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Biometric risk transfers in life annuities and pension products: A survey.

Ermanno Pitacco*

*Professor, University of Trieste, Italy and Associate Investigator, ARC Centre of Excellence in Population Ageing Research (CEPAR), UNSW, ermanno.pitacco@deams.units.it.

Biometric risk transfers in life annuities and pension products: a survey *

Ermanno Pitacco
University of Trieste (Italy)
ermanno.pitacco@deams.units.it

Abstract

We address life annuities and pensions, looking in particular at transfers/sharing of biometric risks, i.e. risks related to the annuitant's lifetime and health status, involved by the policy conditions or the pension plan rules.

First, we focus on various arrangements which aim at building the post-retirement income, and involve either the accumulation phase, or the payout phase, or both. Various products are available on financial and insurance markets, each product with a specific guarantee structure (Conventional Life Annuities either immediate or deferred, Variable annuities, withdrawal plans, etc.).

We then shift to a range of specific annuity products, stressing the relevant features: Advanced Life Delayed Annuity (ALDA), Ruin Contingent Life Annuity (RCLA), Variable Annuities (VA). Finally, we focus on some arrangements for the payout phase: the life annuity with a guarantee period, the value-protected life annuity (that is, providing "capital protection"), progressive annuitization schemes, life annuities combined with Long Term Care (LTC) benefits. We conclude with a short introduction to the longevity-linked life annuities.

Keywords: Life annuities, Annuitization, Variable annuities, Advanced Life Delayed Annuity (ALDA), Ruin Contingent Life Annuity (RCLA), Long Term Care (LTC), Life Care annuity, Enhanced pension, Longevity-linked life annuities

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1 Introduction

Each insurance and pension product can be interpreted as a package of guarantees and options, implying risk transfers between the insurer or annuity provider on the one hand, and the insured or annuitant on the other.

In this paper we address both life annuities provided by occupational pension schemes and purchased life annuities, focusing in particular on the guarantees provided by different types of life annuities. Indeed, each annuity product design determines a specific guarantee structure, which should carefully be considered when pricing the product itself.

Special attention will be placed on biometric risks (and, in particular, the longevity risk) which can be shared between annuity provider and annuitants according to the product design.

The paper is structured as follows. In Sect. 2 some option/guarantee structures are briefly described, while in Sect. 3 several examples of products aiming at building the post-retirement income are provided. Specific annuity products are described in Sect. 4. In Sect. 5 some possible arrangements for the payout phase (that is, the retirement period) are discussed, while the time profile of the annuity benefits is dealt with in Sect. 6, where basic concepts underlying the longevity-linked life annuities are also sketched. Final remarks in Sect. 7 conclude the paper.

In this paper, there is nothing original from a scientific point of view: we only aim at providing a review of products available on insurance and pension markets, with a special focus on features related to biometric risk sharing.

2 Options and guarantees in life insurance and annuities

In this Section some guarantees provided by life insurance and annuity products and some options which can be included in the products themselves are briefly described. We focus on those guarantees and options which are particularly relevant to the construction of the post-retirement income, while the reader can refer to Black and Skipper (2000), Gatzert (2009), Hardy (2004), and Pitacco (2012) for the meaning of other guarantees and options.

In an endowment insurance policy, the *mortality guarantee* implies that, whatever the number of deaths in the portfolio, the insurer has to pay the death benefit amount as stated in the policy. It is worth noting that the impact of this guarantee decreases as time from policy issue increases because

of the progressive decrease in the sum at risk.

According to the *interest guarantee*, the policy reserve must be annually credited with an amount calculated with the specified interest rate, whatever the investment yield obtained by the insurer.

Several options can be included in an endowment insurance policy. The following ones are of particular interest (see Fig. 2.1).

If the *surrender option* is exercised, the contract terminates and the surrender value (that is, the policy reserve minus the surrender fee) is paid to the policyholder. Several risks are implied by this option; for example, the market risk (when the insurer is forced to sell bonds with an interest rate lower than the current rate), the liquidity risk, etc.

Various *dividend options* can be available, which allow the policyholder to participate in insurer's profits (which arise from investment returns, mortality, expenses); in particular:

1. dividends can be paid in cash, usually via reduction of future premiums;

2. as an alternative, adopted in many European policies, dividends can be used to finance increments in the sum insured (either in the case of survival at maturity, or in the case of death, or both);

3. another alternative consists in a financial accumulation of the dividends, with a guaranteed interest rate.

Alternatives 3 and, possibly, 2 (according to the mechanism adopted for increasing the sum at maturity) imply a financial risk borne by the insurer, in addition to the risk borne because of the interest guarantee on the "basic" reserve.

Several *settlement options* are available as regards the death benefit. In particular:

- usually the benefit is paid to the beneficiary as a lump sum;

- as an alternative, the benefit can be paid during a fixed period as a sequence of instalments;

- another alternative consists in paying the benefit as a life annuity to the beneficiary, as long as he/she is alive; it is worth stressing that, in this case, a longevity risk is taken by the insurer.

Also the survival benefit can be paid according to various arrangements. In particular, if the *annuitization option* is exercised, then the benefit is paid as a life annuity, i.e. as long as the beneficiary is alive. A crucial problem is

related to the time at which the annuitization rate is stated; this time can vary from the date of policy issue to policy maturity: the sooner this rate is fixed, the higher is the aggregate longevity risk, due to the uncertainty in future mortality trend, taken by the insurer (see also Sect. 3 and, in particular, 3.2). Anyway, whatever the time at which the annuitization rate is stated, if the annuitization option is exercised, various risks are taken by the insurer, and in particular:

- the adverse selection risk, caused by the likely good health conditions of the beneficiary who annuitizes, and hence by a presumably long expected lifetime;
- the longevity risk (in particular, its aggregate component; see below);
- the financial risk, originated by the minimum interest guarantee usually provided by the life annuity.

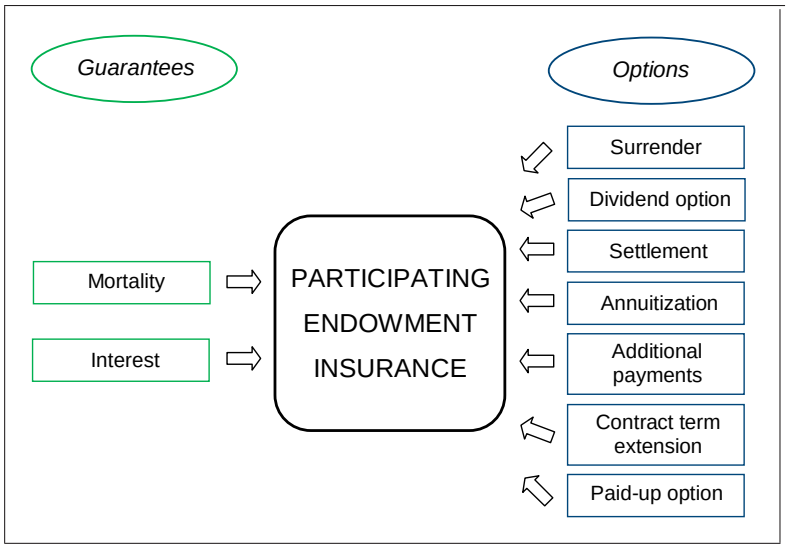

Figure 2.1: Guarantees and options in a (participating) endowment insurance

By exercising the *additional payments* option, the policyholder can increase the sum insured. As regards the death benefit, this option implies the "guaranteed insurability".

Thanks to the *contract term extension*, the policyholder can take advantage from the guaranteed interest rate; thus, the value of this option depends on the current interest rate.

The *paid-up option* is exercised when the policyholder stops the premium payment without terminating the insurance contract. Thus, the contract remains in force with properly reduced benefits.

