ARC Centre of Excellence in Population Ageing Research

Working Paper 2012/14

Solvency Capital, Pricing and Capitalization Strategies of Life Annuity Providers

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May 28, 2012

Abstract

This paper provides a detailed quantitative assessment of the impact of solvency capital requirements on product pricing and shareholder value for a life insurer. A multi-period firm value maximization model for a life annuity provider, allowing for stochastic mortality and asset returns, imperfectly elastic product demand, as well as frictional costs, is used to derive optimal capital and pricing strategies for a range of solvency levels reflecting differences in regulatory regimes. The model is calibrated using realistic assumptions and the sensitivity of results assessed. The results show that value-maximizing insurers should target higher solvency levels than the Solvency II regulatory 99.5% under assumptions of reasonable levels of policyholder's aversion to insolvency risk. Even in the case of less restrictive solvency regulation, policyholder price elasticity and solvency preferences are shown to be important factors for a life insurer's profit maximizing strategy.

Keywords: life annuity, insurance regulation, solvency, longevity risk
JEL Classifications: G22, G23, G28, G32

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1 Introduction

International demographic change has highlighted the significance of sustainable products to manage longevity risk - the uncertainty surrounding the risk of people living longer. Developed countries require retirement solutions for an ageing population, whilst mortality continues to improve. Effective longevity risk management solutions that transform retirement savings into reliable retirement income sources need to be provided at an efficient cost.

Life annuities provide an ideal hedge against longevity risk (Brown and Orzag, 2008 [10]), and risk averse individuals should value these annuities even more than the amount paid, since the probability of outliving individual retirement savings is significant (Mitchell, 2001 [20]). However, in reality, consumer demand for annuities is limited. This *annuity puzzle* is attributed to a variety of reasons, including bequest motives (Piggott and Purcal, 2008 [23]), the poor value for money of annuities (Brown et al., 1999 [9]), and the loss of liquidity and control over their finances in the case of unexpected and uninsured events (Piggott and Purcal, 2008 [23]).

Effective regulatory controls help to develop and enhance market participation in longevity insurance products by ensuring that providers of these products will deliver on consumer contracts to a high degree of certainty. Higher capital requirements will lower insolvency risks, reducing insolvency costs to policyholders and shareholders, while at the same time increasing capital costs leading to higher frictional costs and premiums. Welfare losses arise from higher premiums since fewer individuals purchase longevity insurance, eschewing the longevity risk benefits associated with these products.[1] That is, effective solvency regulation needs to consider the trade-off between prudential security and consumer affordability.

This paper provides a detailed quantitative assessment of the impact of solvency capital requirements on product pricing and shareholder value. A multi-period value maximization model for a life insurer offering lifetime guaranteed annuities is developed and calibrated using realistic assumptions and market data. The model incorporates a stochastic mortality model, stochastic investment returns, and an imperfectly elastic demand function capturing consumer preferences for financial quality. The model is used to derive optimal pricing and capitalization strategies for a range of solvency levels reflecting differences in regulatory regimes.

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[1]Rees, Gravelle and Wambach (1999) [24] examine the arguments for solvency regulation and find that solvency regulation is unnecessary if consumers are fully informed about the risks of the insurer's insolvency. In reality, consumers cannot adequately inform themselves of insurer insolvency and its implications, hence prudential regulation is justified.

The results show that value-maximizing insurers should target higher solvency levels than the Solvency II regulatory 99.5% under assumptions of reasonable levels of policyholder's aversion to insolvency risk. Even in the case of less restrictive solvency regulation, policyholder price elasticity and solvency preferences are shown to be important factors for a life insurer's profit maximizing strategy.

The structure of this paper is as follows. Section 2 presents the insurer value maximization model. Section 3 presents the calibration of the model. Section 4 presents the results of a quantitative study of an annuity provider's capitalization and pricing strategies under different regulatory requirements. Section 5 concludes.

2 Insurer Value Maximizing Model

Value maximization models used to study the effect of solvency requirements on insurers should incorporate consumer demand and frictional costs. The costs of holding high levels of capital impact both shareholders and policyholders. High levels of capital result in higher costs of capital and higher premiums. The effect of this on insurer profitability depends on consumer preferences for solvency and price elasticity.

Much of the analysis of the impact of solvency, capital and the links with pricing has been for non-life insurers. Optimal insurer capitalization was considered by Munch and Smallwood (1981) [21] to assess the effect of solvency regulation on the property and casualty insurance industry. Optimal capitalization strategies were determined by maximizing the market value of the insurance firm. The firm value maximization model of Rees, Gravelle and Wambach (1999) [24] included the consumer's willingness to pay for insurance depending on the insurer's insolvency risk and consumers were assumed to be fully informed of insurer insolvency risk.

Imperfectly elastic demand for insurance and frictional costs of capital were incorporated in a single-period value maximization model for a multi-line non-life insurance company by Zanjani (2002) [31]. Yow and Sherris (2008) [30] used a single period model based on Zanjani (2002) [31] to assess the effects of frictional costs on a multi-line non-life insurance company's pricing and capitalization strategies. The model includes frictional costs of agency, bankruptcy and taxation and consumer preferences for solvency. Zimmer, Gründl and Schade (2011) [33] incorporated a demand curve into a single-period shareholder value maximization model similar to the model

developed in Zanjani (2002) [31] and used the model to analyze the impact of consumer reactions to default risk on an insurer's optimal solvency level.

A single-period shareholder value maximization model for a life insurance company offering term life insurance and life annuities to insolvency-averse consumers was developed in Gründl, Post and Schulze (2006) [16] and used to study the impact of demographic risk on the optimal risk management mix of the insurer.

2.1 A Value Maximization Model for a Life Annuity Provider

A model is developed to assess for different default levels optimal capital and pricing strategies for a life insurer offering life annuities. Optimal strategies are those that maximize economic value added by the insurance business for shareholders over a one-year horizon. A one-year horizon is chosen to reflect international solvency requirements (including Solvency II). Although a one-year horizon is used, cash-flows are multi-period and stochastic to reflect changes in the reserves and in the asset values.

