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Postcode-Level Reverse Mortgages: Longevity Risks, House Price Risks, and Welfare Gain

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Postcode-Level Reverse Mortgages: Longevity Risks, House Price Risks, and Welfare Gain

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Abstract

This paper evaluates the Home Equity Access Scheme (HEAS) versus downsizing for older Australians, factoring in elements such as means tests, health expenditures, taxes, and home maintenance. It builds on a utility approach, considering region-specific house prices and longevity risks. Findings reveal that HEAS enhances healthy aging for healthy and mildly disabled retirees more than downsizing. This scheme benefits cash-poor but asset-rich retirees who have lower bequest motives, derive higher satisfaction from spacious homes, and prioritise long-term gratification. However, spatial disparities in housing prices and life expectancy decrease the uptake of HEAS, offering new perspectives on housing decisions among seniors in Australia.

Keywords: Home Equity Release, Reverse Mortgage, Downsizing, Healthy Ageing, Utility Approach.

1 Introduction

Mortality rates vary substantially across Australia. Focusing on areas on the Statistical Area Level 4 (SA4s) level [1], provisional mortality statistics show that life expectancy at birth ranges from a high of 85.6 in Sydney - Baulkham Hills and Hawkesbury for males and 88.4 in Sydney - Ryde for females to 71.7 and 77.1 for both males and females in Northern Territory - Outback(Australian Bureau of Statistics, 2023). The demand for amenities, health and aged care, and social support are of concern to rapidly ageing communities, and places with favourable life expectancy have experienced population inflow. According to Hugo (2013), population growth is concentrated in the urban periphery, high-amenity regions on the east and southwest coasts, resource-rich areas, and regions supported by tourism, while corresponding decline occurs in the wheat-sheep belt, pastoral areas, and former

[1] Statistical area level 4 (SA4) is one of the spatial units defined under the Australian Statistical Geography Standard (ASGS). The ASGS is a hierarchical geographical classification, defined by the Australian Bureau of Statistics (ABS), which is used in the collection and dissemination of official statistics. SA4s are the largest sub-state regions in the Main Structure of the ASGS and are designed for the output of a variety of regional data.

mining towns. Borsellino (2020) specifically investigates the migration patterns of the 65+ population in Australia and finds that new destinations of migrations emerged in regional Victoria and New South Wales over the past 40 years. However, they also notice an overall decline in migration propensity between regions, and the possibility of the 65+ age group moving was approximately half that observed in the 0-64 and total populations, which leads to the further comparison between the benefits of migration to a place with favourable life expectancy and ageing in place benefits of a retiree (Cocco and Lopes, 2020).

When assessing the advantages of the relocation for a retiree, it is crucial to accurately describe the potential changes in their life expectancy after the move. Possible causes of longevity disparities can be separated into two main categories: differences in residents' stocks of health capital (Grossman, 1972) and the environment associated with their current location. Health capital includes genetic endowments, as well as the persistent effects of health behaviours, medical care, and other past experiences that impact current mortality. Potentially mortality-relevant aspects of current locations include the quality and quantity of available medical care, local climate and pollution, and risk factors such as crime and traffic accidents. Chetty et al. (2016) find that the main correlates of area mortality are health capital factors such as smoking, obesity, and exercise, and that correlations with place factors such as health care spending or local environmental conditions are weak. Health Affairs (2014)'s review attributes the largest importance for mortality to health capital — specifically to behaviours (35–50%) and to genetics (20–30%). Among potential place effects, it attributes between 5–20% of the determinants of mortality to the environment and around 10% to medical care. However, neither they nor other existing works isolate the causal impact of place effects. Empirical studies of survivors of Hurricane Katrina find that moving to a place with a one percentage point higher mortality rate is associated with an increase in migrant mortality of approximately one percentage point (Deryugina and Molitor, 2018). Finkelstein et al. (2021) use mortality outcomes of migrants to separately identify the effects of health capital and current location on mortality in the United States and find that one unit increase in average life expectancy is associated with a 0.23 units increase in the causal effect of migration on life expectancy for individuals.

Reverse mortgages can be utilised by elderly individuals who choose to age in their own homes. This allows them to access the built-up equity in their property for daily expenses and essential health and elderly care, safeguarding them from financial instability in their later years. However, the actual market size is still far below the theoretical estimates around three decades after its formal launch in Australia. The Australian government's Pension Loan Scheme [2] had only 710 loans outstanding, and

[2] The focus of this paper centres around the Home Equity Access Scheme (HEAS), formerly known as the Pension Loan Scheme.

many financial institutions offering reverse mortgages in Australia are exiting the market because of the increased scrutiny and regulation coupled with a lack of understanding on the part of borrowers regarding the financial consequences of taking out a reverse mortgage (Australian Securities and Investment Commission, 2019). Furthermore, in a major study in Australia that involved a survey of 7,000 men and women aged over 50 specifically explored issues of bequests, Australian Housing and Urban Research Institute (2019) find that a large portion of homeowners would rather release the equity in their homes by selling and downsizing than taking up reverse mortgages. Therefore, the small size of the reverse mortgage market is not explained by a lack of supply of loanable funds or constraints imposed by lenders; rather, it is low demand from the borrowers (Haurin and Moulton, 2017). The survey conducted by Whait et al. (2019) shows that the reasons dominating negative views of reverse mortgages in Australia are that some of the elderly are concerned with the loss of an asset, mainly for the purpose of passing it onto children through inheritance or the worry that recipients would inherit the reverse mortgage and interest payments. In order to alter the negative view from the borrower's side, the Australian Securities and Investment Commission introduced several enhanced consumer protections, restricting how a reverse mortgage can be enforced, and introducing the "no negative equity guarantee" (Australian Securities and Investment Commission, 2019).

The Home Equity Access Scheme is a voluntary scheme for older Australians, who can use this scheme to supplement retirement income through a non-taxable loan from Services Australia or the Department of Veterans' Affairs (DVA). This paper will consider the roles of HEAS, which is a representative of reverse mortgage products, and selling and downsizing in retirement planning, taking into account the geographic variation of risk factors associated with mortality and house prices. Nakajima and Telyukova (2017) use a lifecycle model to examine the welfare gains of individuals when a reverse mortgage is available in the market. They find that individuals with low income, low wealth, and poor health have a higher take-up rate of the reverse mortgage. Therefore, the model supports the efficacy of the reverse mortgage as a primary funding method in retirement. Nakajima and Telyukova (2017) do not take into account other realistic factors for retirees releasing locked residential assets. Several studies analyse the role of housing wealth in the retirees' portfolio in a lifecycle or utility framework without considering reverse mortgages or other home equity release products. Xu et al. (2023) study the impact of housing wealth and individual preferences on demand for annuities and long-term care insurance (LTCI), using a multi-state lifecycle model that includes longevity risk and health shocks. They show that the presence of home equity has a crowding-out effect on LTCI demand. The time for releasing home equity is assumed to be equivalent to the first time entering the severely disabled state later in life. Several other studies compare different reverse mortgages and other home equity release methods. Cocco and Lopes (2020) focuses on the maintenance requirement of reverse mortgages, which is assumed to be a home equity release strategy. In each period, the borrower chooses the maintenance payment, which decides how much to keep in their home equity. Ho (2021) investigate four different approaches to using housing wealth in Australia, which include the downsizing options. The downsizing amount is fixed and calculated as an actuarial equivalent value of the total withdrawal amount from the reverse mortgage.

In this paper, we consider the effects of changing residence on the mortality outcomes of migrants in the elderly population. A retiree aged 65+ is entirely obtain the life expectancy of the destination after migration. The mortality rate is only partially altered, and the prior reason is the causal effect of the target place on longevity, such as the environment and the medical and aged care facilities. In addition to the geographically various mortality rates, we take into account geographically idiosyncratic house prices and downsizing across areas when analysing the optimal strategies for home equity release. Furthermore, means testing for pension income, various taxes associated with the purchase and sale of real estate, the agent's and attorney's fees, the home maintenance costs required by reverse mortgage contracts, and the cost of home insurance provided by lenders to borrowers of reverse mortgages are incorporated into the model. In addition, we propose a Markov process as a modelling approach to capture health state transitions and fit the data collected by the U.S. Health and Retirement Study. The health states under consideration include healthy, mildly disabled, and severely disabled states, which are defined based on the number of difficulties encountered in performing activities of daily living (Xu et al., 2023). To explore the relationship between home equity liquidation and health shocks, we assume that the borrower of a reverse mortgage product opts to sell their housing assets and transition to a residential aged care facility upon reaching a state of severe disability. Moreover, adjustments are carefully made to the multistate model to accommodate the specific Australian context.

We endeavour to make contributions to the existing body of research on retirement financial planning in the context of home equity release. Our research introduces a perspective by integrating the concept of downsizing as an alternative to reverse mortgages. Our work considers a range of downsizing options across various sizes and locations. This flexibility offers a more realistic and diverse set of choices for retirees. Furthermore, in line with Nakajima and Telyukova (2017)'s insights on the financial and risk profiles of Reverse Mortgage borrowers, our study extends the analysis to understand the potential opportunity costs associated with the decision to retain one's primary residence after retirement. By exploring strategies like relocating to areas with lower house price index levels, we aim to shed light on how retirees can maximise the financial benefits from their home equity. Our study offers a varied perspective on the decisions concerning home equity faced by retirees, delivering information for both policymakers and individuals to efficiently manage retirement resources; while the Home Equity Access Scheme (HEAS) is more advantageous than downsizing at a national level, variations in house prices and life expectancies lead to a stronger preference for downsizing in certain areas.

The subsequent sections of this paper are structured as follows: Section 2 narrates the problem formation, encompassing models and empirical tactics. Section 3 showcases our data and provides a summary of statistics. Section 4 elucidates the principal results concerning the influence of various variables on post-retirement strategies. Lastly, Section 5 concludes and discusses potential directions for future research.

2 Problem formation

This section introduces a non-stochastic model aimed at describing regional mortality rates. By incorporating area-specific mortality rates, it becomes feasible to compare and evaluate reverse mortgage products and downsizing options across diverse geographical areas. To delve deeper into borrowers' behaviour and analyse welfare implications from their perspective, a utility approach is employed, complemented by a multi-state model that illustrates varying levels of health expenditures associated with different health states. This comprehensive approach enables a thorough investigation of borrowers' decision-making processes and facilitates the assessment of potential welfare gains linked to different choices. The integration of the regional mortality rates model with the utility approach provides valuable insights into the dynamics of reverse mortgage products and downsizing options within different states, leading to a more informed evaluation of their respective advantages and disadvantages.

