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Abstract

The insurance linked securities (ILS) market is an increasingly important alternative asset class for which risk and return analysis differs from other asset classes. Measures of portfolio risk and return for an ILS portfolio are based on the expected losses and expected excess returns over the risk free rate. Multiple criteria decision making (MCDM) has found successful applications to many real world decision problems. This paper examines the application of two popular MCDM methods, Analytical Hierarchy Process (AHP) and ELECTRE III, to ILS portfolios. These methods are used to screen the securities before constructing portfolios using linear optimisation with constraints. The objective function is to minimise the portfolio expected loss for a given level of expected excess return. Upper and lower bounds are also placed on the investment in each individual ILS. The results demonstrate the benefits from applying MCDM to ILS portfolio selection.

Keywords: portfolio selection, insurance linked securities, multiple criteria decision making, Analytical Hierarchy Process, ELECTRE

JEL Classifications: G11, C61, G22, G32

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1 Introduction

Modern portfolio theory was developed on the assumption that investors trade-off the criteria of risk and return in investment decisions (Spronk et al., 2005; Bodie et al., 2008; Maginn et al., 2007). Markowitz (1952) provided the foundation for subsequent developments such that the mean variance model for portfolio selection has become a standard approach (Maginn et al., 2007). Risk and return, captured using the expected return and variance, along with a risk aversion for an individual, are used for portfolio optimisation. This approach has limitations including being overly simplistic and not reflecting investment decision problems in reality (Zopounidis, 1999; Zopounidis & Doumpos, 2002; Hallerbach & Spronk, 2002).

Multiple criteria decision making (MCDM) is a growing field which has found numerous successful applications in a wide range of areas including engineering, management, business, finance and environment (Zopounidis & Doumpos, 2002; Hallerbach & Spronk, 2002; Steuer & Na, 2003; Spronk et al., 2005; Wallenius et al., 2008). MCDM has shown promise in integrating investors' preferences and value systems into portfolio selection decisions (Steuer & Na, 2003; Steuer et al., 2008; Xidonas et al., 2009a). MCDM has found application to financial portfolios (Zopounidis & Doumpos, 2002; Hallerbach & Spronk, 2002; Steuer & Na, 2003). Increasingly, additional criteria are incorporated into the portfolio selection process (Zopounidis & Doumpos, 2002; Hallerbach & Spronk, 2002; Steuer & Na, 2003).

Insurance linked securities (ILS) are an alternative investment asset class that provides risk premiums from capital exposure to insurance risk mainly from catastrophe risk and longevity risk (Banks, 2004; Mocklow et al., 2002). ILS are structured products, a hybrid of finance and insurance instruments, that transfer risk to investors in capital markets. ILS risk and return characteristics are modeled in detail by specialised risk assessment companies. A broader range of risk criteria is used for insurance risks since the variance risk measure used in portfolio selection models does not effectively capture the risk of these ILS securities.

There are a number of recognised MCDM methods. Applications of the different MCDM methods can lead to different results for the same problem. In the literature, there exist only a small number of papers on comparison and analysis of results obtained from different techniques. Little is known about how these methods compare in financial and insurance applications. AHP and ELECTRE III are the two most prominent techniques amongst these numerous methods and these are used in our study.

Selection of ILS portfolios using a multiple criteria decision making framework has not been considered previously. Kreuser & Lane (2006) is one of the only papers considering insurance portfolio optimisation from an underwriters' perspective. Portfolio selection uses linear optimisation with the objective function of maximising expected return of the portfolio among available deals (assets) subject to expected shortfall risk preference constraints, practical deal size constraints, special constraints on retrocession, and a capital limit. Scenarios of risk events are used to determine the loss/gain for different portfolios. A similar linear optimisation is used in our portfolio selection approach for ILS.

This paper aims to:

- explore the application of MCDM techniques to the portfolio selection problem for ILS securities using two popular MCDM techniques: Analytic Hierarchy Process (AHP) and ELECTRE III.

- assess the methods by comparing the results obtained from the two techniques applied to ILS portfolio selection including an assessment of the robustness of the results to the assumptions used.

The paper shows that AHP and ELECTRE are effective in screening ILS securities in the portfolio selection process before portfolio optimisation. The optimised portfolios from the screened ILS have improved risk and return characteristics based on expected excess return and expected loss. Portfolios constructed using AHP screening dominate those from ELECTRE screening. This result reflects differences in the methods as well as in the preferences and other subjective decision criteria. The importance of these criteria are also assessed by considering alternative assumptions.

The paper is structured as follows. Section 2 provides a background on MCDM. Section 3 gives a brief background to the ILS market development and the ILS securities used in the analysis. Section 4 outlines the standard portfolio management process for actively traded securities. Section 5 describes the MCDM methodology. Section 6 gives the details of the assumptions and methods used for the ILS portfolio selection application. Section 7 presents the results together with a an assessment of the robustness of the results to the model assumptions. Section 8 concludes.

2 MCDM Overview

MCDM is a group of methods or techniques that help decision makers aggregate several criteria in order to evaluate a set of predefined alternatives (Zopounidis, 1999). The final objective is to select or rank a preferred subset of the alternatives in a structured and meaningful way, taking all evaluation criteria into consideration.

Two main categories of multiple criteria decision making problems have been identified (Kahraman, 2008; Triantaphyllou, 2000):

1. Multiple attribute decision making (MADM) problems: involve a predefined, limited number of known alternatives. For example, choosing a car amongst available models in the market or selecting stocks to be considered in an investment portfolio. MADM are also known as discrete multicriteria decision problems. MADM use sorting or ranking techniques. Examples of MADM methods include multiple attribute utility theory, AHP and ELECTRE.

2. Multiple objective decision making (MODM) problems: involve an infinite number of alternatives such as in portfolio optimisation, or in engineering design problems. A mathematical framework expressed in terms of continuous functions is used to define a set of alternatives. Tradeoffs between multiple criteria are specified through the use of objective and constraint functions. This class of problems is also called continuous multicriteria decision problem. Examples of MODM methods are goal programming and multiple objective programming techniques.

The ultimate goal of MCDM is to aid decision makers in the decision making process to make "better" decisions (Roy, 2005). In the presence of multiple and often conflicting criteria, the "optimal" decision is determined that is most satisfactory in terms of the decision maker′s value system and not dominated by any other possible decisions. Wallenius et al. (2008) emphasised that an important contribution of MCDM is to support decision makers in investigating and understanding the problem in a structured and systematic way.

Some fundamental concepts of MCDM problems that play critical roles for structuring and analyzing a decision problem are (Figueira et al., 2005; Triantaphyllou, 2000):

- Alternatives or options: these represent potential actions or choices available to a decision problem, they are the objects of MCDM process.
- Criteria or attributes: these are attributes, characteristics or values that are used to compare and evaluate alternatives. They are the different dimensions from which alternatives are evaluated Triantaphyllou (2000).
- Decision or importance/priority weights: in MCDM, criteria and alternatives are usually assigned numerical values to represent importance or priority with regards to the decision maker's system of values. Weights are usually normalised to add up to one.

2.1 Financial Decisions Applications

A literature review of MCDM applications in finance by Steuer & Na (2003) found a total of 265 published papers up until 2002. Goal programming and multiple objective programming were the most widely used methodology, accounting for 36% and 31% of the papers respectively, followed by outranking techniques (17%) and AHP (7%).

There were a wide range of applications in a variety of areas including portfolio selection and management, financial planning, capital budgeting, interest rate risk analysis and management, working capital and commercial banking management, auditing, accounting and insurance management, strategic planning including merger and acquisitions. Steuer & Na (2003) observed that despite the popularity of the over-simplified single-criterion "bottom line" or the bi-criteria "risk-return", many complex decision problems in finance are better resolved in a conflicting multiple criteria environment and there is a growing trend of resolving financial problems under multiple objectives beyond risk and return.