The model in Yow and Sherris (2008) [30] was used as a framework to develop a multiple period cash flow life insurer model depending on stochastic mortality. The insurer model includes, in addition, a stochastic term structure model to value future expected annuity cash flows, stochastic investment returns and a price-default risk demand curve based on that estimated by Zimmer, Gründl and Schade (2011) [33].

Enterprise Value Added (EVA), as in Yow and Sherris (2008) [30], is used as the measure of economic value added by the insurance business for shareholders. This is determined as the expected profit over the first year after allowing for the establishment of reserves for future labilities for survivors at the end of the first year. To allow for the initial capital subscribed an allowance is also made for the cost of capital (CoC) to determine an EVA adjusted for CoC. Both EVA and EVA adjusted for CoC measure the economic value added for the life insurer.

A single cohort of males aged 65 is simulated in the model. Stochastic values of assets and liabilities are determined at the end of the period using simulation of stochastic future mortality rates and yield curves. These are used to determine stochastic profit realizations over the year.

Premiums: Single premiums include a loading on the best estimate annuity prices determined

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using the future expected survival rates and the current market yield curve. Policyholder demand is assumed to depend on the price per contract and the solvency, or default risk, of the insurance company. Total premium income P at time 0 is the number of policies sold by the per policy single premium:

$$P = Q(\pi, d) \cdot \pi, \tag{1}$$

where Q is the number of annuities sold at time 0. Q depends on the default probability d and on the per policy single premium π. For each policy

$$\pi = (1 + k) \cdot A \cdot \sum_{t=1}^{45} {}_t p_{65,0} \cdot \nu(0, t), \tag{2}$$

where k is the premium loading, A is the fixed annual payment, ${}_t p_{65,0}$ is the expected probability of a male aged 65 to survive another t years at time 0, and $\nu(0, t)$ is the discount factor for a payment at time t. The discount factor is derived from the expected yield curve at time 0, fitted with a Vasicek model described below.

Stochastic Mortality Model: Survival probabilities are derived from the stochastic mortality model presented in Wills and Sherris (2010) [29]. This model extends the traditional Lee-Carter mortality model (Lee and Carter, 1992 [19]) to incorporate age and cohort effects as well as multiple risk factors. The force of mortality $\mu(x, t)$ for age x at time t is modeled as a discrete approximation to a stochastic diffusion process:

$$d\mu(x, t) = (a(x_0 + t) + b)\mu(x, t)dt + \sigma\mu(x, t)dW(x, t), \tag{3}$$

where a, b and σ are constants and $dW(x, t)$ is a multivariate Wiener process. x_0 is the initial age at the start of the contract.

The Wills-Sherris model provides a very good fit to Australian mortality data for lives aged 50 to 99 and includes a simulation procedure for projecting future mortality rates incorporating cohort longevity improvements over time. The model was re-calibrated to incorporate ages up to 110.

Demand Curve: Product demand with respect to solvency and price is a critical component of a model that aims to assess the solvency trade-off. Estimates of consumers' reactions to insurance default risk are reported in Zimmer, Schade and Gründl (2009) [32] and Zimmer,

Gründl and Schade (2011) [33] based on experiments used to elicit an individuals' willingness to pay for theft insurance contracts for differing levels of default risk.

Four levels of default (0%, 1%, 2% and 3%) were used to calibrate the demand function. Monetary incentives and the secret price mechanism developed by Schade et al (2009) [25], which provides an accurate reflection of maximum willingness to pay, increased the experiment's reliability. A range of different demand functions were fitted to the data and the exponential demand function was the overall best fit. The price-default risk demand curve from Zimmer, Gründl and Schade (2011) [33] captures both the default risk aversion and the price sensitivity of policyholders. The functional form is given in the equation below.

$$\phi(\pi, d) = e^{(\alpha \cdot d + \beta \cdot \pi + \gamma)}, \tag{4}$$

where $\phi(\pi, d)$ represents the percentage of individuals willing to buy annuities at price π from an insurer with default probability d, α is the default sensitivity parameter ($\alpha < 0$), β is the price sensitivity factor ($\beta < 0$) and γ is a constant.

Given the demand function and the maximum potential market size, M, the number of annuities Q sold at time 0 is:

$$Q(\pi, d) = M \cdot \phi(\pi, d). \tag{5}$$

Initial Capital: Initial capital, R, is subscribed from the shareholders at time 0 in order to achieve a target solvency level d promised to the policyholders.

Expenses: Expenses are assumed to be a fixed percentage of the single annuity premium π and are a one-off, paid at the end of the first year. Total expenses are given by:

$$c = P \cdot expense\ factor. \tag{6}$$

Although in practice there are other expenses, these are the major form of expenses.

Claims and Reserves: Claims at the end of the first period are random. They are given by:

$$\tilde{L}_1 = Q(\pi, d) \cdot A \cdot {}_1\tilde{p}_{65}, \tag{7}$$

where Q is the number of annuities sold at time 0, A is the fixed annual payment per annuity

contract, and $_1\tilde{p}_{65}$ the random probability of a 65-year old to survive to age 66.

The policy liability reserves are set up to include the net present value of future liabilities as well as premium loadings weighted by the proportion of survivors, valued using the government bond yields to maturity. The reserve at time 0 is given by:

$$Reserve_0 \quad = \quad Q(\pi, d) \cdot (1 + k) \cdot A \cdot \left[\sum_{t=1}^{45} {}_t p_{65,0} \cdot \nu(0,t) \right]. \tag{8}$$

The reserve at time 1 is calculated using random time 1 survival probabilities $_t\tilde{p}_{66,1}$ and discount factors $\tilde{\nu}(1,t)$. These differ across simulation scenarios. The reserve is established for the random number of survivors in each scenario $_1\tilde{p}_{65} \cdot Q(\pi, d)$:

$$\widetilde{Reserve}_1 = {}_1\tilde{p}_{65} \cdot Q(\pi, d) \cdot (1 + k) \cdot A \left[\sum_{t=1}^{44} {}_t\tilde{p}_{66,1} \cdot \tilde{\nu}(1,t) \right]. \tag{9}$$

The reserves include the value of the loadings in the single premium. Since expenses are assumed to be incurred at the end of year 1 as a single up front payment, the premium loadings are included in the determination of economic profit to offset the initial expenses in the first period.