2.1 Mortality model

A continuous-time survival model is proposed to describe the i^{th} individual's mortality rate at a given age x, depending on the accumulated stock of health capital ϑ_i and the place effect γ_j associated with the retiree's current location indexed by j. Chetty et al. (2016) adopts a Gompertz specification where the logarithmic mortality rate $m_{ij}(x)$ that individual i living in location j would experience at age x is

$$\log\left(m_i^{@j}(x,t)\right) = g_t(x) + \gamma_j + \vartheta_i, \tag{1}$$

where $g_t(x)$ is a function on x and t. The individuals of interest in this research are generally separated into two categories: (i) movers who live in an original location o move to j and then stay in the destination location j; (ii) non-movers who stay in location j all the time. Both γ_j and ϑ_i are assumed to be time constant, and the only systematic changes in mortality risk over time originate from ageing and changes in location.

Tab. 1. Average mortality rates and associated life expectancies

Average Mortality Rate	Associated Average Life Expectancy
$\bar{m}^{@j}(x,t) = \exp[g_t(x) + \gamma_j + \bar{\vartheta}_j]$	L_j: average life expectancy in area j.
$\bar{m}(x,t) = \exp[g_t(x) + \bar{\gamma} + \bar{\vartheta}]$	$\bar{L}$: population-weighted average life expectancy

Two different kinds of average mortality rates of non-movers, derived from Equation (1) and associated life expectancies are shown in Table 1. In the table, $\bar{\vartheta}_j$ denotes the average health capital of non-movers in j, $\bar{m}^{@j}(x)$ denotes the mortality rate of an average non-mover in j at age x, $\bar{\vartheta}$ denotes the average health capital over the full population of non-movers. $\bar{\gamma}$ denotes the population-weighted average of the γ_j, and $\bar{m}(x)$ denotes the mortality rate of an average non-mover.

According to Equation (1) which assumes that age, place effects, and health capital are additively separable, the new expression of the logarithmic average mortality rate of a mover from area o to area j is

$$\log\left(\bar{m}^{oj}(x)\right) = g_t(x) + \gamma_j + \bar{\vartheta}_o. \tag{2}$$

To deduce the mortality rate at the SA4 level, we posit that $\bar{\gamma} + \bar{\vartheta}$ from Table 1 equals zero. We then calculate $\gamma_j + \bar{\vartheta}_j$ based on the 10-year average of life expectancy L_j of the j^{th} area. Finkelstein et al. (2021) find that one unit increase in average life expectancy is associated with a 0.23 unit increase in the causal effect of migration on life expectancy for individuals. Therefore, we introduce a calibrated parameter, ρ, confined to the interval $[0, 1]$, to assign the relative weight of γ:

$$\rho = \frac{\gamma}{\gamma + \bar{\vartheta}}, \tag{3}$$

where a higher value of ρ indicates that relocating to another area exerts a more pronounced effect on mortality. We operate under the presumption that ρ is uniformly consistent across different areas once calibrated. Given this premise, we can re-express Equation (2) as:

$$\log\left(\bar{m}^{oj}(x)\right) = g_t(x) + \rho(\gamma_j + \bar{\vartheta}_j) + (1-\rho)(\gamma_o + \bar{\vartheta}_o). \tag{4}$$

2.2 Health expenditures

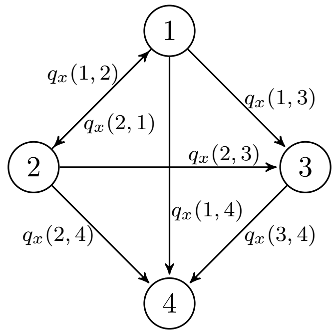


Fig. 1. Diagram illustrating a four-state Markov process for health state transitions. States 1 through 4 represent the health conditions: healthy, mildly disabled, severely disabled, and deceased. The notation $q_x(j,k)(j \in \{1,2,3\}, k \in \{1,2,3,4\})$ signifies the transition intensity from health state j to state k at a given age x.

Following Xu et al. (2023), we model the health status of retirees with a four-state Markov chain: `1` (healthy), `2` (mildly disabled), `3` (severely disabled), and `4` (dead). Being severely disabled or dead is assumed to be the cause of home equity liquidation. The transition intensity between two different states is denoted by $q_x(\cdot,\cdot)$, and the single-period (annual) transition probabilities $\pi_x(\cdot,\cdot)$, which are calculated using the transition intensities as follows:

$$\begin{pmatrix} \pi_x(1,1) & \pi_x(1,2) & \pi_x(1,3) & \pi_x(1,4) \\ \pi_x(2,1) & \pi_x(2,2) & \pi_x(2,3) & \pi_x(2,4) \\ \pi_x(3,1) & \pi_x(3,2) & \pi_x(3,3) & \pi_x(3,4) \\ \pi_x(4,1) & \pi_x(4,2) & \pi_x(4,3) & \pi_x(4,4) \end{pmatrix} = \exp\left[\begin{pmatrix} q_x(1,1) & q_x(1,2) & q_x(1,3) & q_x(1,4) \\ q_x(2,1) & q_x(2,2) & q_x(2,3) & q_x(2,4) \\ 0 & 0 & q_x(3,3) & q_x(3,4) \\ 0 & 0 & 0 & 0 \end{pmatrix}\right], \tag{5}$$

where

$$\begin{aligned} q_x(1,1) &= -\big(q_x(1,2) + q_x(1,3) + q_x(1,4)\big), \\ q_x(2,2) &= -\big(q_x(2,1) + q_x(2,3) + q_x(2,4)\big), \\ q_x(3,3) &= -q_x(3,4). \end{aligned}$$

In line with the approach presented in Fong et al. (2015), this paper estimates health state transitions by employing a generalised linear model (GLM) featuring a logarithmic link function. It is posited that the quantity of transitions at age x adheres to a Poisson distribution characterised by a mean of (μ_x):

$$\mu_x(s_1,s_2) = e_x(s_1)q_x(s_1,s_2) = e_x(s_1)\exp(\sum_{i=0}^{K}\zeta_i(s_1,s_2)x^i), \tag{6}$$

where e_x represents the central exposure to risk for individuals at age x, K signifies the polynomial's degree, and ζ_i refers to the coefficients associated with the polynomial.

2.3 Utility preferences

In this paper, an individual benefits from the consumption of home equity (S_t), which is linear in the size of the property, and nondurable goods (C_t). As in Cocco and Lopes (2020), we assume that the individual's preference is modelled by the Constant Relative Risk Aversion (CRRA) utility function:

$$U_1\left(C_t, S_t\right) = \frac{\left\{\left[\theta^{\frac{1}{\epsilon}} C_t^{\frac{\epsilon-1}{\epsilon}} + (1-\theta)^{\frac{1}{\epsilon}}\left(S_t\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}\right\}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \tag{7}$$

where θ is the expenditure share in non-durable consumption, ϵ is the intra-temporal elasticity of substitution, and σ is the intertemporal elasticity of substitution. This utility function captures an individual's preferences for the consumption of nondurable goods and home equity by incorporating relative risk aversion and the weighting of different consumption.

In addition, the retiree aged x derives utility from bequeathed wealth (W_{τ_x}):

$$U_2\left(W_{\tau_x}\right) = b\frac{\left(W_{\tau_x} + \kappa\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \tag{8}$$

where τ_x denotes the time of death, b measures the intensity of the bequest motive, and κ is the extent to which bequests are a luxury good.

2.4 Home Equity Access Scheme

The Home Equity Access Scheme (HEAS), previously known as the Pension Loans Scheme (PLS), allows older Australians to unlock the illiquid wealth tied up in their house as an income stream to supplement their existing retirement income. A retiree has the option to obtain a loan through a fortnightly payment plan, a lump sum payment, or a combination of both. In this paper, a retiree is assumed to withdraw the entire loan amount available in a single advance payment at the start of each time period t.

The ongoing mortgage interest is calculated as an annual rate against the outstanding balance, BAL_t. The outstanding balance at the beginning of the time period t is defined as follows:

$$\mathrm{BAL}_t = \left(1+\psi_{t-1}\right)\mathrm{BAL}_{t-1} + A_t, \quad t = 1, 2, \ldots, \tag{9}$$

where A_t is the maximum annual payment from the HEAS, which is paid at the beginning of each year, and ψ_t is the annual HEAS interest rate. The interest rate is published as fixed in the Australian Government Gazette (Services Australia, 2023b).

The combined loan and pension payment each fortnight cannot be more than 150% of the maximum pension rate (MPR_t). It is important to note that the maximum pension rate is subject to inflation adjustments in the future. In addition, the outstanding balance must not exceed the maximum loan amount ($\mathrm{MLA}_{x,t}$), which ensures that the loan amount remains within acceptable limits and aligns with the borrower's circumstances. The MLA is based on both the borrower's age x and the value of the home equity used as collateral for the loan:

$$\mathrm{MLA}_{x,t} = \lfloor 10^{-4} H_t \rfloor \times \mathrm{ACP}_x, \tag{10}$$

where the details of the age component ACP_x will be provided in Appendix B. It is assumed that the age component remains constant over time, ensuring its stability throughout the analysis. Additionally, the house price (H_t) is influenced by the size of the home equity (S_t) and changes in the house price index. The relationship between the house price index, inflation rates, and other relevant risk factors will be discussed in detail in Section 3.

Based on the specified restrictions, the maximum annual payment that an individual can withdraw is determined as follows:

$$A_t = \Big\{ \min \Big[150\%\mathrm{MPR}_t - Y_t, \mathrm{MLA}_{x,t} - (1 + \psi_{t-1})\, \mathrm{BAL}_{t-1} \Big] \Big\}^+, \tag{11}$$

where Y_t is the annual pension income of an individual. The retiree is assumed to rely solely on their pension income and payments from the HEAS, apart from interest income.

