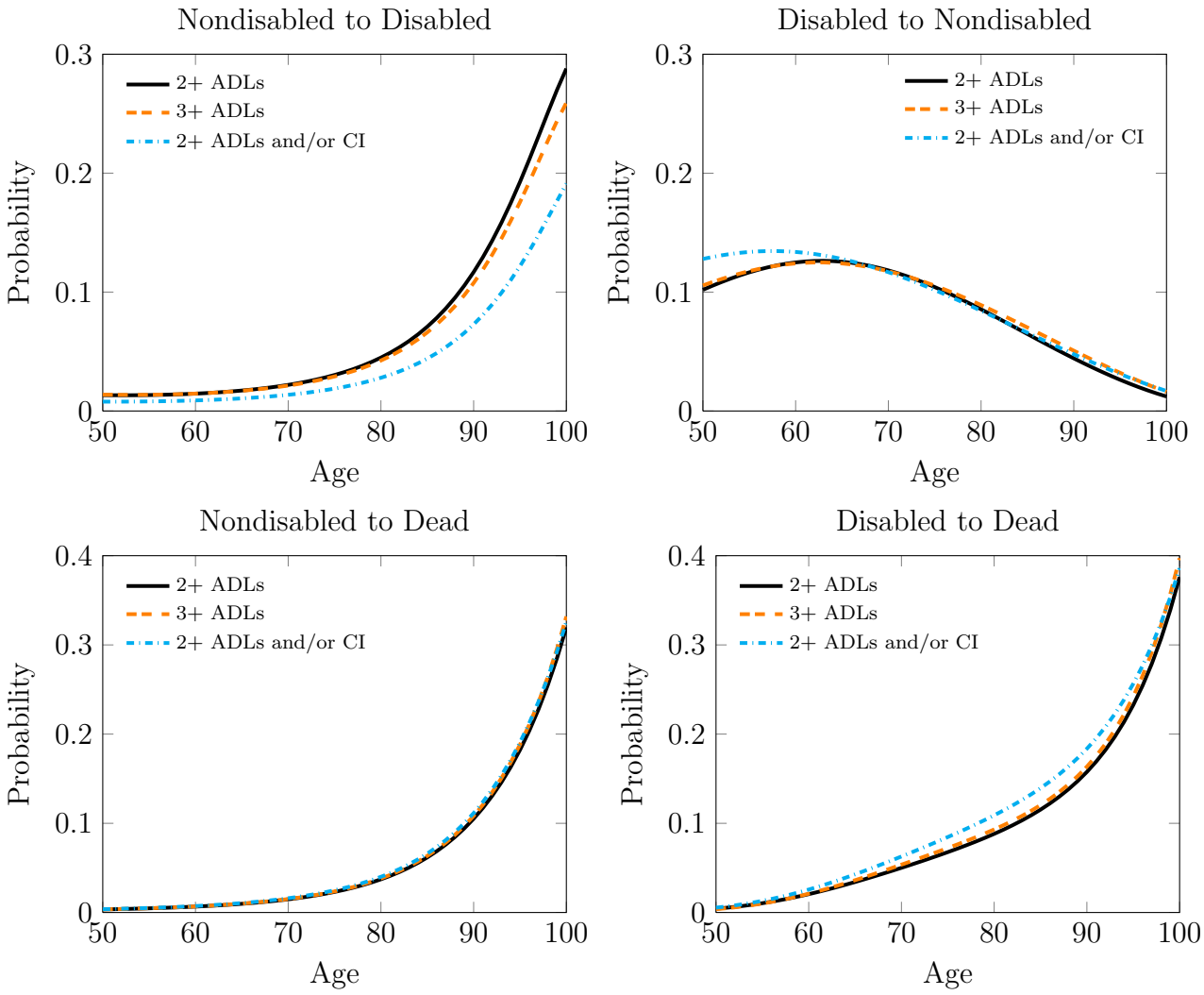


Fig. 9. Different LTC insurance benefit triggers (female).

tion process, the SOA/Robinson model is used as the basis for comparison with our GLM approach. We find that the disability probabilities obtained from the GLM method are slightly more optimistic; in particular, individuals face a fairly high chance of becoming LTC disabled only at about age 90 (compared to age 80 using the Robinson method). Also, the recovery probabilities from the GLM method appear to capture more accurately sex-specific age patterns in recovery from disability as compared to those from the Robinson method. One reason is due to differences in the parametric specifications assumed under each method. Another critical difference is that, unlike the GLM method, the Robinson method does not directly yield age-specific transition rates; one needs to exponentially interpolate between the transition rate estimates for a 70-, 80-, and 90-year old.