The study by Steuer & Na (2003) found that portfolio analysis is the most popular application of MCDM in finance. Their study also indicated that AHP and ELECTRE are the most widely used MADM techniques in this area. Applications of AHP and its variants in portfolio management can be found in a number of papers including Saaty et al. (1980); Bahmani et al. (1987); Saraoglu & Detzler (2002); Tiryaki & Ahlatcioglu (2009); Gupta et al. (2010). Similarly, ELECTRE is used in a number of papers, including Martel et al. (1988); Khoury et al. (1993, 1994); Xidonas et al. (2009a, 2009b, 2010).

3 ILS market

Insurance linked securities (ILS) are tradable capital market products with returns contingent on insurance risk events. Mocklow et al. (2002) notes that ILS have many similar characteristics to other fixed income securities including a fixed annual return (coupon), usually a margin over LIBOR, for the capital invested (principal), a fixed maturity date when principal is expected to be return and a rating to indicate the likelihood of timely payment of coupon and repayment of principal.

Risk analysis of ILS is performed at a much more detailed level compared to other bonds by specialised insurance risk assessment companies. Due to the lack of reliable historical information, computer simulation for losses from rare catastrophic event is used. The most important output from the simulation is the loss exceedance curve, detailing the estimated level of losses for varying probabilities of occurrence of a risk event. A rating agency rates the ILS before it is offered in the market.

ILS is regarded as an attractive investment class to investors from a portfolio standpoint as they are likely to have little, or no, correlation with other risky assets in their portfolios (Banks, 2004). ILS also provide extra risk premiums due to the "newness factor" to entice investors. These two important characteristics make ILS appealing (Lane & Beckwith, 2011). From an issuer's viewpoint, the ILS product is also attractive as it can be highly customised and structured to meet an issuer's needs. It is an alternative source for risk transfer in addition to traditional insurance/reinsurance contracts.

The ILS market is largely a private placement market for institutional investors including hedge funds. The market is also relatively illiquid due to the lack of an active secondary market. The most common investment strategy for ILS is to buy and hold. A recent overview of the ILS market is provided in Lane & Beckwith (2011). Market size at the end of June 2011 was about US$10.5B. Annual issuance size ranging from US$0.9B to US$7.5B depending on whether the traditional insurance market was in a hard or soft cycle. The market is dominated by CAT, or catastrophe, bonds, with mortality bonds following a distant second.

The investment universe of 31 ILS used in the paper are as listed in Lane & Beckwith (2010) (Table 2 in page 9 of the article). These are ILS issued in the year 2009. A summary of the data set statistics is provided in Table 1.

Table 1: ILS data set statistics. Source: Lane & Beckwith (2010)

No. of ILS:	31			
-Cat ILS	30			
-Life ILS	1			
Characteristics	**Max**	**Min**	**Mean**	**Sd**
Rating	BB+	Not rated	-	-
Adj. spread to LIBOR (bps)	2180	548	1135	421
Expected loss (%)	10.12%	0.46%	3.13%	2.47%
Probability of first loss	12.45%	0.59%	3.78%	2.97%
Probability of exhaust	8.25%	0.32%	2.61%	2.09%
Expected excess return (%)	14.14%	4.65%	8.23%	2.84%
Conditional expected loss (%)	100.00%	63.30%	82.48%	10.50%

4 Portfolio management process

The portfolio management process consists of three elements similar to many other business processes (Maginn et al., 2007):

- Planning: Investor's inputs are used to identify investment objectives, preferences and constraints. This information is then formulated into an investment policy statement (IPS). Next, relevant market information (economics, social, political, industry data and views) is collected and analysed to form capital market expectations. These expectations are combined together with the IPS to create the strategic asset allocation.

- Execution: in this step, portfolio managers decide to select specific assets or securities for the portfolio and the amount of funds committed to each individual security. Portfolio selection and composition decisions are made based on inputs from asset analysts. Quantitative tools such as portfolio optimisation can be used to construct select the portfolio.

- Feedback: this step involves 2 components: (1) monitoring and rebalancing, and (2) performance evaluation.

The execution step can often involve a two stage process (Spronk et al., 2005; Xidonas et al., 2009a):

- Screening of available securities: evaluate assets in the market under the investor's preferences and system of values in order to select the ones that best satisfy investor requirements

- Portfolio optimisation: allocate specific amounts of capital to be invested in each of the securities chosen in the first stage.

4.1 Mean-variance portfolio optimisation

Portfolio selection involves selecting efficient portfolios that trade off risk and return (Steuer et al., 2008; Spronk et al., 2005; Maginn et al., 2007). Portfolio returns, r_p, are a weighted average of the individual security returns given by

$$r_p = \sum_{i=1}^{n} x_i r_i$$

where x_i is the proportion of the initial capital (investment proportion or weight) invested in security i at the beginning of the period and r_i is a random variable denoting the percentage return of security i over the holding period. The means of r_i (their expected values), variances (σ_i^2) and covariances (σ_{ij}) of r_i's distribution are assumed known. Investors are assumed risk averse, preferring higher return and lower risk, where risk is measured by variance.

The portfolio selection problem can be specified as:

$$\text{Max}\{E(r_p) = \sum_{i=1}^{n} x_i E(r_i)\} \text{ for a given variance}$$

or

$$\text{Min}\{\text{Var}(r_p) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}\} \text{ for a given mean}$$

Subject to the constraints:

$$\sum_{i=1}^{n} x_i = 1, \text{ and optionally } l_i \leq x_i \leq u_i$$

where l_i and u_i are minimum and maximum permissible exposures for each security.

The set of efficient portfolios when plotted as a curve in mean-variance space is referred to as the efficient frontier. The investor selects the preferred combination of expected return and variance from this efficient frontier.

The MV model is a special case of a multiple criteria decision problem with bi-criteria: mean and variance (Zopounidis, 1999; Spronk et al., 2005; Steuer et al., 2008). For any given level of expected return or variance the problem becomes a classical mono-criteria optimisation with constraints.

The Markowitz MV model has a number of limitations (Bodie et al., 2008). As the number of securities under consideration increases, the variance-covariance matrix becomes very large. Specifically, the number of parameters to be estimated in MV optimisation is $n(n+3)/2$, consisting of n expected returns, n variances and $n(n-1)/2$ covariances. The model does not provide a basis for determining forecast expected returns and ensuring these are consistent with the covariance matrix, which is crucial for the construction of an efficient frontier. The MV model is also very sensitive to the quality of inputs, especially to errors in estimation of expected return. Maginn et al. (2007) highlights how small changes in input values can result in a significant proportion of the assets not being included in the optimal portfolio.

4.2 Multiple factor models

Asset returns are known to be determined by many factors. The multiple factor model introduced by Ross (1976) assumes a set of return drivers or risk factors affect all assets to a greater or lesser degree. A linear multi-factor model for k factors has the form:

$$r_i = a_i + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{ik}F_k + e_i$$

where a_i is an intercept term; $F_j, j = 1 \ldots k$ is the portion of return attributable to factor j; β_{ij} represents the sensitivity of return of asset i to factor return j, also called factor loadings, factor sensitivities or factor betas, and e_i is the residual or error term with zero mean. It is assumed that e_i is uncorrelated with the k factors as well as the residuals of other assets.

In the case of a single factor model, such as an index model underlying the CAPM, the number of estimates required is $3n + 2$, including n intercept terms, n betas, n asset's specific variances ($\sigma_i^2(e_i)$), 1 expected factor return and 1 factor variance. The common factors produce correlations between asset returns for different assets. This makes the portfolio selection process computationally efficient.

The multi-factor model is useful for modeling asset returns and covariances since it is simpler to estimate the variance-covariance matrix as the number of factors chosen is small compared to number of assets, it filters out noise (random variation in data) and facilitates verification of the consistency of covariance matrix (Maginn et al., 2007).

These models allow multiple factors to be taken into account in portfolio selection but still assuming a mean-variance framework.

5 Methodology

5.1 ILS Portfolio selection

The assumptions for the ILS portfolio selection is a single period investment horizon of 1 year. The portfolio selection process consists of two stages. Initially there is a screening of available assets to select the highest ranked 20 ILS. The portfolio optimisation is applied to these highest ranked 20 ILS individual assets after screening. For the screening step both AHP and ELECTRE III are used to rank the ILS securities using the criteria rating, adjusted premium spread, probability of first dollar loss and conditional expected loss.