2.5 Strategies for equity release in the Australian context

In Australia, more than 20% of individuals aged 50 to 75 possess dwellings with three or more bedrooms, suggesting that their owned dwellings are frequently larger than needed (Australian Institute of Health and Welfare, 2021). This overconsumption of housing may have negative implications for later-life expenditures related to house-related expenditures, prompting the belief that some retirees could benefit from home reversion behaviour. Reduction in property maintenance costs is another possible benefit of downsizing, which costs 1% to 4% of home value per year in Australia.

Downsizing does not guarantee an increase in the welfare gain for all retirees. Purchasing property is subject to stamp duty, which is a transaction tax of home equities. Such taxes are economically inefficient, which limits mobility and inhibits downsizing. However, diverse approaches have been taken to support pensioners and elderly individuals in the Australian property market, and each state has its specific eligibility criteria and thresholds for these exemptions and concessions. In Victoria, pensioners are eligible for a one-time stamp duty exemption or concession on properties valued up to AUD 750,000, provided the property serves as their principal place of residence (State Revenue Office Victoria, 2021). In the Australian Capital Territory (ACT), pensioners who downsize receive a reduction on the conveyance duty payable on the purchase of residential homes or residential vacant land. No duty is payable for properties valued at AUD 440,000 or less, while concessions are available for properties below AUD 570,000 (ACT Revenue Office, 2023). In Tasmania, pensioners who sell their home and downsize by purchasing another home in the state may qualify for a 50% concession on the transfer duty of their new home, as long as its dutiable value is AUD 400,000 or less and lower than the former home's dutiable value (State Revenue Office Tasmania, 2022). In the Northern Territory (NT), pensioners may be eligible for a reduction of up to AUD 10,000 in stamp duty when purchasing a home or land for building a home. To qualify for this concession, the home must have a dutiable value of AUD 750,000 or less, or AUD 385,000 or less when purchasing land (Northern Territory Government, 2011).

Furthermore, the optimal timing of equity release is an important and complicated issue, which has remained largely unexplored, especially in Australia (Lamarra et al., 2022). The influence of the means-tested Age Pension on taking strategies related to home equities should be carefully considered. The government pension provides income support to eligible seniors based on residency requirements, the outcome of an income and assets test, and other personal information. In the assets test, the value of an individual's assets, which may include property, savings, and investments, is evaluated to determine their eligibility for the Age Pension differently for homeowners and non-homeowners. Currently, the asset value limit to get a full pension is AUD 280,000 for a single homeowner and is AUD 504,500 for a single non-homeowner (Services Australia, 2021). Alternatively, the income test considers the entirety of an individual's income, ranging from employment and investment earnings to rental income, in determining their pension eligibility. Both tests are employed, with the individual subject to the lower rate of pension. When the sale proceeds are utilised to secure a new principal home, the proceeds will be deemed at a reduced rate (0.25%) in the income test. However, any additional sale proceeds held in a financial asset will be subject to the standard deeming rates (Services Australia, 2023a). The details of asset and income tests are listed in Section 3.

As previously discussed, homeowners have the option to utilise reverse mortgages to extract equity from their properties. The resulting property following the arrangement of a reverse mortgage is still considered home equity. The utilisation of reverse mortgages can impact an individual's eligibility for

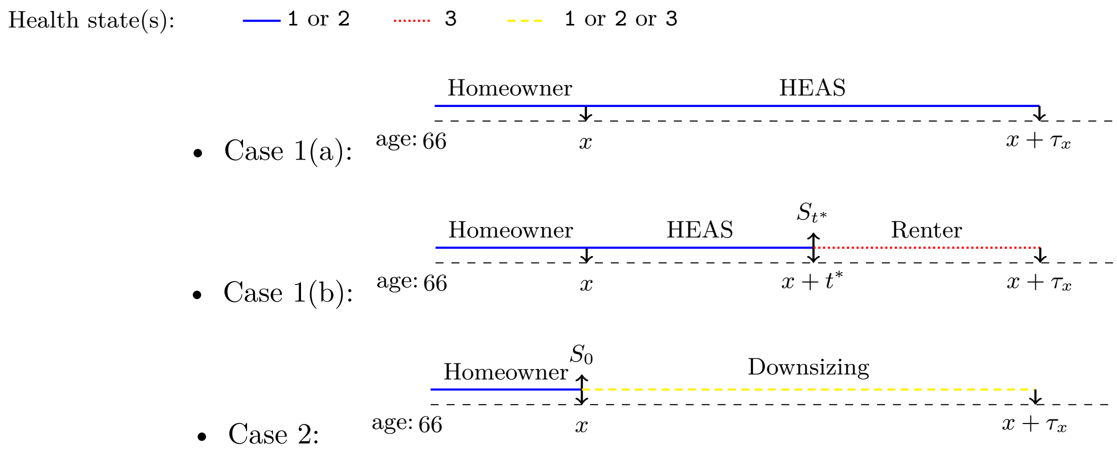


Fig. 2. Strategies for Home Equity Release and Health Status. Case 1(a): One starts as a homeowner. At some point, one starts using HEAS. One does not experience a decline in health to State 3 before death. Case 1(b): Similar to Case 1(a), but in this scenario, one experiences a decline in health to State 3 before death. If one's health worsens to State 3 at time t^*, one shifts to a rented property. Case 2: Here, one starts as a homeowner. Then, one downsizes - moving to a new property to free up some of the home's value. This strategy is assumed to be followed regardless of changes in health.

government benefits, as it increases the assessable value of their assets, while they remain identified as homeowners. The downsizing option is an alternative to reverse mortgages, and sale proceeds generated from downsizing are considered part of an individual's asset and income tests, which can reduce future cash flows from the Age Pension. Both downsizing and using reverse mortgages before retirement will impact an individual's total assets at the start of retirement; therefore, one assumption is made that one will choose to be an RM borrower after retirement.

Residence-related strategies are available for selection by individuals in different health states. The varying expected utility of an investor associated with the implementation of these strategies at distinct time points underscores the criticality of optimal decision-making by the investor. Figure 2 shows different strategies for using home equity based on health and age. Each line on the charts represents transitions of health states over the lifetime, from the age of retirement at 66 until death (represented by τ_x). Different styles of lines represent different combinations of health states. The expected utility for Case 1(b) and Case 2 will be compared. Details for describing the mechanism of the relationship between strategies and utility will be discussed later in this section.

For the sake of clarity, the logic behind notations regarding property variables is introduced before the exploration of the driving forces behind these behaviours and their potential impacts on the product holder's utility. The variables of interest in this section are generally separated into two categories: (i) the variables related to the old residential property in the original location o and to the residential property/rented space in the new location j; (ii) variables related to the residential property belonging to a non-mover in location j. For example, a mover can sell the old residential property at size $S^{@o}$, and buy/rent a new place at size S^{oj}. However, for a non-mover, the size of the living space is always $S^{@j}$. The extent of the change in the size of the house is computed according to the following equation:

$$\Delta S_t = S_t^{oj} - S_t^{@o} = \frac{H_t^{oj}}{\mathrm{HPI}_{j,t}} - \frac{H_t^{@o}}{\mathrm{HPI}_{o,t}}, \tag{12}$$

where HPI_t are house price indices across different areas at time t, illustrating retirees can move and unlock wealth without reducing home consumption too much. If the retiree chooses to downsize but the new residential property is still located in the original area, $\mathrm{HPI}_{o,t} = \mathrm{HPI}_{j,t}$.

2.5.1 HEAS

In this section, we focus on individuals who are already HEAS users, which describe options related to Case 1(b) in Figure 2. When entering into the contract, individuals are typically in one of the two initial health states: healthy or mildly disabled, and they are allowed to maintain their homeownership while accessing their home equity. However, as their health deteriorates, particularly when transitioning to the severely disabled or dead state, the contract terminates. In the case of severe disability, the individual is likely to become a renter, often moving into specialised facilities such as nursing homes to receive appropriate care and support. This transition highlights the importance of considering health state transitions in the context of HEAS users to better understand and manage the risks associated with the contract.

The remaining utility for a female retiree who is a HEAS user at time t is expressed as:

$$\begin{aligned} V_{RM}^{HO}(t, s_{x+t}) = \max_{C_t} \Big\{ & U_1(C_t, S_t^{@j}) + \beta \mathbb{E}\Big[\sum_{s_{x+t+1}\in\{1,2\}} \pi_{x+t}^{@j}(s_{x+t}, s_{x+t+1}) V_{RM}^{HO}(t+1, s_{x+t+1}) + \\ & \pi_{x+t}^{@j}(s_{x+t}, 3) V_{RM}^{Renter}(t+1) + \pi_{x+t}^{@j}(s_{x+t}, 4) U_2\left(W_{t+1} + ((1-\lambda_1)H_{t+1}^{@j} - BAL_{t+1})^+\right)\Big]\Big\}, \end{aligned} \tag{13}$$

where the subscript RM of V_{RM}^{HO} indicates that the individual is a borrower of reverse mortgages, and the superscript HO of V_{RM}^{HO} signifies that the borrower remains a homeowner. The transition rate π_{x+t}^{oj} is annual transition probability for a $(x+t)$-year-old mover from area o to area j. If this individual opts to move to a new place in the same area, $\pi_{x+t}^{oj} = \pi_{x+t}^{@j}$.

The equation describing the evolution of cash-on-hand is:

$$W_{t+1} = \left(W_t + A_t^{@j} - C_t - HC_t(s_{x+t}) - HE_t^{@j}\right)(1+r_t) + Y_t, \tag{14}$$

where Y_t represents constant permanent income, $HC_t(\cdot)$ signifies the dollar value of health care expenditure in period t, which varies with time and the current health state, and $HE_t^{@j}$ denotes the dollar value of the housing expenses incurred in period t, including maintenance expenses and insurance costs.

The expected utility for a HEAS user forced to become a renter at time $t \in [0, T_x]$ is expressed as:

$$V_{RM}^{Renter}(t) = \max_{O_t}\Big\{U_1(C_t, S_t^{oj}) + \beta\mathbb{E}\Big[\pi_{x+t}^{oj}(3,3)V_{RM}^{Renter}(t+1) + \pi_{x+t}^{oj}(3,4)U_2\left(W_{t+1}\right)\Big]\Big\}, \tag{15}$$

where O_t is also an optimal strategy set. When t is the first instance of an individual entering health state 3, $O_t = \{C_t, S_t^{oj}\}$. After this period, $O_t = \{C_t\}$.