Term Structure Model: The Vasicek model (Vasicek, 1977 [27]) is used for the term structure model. This is a one-factor short rate model that incorporates mean reversion of interest rates.[2] The stochastic process for the short rate r_t is:

$$dr_t = \alpha(\mu_r - r_t)dt + \sigma_r dW_t, \tag{10}$$

where α, μ_r and σ_r, together with the initial condition r_0, characterize the dynamics of the instantaneous interest rate. Discount factors $\nu(t,T)$ for the value at time t of a payment at time T assuming continuously compounded zero-coupon bond yields are given by (see Van Deventer, Imai, and Mesler, 2005 [28], pp. 209-212):

$$\nu(t,T) = e^{-F(t,T)r_t - G(t,T)}, \tag{11}$$

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[2]The Vasicek model can generate negative interest rates which are unrealistic. This problem is dealt with by eliminating simulation runs where the short rate becomes negative and the same number of simulations that attain the highest returns are also removed to avoid bias.

Balance Sheet		Profit and Loss Account	
Assets	Liabilities	Income	Expenses
Stocks	*Reserve*	*Premiums*	*Claims*
Bonds	*Equity*	*Investment income*	*Expenses*
Cash	*Capital subscribed*		*Frictional costs*
			Change in the reserve

Figure 1: Insurer balance sheet and profit and loss account over the first year.

where (using τ to denote the maturity):

$$F(t,T) = F(\tau) = \frac{1}{\alpha}(1 - e^{-\alpha\tau}) \tag{12}$$

$$G(t,T) = G(\tau) = \left[\mu + \frac{\lambda\sigma}{\alpha} - \frac{\sigma^2}{2\alpha^2}\right][\tau - F(\tau)] + \frac{\sigma^2}{4\alpha}F^2(\tau). \tag{13}$$

When calibrating the model's parameter, a market price of risk λ is calibrated to observed market yield curve data (see Van Deventer, Imai, and Mesler, 2005 [28], pp. 209-212 and 222-225).

Assets and Investment Returns: Assets at time 0 comprise the total premium income P and the capital R subscribed from the shareholders: $V_0 = P + R$. Returns on the assets are random and denoted by $\widetilde{return_t}$, a weighted average of the per period returns from different investments. Asset classes included in the model are bonds, stocks and a cash account.

The short rates generated by the Vasicek model are used for single period bond returns. Stock prices and cash rates are modeled as Geometric Brownian motions with drift terms μ_s and μ_c, and volatility parameters σ_s and σ_c, respectively (see, e.g., Hull, 2009 [17]).

Frictional Costs: Three types of frictional costs are included in Yow and Sherris (2008) [30]: taxation, agency and bankruptcy costs. The taxation rate is denoted as τ_1. Total agency costs are assumed to be proportional to the initial capital subscribed, $\tau_2 R$. Bankruptcy costs, f, reflecting financial distress, are included whenever profit is negative and are larger in absolute amount for larger losses.

Insurer Profit and Enterprise Value Added: Insurer profit is calculated using an economic valuation approach. Balance sheet amounts are market value based in accordance with International Financial Reporting Standards.[3] The balance sheet and the profit and loss account, used to determine economic profit, are shown in Figure 1.

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[3] Balance sheet amounts are also in accordance with the Australian Accounting Standards set by AASB.

Enterprise Value Added (EVA) is defined as the expected present value of profits to shareholders in excess of the initial capital subscribed. This is determined based on the random profit over the first year:

$$
\widetilde{Profit}_1 = \begin{cases} Reserve_0 + \tilde{K} + (P + R) \cdot \widetilde{return}_1 \cdot (1 - \tau_1) - \tilde{L}_1 - c - \tau_2 \cdot R - Reserve_1 & if\ \widetilde{Profit}_1 > 0 \\ \left(Reserve_0 + \tilde{K} + (P + R) \cdot \widetilde{return}_1 \cdot (1 - \tau_1) - \tilde{L}_1 - c - \tau_2 \cdot R - Reserve_1 \right)(1 + f) \\ if\ \widetilde{Profit}_1 < 0. \end{cases}
$$

(14)

The profit is determined as the premium, which is used to establish the time 0 initial reserve, plus the present value of future premium loadings for the survivors, $\tilde{K} = {}_1\tilde{p}_{65} \cdot \frac{k}{1+k} \cdot P$, and asset returns, less claims, expenses, frictional costs as well as the amount required to establish a reserve at the end of the year for the survivors. The simulated EVA then depends on the amount of capital subscribed and, where this is positive, the extent to which it offsets losses. For lower levels of solvency and higher levels of premium loadings it is possible that the amount of capital required to establish the target solvency is negative. This reflects the fact that the premium loadings alone are sufficient to ensure the target solvency level, suggesting a mutual structure for the life insurer.

That is, there are two main cases for EVA. In the first case, a positive amount of capital R is subscribed from shareholders to establish the target default probability d. In this case, shareholders can either receive a profit (EVA $= \widetilde{Profit}_1$) or, if the losses are large, lose their initial capital (EVA $= -R$). In the second main case, shareholders can withdraw capital at time 0 because the total premium income is more than enough to establish the target default probability. In this case, shareholders either receive a profit (EVA $= \widetilde{Profit}_1$) or the company defaults but EVA $= 0$ because shareholders did not have to invest capital at time 0.[4]

The EVA is the expected value of this simulated EVA across all the simulation scenarios. The EVA adjusted for cost of capital is the EVA minus the initial subscribed capital times the cost of capital.

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[4]In this model, reserves are calculated assuming no default of the insurer as required by accounting and solvency requirements. As a result, the reserves include the default put option (DPO) value (which is given by the expected value of the payments policyholders will not receive in the case of insolvency, that is if $R + Profit < 0$).