After screening, portfolio optimisation uses linear optimisation to minimise the portfolio expected loss with a specified level of expected excess return as a constraint. The Matlab linear optimiser is used. The linear optimisation model is:

$$\text{Min}\{E(l_p) = \sum_{i=1}^{20} x_i E(l_i)\}$$

$$\text{subject to: } E(r_p) = \sum_{i=1}^{20} x_i E(r_i) = c, \sum_{i=1}^{20} x_i = 1 \text{ and } 0.01 \leq x_i \leq 0.3$$

where l_i is ILS expected loss, r_i is ILS expected excess return and c is a target level of portfolio excess return, ranging from 6% to 12%.

In practice, investors hold portfolios of a smaller subset of available assets. This reflects the computational and other difficulties in optimizing portfolios with a large numbers of assets, especially under the multiple objective programming framework. Trading and monitoring a smaller number of preferred assets is also more cost and time efficient. The screening process reduces the number of securities to a number that ensures the portfolio will be well diversified. Although diversification benefits increase with the number of assets when the number of assets increases beyond 20 the increase in diversification benefit is small. This is shown for a portfolio of equally weighted stocks from the New York Stock Exchange in Statman (1987).

5.2 Analytical Hierarchy Process (AHP)

The Analytical Hierarchy Process, first introduced by Saaty in the 1970s, is one of the most popular MADM methods. It has been used extensively in many decision problems across different fields including economics, medical, manufacturing, engineering and environment (Saaty, 2006; Fulop, 2005; Triantaphyllou, 2000). AHP is based on the observation that humans are more capable of making relative judgments than absolute judgments (Saaty, 2006).

A description of the steps in applying AHP based on Saaty (2006) is included here for completeness and Appendix A illustrates the method using a multiple criteria decision example.

The steps required are:

1. Problem formulation:
   Define the problem, set the goal and possible alternatives or solutions.

2. Building a hierarchy:
   Analyze and decompose the problem into smaller components, constructing an hierarchy structure that adequately represents the problem consisting of the goal, criteria, subcriteria and alternatives in multiple levels.

3. Perform pairwise comparison
   At each level, construct a pairwise comparison matrix to compare elements with each another based on each criterion using AHP's fundamental scale for comparative judgments (see Table 2 from Saaty (2006)).

Table 2: AHP Fundamental 9 point scale for pairwise comparison

Importance Intensity	*Definition*	Explanation
1	Equal importance	Two activities contribute equally to the objective
2	Weak	
3	Moderate importance	Experience and judgment slightly favour one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favour one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favoured very strongly over another; itŠs dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation
Reciprocals of above	If activity i has one of the above non zero value assign to it when compared to j, then j has the reciprocal value when compared to i	A reasonable assumption (consistency)

For a level in the hierarchy having n elements, the total number of comparisons needed is $n(n-1)/2$. A comparison matrix is used with the main diagonal elements equal 1 and half of the elements are the reciprocals of the other comparisons (principle of consistency). See Table 3.

Table 3: Pairwise comparison matrix

Importance	A_1	..	A_j	..	A_n
A_1	1	..	a_{ij}	..	a_{1n}
:	..	1	..	..	..
A_j	$1/a_{ij}$	..	1	..	..
:	..	..	..	1	..
A_n	$1/a_{1n}$		..	..	1

4. Evaluate consistency of pairwise comparisons
   Consistency for a comparison matrix is measured by calculating the consistency index (CI).

$$CI = (\lambda_{max} - n)/(n-1)$$

n is the number of elements and λ_{max} is the maximum eigenvalue of the comparison matrix.

This consistency index is then compared to a random index (RI). The RI is the average CI of randomly generated reciprocal matrices using the scale 1/9, 1/8,. . . , 8, 9. The random consistency index for different dimensions n is given in Table 4 (from Saaty (2006)).

Table 4: Random consistency index

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49	1.51	1.54	1.56	1.57	1.58

The consistency ratio is defined as $CR = CI/RI$. For $n = 3$ and $n = 4$, conventionally it is required that $CR \leq 0.05$ and 0.08 respectively to be acceptable. For $n \geq 5$, a consistency ratio of 0.10 or less is acceptable.

5. Determine the local priority (local weight)
Based on the comparison matrix, a priority vector is derived using an eigenvector method. Eigenvectors for the matrix are given by:

$$A.\mathbf{w} = \lambda_{max}.\mathbf{w}$$

where A is the pairwise comparison matrix, $\mathbf{w} = (w_1, w_2, ..., w_n)$ is the right principal eigenvector and λ_{max} is the maximum eigenvalue of matrix A. Normalising **w** by dividing each element by the sum of the elements, ensures they are in the range between 0 and 1. This is referred to as the local priority vector of one level with respect to the upper level.

6. Calculate global priorities (total performance scores)

Each alternative is assigned a global score by aggregating local priorities weighted by the importance of the respective criteria:

$$x_i = \sum_{j=1}^{m} k_j w_{ij}, \text{for } i = 1, \ldots, n$$

where x_i is the global priority score of alternative i, w_{ij} is the local priority of i with respective to criterion j, k_j is the importance weight (local priority) of criterion j, $j = 1, \ldots, m$. Global priority values are ranked in the range between 0 and 1 and measured in (dimensionless) priority units.

AHP has some limitations that need to be recognised. AHP uses a weight allocation technique without any reference to the ranges of performance of the alternatives under consideration (Lenzen, 2006). For the AHP fundamental scale, Triantaphyllou (2000) notes there is no particular reason that the scale should be evenly distributed from 1 to 9 but not between 1/9 and 1. An important issue with AHP is rank reversal. This occurs where the ranks of an existing set of alternatives are changed when other alternatives are added or deleted. This happens under the normal or distributive version of AHP where the normalisation is done by dividing the local eigenvector by the sum.

To avoid rank reversal, Saaty (2006) proposes a variation of AHP referred to as ideal mode or the revised AHP method. For this method the right eigenvector of the comparison matrix is normalised by dividing by the largest element instead of the total sum. This ensures that no rank reversal occurs. Another version of AHP uses a direct rating technique. In this method, alternatives are directly rated under each criterion based on a common, predefined scale instead of using a pairwise comparison and eigenvector values. This helps reduce the number of comparisons significantly and is useful in problems with a large numbers of alternatives. It also avoids the rank reversal problem. This is the method used in this paper.

5.3 Elimination and Choice Translating Reality technique (ELECTRE) III

ELECTRE has a number of different versions which have been developed and improved over the years:

- ELECTRE I, ELECTRE Iv, ELECTRE IS: choice/ selection problems
- ELECTRE II, III, IV: ranking problems
- ELECTRE Tri: sorting problems

ELECTRE III is a version of ELECTRE which belongs to the broader group of the outranking family. The concept of outranking was proposed by Bernard Roy in the late 1960s (Fulop, 2005). The outranking relation S is a binary relation defined on the set of alternatives by using pairwise comparison under each criterion. Alternative a outranks b if on most of the criteria a performs at least as good as b (concordance condition), and for those criteria where a has worse performance than b, it is still considered acceptable (non-discordance condition).

The outranking method finds all alternatives that dominate others while they are not dominated by any other alternative. Dominated alternatives have another alternative performing better in one or more criteria and performing equally for the remaining criteria. Since the method can result in an incomplete or partial ranking, there may be a smaller set of non-dominated alternatives.

ELECTRE III has been successfully applied in a broad range of decision problems (Figuiera et al., 2005; Belton & Stewart, 2002; Rogers et al., 2000). Further details on the ELECTRE III technique can be found in Buchanan et al. (1999); Buchanan & Vanderpooten (2007); Roy (1991); Rogers et al. (2000); Belton & Stewart (2002). An example is given in Appendix A.

Preference modeling in ELECTRE III

Consider comparing a set of alternatives A under a predefined set of criteria $F = g_1, \ldots, g_m$. ELECTRE III allows for imprecision and uncertainty in judgements by making use of the concept of an indifference threshold q and preference thresholds p.