The range of guarantees and options provided by life annuities and the relevant features are strictly related to the type of the life annuity product. For example, in a deferred life annuity both the "accumulation" and the "payout" (or "decumulation") phases are involved, so that some guarantees (e.g. the interest rate guarantee) can extend over a period of several decades. Moreover, the amount of longevity risk borne by the insurer (or, in general, by the annuity provider) depends on the time at which the annuitization rate is stated.

Section 3.2 is specifically devoted to guarantee structures in life annuities. Hence, now we only focus on the payout phase, thus addressing immediate life annuities (see Fig. 2.2). The *interest guarantee* has been already discussed while referring to the endowment insurance. Of course, in a life annuity the importance of this guarantee is a consequence of the average long duration of the annuity itself.

Thanks to the *longevity guarantee*, the annuitant has the right to receive the stated annuity benefit as long as he/she is alive, and hence:

1. whatever his/her lifetime;

2. whatever the lifetimes of the annuitants in the annuity portfolio (or pension fund).

Because of feature 1, the annuity provider takes the *individual* longevity risk, originated by *random fluctuations* of the individual lifetimes around the relevant expected values. Feature 2 also implies the *aggregate* longevity risk: if the average lifetime in the portfolio is higher than expected, the annuity provider suffers a loss, because of *systematic deviations* of the lifetimes from the relevant expected values.

Various options can be added to the life annuity product. These options can be exercised before the start of the payout period, that is, at policy issue or, in the case of deferred annuities, before the end of the deferment period (usually with some constraints, e.g. 6 months before the end of this period, to reduce the possible adverse selection). By exercising these options, other benefits are added to the basic life annuity product.

The *capital protection* (or *money-back*) option and the *LTC (Long Term Care) uplift* option are described in Sect. 5. Now we only focus on the *last-survivor annuity*, that is, an annuity payable as long as at least one of two individuals (the annuitants), say (x) and (y), is alive. It can be stated that the annuity continues with the same annual benefit, say b, until the death of the

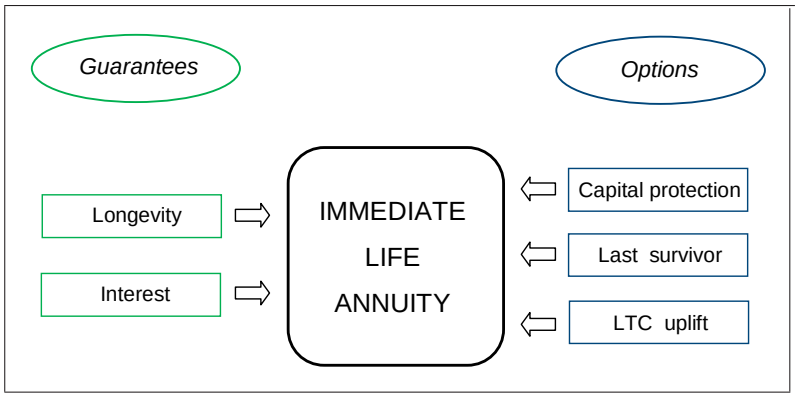

Figure 2.2: Guarantees and options in an immediate life annuity

last survivor. According to a modified form, the benefit amount, initially set to b, will be reduced following the first death: to b' if individual (y) dies first, and to b'' if individual (x) dies first, clearly with $b' < b$, $b'' < b$. Conversely, in many pension plans the last-survivor annuity provides that the annual benefit is reduced only if the retiree, say individual (x), dies first. Formally, $b' = b$ (instead of $b' < b$) and $b'' < b$. Whatever the arrangement, the expected duration of a last-survivor annuity is longer than that of an ordinary life annuity (that is, with just one annuitant), and a higher longevity risk (both individual and aggregate) is borne by the annuity provider.

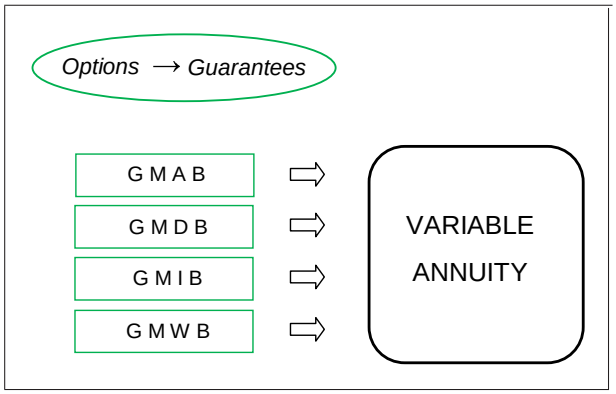

Figure 2.3: Options in Variable Annuity products

The main features of Variable Annuities are described in Sect. 4.3. Here we just introduce the topic. The term Variable Annuity is used to refer to a

wide range of life insurance products, whose benefits can be protected against investment and mortality / longevity risks by selecting one or more guarantees out of a broad set of possible arrangements (see, for example, Kalberer and Ravindran (2009) and Bacinello et al. (2011)). Hence, in variable annuity products the presence of guarantees is a consequence of policyholder's choices via the exercise of specific options (see Fig. 2.3). Available guarantees are referred to as GMxB, where 'x' stands for the class of benefits involved. Whatever the arrangement chosen by the policyholder, a variable annuity is a long-term, tax-deferred investment, designed for obtaining a post-retirement income.

3 Building the post-retirement income

We describe various arrangements, involving either the accumulation phase, or the payout phase, or both.

3.1 Introduction

Various products are available on financial and insurance markets, each product with a specific guarantee structure (accumulation plans, conventional life annuities either immediate or deferred, etc.).

In what follows we focus on guarantees provided by each arrangement. Risks taken by the intermediary, in particular the annuity provider (either insurer or pension fund) can immediately be identified looking at the guarantee structure.

In the following figures:

- x denotes the age at policy issue, or at entering the pension scheme;
- $x + r =$ denotes the age at retirement.

In each figure, the graphical notation shown in Fig. 3.1 is adopted.

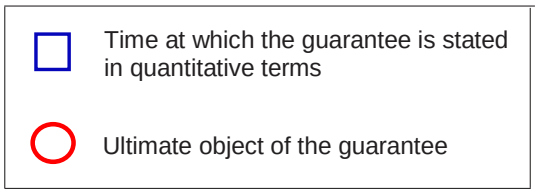

Figure 3.1: Defining the guarantee

For more general issues on life annuities, the reader can refer, for example, to Milevsky (2006), Milevsky (2013), Pitacco et al. (2009), Rocha et al. (2011), Wadsworth et al. (2001). An extensive literature review of post-retirement financial strategies is provided by Shapiro (2010).

3.2 Some basic structures

Structure 1 only involves the accumulation phase. For any given sequence of (annual) contributions / premiums / savings, $c_0, c_1, \ldots, c_{r-1}$, the amount S is guaranteed; see Fig. 3.2.

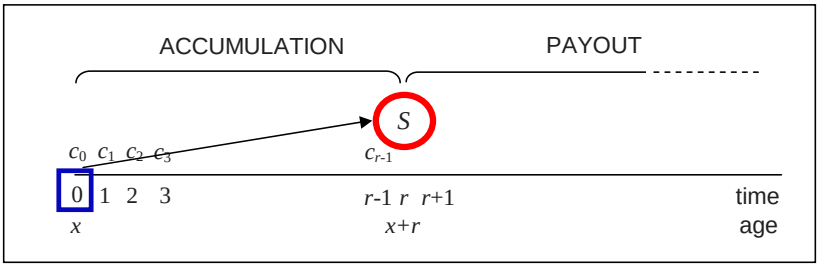

Figure 3.2: Structure 1 - Accumulation phase only

We consider the following examples.