Parameter	MLE Value
$\hat{a}$	$4.5089e-04$
$\hat{b}$	-0.1011
$\hat{\sigma}$	0.0605

Table 1: Parameter estimates for the mortality model using MLE techniques.
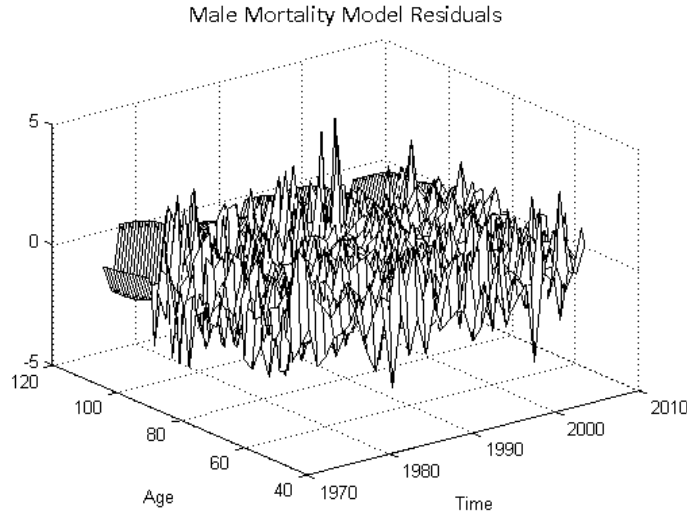

Figure 2: Standardized residuals from the mortality model.

3   Calibration of the Life Insurer Model

The model was calibrated to market and other relevant Australian data for yield curves, asset returns, expenses, mortality and frictional costs. The demand function and the frictional costs are the most challenging to calibrate to market data. There are no studies or industry information that can be readily used for this purpose. We follow a similar process as in Yow and Sherris (2008) [30] to do this calibration. The robustness of these calibrations are assessed in the results.

Stochastic Mortality Model: The stochastic mortality model was estimated using Australian male mortality rates for ages 50 to 110 from 1971 to 2007 obtained from the Human Mortality Database (2011), [18]. The maximum likelihood parameter estimates are shown in Table 1.

Figure 2 shows the age-specific (standardized) model residuals. The model fits Australian mortality data well. Prior to 1960, mortality data in the Human Mortality Database for older ages was smoothed which results in smooth residuals past the age of 96. Prior to this age, the model residuals fluctuate randomly around a mean of zero without age or time trends.

Table 2 gives the descriptive statistics of the standardized residuals. The standard error of the

mean estimate is small and the standard deviation is very close to one.

Parameter	Value
Mean	$4.5797e - 016$
Standard Error	0.0215
Standard Deviation	1.0002
Minimum	-4.3935
Maximum	4.6108

Table 2: Residuals descriptive statistics for the standardized residuals from the mortality model.

Pearson's chi-square goodness of fit test between observed mortality rates and expected mortality rates has a value of $\chi^2 = 124.42$. This statistic approximately has a chi-square distribution with 326 degrees of freedom.[5] The critical value is χ^2_{326} at $99\% = 269.55$ and the test statistic is less than the critical value, confirming the model provides a good fit to the data.

Mortality scenarios are simulated using the procedure in Wills and Sherris (2010) [29]. Figure 3 plots the expected survival curve for a 65 year old at time $t = 1$ together with confidence intervals.
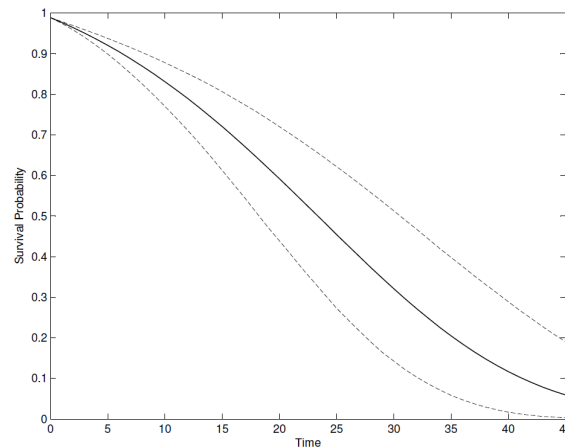
Figure 3: Simulated survival probabilities at time $t = 1$, mean values and 95% confidence intervals.

Premiums: With a zero loading, the average annual payment of $A = \$5,149.70$ was based on a single premium of \$70,000 using expected survival rates and the fitted initial yield curve. This is consistent with the average retirement savings of approximately \$71,000 for a 65 year old (Australian Bureau of Statistics, 2010, [2]).

Expenses: Expenses were assumed to be 3.3% of the total annuity premium (including the loading). Challenger Life Company Ltd is the major company writing lifetime guaranteed an-

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[5]There are 2160 observations, 3 parameters in the main model and 1830 parameters in the 60×60 correlation matrix of $dW(x,t)$: $df = number\ of\ observations - number\ of\ independent\ parameters - 1 = 326$.

nuities in Australia. Their product disclosure statement reports an upfront adviser service fee of up to 3.3% of the purchase price (Challenger, 2011 [11]).

Assets Allocation: The insurer's asset mix was based on investment strategies of insurers offering annuities with longevity risk. The Australian Prudential Regulation Authority (APRA) publishes assets backing policy liabilities in their Half Yearly Life Insurance Bulletin. An allocation of 5.5% in cash, 86.8% in bonds, and 7.7% in stocks was used (APRA, 2010a [4]). A portfolio consisting largely of bonds provides a matching investment strategy for a life insurer issuing life annuities. However the maturity of available government bonds in Australia are not long enough for full asset liability matching. As a result, interest rate risk is captured in the model over the one year horizon with a stochastic yield curve model.

Yield Curve and Asset Returns: The data used to calibrate the yield curve and asset return models came from 'Australian Government Bonds Yields and Interest Rates' obtained from Bloomberg (accessed September 2011), time series for the period 1990-2010 for the 'Cash Rate - Interbank Rate' (accessed August 2011) and capital market yields of 10-year Australian Government Bonds (accessed July 2011) from the Reserve Bank of Australia.