Our empirical results demonstrate that transition probability estimates are sensitive to

the definition of 'long-term care disability'. This has implications for the design of benefit triggers for private and public long-term care insurance programs. In particular, insurers who use a more stringent 3+ ADLs criterion rather than a 2+ ADL trigger will observe considerably lower disability transition probabilities for insureds at the advanced ages and higher recovery transition probabilities for insureds at the younger ages. This implies lower premiums in LTC insurance policies or programs which feature the stricter 3+ ADL trigger. We find no compelling grounds for differential premiums between an insurer who incorporates cognitive impairment as an alternative trigger (i.e. 2+ ADLs and/or CI criterion) and an insurer who uses the 2+ ADL trigger.

AppendixA. Residual deviance and variance formula

For the Poisson GLM, we specify the residual deviance statistic, the variance of the fitted transition rates, and the variance of the responses. The residual deviance (D_c) is:

$$D_c = 2[\log(l_s) - \log(l_c)], \tag{A.1}$$

where $\log(l_s) = \sum_x \{-i_x + i_x \log(i_x)\} + C$ and $\log(l_c) = \sum_x \{-\hat{m}_x + i_x \log(\hat{m}_x)\} + C^*$.

In this set of equations, $\log(l_c)$ is the maximized likelihood under the current model (denoted using subscript c), $\log(l_s)$ is the maximized likelihood under the saturated model (denoted using subscript s), $\hat{m}_x$ are the fitted mean values of the response variables, and C is a constant. The response variable (i_x) is given by the number of transitions at age x, and has mean $m_x \equiv E(i_x)$. The residual deviance is a measure of the overall goodness-of-fit of the model to the data. The variance of the fitted transition rates is:

$$\begin{aligned} Var(\hat{\sigma}_x) &= \frac{Var(\hat{m}_x)}{e_x^2} \\ &= \frac{Var\left(e_x \exp\left(\sum_{s=1}^{k} x_s \hat{\beta}_s\right)\right)}{e_x^2}, \end{aligned} \tag{A.2}$$

where $\hat{\sigma}_x$ are the fitted values of the transition intensity, e_x is the corresponding central exposed to risk recorded over the observation window, and $\hat{\beta}_s$ are the estimated values of the parameters via maximum likelihood estimation. The covariance matrix of the $\hat{\beta}_s$'s is employed to derive the variance using the delta method.

The dispersion parameter (ϕ) is typically estimated using Pearson residuals and for the Poisson distribution, is given by:

$$\tilde{\phi} = \frac{1}{n-k-1} \sum_{x} \frac{(i_x - \hat{m}_x)^2}{\hat{m}_x}, \tag{A.3}$$

where $n-k-1$ is the degrees of freedom of the model (number of observations less number of parameters and a constant).

The overall variance of the responses is:

$$\begin{aligned} Var(i_x) &= E\left[Var(i_x \mid \hat{m}_x)\right] + Var\left[E(i_x \mid \hat{m}_x)\right] \\ &= \tilde{\phi} E[\hat{m}_x] + Var[\hat{m}_x]. \end{aligned} \tag{A.4}$$

Here, $\tilde{\phi}$ is set to unity (statistical tests reveal that over-dispersion is not significant) and simulations are used to generate the unconditional distribution of the responses. The overall variance of the responses captures both the variations from the residuals in the first term, and uncertainty from the parameters in the second term.

AppendixB. Tabulations of raw transition counts and exposure years

N = nondisabled state
F = functionally/LTC disabled state
D = Dead

Table B.1. Tabulations of raw transition counts and exposure years, male.

Age Band	i_x = # of transitions				e_x = exposure years	
	σ : N $\mapsto$ F	φ : F $\mapsto$ N	μ : N $\mapsto$ D	ν : F $\mapsto$ D	in state N	in state F
50-54	13	10	5	2	1,742.9	111.5
55-59	80	61	60	20	7,039.6	454.5
60-64	160	143	192	37	13,611.0	870.4
65-69	222	148	325	85	15,504.6	1,096.80
70-74	282	141	405	120	13,182.1	1,247.10
75-79	310	113	447	143	10,011.6	1,254.50
80-84	274	108	446	192	6,427.7	1,159.50
85-89	236	56	325	203	3,089.7	921.1
90-94	126	28	186	140	977.9	414.6
95-100	40	4	50	52	169.0	119.9
Total	**1,743**	**812**	**2,441**	**994**	**71,756.2**	**7,649.9**
N	**8,496**					

Table B.2. Tabulations of raw transition counts and exposure years, female.