- in a financial product, with guaranteed interest rate i, the guaranteed amount is given by:
$$S = \sum_{h=0}^{r-1} c_h \, (1+i)^{r-h} \tag{3.1}$$
- in an insurance product, e.g. a pure endowment insurance or an endowment insurance, the sum S at maturity is guaranteed if an interest guarantee is provided (and a longevity guarantee as well, in the case of the pure endowment).

Remark 1

The amount which will actually be available at time r as the result of the accumulation or the reserving process can be higher than S, thanks to a very good performance of the fund or the assets backing the policy reserve in a participating policy. Hence, S must be interpreted as the *minimum guaranteed amount*. The same remark also applies to other structures described in this Section.

Structure 2 involves the payout phase only. For any given amount S, the annual benefit b (assuming a flat payment profile) is guaranteed; see Fig. 3.3.

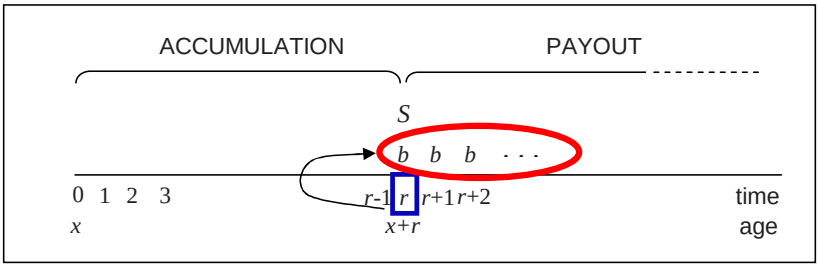

Figure 3.3: Structure 2 - Payout phase only

Examples are as follows.

- In a financial product, the annual benefit b is guaranteed up to (possible) fund exhaustion, thanks to an interest rate guarantee.
- In an immediate life annuity, the annual benefit b is guaranteed lifelong thanks to the interest guarantee and the longevity guarantee; the relation between the annuitized amount S and the benefit b is given, in quantitative terms, by the following relation:

$$b = \frac{1}{\ddot{a}_{x+r}^{[\text{curr}]}} S \tag{3.2}$$

  where $\frac{1}{\ddot{a}_{x+r}^{[\text{curr}]}}$ is the current annuity rate (CAR), i.e. stated at annuitization time r. This life annuity is briefly called a CAR immediate life annuity.

Structure 3 involves both the accumulation phase and the payout phase, and combines structure 1 and 2; see Fig. 3.4. We note that the interest guarantee working throughout the accumulation phase is stated at time 0, whereas the guarantee concerning the payout phase is state at time r, that is, at the beginning of the decumulation.

Examples are as follows.

- A financial product or an insurance product provides the guaranteed amount S at time r.
- A CAR immediate life annuity for the payout phase guarantees lifelong the annual benefit b.

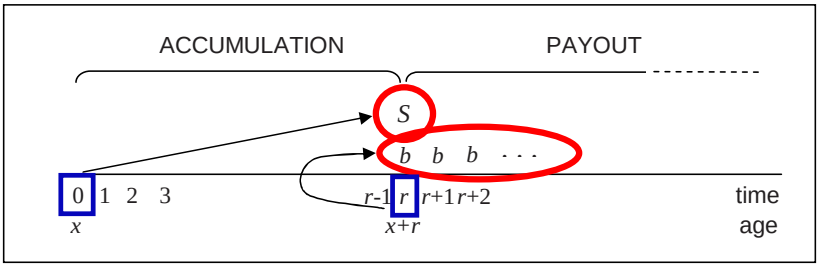

Figure 3.4: Structure 3 - Accumulation phase + Payout phase (1)

Also **Structure 4** embraces the accumulation phase and the payout phase. Unlike in Structure 3, all the guarantees are stated at time 0 (a challenge for the annuity provider!); see Fig. 3.5.

We consider the following examples.

- A GAR deferred life annuity, i.e. an annuity with a guaranteed annuity rate stated in particular at time 0, provides, for any given sequence $c_0, c_1, \ldots, c_{r-1}$, the lifelong benefit b. We note that, assuming $c_0 = c_1 = \cdots = c_{r-1} = P$, this structure is implied in particular by the classical actuarial formula

$$P\,\ddot{a}_{x:r\rceil} = b\,{}_{r|}\ddot{a}_x^{[\text{guar}]} \tag{3.3}$$

  according to which S represents the policy reserve at time r, that is, $S = b\,\ddot{a}_{x+r}^{[\text{guar}]}$; the value ${}_{r|}\ddot{a}_x^{[\text{guar}]}$ is, of course, stated at time 0.

- Combining a financial product with interest guarantee over the accumulation phase and a GAR immediate life annuity for the payout phase yields a similar result.

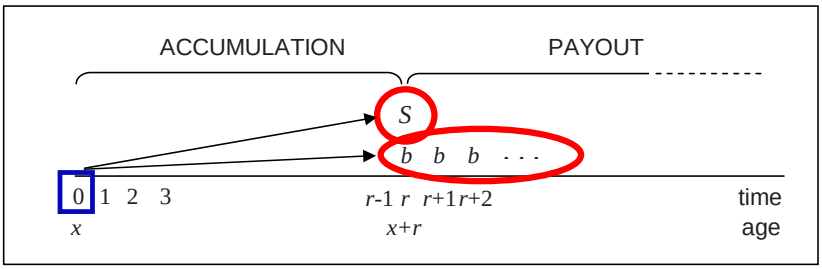

Figure 3.5: Structure 4 - Accumulation phase + Payout phase (2)

Also **Structure 5** involves both the accumulation phase and the payout phase. The annuity rate is stated at time 0. See Fig. 3.6.

An example is provided by the following combined product:

1. a financial product for the accumulation phase (possibly providing a guaranteed interest rate);

2. an immediate life annuity for the payout phase, whose benefit b is determined according to a guaranteed annuity rate (stated at time 0).

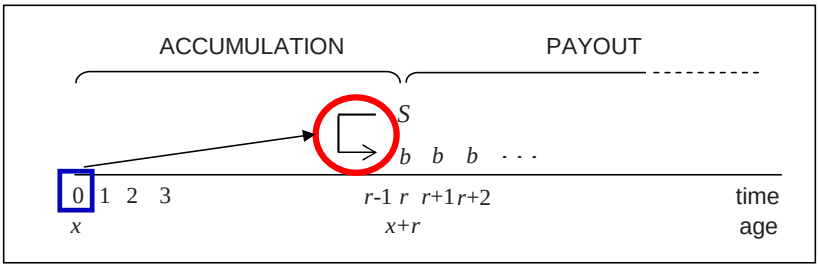

Figure 3.6: Structure 5 - Accumulation phase + Payout phase (3)

In particular, the GAO product (where GAO means "guaranteed annuity option") implies the implementation of the above structure. Actually the GAO product provides the following options (at retirement), that is the choice among:

- ▷ lump sum;
- ▷ annuitization according to the CAR (with annuity value $\ddot{a}_{x+r}^{[\text{curr}]}$);
- ▷ annuitization according to the GAR (with annuity value $\ddot{a}_{x+r}^{[\text{guar}]}$).

Remark 2

> Assume that the accumulation phase is implemented via an insurance product (e.g. a pure endowment with S as the sum at maturity), and works according to the logic of single recurrent premiums (that is, a particular progressive funding of S).
> Then, guarantees in both Structure 4 and Structure 5 can be weakened by linking the guarantee specification (the accumulation guarantee and/or the conversion rate) to each single recurrent premium. Thus, the guarantee specified at time 0 only pertains to the first single

recurrent premium and the corresponding share of the amount at maturity; in general, the guarantee specified at time h ($h = 0, 1, \ldots, r-1$) only pertains to the single recurrent premium paid at time h and the corresponding share of the amount at maturity.