Preference relations under a single criterion g are defined as follows (assuming an increasing performance scale):

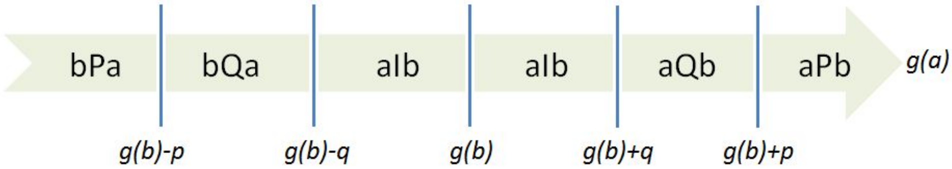
- a is strictly preferred to b (aPb): $g(a) - g(b) \geq p$
- a is weakly preferred to b (aQb): $q < g(a) - g(b) < p$
- a is indifferent to b (aIb): $|g(a) - g(b)| \leq q$

This double threshold avoids the need for a clear distinction between indifference and strict preference. This is illustrated in Figure 1.

ELECTRE III aims to produce an outranking relation S between every ordered pair (a, b) in A when evaluated under the whole set of criteria F. aSb means that according to the decision maker's preferences, "a is as least as good as b" or, equivalently, "a is not worse than b" when considering all criteria. The assertion aSb is tested using two principles:

- Concordance principle: a sufficient majority of criteria support this assertion.

Figure 1: ELECTRE III double thresholds



- Non-discordance principle: for the minority of criteria that do not support the assertion, none of them is strongly against the assertion.

The criteria g_j is in concordance with aSb if and only if a is as least as good as b when compared under g_j, that is aS_jb. Even if $g_j(a)$ is less than $g_j(b)$ by an amount up to q_j, it still fully supports the assertion aSb. The g_j criterion does not support aSb only when bPa, that is, $g_j(b)$ is larger than $g_j(a)$ by an amount at least equal to p_j.

With all criteria to be maximised (increasing performance scale), the strength of support for aSb (concordance force) is measured by a global concordance index:

$$C(a,b) = \sum_{j=1}^{m} k_j c_j(a,b) / \sum_{j=1}^{m} k_j$$

with k_j is the important weight for g_j and $c_j(a,b)$ is the partial concordance index defined as:

$$c_j(a,b) = \begin{cases} 1 & \text{, if } g_j(a) + q_j \geq g_j(b) \\ 0 & \text{, if } g_j(a) + p_j \leq g_j(b) \\ \dfrac{p_j + g_j(a) - g_j(b)}{p_j - q_j} & \text{, otherwise.} \end{cases}$$

To calculate a discordance force (the strength of evidence against aSb), a veto threshold v is introduced. Complete dismissal of the assertion aSb occurs if, for any one criterion g_j, $g_j(b)$ is larger than $g_j(a)$ by at least v_j. The g_j criteria is not against aSb if b is not strictly preferred to a, that is even when $g_j(b)$ is larger than $g_j(a)$ by an amount smaller than p_j.

A partial discordance index is defined as follow:

$$d_j(a,b) = \begin{cases} 1 & \text{, if } g_j(a) + v_j \leq g_j(b) \\ 0 & \text{, if } g_j(a) + p_j \geq g_j(b) \\ \dfrac{g_j(b) - g_j(a) - p_j}{v_j - p_j} & \text{, otherwise.} \end{cases}$$

Unlike the concordance index, no global discordance index is defined. If no veto thresholds is specified for g_j then $d_j(a,b) = 0$ for all pairs of alternatives (a,b).

Finally, the degree of outranking is measured by combining the concordance and discordance index. A credibility index $S(a,b)$ is defined as:

$$S(a,b) = \begin{cases} C(a,b) & \text{, if } d_j(a,b) \leq C(a,b) \forall j \\ C(a,b) \displaystyle\prod_{j \in J(a,b)} \dfrac{1 - d_j(a,b)}{1 - C(a,b)} & \text{, otherwise.} \end{cases}$$

where $J(a,b)$ is the subset of criteria for which $d_j(a,b) > C(a,b)$.

The credibility matrix S has the credibility indices for all ordered pairs (a,b) of the alternatives in A as its elements.

The credibility matrix S is then used to establish outranking relations and to rank alternatives. In ELECTRE III, this procedure is called distillation. There are two ways to perform the procedure.

Descending distillation:

The steps are (Rogers et al., 2000; Belton & Stewart, 2002):

1. A minimum acceptable value of the credibility index is defined and used to determine if the credibility index is compatible with the assertion aSb. Denoting by $\lambda_0 = \text{Max}\ \{S(a,b), a \neq b\ \}$, the smallest value of $S(a,b)$ that is still considered acceptable must be sufficiently close to λ_0. A cut-off level is defined λ^* as:

$$\lambda^* = \text{ Max}\{S(a,b),\ S(a,b) < \lambda_0 - s(\lambda_0), a \neq b\}$$

   s is known as the discrimination threshold. In ELECTRE III, s is usually set at $s(\lambda) = 0.3 - 0.15\lambda$.

2. At cut-off level λ^*, a outranks b if and only if $S(a,b)$ exceeds the cut off level and $S(a,b)$ is greater than $S(b,a)$ by more than the discriminant threshold. The credibility matrix S is converted into an outranking relation matrix T with entries as follows:

$$aS^{\lambda^*}b = T(a,b) = \begin{cases} 1 & \text{, if } S(a,b) > \lambda^* \text{ and } S(a,b) - S(b,a) > s((Sa,b)) \\ 0 & \text{, otherwise} \end{cases}$$

3. Each alternative is assigned a qualification $Q(a)$, defined as the difference between number of alternatives outranked by a and number of alternatives outrank a. $Q(a)$ is the row sum minus the column sum of T for alternative a.

4. The set of alternatives having the largest Q is the first distillation D_1 of A.

5. If D_1 has more than one member, repeat the process inside D_1 until D_1 has only one member or if it still has more than one member but is no longer reducible. As we proceed, λ_0 is subsequently reduced from maximum of $S(a,b)$ to λ^* of the previous step. Thus the cut off level is reduced accordingly toward 0. Once D_1 is reduced to only one member or becomes irreducible, we then repeat the process with the original set of alternatives A excluding D_1, until all alternatives are ranked.

Ascending distillation:

The process is similar to descending distillation except in step 4 the alternative(s) with smallest qualification Q is retained first.

The rankings from both distillations are combined to get a final overall ranking for all alternatives.

Outranking techniques allow for situations where not all alternatives or actions are comparable (incomparability cases). They can also be structured as a non-compensatory approach, where good performance under one criterion cannot make up for poor performance on another criterion, through the use of veto thresholds.

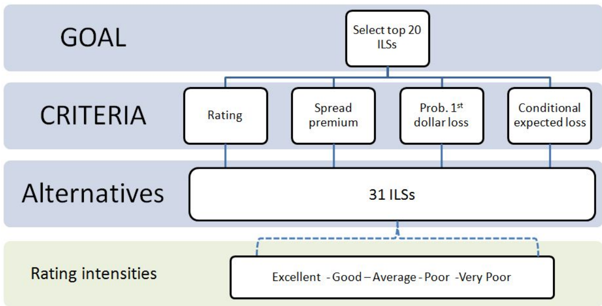
The technique may not provide a complete order and thus a single best alternative for a choice problem. Concepts, such as threshold and veto levels, can be difficult to understand and difficult to assign values. The aggregation procedure and ranking algorithm can appear at first to be complicated and not very transparent Belton & Stewart (2002).

6 Application to ILS Portfolio Selection

AHP

AHP is applied in ratings mode. The hierarchy model for our ILS screening phase is shown in Figure 2.

Figure 2: AHP Hierarchy for ILS screening



The 5 point intensity scale used is the same as in the example in Appendix A Table 18. The same 5 point scale is used for all 4 criteria. For each criterion, the range between maximum and minimum performance scores is split into five equal intervals and each alternative is assigned into the corresponding rating, depending on their raw performance scores. Table 5 shows the rating scale for the ILS criteria.