Least squares was used to estimate the parameters of the Vasicek model. The fitted initial yield curve and simulated yield curves at time 1 are shown in Figure 4. The initial curve fits the current Australian yield curve well apart from the very short maturity. The Vasicek model parameters are shown in Table 3.
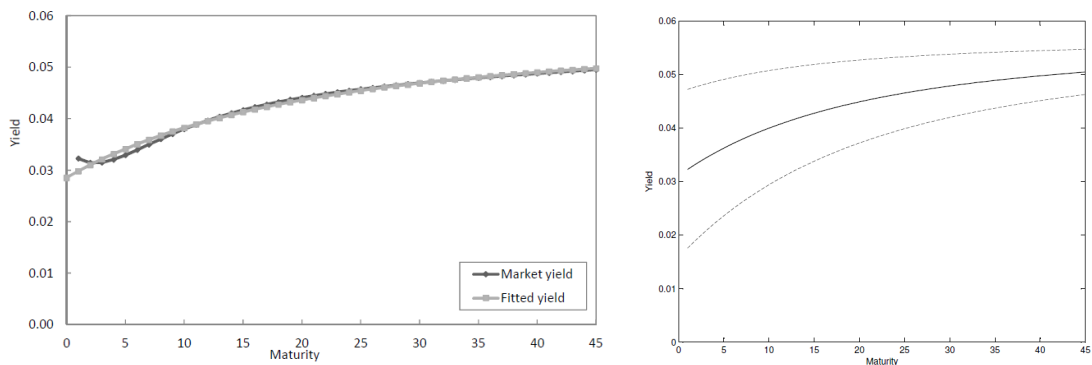

Figure 4: Fitted yield curve at time $t = 0$ and simulated yield curves at time $t = 1$, mean values and 95% confidence intervals.

Parameters of the stochastic processes for the stock returns and the cash rate were estimated based on 1990-2010 data for the 'S&P/ASX 200 Accumulation Index' and for 'Cash Rate - Interbank Rate' provided by the Reserve Bank of Australia and are shown in Table 3. The time

series used for the 'S&P/ASX 200 Accumulation Index' covering this period was constructed by linking 2006-2010 data from the Reserve Bank of Australia and with a longer time series from a free internet resource[6] (both accessed June 2011).

The correlation matrix between cash, bond, and stock returns over the period 1990-2010 is shown in Table 4.

Yield Curve		Stock Returns		Cash Rate	
Parameter	Value	Parameter	Value	Parameter	Value
$\hat{\alpha}_r$	0.0790	$\hat{\mu}_s$	0.0981	$\hat{\mu}_c$	0.0613
$\hat{\mu}_r$	0.0608	$\hat{\sigma}_s$	0.1239	$\hat{\sigma}_c$	0.0222
$\hat{\sigma}_r$	0.0079				
$\hat{r}_0$	0.0285				
$\hat{\lambda}$	0.0102				

Table 3: Parameter estimations for yield curve and asset return models, annual data for the period 1990-2010.

	Cash	Bond	Stock
Cash	1.0000	0.9490	0.0167
Bond	0.9490	1.0000	0.0013
Stock	0.0167	0.0013	1.0000

Table 4: Correlation matrix between asset returns, annual data for the period 1990-2010.

Frictional costs: For the shareholder value we assume a tax rate of 0% allowing for the benefits from the imputation credit system in Australia.[7] The taxation rate, τ_1, for life annuities owned as superannuation is normally 15% and applies to investment income. The shareholder agency cost of capital, τ_2, was assumed to be 2% based on Swiss Re (2005) [26]. The bankruptcy cost factor, f, was assumed to be 15%.

Cost of capital: The one-year bond rate is used for the cost of capital. This is 3.3% from the market yield curve used to calibrate the yield curve model.

Market size: A maximum potential market size of $M = 25,000$ was assumed for the representative life annuity provider. The current Australian male population aged 65 is 102,857 (Australian Bureau of Statistics, 2010 [2]). In 2010, the largest life insurer in Australia had a market share of 25-30% (APRA, 2011a [5]).

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[6]http://www.economagic.com/em-cgi/data.exe/rba/fsmspasx2ai

[7]The Australian imputation system allows corporate tax entities to distribute to their members franking credits for taxes paid by these corporations in order to avoid double taxation of the same income earned. These franking credits act as a tax offset on assessable income for the shareholders of these corporations (Australian Tax Office, 2011 [7]).
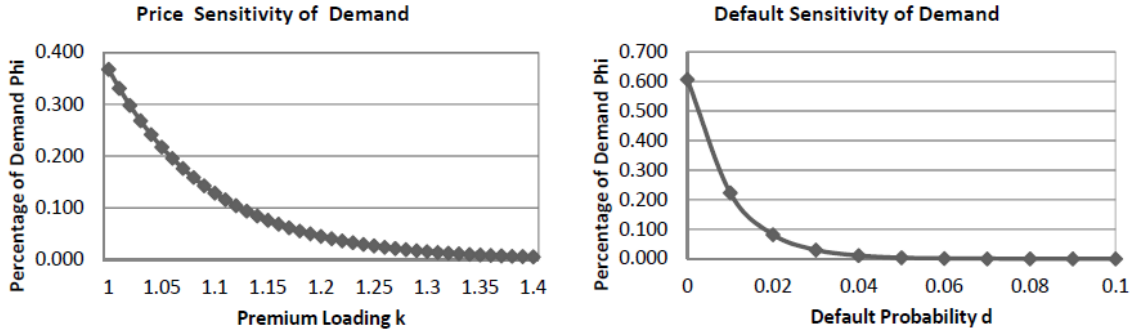
Figure 5: Assumed price and default sensitivity of the demand for annuities.

Demand curve: There is no empirical study that we are aware of that provides estimates of the price and default risk sensitivity of the demand for life annuities.[8] Annuity markets sizes and premium loadings observed internationally provide only limited insight because of differences in solvency regulations and social security systems.