Remark 3

Starting from the basic structures we have described, it is possible to conceive specific product designs by moving in various directions. In particular:

- ▷ by reducing the "scope" over time of some guarantees, viz the longevity guarantee;
- ▷ by designing a "basic" non-guaranteed product, which can be shaped according to client's choices by including one or more guarantees.

Some interesting examples are provided in Sect. 4.

4 A range of annuity products

In this Section we describe two specific products, the Advanced Life Delayed Annuity (ALDA) and the Ruin Contingent Life Annuity (RCLA), and one "category" of products, that is, the Variable Annuities. All these products involve, to some extent, both the accumulation phase and the payout phase.

4.1 Advanced Life Delayed Annuity (ALDA)

The ALDA product was proposed by Milevsky (2005). See also: Gong and Webb (2010), Stephenson (1978).

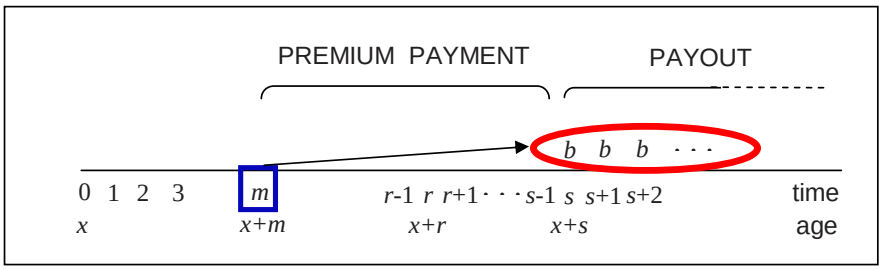

Figure 4.1: The ALDA model

The premium payment period does not necessarily coincide with the (traditional) accumulation phase, being possibly shifted towards older ages. The payout period starts after retirement time (age 80 or 85, say). See Fig. 4.1.

We note that the ALDA product can be interpreted as an implementation of Structure 4, adapted by shifting: $0 \rightarrow m$, $r \rightarrow s$.

The payout period delayed to time s implies withdrawals from a fund throughout the time interval $(r, s-1)$ in order to get post-retirement income.

The main purposes of ALDA are the following ones:

- to provide an insurance cover of the longevity risk at old ages only; hence, ALDA results in an insurance product with a deductible (the time interval $(r, s-1)$);
- to reduce the premium amount (with respect to conventional life annuities), so to enhance rates of voluntary annuitization.

4.2 Ruin Contingent Life Annuity (RCLA)

The RCLA product was proposed by Huang et al. (2009). According to the features of this product, the post-retirement income is provided by:

1. withdrawals from a fund from time r onwards, up to (possible) exhaustion of the fund;
2. a life annuity paid to the retiree from (random) time T of fund exhaustion because of "adverse" scenario, which can result from:
   (a) poor performance of the fund
   (b) long lifetime

See Fig. 4.2.

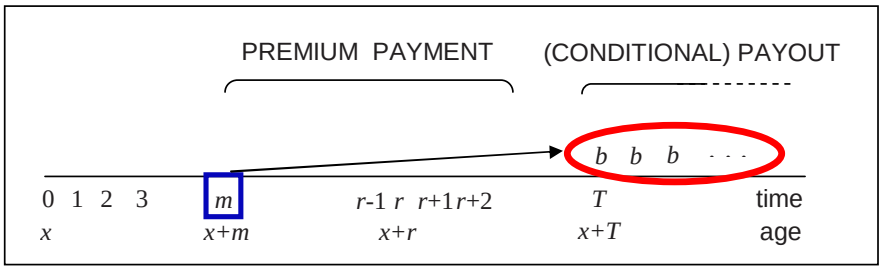

Figure 4.2: The RCLA model

We note that the RCLA can be thought either as an ALDA with random delay $T - r$, where the random time T is defined by a trigger expressing the scenario, or an insurance product which generates an annuitization as a "worst case" scenario.

The pricing procedure for a RCLA product must rely on a properly defined pseudo-index accounting for:

- the behavior of a market performance index, which should represent the performance of the fund used by the retiree during the withdrawal phase;
- a set of reasonable benefit amounts (providing the post-retirement income) throughout the withdrawal phase.

4.3 Variable Annuities (VA)

The term variable annuity is used to refer to a wide range of life insurance products, whose benefits can be protected against investment and mortality/longevity risks by selecting one or more guarantees out of a broad set of possible arrangements. Originally developed for providing a post-retirement income with some degree of flexibility, nowadays accumulation and death benefits constitute important components of the product design. Indeed, the variable annuity can be shaped so as to offer dynamic investment opportunities with some guarantees, protection in case of early death and/or a post-retirement income (see Olivieri and Pitacco (2011)).

We stress that no guarantee is implicitly embedded in a variable annuity product, whereas one or more guarantee can be chosen by the client and then added to the product. Guarantees are usually denoted by GMxB, that is Guaranteed Minimum Benefit of type x. As we will see, including guarantees logically results in structures we have defined above.

For more information, the reader can refer to Bacinello et al. (2011) and Kalberer and Ravindran (2009), and references therein.

In what follows we refer for simplicity to a product financed by a single premium Π, and assume that no withdrawals occur prior to retirement time r. Let F_t denote the balance (fund value) at time t.

The **Guaranteed Minimum Accumulation Benefit (GMAB)** is usually available prior to retirement. At some specified date, the insured (if alive) is credited the greater between the policy account value and a guaranteed amount. Assuming that the guarantee refers at retirement time r, the guaranteed amount, $G_r^{[\mathrm{A}]}$, can be stated as follows.

- *Return of premiums*:
$$G_r^{[\mathrm{A}]} = \Pi \tag{4.1}$$

- *Roll-up guarantee*:
$$G_r^{[\mathrm{A}]} = \Pi\,(1+i)^r \tag{4.2}$$
where i is the guaranteed interest rate;

- *Ratchet guarantee*:
$$G_r^{[\mathrm{A}]} = \max_{t_h<r}\{F_{t_h}\} \tag{4.3}$$
where t_h, $h = 1, 2, \ldots$ are stated times;

- *Reset guarantee*:
$$G_r^{[\mathrm{A}]} = F_{\max\{t_j:\, t_j<r\}} \tag{4.4}$$
where t_j, $i = 1, 2, \ldots$ are the stated reset times.

In principle, guarantees can be combined; for example

- *Roll-up & Ratchet guarantee*:
$$G_r^{[\mathrm{A}]} = \max\left\{\Pi\,(1+i)^r,\, \max_{t_h<r}\{F_{t_h}\}\right\} \tag{4.5}$$

As a result of the guarantee mechanism, the amount acknowledged at time r, $B_r^{[\mathrm{A}]}$, is defined as follows:

$$B_r^{[\mathrm{A}]} = \max\{F_r, G_r^{[\mathrm{A}]}\} \tag{4.6}$$

We note that $G_r^{[\mathrm{A}]}$ corresponds to the amount denoted by S in Structures 1, 3 and 4 defined in Sect. 3.2.

Similarly to the GMAB, also the **Guaranteed Minimum Death Benefit (GMDB)** is available during the accumulation period; some insurers are willing to provide a GMDB also after retirement, up to some maximum age (say, 75 years). The structure of the guarantee is similar to the GMAB: in case of death prior to the stated maturity r, the insurer will pay the greater between the account value and a stated amount $G_t^{[\mathrm{D}]}$. Hence, the death benefit at time t is given by:

$$B_t^{[\mathrm{D}]} = \max\{F_t, G_t^{[\mathrm{D}]}\} \tag{4.7}$$

The guaranteed amount $G_t^{[\mathrm{D}]}$ can be defined according to formulae similar to those adopted for the GMAB, that is:

- *Return of premiums*:
$$G_t^{[\mathrm{D}]} = \Pi \tag{4.8}$$

- *Roll-up guarantee*:
$$G_t^{[\mathrm{D}]} = \Pi\,(1+i)^t \tag{4.9}$$

- *Ratchet guarantee*:
$$G_t^{[\mathrm{D}]} = \max_{t_h<t}\{F_{t_h}\} \tag{4.10}$$
where t_h, $h = 1, 2, \ldots$ are stated times;

- *Reset guarantee*:
$$G_r^{[\mathrm{D}]} = F_{\max\{t_j:\, t_j<t\}} \tag{4.11}$$
where t_j, $i = 1, 2, \ldots$ are the stated reset times.