Age Band	i_x = # of transitions				e_x = exposure years	
	$\sigma : \mathrm{N} \mapsto \mathrm{F}$	$\varphi : \mathrm{F} \mapsto \mathrm{N}$	$\mu : \mathrm{N} \mapsto \mathrm{D}$	$\nu : \mathrm{F} \mapsto \mathrm{D}$	in state N	in state F
50-54	34	19	9	1	2,689.7	210.6
55-59	153	132	58	14	10,009.9	973.1
60-64	293	260	137	48	17,358.0	1,799.3
65-69	417	285	236	113	18,758.0	2,131.4
70-74	422	274	272	127	15,842.8	2,098.8
75-79	443	227	319	185	12,619.5	2,116.7
80-84	542	222	413	269	9,099.1	2,371.5
85-89	498	168	413	341	5,139.6	2,302.6
90-94	308	100	247	313	1,761.3	1,505.5
95-100	107	21	75	177	355.1	519.6
Total	**3,217**	**1,708**	**2,179**	**1,588**	**93,633.0**	**16,029.0**
N	**11,051**					

Acknowledgement

The authors acknowledge the financial support of the Australian Research Council Centre of Excellence in Population Ageing Research (project number CE110001029). Shao also acknowledges the financial support from Australian School of Business and China Scholarship Council. Opinions and errors are solely those of the authors and not of the institutions providing funding for this study or with which the authors are affiliated.

References

Alai, D. H. and Sherris, M. (2012). Rethinking age-period-cohort mortality trend models. *Scandinavian Actuarial Journal*, DOI: 10.1080/03461238.2012.676563.

Beckett, L. A., Brock, D. B., Lemke, J. H., de Leon, C. F. M., Guralnik, J. M., Fillenbaum, G. G., Branch, L. G., Wetle, T. T., and Evans, D. A. (1996). Analysis of change in self-reported physical function among older persons in four population studies. *American Journal of Epidemiology*, 143(8):766–778.

Bell, F. C. and Miller, M. L. (2005). *Life tables for the United States social security area 1900-2100*. Actuarial Study No. 120. Social Security Administration, Office of the Chief Actuary.

Burnham, K. P. and Anderson, D. R. (2004). Multimodel inference: understanding aic and bic in model selection. *Sociological Methods and Research*, 33(2):261–304.

Census Bureau (2008). National population projections - percent distribution of the projected population by selected age groups and sex for the United States: 2010 to 2050. Available at `http://www.census.gov/population/projections/data/national/2008/summarytables.html`, retrieved on Dec 3rd, 2012.

Congressional Budget Office (2004). Financing long-term care for the elderly. *The Congress of the United States*.

Crimmins, E. M. (2004). Trends in the health of the elderly. *Annual Review of Public Health*, 25:79–98.

Cutler, D. M. (2001). Declining disability among the elderly. *Health Affairs*, 20(6):11–27.

Fauth, E. B., Zarit, S. H., Malmberg, B., and Johansson, B. (2007). Physical, cognitive, and psychosocial variables from the disablement process model predict patterns of independence and the transition into disability for the oldest-old. *The Gerontologist*, 47(5):613–624.

Ferri, S. and Olivieri, A. (2000). Technical bases for LTC covers including mortality and disability projections. *Proceedings of the XXXI International ASTIN Colloquium*, Porto Cervo, Italy:295–314.

Fonda, S. J., Clipp, E. C., and Maddox, G. L. (2002). Patterns in functioning among residents of an affordable assisted living housing facility. *The Gerontologist*, 42(2):178–187.

Fong, J. H., Koh, B. S., and Mitchell, O. S. (2012). Functional disabilities and nursing home admittance. *Pension Research Council Working Paper 2012-19*.

Forfar, D. O., McCutcheon, J. J., and Wilkie, A. D. (1988). On graduation by mathematical formula. *Journal of the Institute of Actuaries*, 115:1–149.

Freedman, V. A., Crimmins, E., Schoeni, R. F., Spillman, B. C., Aykan, H., Kramarow, E., Land, K., Lubitz, J., Manton, K., Martin, L. G., Shinberg, D., and Waidmann, T. (2004). Resolving inconsistencies in trends in old-age disability: Report from a technical working group. *Demography*, 41(3):417–441.

Haberman, S. and E., R. A. (1989). Fitting loss distributions using generalized linear models. *Proceedings of the XXI International Astin Colloquium*, New York:149–160.