Table 5: AHP rating scale for criteria set

Criteria	Max	Min	Excellent	Good	Average	Poor	Very Poor
Rating	BB+	Not rated	BB+	BB, BB-	B+, B	B-	Not rated
Adjusted spread (bps)	2180	548	2180-1853.6	1853.6-1527.2	1527.2-1200.8	1200.8-874.4	874.4-548
Probability first $ loss (%)	12.45	0.59	0.59-2.96	2.96-5.33	5.33-7.71	7.71-10.08	10.08-12.45
Conditional expected loss (%)	100.00%	63.30%	63.30-70.64	70.64-77.98	77.98-85.32	85.32-92.66	92.66-100.00

The next step is to convert the intensity level or rating into a local priority (preference weight). AHP pairwise comparison is used to get a local priority corresponding to each performance level based on the eigenvector method. The same comparison for all four criteria is used as in the AHP example in Appendix .

Eigenvector values and consistency checking are calculated using the AHP software called SuperDecision. Local priority is standardised by dividing the eigenvector element by the largest value. This is to ensure that an alternative with the highest ratings under all criteria will have a global priority score of 1. This helps avoid the problem of rank reversal. We use an equal weighting for the criteria set, with an importance weight for each criterion of 0.25.

ELECTRE

For the ILS portfolio selection problem, Matlab is used to carry out the ELECTRE III computations. Most of the scripts are taken from the Matlab Decision Theory Toolbox with some modifications to obtain the final ranking. Equal importance weight is given to the four criteria, each with 0.25 weight. The ILS rating is converted into numerical scores from 1 to 7 as in Table 6.

Table 6: Rating scores

BB+	BB	BB-	B+	B	B-	Not rated
7	6	5	4	3	2	1

The ELECTRE III inputs are indifference, preference, and veto thresholds for each individual criterion. These thresholds reflect both standard errors, or level of imprecision, associated with the performance scores of the alternatives and subjective inputs from decision makers.

Preference thresholds can be set at twice the indifference thresholds, and veto thresholds are usually set between 3 to 10 times preference thresholds (Rogers et al., 2000). In our case, no veto threshold is used and the preference thresholds are set at twice the indifference thresholds. Threshold values used for the four criteria are shown in Table 7. Final ranking for each ILS is based on averaging ranks obtained from descending and ascending distillations.

Table 7: ELECTRE III thresholds

	Rating	Adj. Spread premium	Prob 1st loss	Cond. expected loss
q	1	100bps	0.59%	10%
p	2	200bps	1.18%	20%
v	inf	inf	inf	inf

7 ILS Portfolio Results

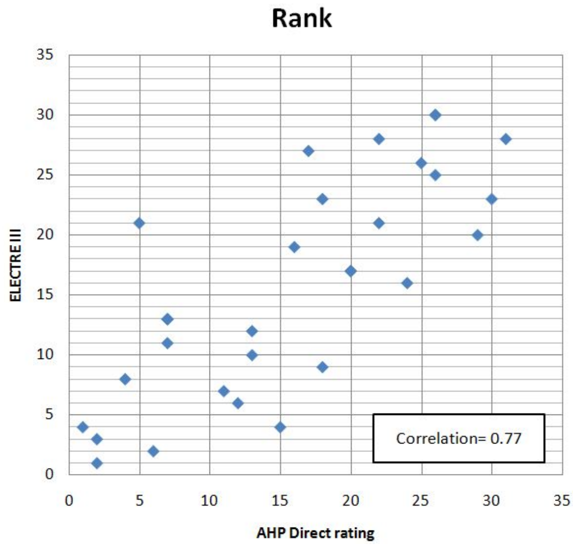
The rankings of the 31 ILS from the screening using AHP and ELECTRE are shown in Figure 3. The top 20 ILS are used in the optimisation. For tie-breaks for rank 20 we select the security that comes first alphabetically. Those ILS having rank highlighted in blue (shaded) are selected for the portfolio optimisation step. The top ranked ILS by AHP is Vita IV E which was ranked 4^{th} by ELECTRE III. The highest ranked ILS by ELECTRE is Residential Re 2009-3 which was ranked 2^{nd} by AHP. The top 20 ILS from AHP and ELECTRE differ by only 3 ILS. A scatter plot for ILS ranks based on AHP and ELECTRE III is shown Figure 4. The rank correlation is 0.77, which could be viewed as moderate.

The optimal portfolios constructed from the AHP and ELECTRE top 20 list for a given level of expected excess return are shown in Table 8. For the case where the portfolio includes all 31 ILS (market portfolio), the expected excess return and expected loss are 8.23% and 3.13% respectively for an equally weighted market portfolio and 7.68% and 2.34% for a deal size weighted market portfolio. Optimal portfolios from AHP and ELECTRE dominate these portfolios showing that the screening process is

Figure 3: Rankings of ILS by AHP and ELECTRE III

	Atlas Re VI 2009-1	Blue Fin 2 Class A	Calabash Re III A	Calabash Re III B	Eurus II	Foundation Re III 2010-1	Ianus Capital	Ibis Re 2009-1 A	Ibis Re 2009-1 B	Lakeside Re II	Longpoint Re II 2009-1 A	Longpoint Re II 2009-1 B	Montana Re A	Montana Re B	MultiCat Mexico A	MultiCat Mexico B	MultiCat Mexico C	MultiCat Mexico D	Parkton Re	Redwood XI 2009-1 A	Residential Re 2009 1	Residential Re 2009 2	Residential Re 2009 3	Successor II F-IV	Successor X I-S1	Successor X I-U1	Successor X I-X1	Successor X II-CN3	Successor X II-CL3	Successor X II-BY3	Vita IV E
AHP	20	12	6	7	13	7	18	4	15	5	7	7	13	20	26	26	26	16	17	22	2	11	2	18	22	24	29	25	31	30	1
ELECTRE	17	6	2	11	10	13	23	8	4	21	13	13	12	17	25	30	30	19	27	28	3	7	1	9	21	16	20	26	28	23	4

Figure 4: Scatter plot AHP vs ELECTRE III rankings



effective and adds value. As long as the criteria set and input values select better risk and return assets, the resulting optimal portfolios will be superior to holding the market portfolio.

Figure 5 shows the efficient frontier for the AHP and ELECTRE optimised portfolios in expected loss and excess return space. Optimal portfolios from AHP screening dominate those from ELECTRE screening. At a 6% expected excess return level, the difference in expected loss is 0.11 percentage point or over 11% lower in favour of AHP optimal portfolios. As the level of expected return increases, the difference in expected loss becomes smaller, dropping to just 0.02 percentage point at an expected return of 12%.

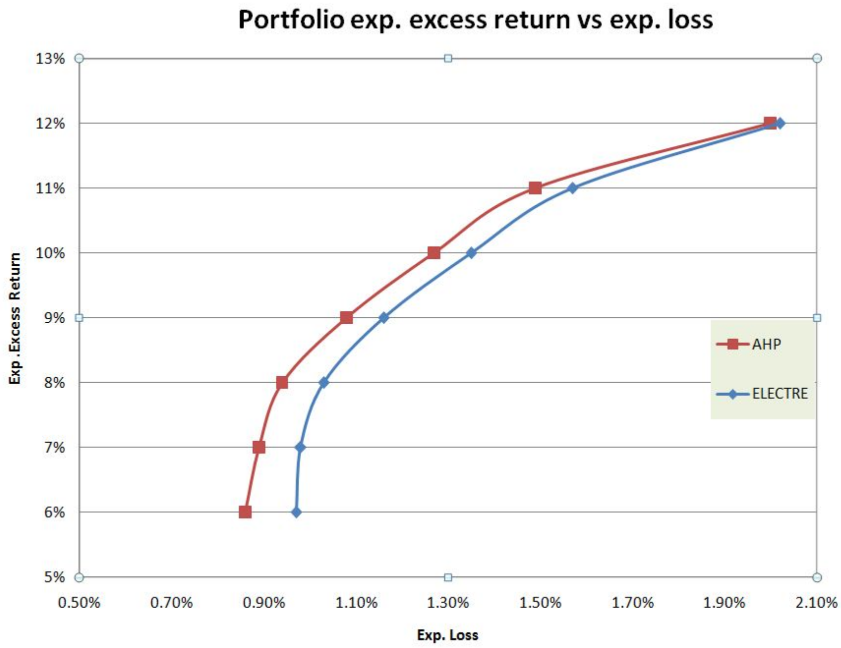
7.1 Robustness of MCDM methods in ILS Portfolio Selection

Subjective factors are used in the screening results and the optimal portfolios depend on the MCDM technique used and the preference inputs/value system used in the decision problem. Results from screening using AHP direct rating are affected by the rating scale used to convert raw performance data into performance/intensity levels as well as the pairwise comparison of intensities to compute local priority. To assess the impact of this, we consider how the pairwise comparison of intensity level affects

Table 8: Optimised Portfolios

Portfolio return	Portfolio expected loss	
	AHP	ELECTRE III
6%	0.86%	0.97%
7%	0.89%	0.98%
8%	0.94%	1.03%
9%	1.08%	1.16%
10%	1.27%	1.35%
11%	1.49%	1.57%
12%	2.00%	2.02%

Figure 5: AHP and ELECTRE III efficient frontiers



the AHP final optimal portfolios. The elements in the pairwise comparison matrix are reduced to 75%, 50% and 35% of the original judgments. Table 9 shows the case for the 50% reduction.