The equation for the evolution of cash-on-hand in the period of selling a reverse mortgaged house is:

$$W_{t+1} = \Big\{W_t - C_t - HC_t(3) + \Big[(1-\lambda_1)H_{t+1}^{@o} - BAL_{t+1}\Big]^+\Big\}\left(1+r_t\right) + Y_t, \tag{16}$$

and the equation for the evolution of cash-on-hand in the subsequent periods is:

$$W_{t+1} = \Big(W_t - C_t - HC_t(3) - RC_t^{oj}\Big)\left(1+r_t\right) + Y_t, \tag{17}$$

where $RC_t^{@j}$ denotes the rental costs, determined by the size of the new residence $S_t^{@j}$ and the related house price index $\mathrm{HPI}_{j,t}$.

In order to maintain a minimum standard of living, the consumption level must not fall below a certain threshold denoted by C_f. In cases where an individual's budget cannot support the minimum consumption level due to various expenditures, it is assumed that the government is responsible for providing subsidies to increase consumption to this threshold. The liquid wealth at the beginning of the next period W_{t+1} is set to zero. This assumption holds true for both renters and homeowners who are (were) HEAS users. For simplicity of computation, the transitions to residential aged care facilities will be restricted to the same area at the postcode level.

2.5.2 Another strategy for equity release

This section further investigates the equity release options in the Australian context. It introduces the concept of expected utility, denoted as V^{dHO}, for homeowners who choose to downsize, which is depicted as Case 2 in Figure 2. The associated changes in cash-on-hand are also considered. By comparing V_{RM}^{HO}, which represents the expected utility of homeowners using HEAS at time t, with the newly introduced V^{dHO}, an analysis is conducted to identify the most advantageous equity release strategies for retirees. These strategies are examined in relation to various health states, different time points, and diverse geographic areas.

The expected utility for a homeowner downsizing and buying a new property $V^{dHO}(t)$ is:

$$\begin{aligned} V^{dHO}(t, s_{x+t}) = & \max_{O_t}\Big\{U_1(C_t, S_t^{oj}) + \beta\mathbb{E}\Big[\sum_{s_{x+t+1}\in\{1,2,3\}} \pi_{x+t}^{oj}(s_{x+t}, s_{x+t+1})V^{dHO}(t+1, s_{x+t+1}) + \\ & \pi_{x+t}^{oj}(s_{x+t}, 4) \cdot U_2\left(W_{t+1} + (1-\lambda_1)H_{t+1}^{oj}\right)\Big]\Big\}, \end{aligned} \tag{18}$$

where the superscript of $V^{dHO}(t)$ indicates the downsized homeowner.

The corresponding evolution of cash-on-hand in the period of downsizing is

$$W_{t+1} = \left[W_t - C_t - HC_t - HE_t^{@o} + (1-\lambda_1)\, H_t^{@o}\right](1+r_t) - (1+\lambda_2)\, H_t^{oj} + Y_t. \tag{19}$$

Following this period, the movement law of cash-on-hand is expressed as

$$W_{t+1} = \left(W_t - C_t - HC_t - HE_t^{oj}\right)(1+r_t) + Y_t. \tag{20}$$

In order to maximise the expected lifetime utility, individuals optimise their consumption of goods and the size of their house. The optimisation problem, which involves maximising the utility represented by V_{RM}^{HO} and V^{dHO}, is addressed and explained in Appendix A.

3 Data

3.1 Exploring the macroeconomic scenarios

The variables delineated in Table 2 are employed to simulate national-level economic dynamics in future scenarios. Contrasting with Project 1, the Cash Rate Target and Discounted Variable Mortgage Rate are deliberately excluded from the Principal Component Analysis (PCA) procedure. The rationale behind this decision is that these variables warrant further investigation in the subsequent phases of this research, and omitting some principal components would result in information loss and impede the recovery of the original variables. PCA is employed to process the following macroeconomic variables: Exchange Rate, Gross Domestic Product, Retail Sales, Private Dwelling Approvals, and Australian Securities Exchange. As a result, five principal components are derived from this analysis. By integrating the National House Price Index, Cash Rate Target, and Consumer Price Index with the previously mentioned principal components, a comprehensive set of national-level risk factors is established. These risk factors are subsequently incorporated into a VAR model with lag 2 according to AIC and BIC criteria. Notably, the past two periods of the third principal component (PC3) do not exhibit a significant influence on any of the other risk factors at a 5% level of significance, leading to its exclusion from the model. A summary of the VAR(2) model is presented in Table 3, wherein the statistically significant variables are denoted in bold font for ease of identification.

Tab. 2. Description of Variables

Variable	Description	Pre-processing Method	Denotation
HPI_t	National House Price Index	$\log(\mathrm{HPI}_t) - \log(\mathrm{HPI}_{t-1})$	h_t
I_t	Cash Rate Target	$\log(\mathrm{I}_t) - \log(\mathrm{I}_{t-1})$	i_t
CPI_t	Consumer Price Index	$\log(\mathrm{CPI}_t) - \log(\mathrm{CPI}_{t-1})$	cpi_t
EXR_t	Exchange Rate	$\log(\mathrm{EXR}_t) - \log(\mathrm{EXR}_{t-1})$	exr_t
GDP_t	Gross Domestic Product	$\log(\mathrm{GDP}_t) - \log(\mathrm{GDP}_{t-1})$	gdp_t
RS_t	Retail Sales	$\log(\mathrm{RS}_t) + \log(\mathrm{RS}_{t-2}) - 2\log(\mathrm{RS}_{t-1})$	rs_t
PDA_t	Private Dwelling Approvals	$\log(\mathrm{PDA}_t) - \log(\mathrm{PDA}_{t-1})$	pda_t
AXS_t	Australian Securities Exchange	$\log(\mathrm{AXS}_t) - \log(\mathrm{AXS}_{t-1})$	axs_t

Tab. 3. Summary of VAR(2) Model at the National Level

	h	ir	cpi	PC1	PC2	PC4	PC5
Constant	0.009	−0.167**	0.002	−0.576	−0.678**	−0.284	0.177
L1.h	0.971***	−1.976	0.039	−2.419*	−0.692	1.382*	2.179
L1.ir	−0.003	0.666***	−0.004	−0.398	−2.094**	−0.480	0.411
L1.cpi	0.320	0.817	0.401**	0.622	0.867**	2.959	−1.964
L1.PC1	0.001	−0.016	−0.001	0.208	0.218	0.188*	0.128
L1.PC2	−0.006**	−0.066*	−0.002*	0.495**	−0.845***	−0.130	−0.270***
L1.PC4	−0.012***	0.012	0.000	−0.091	−0.378*	0.181	−0.070
L1.PC5	−0.004	0.004	0.000	0.482	−0.274	0.111	−0.052
L2.h	−0.429***	3.614*	0.034	0.862	−2.924	−2.994	1.344
L2.ir	0.003	−0.420**	0.012**	−1.147	1.283	0.602	−0.532
L2.cpi	−0.726	1.258	0.146	0.701	0.520	−2.589	−1.750
L2.PC1	0.001	0.037	0.002**	−0.368**	0.306**	0.120	−0.038
L2.PC2	−0.001	0.065	−0.001	0.131	−0.657***	−0.021	−0.025
L2.PC4	0.006	−0.075	0.000	−0.348	−0.303	0.274*	−0.087
L2.PC5	0.011***	−0.044	0.000	−0.486*	0.184	0.028	−0.111

This study consciously excludes black swan events from the analysis. Significant, sudden shifts, commonly referred to as "jumps", in the residuals of the VAR model are systematically removed. This elimination is based on the respective z-scores of these fluctuations. This technique is applied iteratively until the residuals of the variables within the VAR model demonstrate compliance with a Gaussian distribution, thereby reinforcing the robustness of the model. Figure 3 presents the forecasts for three key indicators over the subsequent 35-year period: the relative house price index, cash rate targets, and the relative consumer price index. For clarification, the relative indices are calculated by dividing the predicted index value by its current value. This normalization allows a

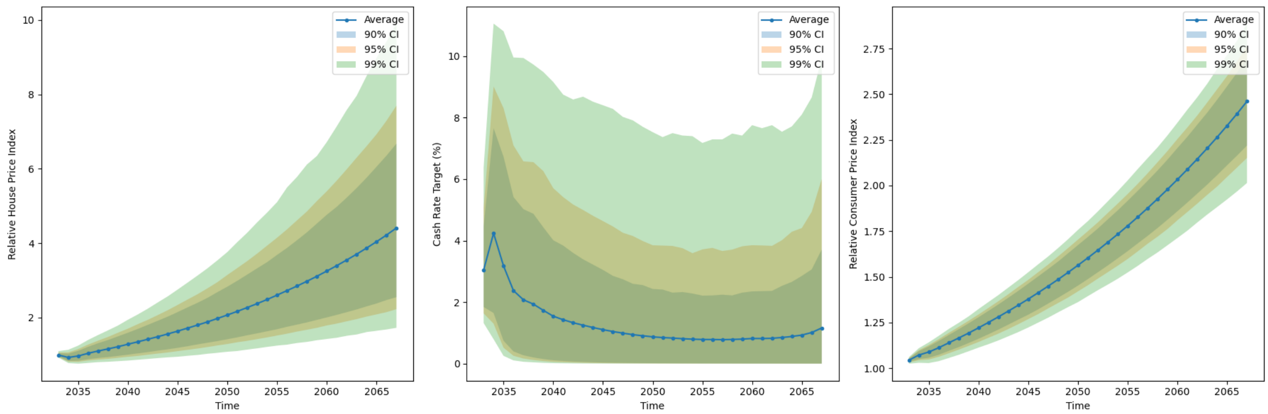


Fig. 3. Projected values of the relative house price index, cash rate targets, and relative consumer price index for the next 35 years. The relative indices are calculated as the ratio of the forecasted index to the current index.

better interpretation of how these indicators over time evolve over time. Moreover, Figure 4 illustrates the anticipated national trends of the outstanding balance and annual payments for a typical user of HEAS starting at the age of 66. Notably, the results indicate that in the most unfavourable scenarios, older HEAS users may reach a point where they are unable to withdraw any additional funds later in life. In these circumstances, forecasted house prices are depressed, and constraints on the outstanding balance restrict the maximum amount that can be withdrawn from HEAS.