Haberman, S. and E., R. A. (1996). Generalized linear models and actuarial science. *The Statistician*, 45(4):407–436.

Haberman, S. and Pitacco, E. (1999). *Actuarial models for disability insurance*. Boca Raton: Chapman and Hall / CRC.

Hatzopoulos, P. and Haberman, S. (2009). A parameterized approach to modeling and forecasting mortality. *Insurance: Mathematics and Economics*, 44(1):103–123.

Health and Retirement Study, HRS (2012). Imputation of cognitive functioning measures: 1992-2010 early release. *University of Michigan, HRS User Guides, February*.

Langa, K. M., Larson, E. B., Karlawish, J. H., Cutler, D. M., Kabeto, M. U., Kim, S. Y., and Rosen, A. (2008). Trends in the prevalence and mortality of cognitive impairment in the United States: Is there evidence of a compression of cognitive morbidity? *Alzheimers Dement*, 4(2):134–144.

Manton, K. G. (1988). A longitudinal study of functional change and mortality in the United States. *Journal of Gerontology: Social Sciences*, 43(5):S153–161.

McCullagh, P. and Nelder, J. A. (1983). *Generalised linear models*. London: Chapman and Hall, 1st edition.

Mendes de Leon, C. F., Beckett, L. A., Fillenbaum, G. G., Brock, D. B., Branch, L. G., Evans, D. A., and Berkman, L. F. (1997). Black-white differences in risk of becoming disabled and recovering from disability in old age: A longitudinal analysis of two EPESE populations. *American Journal of Epidemiology*, 145(6):488–497.

Nuttall, S. R., Blackwood, R. J. L., Bussell, B. M. H., Cliff, J. P., Cornall, M. J., Cowley, A., Gatenby, P. L., and Webber, J. M. (1994). Financing long-term care in Great Britain. *Journal of the Institute of Actuaries (1886-1994)*, 121(1):1–68.

Olivieri, A. and Pitacco, E. (2001). Facing LTC risks. *Proceedings of the XXXII International ASTIN Colloquium*, Washington.

Oman, D., Reed, D., and Ferrara1, A. (1999). Do elderly women have more physical disability than men do? *American Journal of Epidemiology*, 150(8):834–842.

Renshaw, A. E. (1991). Actuarial graduation practice and generalised linear and non-linear models. *Journal of the Institute of Actuaries (1886-1994)*, 118(2):295–312.

Renshaw, A. E. and Haberman, S. (1995). On the graduations associated with a multiple state model for permanent health insurance. *Insurance: Mathematics and Economics*, 17(1):1–17.

Rickayzen, B. D. and Walsh, D. E. P. (2002). A multi-state model of disability for the United Kingdom: Implications for future need for long-term care for the elderly. *British Actuarial Journal*, 8(2):341–393.

Robinson, J. (1996). A long-term care status transition model. *The Old-Age Crisis – Actuarial Opportunities: The 1996 Bowles Symposium, Georgia State University*, pages 72–79.

Rodda, D. T., Lam, K., and Youn, A. (2004). Stochastic modeling of federal housing administration home equity conversion mortgages with low-cost refinancing. *Real Estate Economics*, 32(4):589–617.

Smedley, B. D. and Syme, S. L. (2001). Promoting health: Intervention strategies from social and behavioral research. *American Journal of Health Promotion*, 15(3):149–166.

Society of Actuaries (1995). Long-term care insurance valuation methods. *Transactions of the Society of Actuaries*, 47:599–773.

Society of Actuaries (2000). The RP-2000 mortality tables. Available at `http://www.soa.org/research/experience-study/pension/research-rp-2000-mortality-tables.aspx`, retrieved on Feb 10th, 2013.

Stone, J. L. (2002). Medicaid: Eligibility for the aged and disabled. *Congressional Research Service Reports No. RL31413*, Available at `https://opencrs.com/document/RL31413/`, retrieved on Jan 1st, 2013.

Verrall, R. J. (1996). A unified framework for graduation. *Cass Business School Actuarial Research Reports No. 91.*

Wiener, J. M., Tilly, J., and Goldenson, S. M. (2000). Federal and state initiatives to jump start the market for private long-term care insurance. *The Elder Law Journal*, 8(1):57–99.

Wolfram (n.d.). "Life Transitions" from the Wolfram demonstrations project. Available at `http://demonstrations.wolfram.com/LifeTransitions/`, retrieved on Feb 1, 2013.