Also the GMDB can in principle be defined as a combination of guarantees, e.g. Roll-up & Ratchet (see Eq. (4.5)).

The **Guaranteed Minimum Income Benefit (GMIB)** provides a lifetime annuity from time r on. Let $b^{[\mathrm{I}]}$ denote the guaranteed annual benefit. The guarantee may be arranged in two different ways.

- Guarantee on the *amount to annuitize*; then:
$$b^{[\mathrm{I}]} = \frac{1}{\ddot{a}_{x+r}^{[\mathrm{curr}]}}\ \max\{F_r, G_r^{[\mathrm{I}]}\} \tag{4.12}$$
where $G_r^{[\mathrm{I}]}$ can be defined as $G_r^{[\mathrm{A}]}$ (see Eqs. (4.1) to (4.4)). We recognize Structure 3 with $S = G_r^{[\mathrm{I}]}$ (see Sect. 3.2).

- Guarantee on the *annuitization rate* (stated before time r, in particular at the date the policy is issued); then:
$$b^{[\mathrm{I}]} = F_r\ \max\left\{\frac{1}{\ddot{a}_{x+r}^{[\mathrm{curr}]}}\,,\ \frac{1}{\ddot{a}_{x+r}^{[\mathrm{guar}]}}\right\} \tag{4.13}$$
This guarantee is also known as the GAO; see Structure 5.

In principle, the two guarantees can be combined, with the following result.

- Guarantee on the *amount & annuitization rate*; then:
$$b^{[\mathrm{I}]} = \max\{F_r, G_r^{[\mathrm{I}]}\}\ \max\left\{\frac{1}{\ddot{a}_{x+r}^{[\mathrm{curr}]}},\ \frac{1}{\ddot{a}_{x+r}^{[\mathrm{guar}]}}\right\}$$
See Structure 4.

In practice, the resulting product would be very expensive, because of the huge risk taken by the insurer.

The **Guaranteed Minimum Withdrawal Benefit (GMWB)** guarantees periodical withdrawals from the policy account, also if the account value reduces to zero because of:

- ▷ poor investment performance;
- ▷ insured's long lifetime.

The guarantee affects both

1. the withdrawal amount;
2. the withdrawal duration, which may be:
   (a) fixed;
   (b) fixed provided that the retiree is alive;
   (c) lifelong.

In the case of guaranteed duration (c), we recognize the logical structure of the RCLA (see Sect. 4.2).

The withdrawal amount, $b_t^{[\mathrm{W}]}$, is stated as a given percentage, β_t, of a base amount W_t which is usually the account value at the date t^* the GMWB is selected. Hence:

$$b_t^{[\mathrm{W}]} = \beta_t \, F_{t^*} \tag{4.14}$$

In some arrangements, at specified dates (e.g., every policy anniversary) the base amount may step up to the current value of the policy account, if this is higher. This is a ratchet guarantee, which may be lifetime or limited to some years (10 years, say). In this case:

$$b_t^{[\mathrm{W}]} = \beta_t \, W_t = \beta_t \, \max\{F_{t^*}, F_t\} \tag{4.15}$$

Remark

The GMWB is the real novelty of variable annuities in respect of traditional life insurance contracts; it provides a benefit which is similar to an income drawdown, but with guarantees. When comparing a GMIB to a GMWB, three major differences arise:

- the duration of the annuity (which is lifetime in the GMIB);
- the accessibility to the account value (just for the GMWB);
- the features of the reference fund (which usually is unit-linked in the GMWB, but typically participating in the GMIB).

5 Some arrangements for the payout phase

As seen in the previous Sections, various products are available to construct the post-retirement income. Nevertheless, a weak propensity to annuitize can be observed in many countries. To enhance this propensity, the payout phase can be improved, either adding some flexibility or including benefits other than the standard life annuity.

5.1 Introduction

When planning the post-retirement income, some basic features of the life annuity product should be carefully accounted for. In particular, we note the following aspects.

1. The life annuity product relies on the mutuality mechanism, like the pure endowment insurance. This means that:
   (a) the amounts released by the deceased annuitants are shared among the annuitants who are still alive;
   (b) on the annuitant's death, his/her estate is not credited with any amount, and hence no bequest is available.
2. A life annuity provides the annuitants with an "inflexible" post-retirement income, in the sense that the annual amounts must be in line with the payment profile, as stated by the policy conditions.

Both features 1(b) and 2 can be perceived as disadvantages, and hence weaken the propensity to immediately annuitize the whole amount available at retirement. We will illustrate how these disadvantages can be mitigated, at least to some extent, either by purchasing life insurance products in which other benefits are packaged (see Sects. 5.2, 5.3 and 5.5), or adopting specific annuitization strategies (Sect. 5.4).

Of course, a key point in the choice of an annuity product or an annuitization strategy should be the amount of longevity risk transferred to the annuity provider. Conversely, the longevity risk taken by the annuity provider should constitute a key point in designing the annuity products. Table 5.1 summarizes this aspect.

5.2 Life Annuity with a Guarantee Period

In this type of life annuity the benefit is paid for the guarantee period (5 or 10 years, say) regardless of whether the annuitant is alive or not. For a guarantee

Table 5.1: Longevity risk: where?

Solution		Longevity risk
Income drawdown		borne by annuitant
Annuitization (conventional life annuity)		borne by annuity provider
Combined solutions	partial annuitization delayed annuitization phased annuitization ALDA RCLA	shared between annuitant and annuity provider
Longevity-linked life annuities		

period of m years, and an amount S to be converted into an annuity (so that S represents the single premium), the resulting annual benefit b fulfills the following relation:

$$S = b\, a_{m\rceil} + b\, {}_{m|}a_{x+r} \tag{5.1}$$

(assuming the annuity is payable in arrears) where $a_{m\rceil}$ denotes the present value of a temporary annuity-certain. Thus, the insurance product results in a deferred life annuity combined with a temporary annuity-certain.

Table 5.2 provides some numerical examples. The following data have been assumed:

- interest rate 2%, for both the annuity-certain and the deferred life annuity;
- projected life table with:
  - life expectancy at the birth: $\overset{\circ}{e}_0 = 85.13$;
  - remaining life expectancy at 65: $\overset{\circ}{e}_{65} = 22.35$;
  - Lexis point: $L = 90$.

It is worth noting the small increment in the single premium moving from a standard life annuity ($m = 0$) to a life annuity with a guarantee period of 5 or 10 years. Actually, we have $a_{m\rceil} > a_{x+r:m\rceil}$, but the difference is very small thanks to the low mortality in the age interval $(x + r, x + r + m)$.

Table 5.2: Single premium S at retirement age; $b = 1\,000$

	Guarantee period		
	$m = 0$	$m = 5$	$m = 10$
$x + r = 65$	18 070	18 131	18 386
$x + r = 70$	15 265	15 376	15 832

5.3 Value-Protected Life Annuity

Capital protection represents an interesting feature of some life annuity products, usually called value-protected life annuities or *money-back life annuities*. Consider, for example, a single-premium life annuity. In the case of early death of the annuitant, a value-protected annuity will pay to the annuitant's estate the difference (if positive) between the single premium and the cumulated benefits paid to the annuitant. Thus, capital protection constitutes a counter-insurance for a single-premium life annuity. Usually, capital protection expires at some given limit age ξ (75, say), after which nothing is paid even if the difference above mentioned is positive.