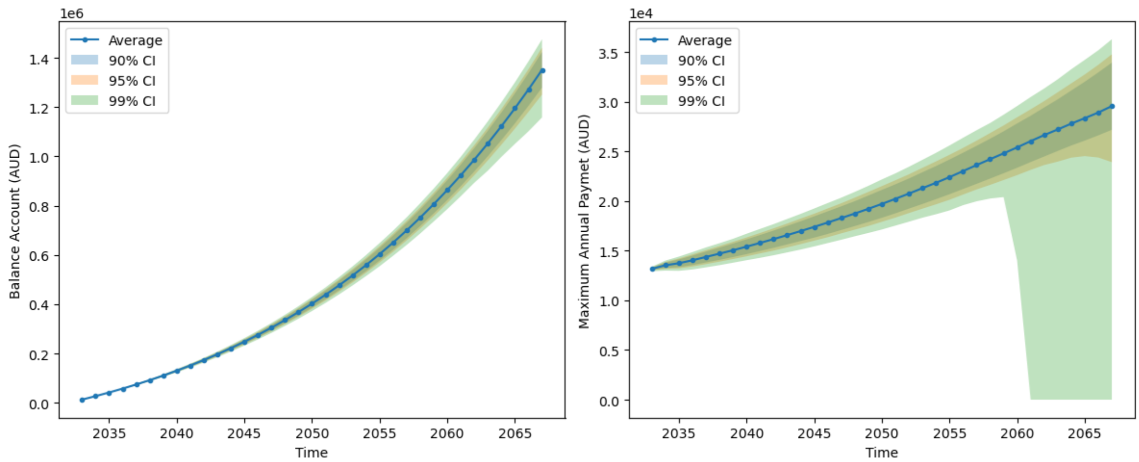


Fig. 4. The predicted outstanding balance and the maximum annual withdrawal for a HEAS user starting at age 66.

We consider the region-specific house price risk based on a hierarchical model. The logarithmic house price growth rate of the i^{th} suburb is modelled as a log-linear function:

$$h_{i,t} = [1 \quad h_t \quad \boldsymbol{f}_t^\top]\boldsymbol{\phi}_i, \tag{21}$$

where h represents the national average of h_i. This model includes the national average of house price growth rates, exogenous risk factors $\boldsymbol{f}$, and coefficients specific to each suburb $\boldsymbol{\phi}_i$. The VAR model of the national average house price growth h and other risk factors $\boldsymbol{f}$ are shown in Table 3.

3.2 Mortality rates

This section investigates the trends and patterns of mortality rates in Australia using life tables provided by the Australian Bureau of Statistics (ABS) from 2009 to 2021. The data is accessible through the ABS website and includes national life tables and SA4-level life expectancies in a two-year period. The reference period for the latest release is from 2019 to 2021 (Australian Bureau of Statistics, 2023). The life tables are period life tables, which are based on death rates for a short period of time when mortality rates have remained largely the same. The mortality rates are calculated based on the occurrence of deaths in the two-year period and the estimated resident population at the midpoint of that period. Therefore, unless it is assumed that changes in mortality will not occur in the future, life expectancy calculated from the mortality rates on the same life table is incomplete for calculating the remaining life expectancy at each age group. In this paper, the last year of the period is used to indicate this phase. For instance, 2021 refers to the period from 2019 to 2021. In addition, nearly 40% borrowers of Home Equity Conversion Mortgage (HECM), which is the most common reverse mortgage program provided by the government in the United States, are specifically single females between 2009 and 2019. Therefore, the analysis will temporarily concentrate on the data of females in this paper.

The national mortality rate model is fitted according to the national mortality rates for females aged 65 to 90 from the years 2011 to 2021. According to the algorithm proposed by Tenachi et al. (2023), which provides a Python package `PhySo` for deep symbolic regression on mortality data (Tenachi, 2023), one possible closed-form expression for $g_t(x)$ in Equation (1) is:

$$g_t(x) = -0.332x + 0.332x\cos^2\left[\exp\left(2.995 - \frac{x}{t}\right)\right], \tag{22}$$

where $66 \leq x \leq 105$. This model is consistent with the Gompertz–Makeham law. The compelling aspect of this model's suitability for this study lies in its not only illustrating the enhancement of longevity over time but also capturing the variations in mortality rates among different cohorts.

This paper simulates health state transitions using data from previous studies on the annual transition between states at different ages. The data and model selection results are shown in Appendix C. The simulation results for the national level are shown in Figure 5. To connect the

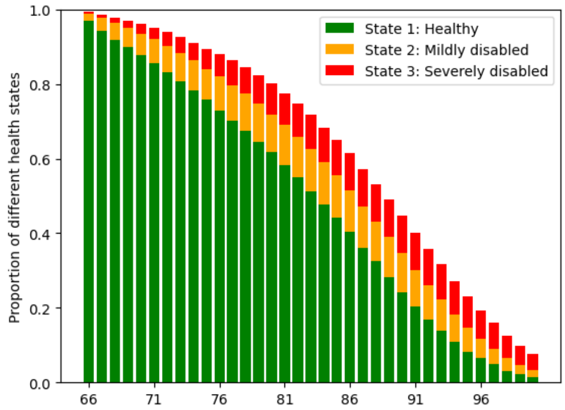


Fig. 5. The predicted proportions of different health states of females who are 65 years old in 2022, based on a simulation of 100,000 trajectories using a multi-state model. The proportions represent the proportion of people in each state among the total number of people alive at the beginning of the simulation.

health transition model and Equation (4), a parameter $\lambda_{x,t}$ is employed to make adjustments to the transition matrix in each area at the SA4 level. The original annual transition matrix is represented by Π_x. The modified annual transition matrix, $\Pi^{oj}_{x,t}$, is

$$\Pi^{oj}_{x,t} = \varsigma^{oj}_{x,t}[\Pi_x - \mathrm{diag}(\Pi_x)] + D^{oj}_{x,t}, \quad \varsigma^{oj}_{x,t} \in [0, 1], \tag{23}$$

where $\varsigma^{oj}_{x,t}$ is the input scaling factor, and D^{oj} is a diagonal matrix whose diagonal elements are

$$D^{oj}_{x,t}(i, i) = 1 - \sum_{j \neq i} \varsigma^{oj}_{x,t} \Pi_x(i, j). \tag{24}$$

The underlying premise in employing adjustments is that the probability of transitions among various states will contract by the same proportion, $\varsigma^{oj}_{x,t}$. A higher value of $\varsigma^{oj}_{x,t}$ indicates a greater life expectancy and a reduced likelihood of transitioning out of the healthy states (Harris and Sharma, 2018). The initial state is denoted as 1 (healthy), with x ranging from 66 to 100, and t spanning from 2023 to 2057. During each iteration, the optimal $\varsigma^{oj}_{x,t}$ is determined by minimising the discrepancy between the predicted mortality rate derived from Equation (4) and the mortality prediction using the transition matrix. In Figure 6, we present the distribution of average life expectancy across various areas. For analytical purposes, these areas are categorised into five groups based on their respective life expectancy ranges: $[72.5 - 77.5), [77.5 - 80), [80 - 82.5), [82.5 - 85)$, and $[85 - 87.5)$. This categorisation facilitates the creation of five unique transition models, each corresponding to a specific life expectancy bracket. The structure of Equation (4) implies that all life expectancies, taking into account migration and derived from mortality rates, should fall within one of the specified five brackets. After assessing future mortality rates, one of these models will be chosen and integrated into the corresponding expected utility.

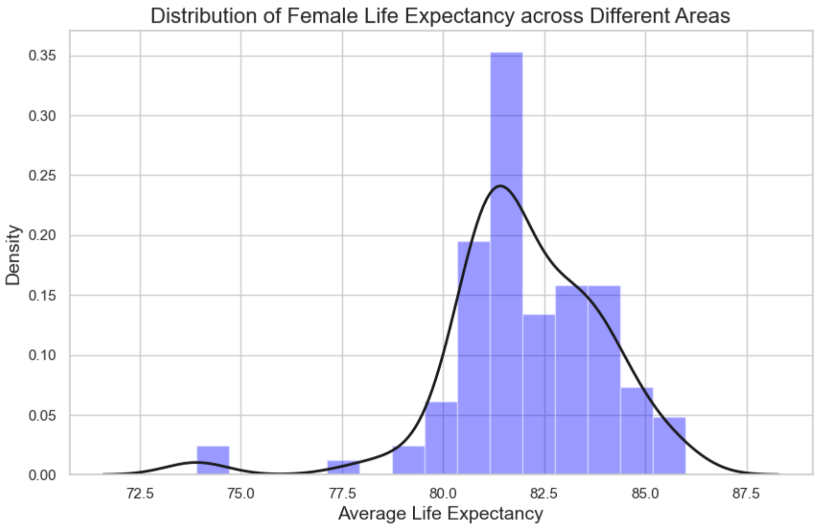


Fig. 6. Distribution of average life expectancy in various areas, categorised into five groups based on life expectancy ranges [72.5 – 77.5), [77.5 – 80), [80 – 82.5), [82.5 – 85), and [85 – 87.5). These categories aid in creating distinct transition models for each range.

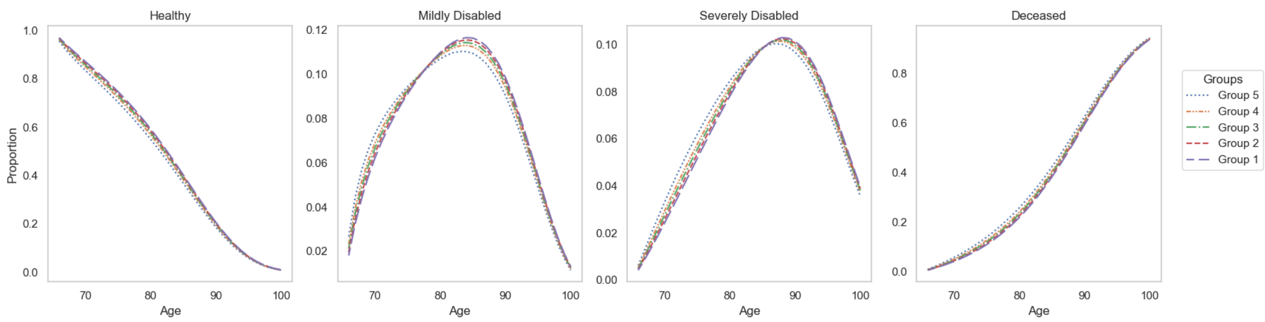


Fig. 7. Health state projections over 35 years for five groups with varying mortality rates, showing a higher proportion of healthy individuals and delayed disability onset with lower mortality. Further analysis of curve areas reveals an obvious change in healthy life expectancy across groups, while time in two disabled states remains stable.