The price-default risk life annuity demand curve was calibrated based on studies of the Australian annuity market along with informed judgement. The sensitivity of the analysis to this assumption is assessed in Section 4.2. The assumed demand curve is:

$$\phi(\pi, d) \;=\; e^{(\alpha \cdot d + \beta \cdot \pi + \gamma)} \tag{15}$$

$$=\; e^{(-100 \cdot d - 0.00015 \cdot \pi + 10)}, \tag{16}$$

where $\phi(\pi, d)$ represents the percentage of individuals willing to buy the annuity contract at the premium of π from an insurer with default probability d. The default sensitivity parameter is set to $\alpha = -100$, the price sensitivity factor is $\beta = -0.00015$, and the constant is $\gamma = 10$.

Figure 5 shows the reduction in demand due to changes in price and the demand curve's responses to changes in default risk. The assumed fair annuity premium is $70,000. The left graph plots annuity demand $\phi(\pi, d)$ against different levels of the premium loading k, assuming a default probability of $d = 0.5\%$. At a zero loading, 37% of individuals would be willing to purchase the annuity contract. At a loading of $k = 24\%$ a very small demand of 3% results, which is consistent with annuity demand in the Australian market.[9] The right graph plots $\phi(\pi, d)$

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[8]Babbel and Merrill (2006) [8] employ a multi-period utility maximization framework to study the impact of an insurer's default risk on annuity demand. The results of this theoretical study suggest that for moderate levels of risk aversion annuity demand is not very sensitive towards premium loadings of up to 30%. However, optimal annuitization levels are shown to drop sharply when the default risk of the annuity provider is stepwise increased from riskless to a 'AAA', 'AA', and 'A' rating.

[9]Ganegoda and Bateman (2008) [15] estimate the loading on a nominal Australian life annuity for a 65 year old male in the general population to be approximately 24%. See Evans and Sherris (2009) [14] for an assessment

against the default probability d for a premium of \$70,000. Annuity demand decreases rapidly as insolvency risk increases. At a default probability of $d = 5\%$ demand is zero.

4   Optimal Solvency and Premium Loadings

4.1   The Life Insurer's Value Optimization

The life annuity insurer maximizes enterprise value added (EVA) over a one-year horizon by choosing the default probability d, the premium loading k and initial capital subscribed from shareholders R. The premium loading and the initial capital determine the default probability d, which has an indirect impact on the EVA via the price-default risk demand curve $\phi(\pi, d)$. A complex optimization problem arises. For a given default probability, the combination of premium loading and capital that results in a higher EVA (or higher EVA adjusted for the cost of capital) is preferred by the shareholders.

The optimal strategy is the combination of default probability d, premium loading k and initial capital R that gives the highest EVA or adjusted EVA value. This optimum is determined by comparing insurer profit and EVA for different combinations of d, k and R. For each combination, 100,000 simulations of the insurer model are used to estimate a profit and EVA distribution.

A given default probability, say d^*, can result from a number of different combinations of the premium loading k and initial capital R. These combinations are determined by considering different levels of the loading k in the range $k = 0, 5\%, 10\%, 20\%, 30\%$ and applying an iterative algorithm that determines for each k the initial shareholder capital R needed to achieve the target default probability d^*.

Different regulated environments are considered including a case where all insurers have the solvency probability of 99.5%, the Australian and European situation, and where insurers have flexibility to choose a target solvency level and credit rating, reflecting the situation in the United States.

EVA for a Target Solvency Probability of 99.5%

Figure 6 shows the EVA for different combinations of premium loading and initial capital sub-
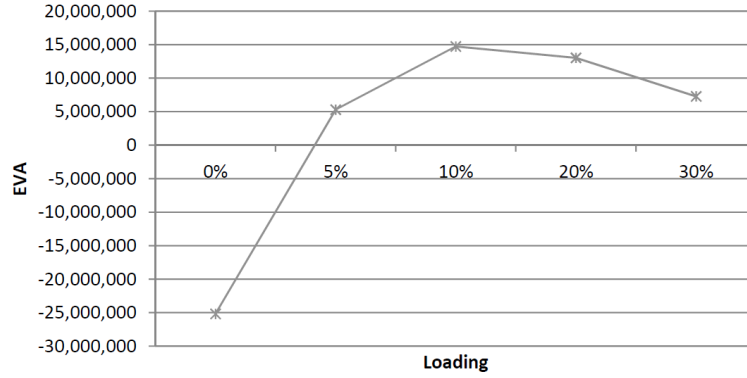
of annuity demand in Australia.

Figure 6: EVA for different combinations of premium loading and initial capital subscribed resulting in a solvency level of 99.5%.

Loading k	Capital R	Premiums P	$P/(P+R)$	Quantity Q	EVA	EVA adj CoC
0%	190.5	643.8	0.228	9,197	-25.2	-31.5
5%	90.9	399.9	0.185	5,441	5.3	2.3
10%	41.1	247.8	0.142	3,218	14.7	13.4
20%	5.9	94.6	0.059	1,126	13.0	12.8
30%	-0.9	35.9	*Note*	394	7.3	7.3

Table 5: Number of annuities sold (Quantity Q), EVA and EVA adjusted for CoC in millions for different combinations of premium loading and initial capital subscribed resulting in a solvency level of 99.5%. Note: When the loading is high enough, the solvency requirement is met wholly from policyholder loadings and no capital subscription is required from shareholders. This is the situation where the insurer would be structured as a mutual.

scribed that provide a solvency probability of 99.5%. There is a hump-shaped relationship between premium loading and EVA reflecting the demand curve elasticity. The highest EVA value, for the loadings considered, occurs for a loading of 10%. The EVA adjusted for CoC occurs for higher loadings where the level of initial capital subscribed by shareholders is lower. At a zero loading EVA is negative because of the up-front expenses.