Table 5.3 shows some results concerning the cost of capital protection. Data are as in the example in Sect. 5.2. We note that, also for this benefit, the increment in the single premium is rather small, even when the protection expires at age $\xi = 80$. Again, this is due to the low mortality in the relevant age intervals.

Table 5.3: Single premium S at retirement age; $b = 1\,000$

	Limit age		
	$\xi = 70$	$\xi = 75$	$\xi = 80$
$x + r = 65$	18 596	19 213	19 807
$x + r = 70$	15 265	16 062	16 936

Remark

From the insurer's perspective, combining a living benefit (the life annuity) with a death benefit (the capital protection) provides a *natural hedging* of the longevity risk. However, the low mortality rates in the

involved age intervals causes a rather poor impact on insurer's cash flows, and hence capital protection does not provide an effective hedge against the (aggregate) longevity risk.

5.4 Annuitization strategies

A *temporary withdrawal* (or *drawdown*) *process* can mitigate both disadvantages 1(b) and 2, mentioned in Sect. 5.1.

Let us assume that an amount S is available at the retirement time, r, and that the retiree can choose between the two following alternatives (see Fig. 5.1):

1. to purchase an immediate life annuity, with annual benefit b (payable in arrears), such that $b\, a_{x+r} = S$, namely to annuitize the available amount;

2. to leave the amount S in a fund, and then

   (a) withdraw the amount $b^{(1)}$ at times $h = r+1, r+2, \ldots, r+k$ (say, with $k = 5$ or $k = 10$);

   (b) (provided he/she is alive) convert at time $r + k$ the remaining amount R into an immediate life annuity with annual benefit $b^{(2)}$.

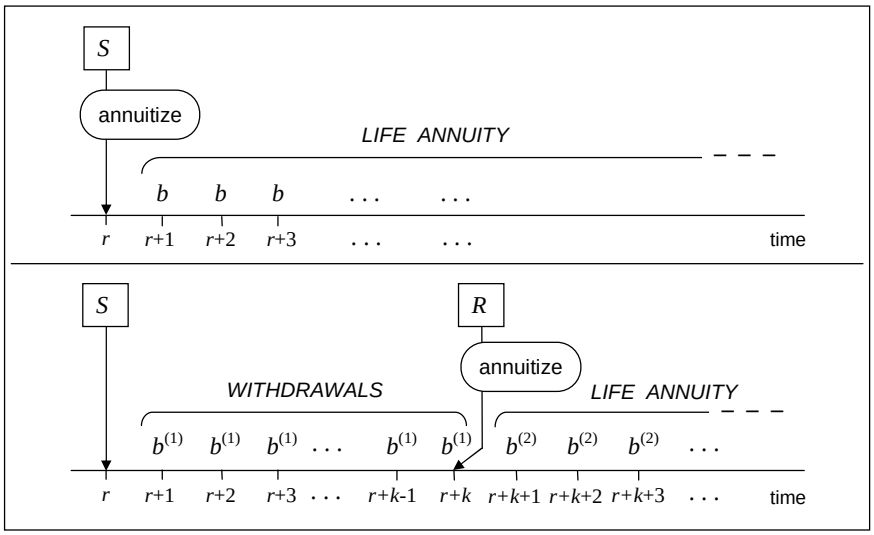

Figure 5.1: Immediate annuitization versus delayed annuitization

If the retiree chooses the second alternative, the amount R available at time $r+k$ to buy the life annuity depends on the annual withdrawal $b^{(1)}$ and the interest rate, g, credited to the non-annuitized fund. If $g = i$, namely the interest rate assumed in the pricing basis of the life annuity, and $b^{(1)} = b$, then the amount R is not sufficient to purchase a life annuity with annual benefit $b^{(2)} = b$, because of the absence of mutuality during the withdrawal period. However, the absence of mutuality can be compensated (at least in principle) by a higher investment yield, namely if $g > i$.

In formal terms, the relations among the quantities g, i, b, $b^{(1)}$, $b^{(2)}$, and k is as follows:

$$S\,(1+g)^k - b^{(1)} \sum_{h=1}^{k} (1+g)^{k-h} = b^{(2)}\, a_{x+r+k} \tag{5.2}$$

Table 5.4 provides an example of the interest rate, $g(k)$, needed in order to recover the mutuality effect (i.e. the "mortality credits") lost during the k-year delay period. Further numerical examples can be found in Olivieri and Pitacco (2011), Sect. 8.5.3. The spread $g(k) - i$ is often called the *Implied Longevity Yield*$^{\text{TM}}$*(ILY)* [1].

Table 5.4: Interest rate $g(k)$, such that: $b^{(1)} = b^{(2)} = b$

k	$g(k)$
5	0.02748
10	0.03009
15	0.03336
20	0.03718

The delay in the purchase of the life annuity has some advantages. In particular:

- in the case of death before time k, the fund available constitutes a bequest (which is not provided by a life annuity purchased at time 0, because of the mutuality effect);
- more flexibility is gained, as the annuitant may change the income profile modifying the withdrawal sequence (however, with a possible change in the fund available at time k).

[1] Registered trademarks and property of CANNEX Financial Exchanges.

Conversely, a disadvantage is due to the risk of a shift to a different mortality assumption in the pricing basis of life annuities, leading to a conversion rate at time $r + k$ which is less favorable to the retiree than that in-force at time r. Further, if k is high, it may be difficult to gain the required investment yield (in particular, avoiding too risky investments) to cover the absence of mutuality.

Remark 1

> We note that, in principle, a delayed annuitization results in an arrangement similar to the life annuity with a guarantee period (see Sect. 5.2, of course provided that $b^{(1)} = b$ and the interest rate used for pricing the k-year annuity-certain is $g(k)$). In practice the situations are rather different, as during the annuity-certain payment period the annuitants has no access to the fund and cannot change the income profile.

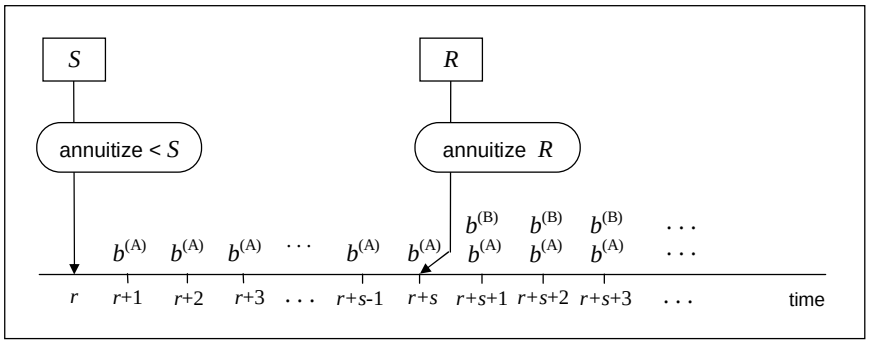

Figure 5.2: Phased annuitization

The ideas underlying the delayed annuitization can be generalized, leading to the so-called *phased annuitization*. This arrangement is illustrated in Fig. 5.2. The phased annuitization can be the result of one of the following choices:

- a *partial annuitization* of the amount S at retirement, and a consequent drawdown process maintained by the non-annuitized part of S, followed, at time $r + s$, by the annuitization of the remaining amount R;
- a *phased retirement*, that is, a two-step shift from full-time work to full-time retirement; thanks to partial retirement, an annual benefit $b^{(\mathrm{A})}$ is chosen, lower than that needed in the case of total retirement; then, the amount R will be annuitized to obtain a further life annuity with annual benefit $b^{(\mathrm{B})}$.