Figure 7 displays the calibrated health state status across five groups for the next 35 years, assuming an initial healthy state for all individuals in the simulation. From Group 1 to Group 5, there is a consistent reduction in mortality rates. This figure demonstrates that a lower mortality rate correlates with a higher proportion of healthy individuals across all ages. Furthermore, the curve shapes for mildly and severely disabled states suggest that a decrease in mortality rates delays the onset of disability. For a more in-depth analysis, we quantify the expected duration in each state using the area under the respective curves. Our findings reveal a decrease in the expected healthy lifespan from 15.9 years in Group 5 to 15.0 years in Group 1. In contrast, the expected durations in mildly or severely disabled states remain relatively stable, approximately 2.7 years and 2.3 years, respectively. These observations indicate that variations in mortality rates predominantly affect the duration of the healthy stage rather than the length of time spent in disabled states by utilising this calibration method.

3.3 Age pension, investment returns and taxation

Table 4 presents the Age Pension means test parameters. All age pensioners are eligible for a non-indexed Energy supplement, which is provided irrespective of income and assets (Services Australia, 2023c). The Pension Supplement may be subject to reduction based on a means test, yet all age pensioners receive a minimum Supplement. The maximum Age Pension rates undergo indexing biannually, in March and September. Both the basic rate and the Pension Supplement of Age Pension income are subject to taxation, while the Energy supplement remains tax-exempt.

Tab. 4. Age Pension means test parameters (2023)

Parameter	Single
Maximum basic rate	\$971.50
Maximum pension supplement	\$78.40
Energy supplement	\$14.10
Asset value limit	\$280,000

The Age Pension calculation is subject to income and assets tests, with the more stringent test being applied. These tests are conducted on a fortnightly basis. The assets test encompasses the net market value of superannuation, financial assets, home contents, and vehicles while excluding the principal home. For every AUD 250 of assets exceeding the limit, the Age Pension is reduced by 75 cents per fortnight. The asset value limits are adjusted annually in July, in accordance with changes in the Consumer Price Index (CPI). Assessable income encompasses deemed income from superannuation and financial investments. It is assumed that HEAS participants are retired and possess no additional income. HEAS income is tax-exempt and excluded from the income test. The Age Pension is reduced by 50 cents for each dollar of income surpassing the income-free threshold. Both the deeming thresholds and income-free thresholds undergo adjustments in July, in line with CPI movements. Deeming rates are presumed to remain constant.

At the age of 66, superannuation is converted into an account-based pension, adhering to minimum drawdown rates (excluding temporary reductions in response to COVID-19). The account-based pension is invested equally in equities and government bonds (Australia), with rebalancing occurring fortnightly. From age 60, superannuation income and investment earnings within superannuation are not subject to taxation. Superannuation fees, based on a typically allocated pension, are charged fortnightly and increase annually in line with the Consumer Price Index. However, in this study, all superannuation funds are assumed to be extracted at the beginning of retirement as financial assets outside superannuation.

Financial assets outside superannuation are presumed to yield the same returns and fees (tax-deductible) as superannuation, though they are taxable at the individual's marginal rate. Investment returns, in the form of dividends and coupons, are reinvested and taxed in the financial year they are earned. Tax brackets are projected to increase annually with CPI. The senior's and pensioner's tax offset (SAPTO) is factored in, with rebate income equating to taxable income.

The retirement age in Australia is progressively rising for both genders. As of September 2021, the eligibility age for Age Pension is 66 years and 6 months, with subsequent increases of six months every two years until the age reaches 67 in July 2023. This study assumes a retirement age of 66 years for females, given that the mortality data utilised in this study predate 2021, and the research sample consists of female homeowners who are not borrowers of reverse mortgage contracts.

3.4 Calibrated parameters

Tab. 5. Calibrated Parameters: Baseline Scenario

Description	Parameter	Value
Non-durable cons exp. share	θ	0.7
Intratemporal elasticity of subs.	ϵ	1.25
Intertemporal elasticity of subs.	σ	0.333
Bequest motive curvature	κ	6,000
Bequest motive intensity	b	12
Discount factor	β	0.97
Housing expenditure rate	HE_t/H_t	0.01
Rental yield rate	RC_t/H_t	0.03
Initial wealth	W_0	AUD40,000
Average living space	S_0	$193m^2$
Initial house price	H_0	AUD881,200
Nominal HEAS rate	ψ_t	3.95%
Transaction cost rate: seller	λ_1	1%
Transaction cost rate: buyer	λ_2	3%
Annual cost: healthy state	$HC(1)$	AUD5,000
Annual cost: mildly disabled state	$HC(2)$	AUD30,000
Annual cost: severely disabled state	$HC(3)$	AUD60,000

The utility parameters employed in this study are in line with the ones utilised in Cocco and Lopes (2020), wherein the monetary and house size units have been adjusted to align with the Australian context. The estimation of health expenditures follows the results in Harris and Sharma (2018), incorporating the use of a multistate model to simulate proportions of individuals in various health states. Other parameters utilised in the analysis are sourced from the Australian Bureau of Statistics.

4 Results

In this section, we present the data-driven results of exploring HEAS and the downsizing option as ways to release home equity in the Australian context separately. Following this, a comparison of these two options is conducted across various financial scenarios. In addition, we take into consideration the geographically idiosyncratic house price and longevity risks, and analyse demand at the postcode level, using the Greater Sydney Area as a reference to evaluate and contrast the two options in the last section.

4.1 HEAS enhances healthy ageing

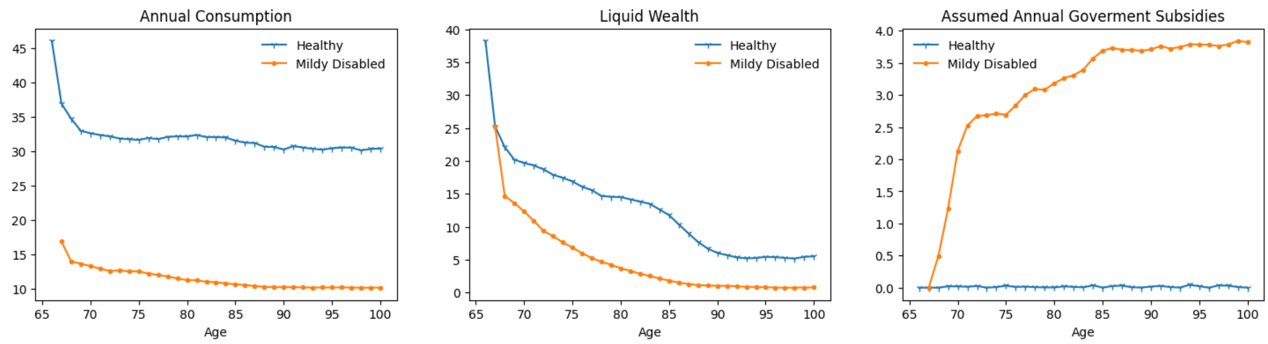


Fig. 8. Simulated average optimal consumption of goods C_t, liquid wealth W_t, and assumed government subsidies for retirees who became HEAS users at age 66. The legend represents two different health states: healthy and mildly disabled

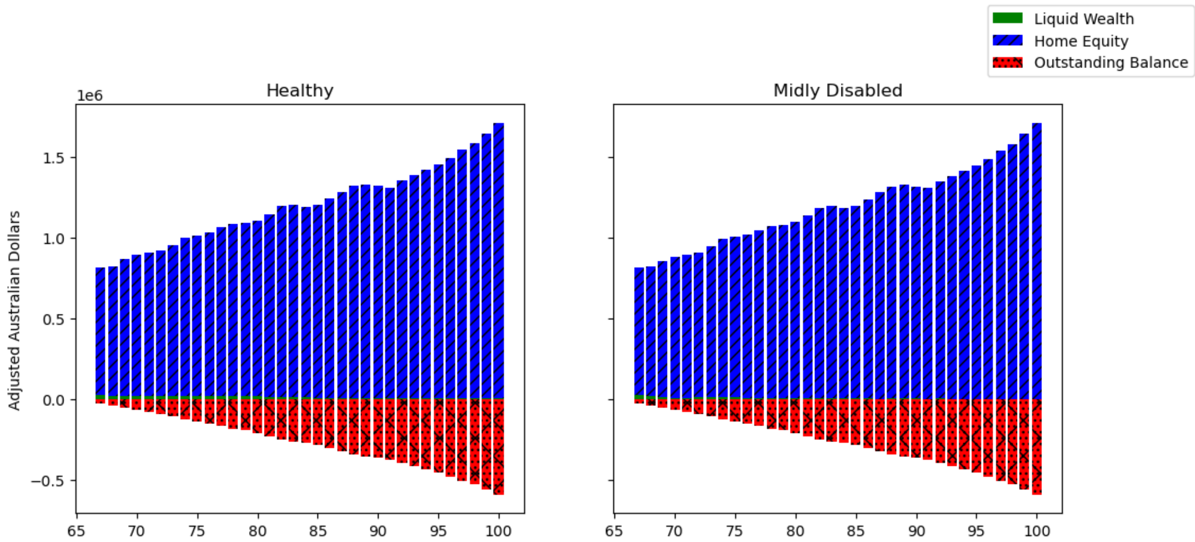
The consumption of non-durable goods and liquid wealth level of female retirees who have utilised the HEAS starting at age 66 will be analysed. Figure 8 shows that optimal consumption C_t and liquid wealth W_t exhibit similar patterns for both healthy and mildly disabled retirees. The variation between these two groups primarily stems from differences in health expenditures. The initial liquid wealth is set to be AUD $40,000$, and the health states are assumed to start as healthy. These retirees are categorised as "asset rich, cash poor", implying they qualify for the maximum pension rate, which is depicted in the baseline case. Consequently, under most economic conditions, their annual payment from the HEAS is confined by the maximum pension rate. Although the HEAS program serves as an income source, it only amounts to 50% of the original pension income in the baseline scenario.