Table 5 shows the numerical results. The higher the premium loading k is, the less initial capital R is required from the shareholders to attain the solvency level of 99.5%. At a loading of 30%, total premiums P are more than enough to ensure the target solvency probability of 99.5% and initial capital R is negative. In this case the policyholder loadings are sufficient to meet solvency requirements and no shareholder funds are required. In these cases, from a policyholder perspective, the insurer would be optimally structured as a mutual. Insurance demand, given by the number of annuities sold Q, decreases as the loading increases.

EVA for Varying Solvency Probabilities

A range of different one-year default probability is considered ($d = 0.1\%, 0.5\%, 1\%, 3\%$) and
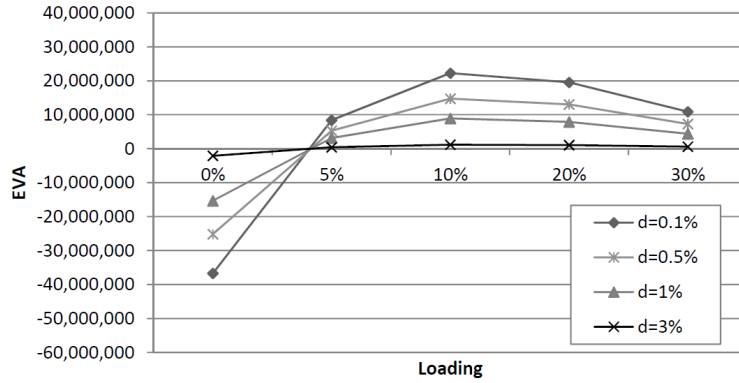
Figure 7: EVA for different combinations of premium loading and initial capital subscribed resulting for different solvency levels.

insurers are assumed to select the default level that maximises EVA. The default probabilities reflect AM Best ratings developed for the insurance sector (AM Best, 2007 [3]).

Figure 7 plots the EVA for different combinations of premium loading and initial capital subscribed resulting for each default probability. Higher solvency levels, or lower default probabilities, result in higher EVA values for positive premium loadings. For each case the highest EVA value is attained at a premium loading of 10%. At a default probability of 3% only very few annuity contracts are sold (for example, 755 at a zero loading and 447 at a loading of 10%).

4.2 Robustness of the Model Assumptions

The calibration of the demand curve and the assumption regarding the frictional costs for bankruptcy are the hardest to calibrate because of a lack of market data. These are important assumptions for our model. In order to confirm the robustness of our analysis, different assumptions are considered for their impact on maximum EVA.

Demand Elasticities

As a first case it is assumed that policyholders are more default risk averse than in the original case. The demand curve $\phi(\pi, d)$ is determined by three parameters, a default sensitivity parameter α, a price sensitivity factor β and a constant γ. The calibration for these parameters was $\alpha = -100$, $\beta = -0.00015$, $\gamma = 10$. Due to the exponential functional form of $\phi(\pi, d)$, changing the default sensitivity parameter α also affects the demand's reaction to price changes. To ensure separate effects, all three parameters are calibrated such that in the first case policyholders' demand reacts to price changes as before and only the reaction to default risk is
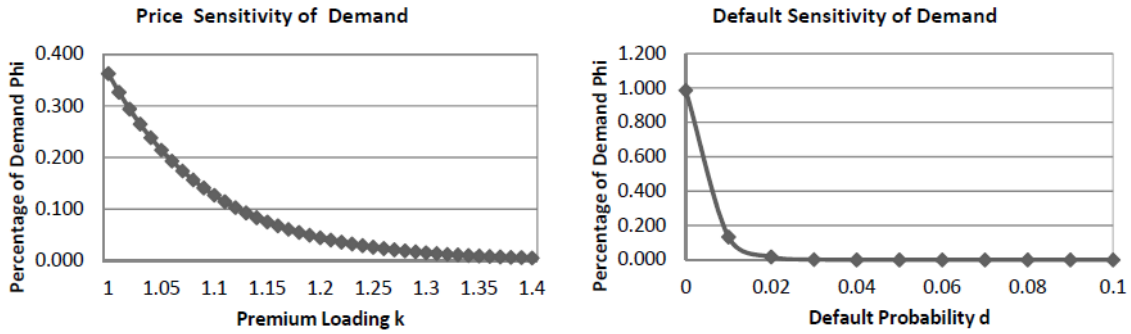
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Figure 8: Sensitivity analysis: Policyholders being more default sensitive. Assumed price and default sensitivity of the demand for annuities.
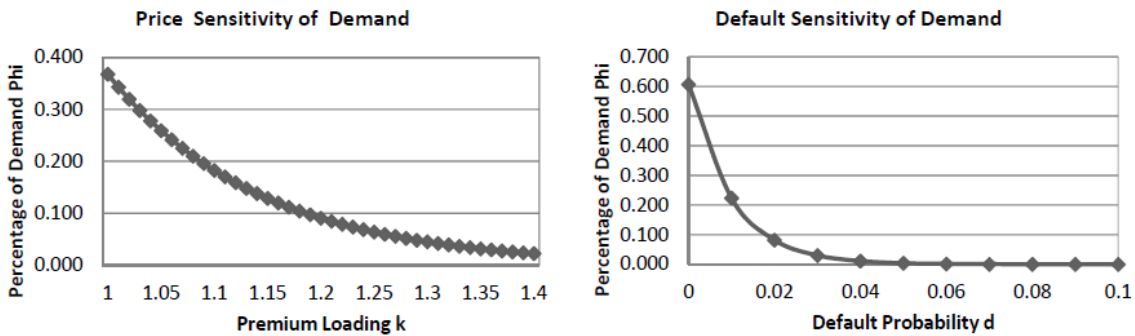

Figure 9: Sensitivity analysis: Policyholders being less price sensitive. Assumed price and default sensitivity of the demand for annuities.

increased. The parameters for this situation are $\alpha = -200$, $\beta = -0.00014997$, $\gamma = 10.48479301$ and the corresponding graphs for the demand function are shown in Figure 8. Annuity demand decreases more sharply with default risk than in the original case. At a default probability of 2% demand is basically zero, whereas in the original case demand was positive up until a default probability of 5%.