Remark 2

> Delayed annuitization and partial annuitization imply a tradeoff between mortality risk and financial risk (and longevity risk as well because of the possible change in the annuitization rate). Indeed, on the one hand non-annuitizing leaves, at the time of death, an amount available as a bequest, then facing to some extent the impact of the mortality risk; on the other hand, a financial risk is taken because of the need for a higher interest rate to recover mortality credits.

For more information on delay in annuitization, the reader can refer to Blake et al. (2003), Horneff et al. (2008), Milevsky and Young (2002), and Milevsky (2004).

5.5 Life annuity products providing LTC benefits

Long Term Care insurance (LTCI) provides the insured with financial support, while he/she needs nursing and/or medical care because of chronic (or long-lasting) conditions or ailments. LTCI can be classified as follows:

- products which provide benefits with *predefined amount* (usually, lifelong annuities); in particular
  - *fixed-amount* benefits;
  - *degree-related* (or *graded*) benefits, i.e. benefits whose amount is graded according to the degree of disability, that is, the severity of the disability itself;
- products which provide reimbursement (usually partial) of nursery and medical expenses, i.e. *expense-related* benefits;
- *care service* benefits (for example provided by the Continuing Care Retirement Communities, briefly CCRCs).

A classification of LTCI products which provide benefits with predefined amount is shown in Fig. 5.3. In this context, we find some products that can be packaged with life annuities. In particular we focus on:

- insurance packages in which LTCI is combined with lifetime-related benefits;
- Life Care annuities and, in particular, Enhanced pensions.

An *insurance package* can include LTC benefits combined with lifetime-related benefits; more precisely, the package can consist of:

1. a lifelong LTC annuity;
2. a deferred life annuity (e.g. from age 80), while the insured is not in the LTC disability state;
3. a lump sum benefit on death, which can be alternatively given by
   (a) a fixed amount, stated in the policy;
   (b) the difference (if positive) between a stated amount and the amount paid as benefit 1 and/or benefit 2.

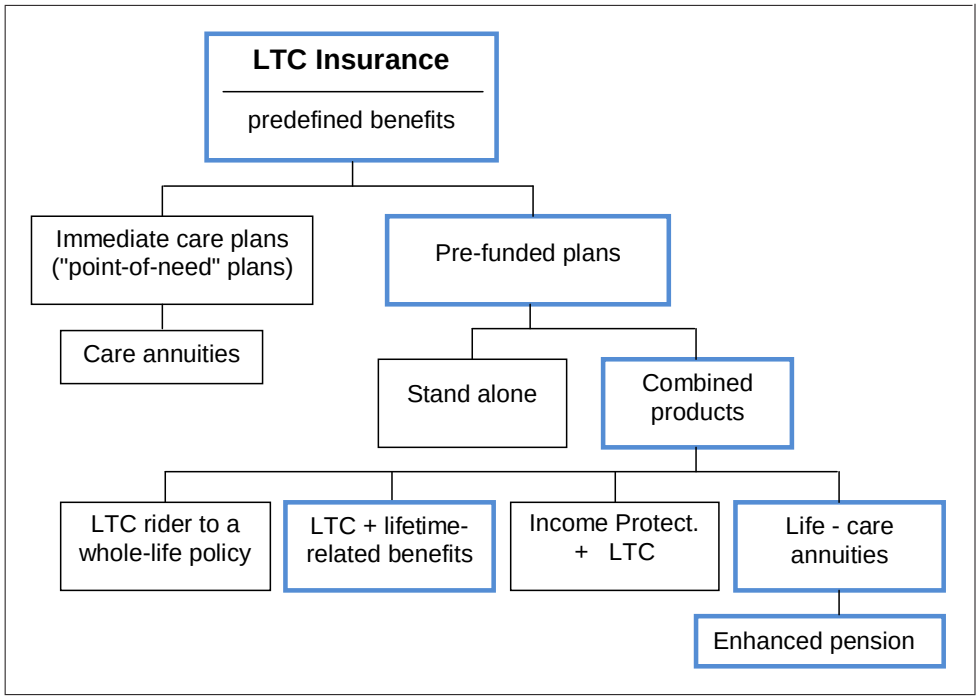

Figure 5.3: A classification of LTCI products

Three possible individual stories and the consequent outcomes in terms of annuity benefits are shown in Fig. 5.4.

This product design clearly aims at a reduction of the prevailing risk feature of the stand-alone LTC annuity.

We note that, apart from the death benefit, this arrangement basically includes the ALDA structure (see Sect. 4.1), as it can provide a deferred life annuity starting at old age (80, say). Conversely, the death benefit defined as in 2(b) aims at capital protection (see Sect. 5.3).

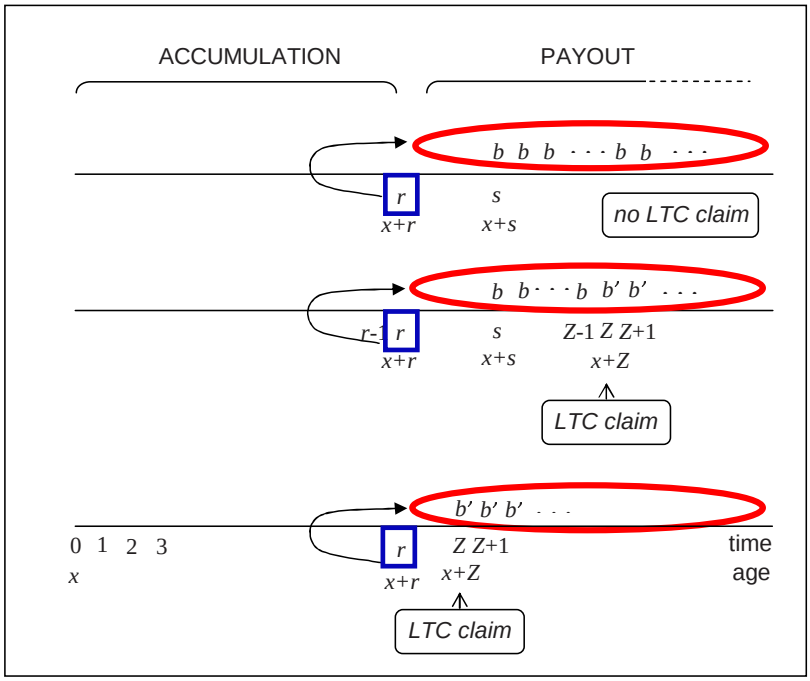

Figure 5.4: Possible outcomes, depending on lifetime and LTC need

Life care annuities are life annuity products in which the LTC benefit is defined in terms of an uplift with respect to the basic pension. The basic pension b is paid out from retirement onwards, and is replaced by the benefit b' ($b' > b$) in the case of LTC claim. See Fig. 5.5. The uplift can be financed either at retirement or during the whole accumulation period by premiums higher than those needed to purchase the basic pension b.

A possible outcome of the annuity payout, according to the Life care structure, is shown in Fig. 5.6.

Remark 1

> We note that the life annuity whose benefit is given by $b' - b$ (that is, the amount of the uplift) has the logical structure of the RCLA (see Sect. 4.2) in which the "scenario" is defined by the health conditions of the insured, the trigger being given by the LTC claim. Of course, the financial structure is different.