Healthy retirees typically maintain their optimal consumption C_t around an approximate level of AUD $30,000$. As illustrated in the plot, a substantial reduction in their liquid wealth is observed over the years, eventually nearing AUD $5,000$. This residual amount could be viewed as precautionary savings for the deterioration of health states and fluctuations in the markets. Based on Figure 8, it appears that these individuals do not necessitate further financial aid. Contrarily, mildly disabled retirees tend to keep both their consumption and liquid wealth at minimal levels. As indicated over time in Figure 8, these retirees require forms of financial assistance like subsidies to manage their basic survival needs, maintaining the lowest consumption level C_f.

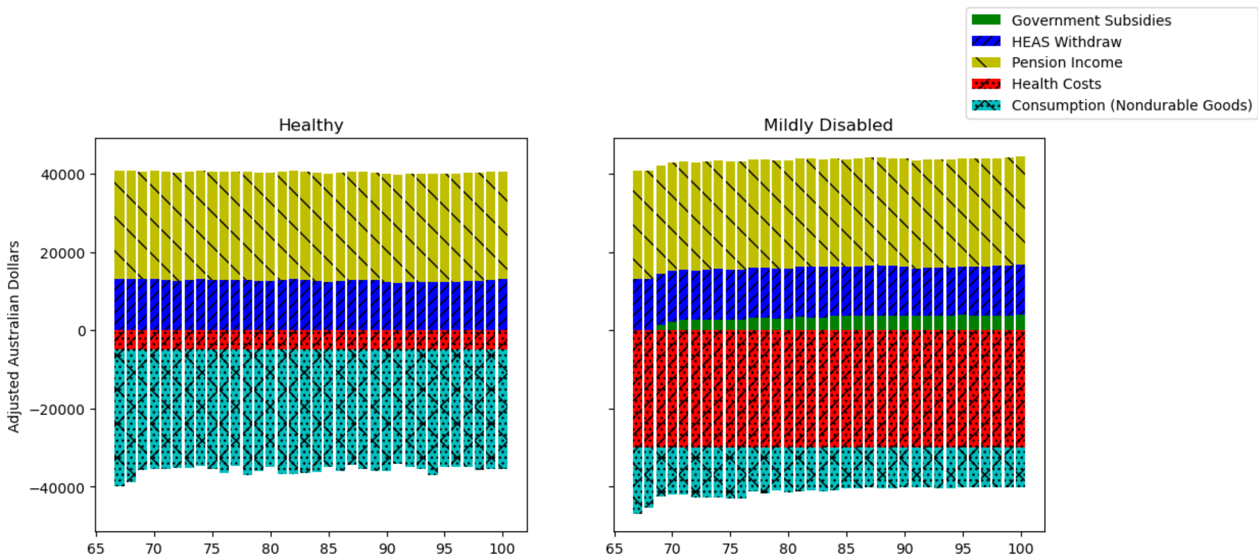
Figure 9 illustrates the typical adjusted financial and income profiles for a retiree who becomes a HEAS user at 66 years old. The financial profile outlines the structure of wealth and debt, while the income profile tracks annual cashflows. In the income profile, expenses such as house expenditures which are identical for both the healthy and mildly disabled HEAS users have been excluded. From the figure, it is evident that both home equity and the outstanding balance of HEAS typically increase over time, with the projected financial gain of home ownership surpassing accumulated interests by the age of 100. As the baseline scenario portrays "asset-rich, cash-poor" retirees, the liquid wealth for both healthy and mildly disabled retirees constitutes only a minor portion of the total wealth. In the income trajectory, pension income and HEAS withdrawal are identical. Government subsidies prove to be essential for mildly disabled retirees in order to sustain the minimum level of consumption.

4.1.1 HEAS benefits "asset-rich, cash-poor" retirees

Figure 10 illustrates various paths of nondurable goods consumption, liquid wealth, and assumed government subsidies, with variations depending on the initial liquid wealth amount. Here, the home equity size remains constant, while the initial wealth is 2, 5 and 10 times of its original value. The consumption trajectory pattern shown in Figure 10(a) reveals that with constant home equity, an increased initial wealth triggers higher consumption in the early years of retirement. This increased wealth not only boosts the consumption level but also prolongs this heightened period of spending. Eventually, this consumption rate stabilises at around AUD 30,000 annually. Contrastingly, an increase in initial wealth leads to a decrease in the pension income for homeowners according to the asset tests. This phenomenon, however, eases the restriction of maximum withdrawals, allowing HEAS users to withdraw more funds during the first few years. When the initial wealth is magnified tenfold from the original amount, consumption levels fall to a lower rate later in life. In such

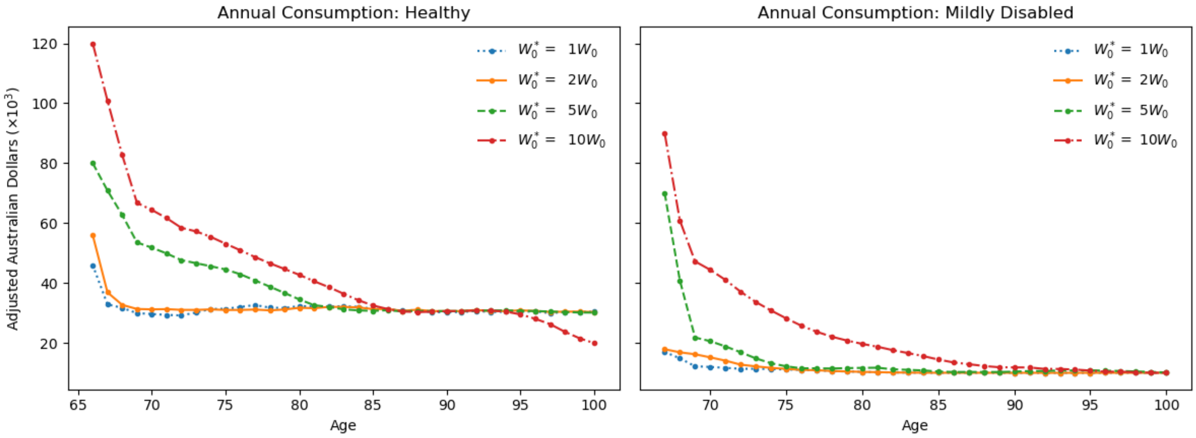


(a) The average financial profile from age 67

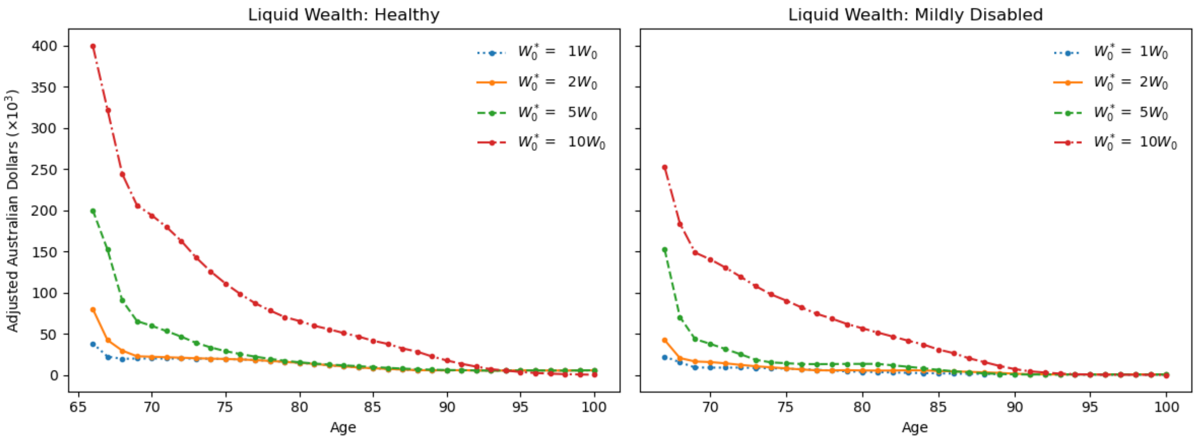


(b) The average income profile from age 67

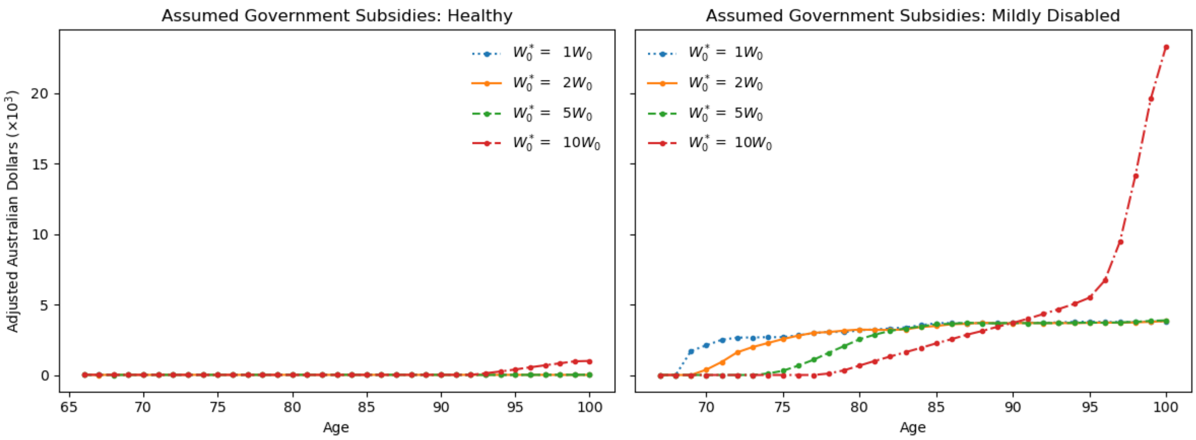
Fig. 9. The typical adjusted financial and income profiles for a HEAS user beginning at 66 years of age are shown. The financial profile outlines the structure of wealth and debt, while the income profile tracks yearly cashflows. In the income profile, expenses that are identical for both the healthy and mildly disabled HEAS users have been excluded.