With the new local priority for each performance level, we reapply AHP calculations to select the top 20 ILS for portfolio optimisation phase. Ranking results for AHP variants are provided in Figure 6.

Rank correlations between AHP screening variants are very high as shown in Table 10. For the top 20 lists, they are the same for AHP 0.75 and AHP 0.5 and this differs from the original AHP by only 1 ILS; the AHP 0.35 top 20 list also has 2 different ILS compared to the original AHP.

Optimal portfolios for AHP and ELECTRE variants are shown in Table 11. The corresponding efficient frontiers for AHP variants are shown in Figure 7. The efficient frontiers are the same for the AHP 0.75 and AHP 0.5, since the top 20 lists are the same, followed by the the AHP 0.35. They dominate the original AHP. The difference in

Table 9: Pairwise comparison for rating scale - AHP 0.5

Rating intensity	E	G	A	P	V	Normalised eigenvector	Local priority
Excellent	1	1.5	2.5	3.5	4.5	0.38	1.00
Good	1/1.5	1	1.5	2.5	3.5	0.26	0.68
Average	1/2.5	1/1.5	1	1.5	2.5	0.17	0.44
Poor	1/3.5	1/2.5	1/1.5	1	1.5	0.11	0.29
Very Poor	1/4.5	1/3.5	1/2.5	1/1.5	1	0.08	0.20

Figure 6: Ranking of ILS by AHP and ELECTRE variants

	Atlas Re VI 2009-1	Blue Fin 2 Class A	Calabash Re III A	Calabash Re III B	Eurus II	Foundation Re III 2010-1	Ianus Capital	Ibis Re 2009-1 A	Ibis Re 2009-1 B	Lakeside Re II	Longpoint Re II 2009-1 A	Longpoint Re II 2009-1 B	Montana Re A	Montana Re B	MultiCat Mexico A
AHP	20	12	6	7	13	7	18	4	15	5	7	7	13	20	26
AHP 0.75	18	12	6	8	13	8	20	4	15	5	8	8	13	19	26
AHP 0.5	17	8	6	10	14	10	20	4	9	5	10	10	14	17	26
AHP 0.35	16	8	5	12	10	12	21	4	9	6	12	12	10	16	27
ELECTRE	17	6	2	11	10	13	23	8	4	21	13	13	12	17	25
ELECTRE 0.75	17	7	3	12	10	11	27	5	9	18	13	13	15	16	24
ELECTRE 0.5	18	8	2	13	8	11	28	3	5	15	12	13	15	17	24
ELECTRE 0.25	24	6	5	11	9	12	27	2	6	14	15	19	13	15	26

	MultiCat Mexico B	MultiCat Mexico C	MultiCat Mexico D	Parkton Re	Redwood XI 2009-1 A	Residential Re 2009 1	Residential Re 2009 2	Residential Re 2009 3	Successor II F-IV	Successor X I-S1	Successor X I-U1	Successor X I-X1	Successor X II-CN3	Successor X II-CL3	Successor X II-BY3	Vita IV E
AHP	26	26	16	17	22	2	11	2	18	22	24	29	25	31	30	1
AHP 0.75	26	26	16	17	22	2	7	2	20	22	24	26	25	31	30	1
AHP 0.5	26	26	16	19	23	2	7	2	20	23	22	26	25	31	30	1
AHP 0.35	27	27	18	19	23	1	7	1	21	23	20	26	25	31	30	3
ELECTRE	30	30	19	27	28	3	7	1	9	21	16	20	26	28	23	4
ELECTRE 0.75	27	29	18	25	29	2	5	1	8	21	21	20	31	25	23	4
ELECTRE 0.5	29	29	19	20	26	6	7	1	8	20	22	25	29	27	22	4
ELECTRE 0.25	29	29	15	21	23	2	8	1	10	21	15	20	28	29	25	4

expected loss compared to the original AHP for a given return level is very small, only 0.04 percentage point at the 6% return level and reducing even further as the expected excess return increases.

The results from screening using ELECTRE III are affected by various threshold levels necessary to derive outranking relations and final rankings. The threshold values are varied from 75%, 50% and 25% of the original values and new optimal portfolios determined. Table 12 shows the threshold values at a 50% reduction.

Ranking results after reapplying ELECTRE screening with the new threshold values are shown in Figure 6. For the top 20 list as compared to the original ELECTRE case, ELECTRE 0.75 and ELECTRE 0.25 differ only by 1 ILS and for ELECTRE 0.5 the top 20 list differs by 2 ILS. Rank correlations with the original ELECTRE case are high and shown in Table 13. But they are less than those for AHP variants.

Efficient frontiers for the optimal portfolios for ELECTRE variants are shown in Figure 8. The efficient frontiers for all ELECTRE variants dominate the original ELECTRE case. The ones for ELECTRE 0.75, 0.25 and original case are very close together ,

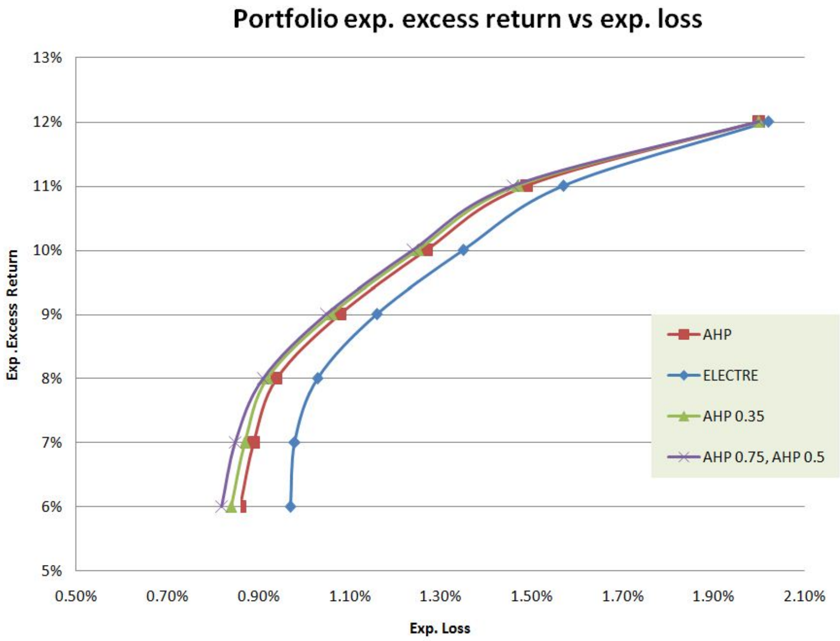
Table 10: AHP rank correlation

Rank correlation	AHP	AHP 0.75	AHP 0.5	AHP 0.35
AHP	1.0000	0.9916	0.9695	0.9430
AHP 0.75	0.9916	1.0000	0.9824	0.9619
AHP 0.5	0.9695	0.9824	1.0000	0.9855
AHP 0.35	0.9430	0.9619	0.9855	1.0000