The second case assumes that policyholders are less price sensitive. All three parameters are calibrated such that only the demand reaction to price changes is changed (lowered) and the reaction to default is the same as in the original analysis. The corresponding parameters are $\alpha = -100.00625552$, $\beta = -0.00010000$, $\gamma = 6.49997956$. Figure 9 shows that the price sensitivity now runs flatter than in the original case and that there is a positive demand for the highest loading shown here (40%).

Figure 10 shows the EVA results for these alternative demand curves. As in Figure 7, default probabilities $d = 0.1\%, 0.5\%, 1\%, 3\%$ are compared. The graph to the left is the case where policyholders are more default sensitive. As before, EVA shows a hump shape pattern for all four cases $d = 0.1\%, 0.5\%, 1\%, 3\%$ when loadings are increased from 0% to 30%. EVA is negative
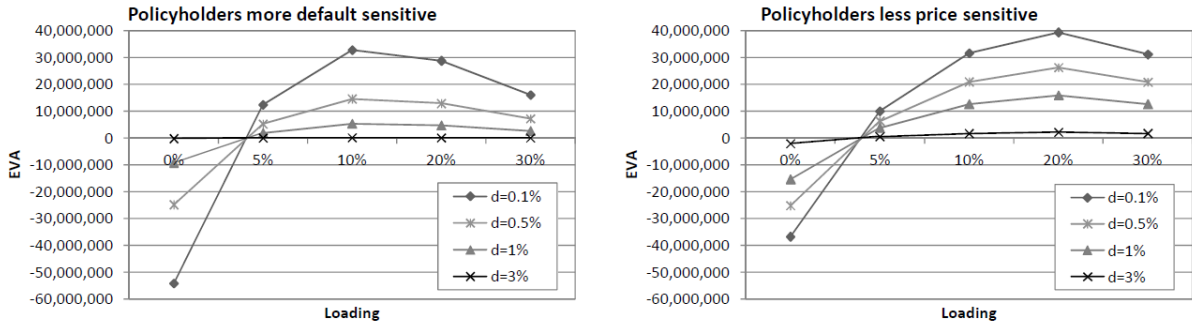
Figure 10: Sensitivity analysis: EVA levels for different assumptions about the demand curve $\phi(\pi, d)$.

for a zero loading and for each default probability the highest EVA value occurs at a loading of 10%. The relative ranking is unchanged. EVA is highest for a default probability of $d = 0.1\%$. There are some differences to the original case. At a target default probability of $d = 0.1\%$, more annuities are sold and EVA is higher for loadings of 5%, 10%, 20% and 30%, and the loss at a zero loading is also higher than in the base case. At a default probability of $d = 0.5\%$, results are very similar to the original case. At $d = 1\%$ and 3%, less annuity business is sold and EVA is lower than originally.

The graph to the right in Figure 10 is the case where policyholders are less price sensitive. As before, the relative ranking of the cases is the same. EVA is highest for a default probability of $d = 0.1\%$. However, the highest EVA value now occurs at a higher loading of 20%. Price sensitivity is an important determining factor for the level of the loading in the premium but not for the optimal solvency level.

The results show that the model's results are robust to assumptions regarding the policyholder's default sensitivity: EVA is highest for higher levels of solvency with lower default probabilities. The optimal loading depends on the policyholders' assumed price sensitivity but this does not impact on the conclusions for the solvency level. Similar conclusions are obtained for the adjusted EVA allowing for the cost of capital.

Frictional Costs for Financial Distress

Frictional costs for bankruptcy f were assumed to be 15% and applied to any losses. Optimal solvency levels for costs of 0% and 30% are consistent with the results for a 15% bankruptcy cost. The premium and demand are not dependent on the level of bankruptcy cost rate.

Figure 11 shows the sensitivity to different frictional costs. The percentage of these costs does
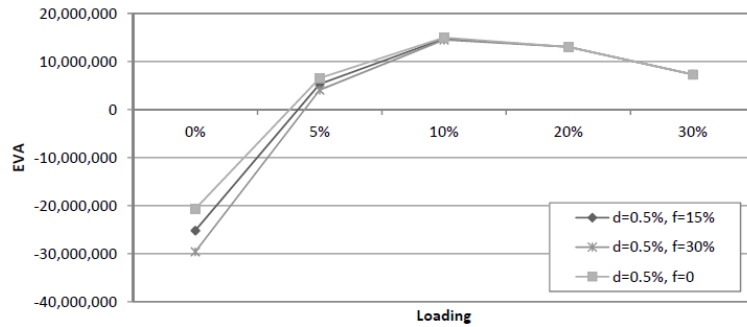
Figure 11: Sensitivity analysis: EVA levels for different frictional costs for bankruptcy f.

not have a significant impact on the results. This is the case since financial distress only occurs with a low probability given the level of solvency assumed.

5   Conclusion

Ensuring that life insurers will be able to deliver on long term life annuity contracts with a very high degree of certainty is important to both regulators and consumers. This is a fundamental requirement to support the development of a viable private sector annuity market. The analysis of the optimal level of solvency, which balances the regulatory trade-off between prudential concerns and consumer attitudes towards purchasing annuities, has shown that higher solvency levels will maximize shareholder wealth and also satisfy consumer preferences for solvency.

The results are based on a realistic calibration of a life insurer model including stochastic mortality, interest rates and consumer preferences. The results were shown to be robust to different levels of default sensitivity. The main impact of price-default elasticity was on the optimal loading in the premium that maximized the shareholder value and not the solvency capital level. Higher levels of solvency than a 99.5% confidence level for a one-year time horizon were found to be optimal for a life insurer based on reasonable assumptions for consumer preferences for solvency.

The paper shows how solvency is critical for a life insurer. A base requirement for policyholders to purchase long term life annuity contracts is a high level of confidence in the life insurer meeting its obligations. To do this, a life insurer needs to hold higher levels of capital than the regulatory requirements under Solvency II. Although policyholders will pay higher premiums, welfare of both shareholders and policyholders can be improved in these circumstances.

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6   Acknowledgements

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