The *enhanced pension* is a particular life care annuity paid as a pension benefit, in which the uplift is financed by a reduction (with respect to the basic pension b) of the benefit paid while the policyholder is healthy. Thus,

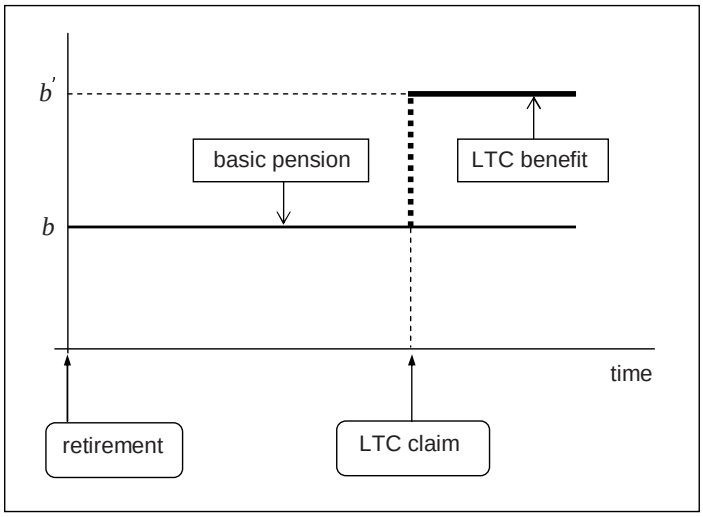

Figure 5.5: The Life care annuity

for a given single premium amount, the reduced benefit b'' is paid out as long as the retiree is healthy, while the uplifted benefit b' will be paid in the case of LTC claim (of course, $b'' < b < b'$). See Fig. 5.7.

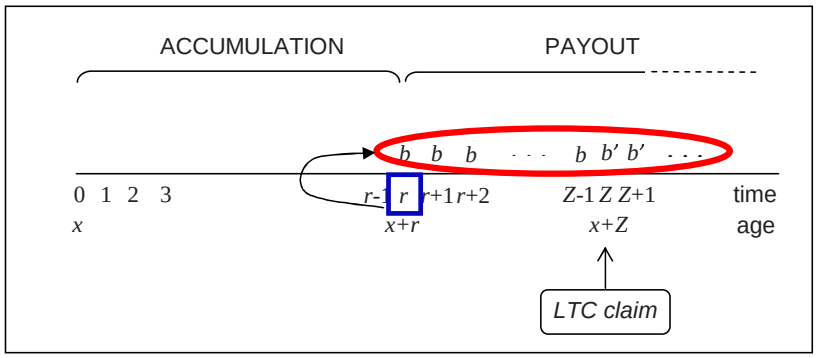

Figure 5.6: The Life care annuity: a possible outcome

Remark 2

It should be stressed that, when a Life Care annuity or a LTC annuity is involved, a specific type of aggregate longevity risk is taken by the annuity provider, inherent the lifetimes of elderly people claiming for LTC. Various theories concerning the relation between trend in expected total lifetime and trend in expected healthy lifetime have been proposed. See, for example, Olivieri and Ferri (2003) and references therein.

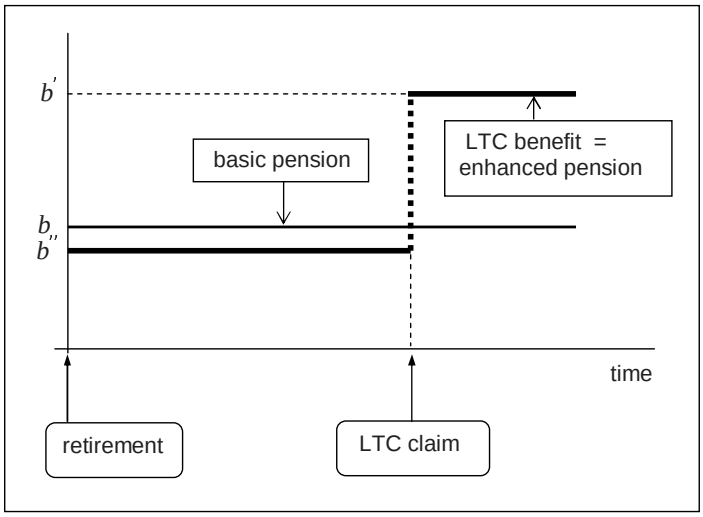

Figure 5.7: The Enhanced Pension: Life care annuity financed via reduction of the basic pension

The actuarial structure of LTCI products (both stand-alone and including lifetime-related benefits) is described by Haberman and Pitacco (1999); see also references therein. As regards Life Care annuities, the reader can refer to Brown and Warshawsky (2013), Warshawsky (2007), Zhou-Richter and Gründl (2011), and references therein.

6 The payment profile

We have so far focussed on *level annuities*, i.e. annuities providing an income which is constant in nominal terms (apart from possible uplifts related to LTC needs).

A number of models of "varying" annuities have been derived, mainly with the purpose of protecting the annuitant against the loss of purchasing power because of inflation. In particular:

1. *fixed-rate escalating annuities* (or *constant-growth annuities*)
2. *index-linked annuities*
   (a) inflation-linked annuities
   (b) equity-indexed annuities
3. *investment-linked annuities*

(a) with-profit annuities (in the UK)

(b) annuities with profit participation mechanisms

(c) unit-linked annuities

(for more information, the reader can refer for example to Pitacco et al. (2009)).

Participation mechanisms (see point 3(b)) can involve both financial and mortality experience. While a mortality higher than expected can originate mortality profits in a life annuity portfolio or a pension plan, and these can be attributed to some extent to the annuitants, according to conventional life annuity and pension design the longevity risk is borne by the annuity provider. Hence, problems may arise from a poor mortality experience because of an unexpected increase in longevity (that is, because of the aggregate longevity risk).

According to alternative product designs, part of the longevity risk can be transferred to the annuitants. This implies the definition of a *longevity-linked life annuity*.

A longevity-linked life annuity involves a benefit adjustment process. The benefit payable at time t is defined as follows:

$$b_t = b_0 \, \alpha_t^{[\mathrm{m}]} \tag{6.1}$$

where b_0 is the benefit amount initially stated, and $\alpha_t^{[\mathrm{m}]}$ denotes the coefficient of adjustment over the time interval $(0, t)$, according to a given mortality trend measure [m].

Coefficient $\alpha_t^{[\mathrm{m}]}$ can incorporate investment profit participation, so that the longevity loss can be offset by the investment profit.

Basic problems in defining the adjustment process are:

- the choice of the age pattern of mortality referred to;
- the choice of the link between annual benefits and mortality.

These choices should be driven by the (reasonable) aim of sharing the aggregate longevity risk (that is, the systematic component of the longevity risk), leaving the volatility (the random fluctuation component) with the annuity provider, as the latter can be diversified by risk pooling, viz inside the traditional insurance - reinsurance process.

The problem of linking annuity benefits to the experienced mortality trend is beyond the scope of this paper. We only stress that considerable attention has been devoted to this topic in the recent actuarial literature. In particular, see: Denuit et al. (2011), Goldsticker (2007), Kartashov et al.

(1996), Lüty et al. (2001), Piggott et al. (2005), Olivieri (2013), Richter and Weber (2011), Rocha et al. (2011), Sherris and Qiao (2011), van de Ven and Weale (2008), and Wadsworth et al. (2001).

7 Concluding remarks

Actuarial mathematics and technique traditionally focussed on benefits in terms of the relevant expected present values. Hence, a "deterministic" approach was only adopted.

Risks implied by guarantees and options provided by the policy conditions and the pension plan rules were usually disregarded (or, at least, not explicitly accounted for).

However, current scenarios, and in particular market volatility and uncertainty in longevity dynamics, require careful consideration of risks inherent in the life annuity and pension structures.

The purpose of this paper is to stress the dramatic importance of focussing (according to risk management guidelines) on risk identification and product design, looking at possible risk sharing between annuitants and annuity provider. Particular emphasis has been placed on biometric risk transfers.

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Remark

> Where links are provided, they were active as of the time this paper was completed but may have been updated since then.

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