Table 11: Optimal portfolios for AHP and ELECTRE variants

Portfolio return	Portfolio expected loss						
	AHP	AHP 0.5, 0.75	AHP 0.35	ELECTRE	ELECTRE 0.75	ELECTRE 0.5	ELECTRE 0.25
6%	0.86%	**0.82%**	0.84%	0.97%	0.95%	**0.88%**	0.96%
7%	0.89%	**0.85%**	0.87%	0.98%	0.96%	**0.90%**	0.97%
8%	0.94%	**0.91%**	0.92%	1.03%	1.01%	**0.95%**	1.02%
9%	1.08%	**1.05%**	1.06%	1.16%	1.14%	**1.09%**	1.15%
10%	1.27%	**1.24%**	1.25%	1.35%	1.33%	**1.28%**	1.34%
11%	1.49%	**1.46%**	1.47%	1.57%	1.55%	**1.50%**	1.56%
12%	2.00%	**2.00%**	2.00%	2.02%	2.01%	**1.99%**	2.02%

Figure 7: Optimal portfolios for AHP variants



differing by only 0.01 to 0.02 percentage point at 6% return level and getting narrower as expected excess return increases. In contrast, the efficient frontier for ELECTRE 0.5 is further apart from the rest and is closer to the ones from AHP variants. Considering only ELECTRE variants, the most efficient portfolios come from ELECTRE 0.5 variant with as much as 0.09 percentage point difference compared to original ELECTRE. In cross comparison to AHP variants, optimised portfolios from ELECTRE variants are still dominated by the ones from AHP even though the ones from ELECTRE 0.5 come very close to the original AHP case.

Optimised portfolios can be compared using their multiples, which is the ratio between expected excess return and expected loss. Portfolios having the highest multiples are at the intersection of the tangent line to the efficient frontiers. Figure 9 shows

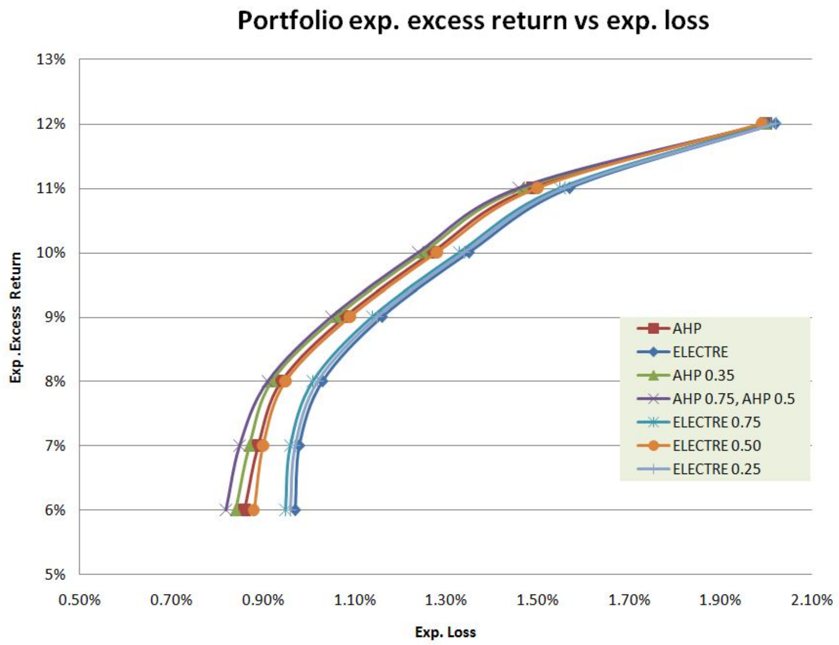
Table 12: Threshold values for the case of ELECTRE 0.5

	Rating	Adj. Spread premium	Prob 1st loss	Cond. expected loss
q	0.5	50bps	0.30%	5%
p	1	100bps	0.59%	10%
v	inf	inf	inf	inf

Table 13: ELECTRE Rank correlation

Rank correlation	ELECTRE	ELECTRE 0.75	ELECTRE 0.5	ELECTRE 0.25
ELECTRE	1.0000	0.9675	0.9479	0.9377
ELECTRE 0.75	0.9675	1.0000	0.9722	0.9418
ELECTRE 0.5	0.9479	0.9722	1.0000	0.9480
ELECTRE 0.25	0.9377	0.9418	0.9480	1.0000

Figure 8: Optimal portfolios for ELECTRE variants



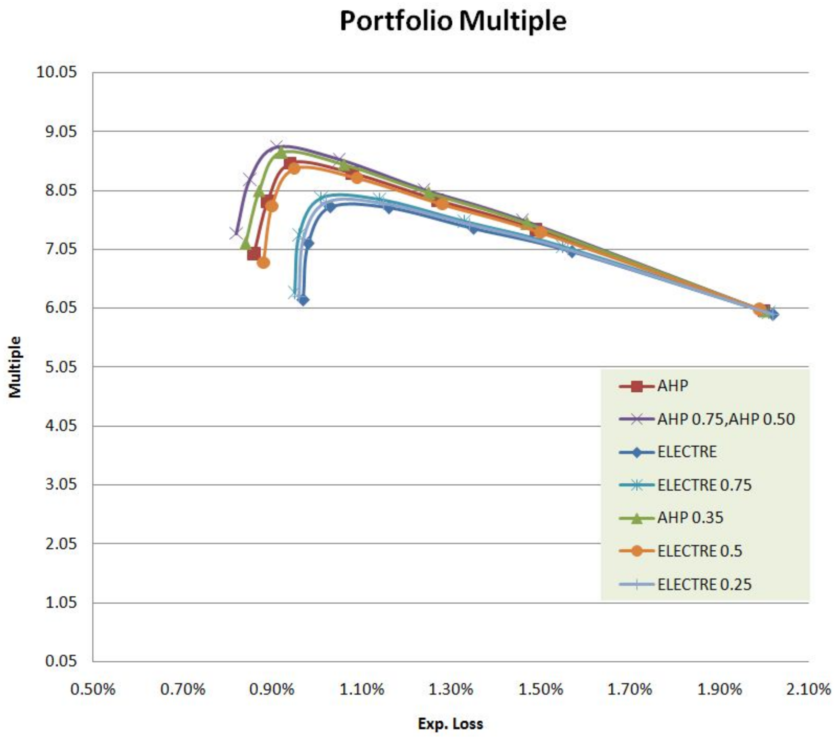
portfolio multiples for optimal portfolios from ELECTRE and AHP from the sensitivity analysis. AHP screening produces higher multiples compared to those from ELECTRE screening at any given level of expected loss. AHP 0.75 and 0.25 portfolios have the highest multiples.

8 Conclusion

The paper applies multi-criteria decision making (MCDM) techniques to ILS portfolio selection by screening assets. The methodology incorporates decision makers/investors value and preference systems as well as expert inputs to rank and select preferred ILS for portfolio optimisation. A comparative analysis of AHP and ELECTRE III in ILS portfolio selection is also provided.

Two popular MCDM techniques, AHP (direct rating mode) and ELECTRE III, are applied to insurance linked security portfolio selection for the first time. MCDM methods are used to screen a subset of ILS from which portfolio optimisation is conducted using linear programming with constraints. The results demonstrate that using MCDM to incorporate investor preferences and value systems selects better performing assets and improves portfolio optimisation.

Figure 9: Portfolio multiples



Rankings of ILS by AHP and ELECTRE are correlated. Optimal portfolios constructed from ILS screened by AHP and ELECTRE dominate portfolios constructed without MCDM screening compared to investing in all assets with equal weighting or deal size weighting. For the ILS data, optimised portfolios from AHP screening dominate those from ELECTRE screening. Sensitivity analysis shows that AHP portfolios are not very sensitive to the subjective pairwise comparison of rating intensities used to rate individual assets. In contrast, ELECTRE portfolios are more sensitive to changes in threshold levels used.

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A Appendix

AHP example

A simple example of AHP (direct rating version) is provided for illustration purposes. Consider the decision problem to select the best car among 3 potential options. The decision maker has analysed the problem and decided that the three main criteria are price, power and safety. Figure 10 presents the hierarchy constructed for this problem. Performance scores for each alternative are given in Table 14. The importance weight for each criterion with respect to the goal are derived from pairwise comparisons as shown in Table 15.

Figure 10: AHP Hierarchy for car selection

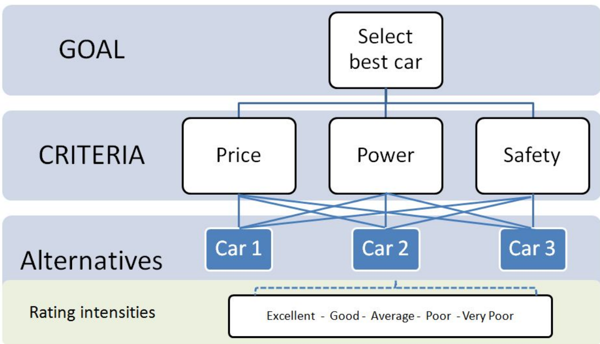

Table 14: Performance scores

	Price ($)	Power (kW)	Safety (1-5)
Car 1	21000	140	4
Car 2	28000	190	3
Car 3	41000	250	5

Table 15: Criteria weight ($CR = 0.037 \leq 0.05$)

Importance	Price	Power	Safety	Local priority
Price	1	3	1/3	0.26
Power	1/3	1	1/5	0.10
Safety	3	5	1	0.64

In the direct rating mode, each alternative is rated according to a predefined intensity or performance scale. We assume that the rating scales for the three criteria are derived from available information in the car market and decision maker inputs as shown in Table 16. Based on these rating scales, the rating for each alternative is shown in Table 17.

The next step is to convert intensity level to priority weight. Different performance levels are compared to one another using the pairwise comparison shown in Table 18. We use the same five point scale for all criteria to simplify the computation. In practice,

Table 16: Rating scales

Criteria	Excellent (E)	Good (G)	Average (A)	Poor (P)	Very Poor (VP)
Price	< 20000	20000-30000	30000-40000	40000-50000	≥ 50000
Power	≥ 300	300-200	200-150	150-100	< 100
Safety	5	4	3	2	1

Table 17: Ratings of alternatives

	Price	Power	Safety
Car 1	G	P	G
Car 2	G	A	A
Car 3	P	G	E

the rating scale and priority weight can be defined differently for each criterion. We normalise the priority weight by dividing eigenvector entries by the largest value.

Table 18: Pairwise comparison for rating scale ($CR = 0.0530 \leq 0.1$)

Importance	E	G	A	P	V	Normalised eigenvector	Local priority
Excellent	1	3	5	7	9	0.51	1.00
Good	1/3	1	3	5	7	0.26	0.51
Average	1/5	1/3	1	3	5	0.13	0.25
Poor	1/7	1/5	1/3	1	3	0.06	0.12
Very Poor	1/9	1/7	1/5	1/3	1	0.03	0.07

Finally, we can calculate global priority scores by summing local performance scores weighted by corresponding criteria importance weight. For example, global score for car 1 is computed as:

$$0.51 \times 0.26 + 0.12 \times 0.10 + 0.51 \times 0.64 = 0.47$$

Results of the AHP application for this problem are summarised in Table 19.

Car 3 is the best option, following by car 1 and car 2 ranked last.

Table 19: Summary of AHP results

	Price (0.26)	Power (0.10)	Safety (0.64)	Global score	Rank
Car 1	0.51	0.12	0.51	0.47	2
Car 2	0.51	0.25	0.25	0.32	3
Car 3	0.12	0.51	1.00	0.72	1

ELECTRE example

We apply ELECTRE III to the same decision problem for car selection as in the AHP example. We assume thresholds values for the three criteria are as in Table 20 based on the decision maker's inputs. No veto threshold is specified.

Table 20: Threshold values for car selection

	Price ($)	Power (kW)	Safety (1-5)
q	5000	50	1
p	10000	100	2
v	inf	inf	inf

Credibility indices for each pair of alternatives are computed, using ELECTRE formulas. As an example, consider the pair (car 3, car 1):

- Under criterion 1 (price, negative scale): $g_1(3) - g_1(1) = 41000 - 21000 = 20000 > p_1 = 10000$, hence, $c_1(3,1) = 0$
- Under criterion 2 (power, positive scale): $g_2(3) = 250 > g_2(1) = 140$, hence, $c_2(3,1) = 1$
- Under criterion 3 (safety, positive scale): $g_3(3) = 5 > g_3(1) = 4$, hence, $c_3(3,1) = 1$

Thus, the global concordance index $C(3,1) = 0.26 \times 0 + 0.10 \times 1 + 0.64 \times 1 = 0.74$.

Because we don't use a veto threshold, the discordance indices are zeros for all criteria. Therefore, the credibility index will equal the global concordance index. The credibility matrix S for the problem is given in Table 21.

Table 21: Credibility matrix

	Car 1	Car 2	Car 3
Car 1	-	1	0.9
Car 2	0.896	-	0.34
Car 3	0.74	0.74	-

In the next step, we perform distillation procedure to rank the 3 cars.

Descending distillation

First distillation:

Step 1.1:

$\lambda_0 = \text{Max } S(a,b) = 1, s = 0.3 - 0.15 \times 1 = 0.15, \lambda_0 - s(\lambda_0) = 0.85, \lambda^* = 0.74$. Outranking matrix T is provided in Table 22. At this cut off level, no alternative outranks one another. $D_1 = \{1,2,3\}$.

Step 1.2:

Lower λ_0 to the previous λ^*, $\lambda_0 = 0.74, s = 0.3 - 0.15 \times 0.74 = 0.189, \lambda_0 - s(\lambda_0) = 0.74 - 0.189 = 0.551, \lambda^* = 0.34$. Outranking matrix T is provided in Table 23. At this cut off level, car 3 has the highest qualification. Thus the result of the first distillation is $D_1 = \{3\}$. Second distillation:

Table 22: First distillation outranking matrix - Step 1.1

	Car 1	Car 2	Car 3	Qualification
Car 1	-	0	0	0
Car 2	0	-	0	0
Car 3	0	0	-	0

Table 23: First distillation outranking matrix - Step 1.2

	Car 1	Car 2	Car 3	Qualification
Car 1	-	0	0	0
Car 2	0	-	0	-1
Car 3	0	1	-	1

Step 2.1:
$\lambda_0 = 1, s = 0.3 - 0.15 \times 1 = 0.15, \lambda_0 - s(\lambda_0) = 1 - 0.0.15 = 0.85, \lambda^* = 0$. Outranking matrix T is provided in Table 24. Since $S(1,2) - S(2,1) = 1 - 0.896 = 0.104 < 0.15 = \min s(\lambda)$, the 2 options don't outrank each other. $D_2 = \{1,2\}$.

Table 24: Second distillation - Step 2.1

	Car 1	Car 2	Qualification
Car 1	-	0	0
Car 2	0	-	0

Ascending distillation

First distillation:

Computations and resulting outranking matrix T are exactly the same as in the descending distillation.

Step 1.1:
All options have the same qualification index, no alternative outranks one another. $D_1 = \{1,2,3\}$.

Step 1.2:
Based on Table 23, car 2 has the lowest qualification. Thus result of first distillation is $D_1 = \{2\}$.

Second distillation:

Step 2.1:
$\lambda_0 = 0.90, s = 0.3 - 0.15 \times 0.90 = 0.165, \lambda_0 - s(\lambda_0) = 0.90 - 0.165 = 0.735, \lambda^* = 0$. Outranking matrix T is provided in Table 25. Since $S(1,3) - S(3,1) = 0.90 - 0.74 = 0.16 < 0.165 = s(S(1,3))$, the 2 options don't outrank each other. $D_2 = \{1,3\}$.

Results of both distillations are combined to get the final rankings as shown in Table 26. The rankings are the same as in AHP application, car 3 ranked first, following by car 1 and car 2.

Table 25: Second distillation - Step 2.1

	Car 1	Car 3	Qualification
Car 1	-	0	0
Car 3	0	-	0

Table 26: Summary table - ELECTRE III ranking

	Descending distillation	Ascending distillation	Final rank
Car 1	2	1	2
Car 2	2	2	3
Car 3	1	1	1