(a) The mean of optimal consumption C_t for healthy and mildly disabled HEAS users



(b) The mean of liquid wealth W_t for healthy and mildly disabled HEAS users



(c) The mean of assumed government subsidies G_t for healthy and mildly disabled HEAS users

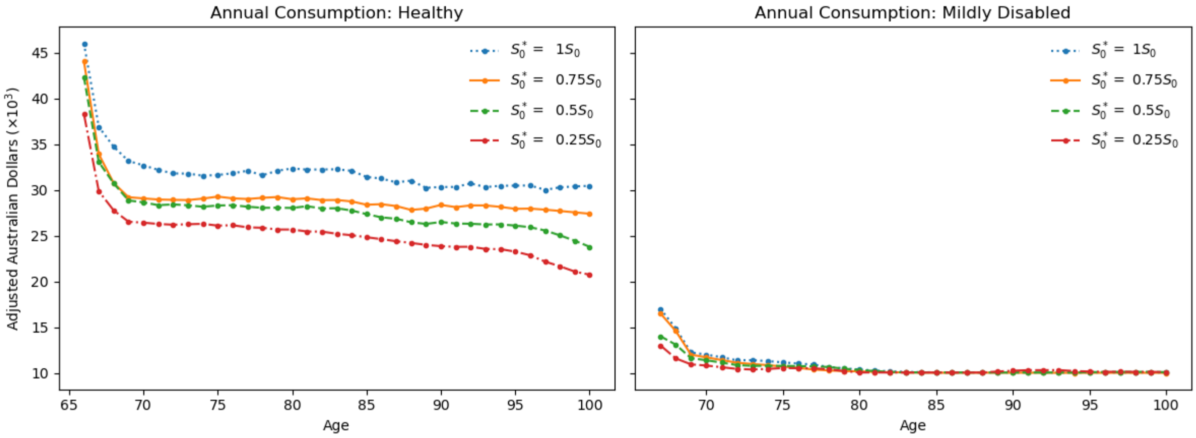
Fig. 10. The influence of different initial wealth on the simulated average optimal consumption of goods, liquid wealth and assumed government subsidies for HEAS users in different health states.

circumstances, due to substantial withdrawals in the earlier years, the outstanding balance is more likely to reach the limit. Once HEAS withdrawals are insufficient, consumption levels drop, and this situation is further aggravated by the reduced pension income. The consumption trajectory for mildly disabled users mirrors this pattern, with one key distinction being that all consumption rates eventually reduce to the lowest level C_f later in life. This pattern is supported by the predicted liquid wealth and assumed government subsidies. As illustrated in Figure 10(b), healthy individuals with the greatest initial wealth will ultimately possess the lowest liquid wealth. Figure 10(c) shows that, regardless of health status, the average assumed government subsidies for individuals with the highest initial wealth tend to surge to a higher level after the age of 95.

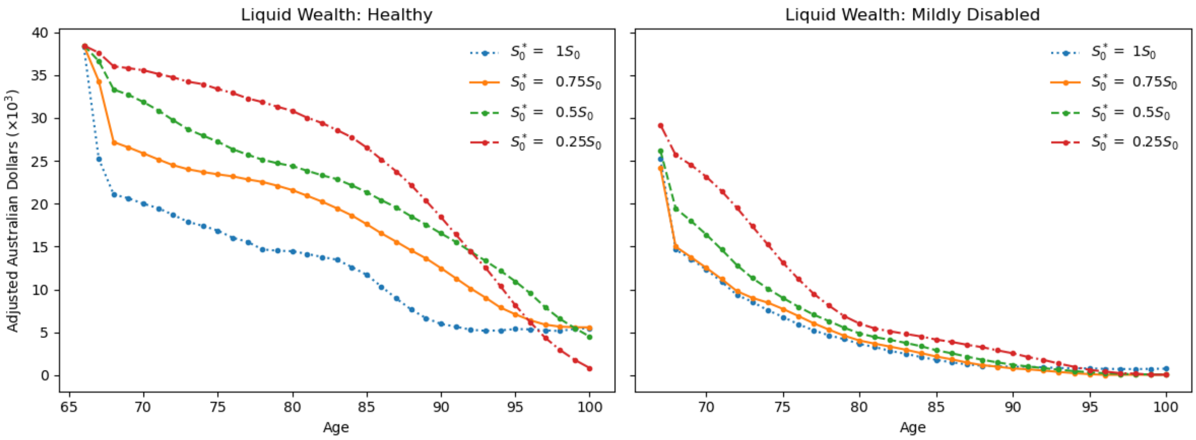
It is assumed that the value and size of home equity are linked via the house price index, which remains constant at a specific time point on a national scale. Figure 11 illustrates how variations in initial living space sizes impact the average optimal consumption of goods, liquid wealth, and assumed government subsidies for retirees of different health states. The size of the home equity is further adjusted to 0.75, 0.5, and 0.25 of the original size, while the initial wealth stays constant. As shown in Figure 10(a), the level of consumption shifts in line with changes in living space for both healthy and unhealthy individuals. HEAS users with a smaller amount of home equity tend to spend less and maintain higher precautionary savings, as evidenced in Figure 11(b). These increased savings can help individuals mitigate risks associated with health deterioration, as depicted in Figure 11(c). However, a smaller home equity size increases the likelihood of restrictions on withdrawals from the HEAS. As a result, healthy HEAS users living in smaller spaces tend to save more money in the first few years post-retirement, but their liquid wealth is likely to diminish to the lowest levels later in life. For mildly disabled retirees, higher levels of precautionary savings early in life might reduce the demand for government subsidies. Nevertheless, smaller payouts from HEAS increase their need for financial assistance in later life stages.

4.1.2 Elasticity of intertemporal substitution (EIS) and bequest motive affect HEAS users

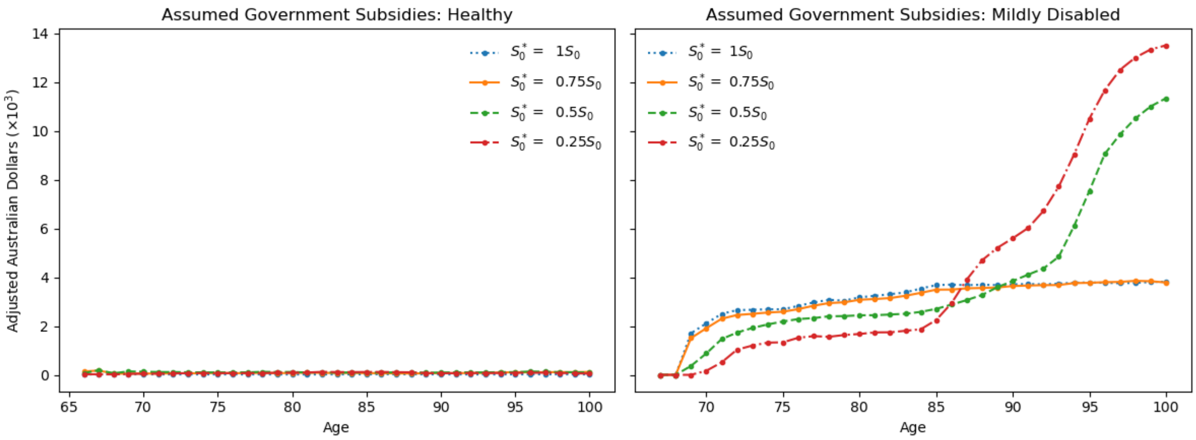
The elasticity of intertemporal substitution (EIS) quantifies the degree of responsiveness and sensitivity of individuals' consumption choices to changes in the intertemporal trade-off between present and future consumption. A higher EIS indicates a greater willingness among individuals to adjust their consumption patterns in response to economic and health risks. In the context of a constant relative risk aversion (CRRA) utility function, the relationship between the EIS and risk aversion is reciprocal.



(a) The mean of optimal consumption C_t for healthy and mildly disabled HEAS users



(b) The mean of liquid wealth W_t for healthy and mildly disabled HEAS users



(c) The mean of assumed government subsidies G_t for healthy and mildly disabled HEAS users

Fig. 11. The influence of different initial living sizes on the simulated average optimal consumption of goods, liquid wealth and assumed government subsidies for retirees in different health states.

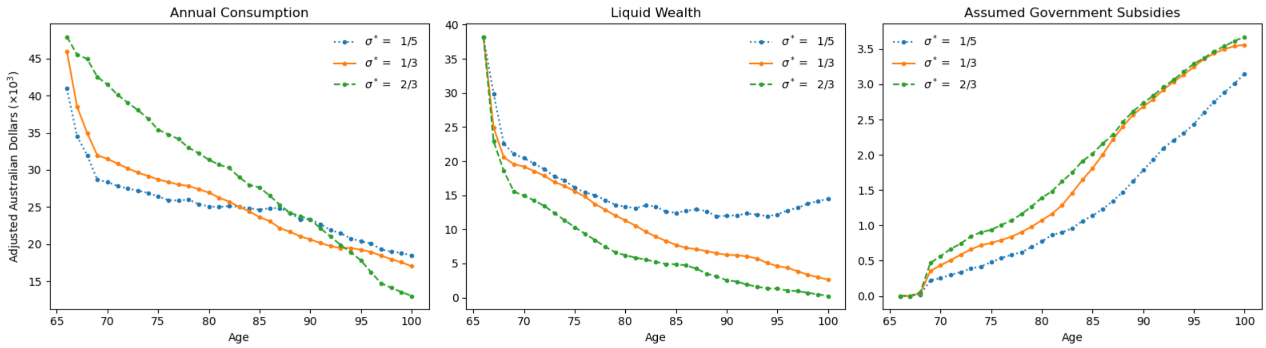


Fig. 12. Simulated average optimal consumption of goods C_t, liquid wealth W_t, and assumed government subsidies of HEAS users with different levels of EIS.

Figure 12 illustrates the impact of different levels of EIS on users of HEAS. During the initial years of retirement, individuals with a lower EIS tend to exhibit a lower level of consumption. However, as time progresses, their consumption level tends to surpass that of individuals with higher EIS. This can be attributed to two factors. Firstly, individuals with lower EIS are more risk-averse, demonstrating a stronger inclination to maintain a stable and secure level of consumption instead of substituting future consumption for present consumption in response to changes. Secondly, individuals with higher EIS initially have a higher level of consumption, which subsequently leads to limitations in liquid wealth and a subsequent decrease in consumption later in life. Users of HEAS with a lower EIS (indicating higher risk aversion) tend to accumulate a greater level of precautionary savings, thereby reducing their need for additional subsidies in later stages of life.

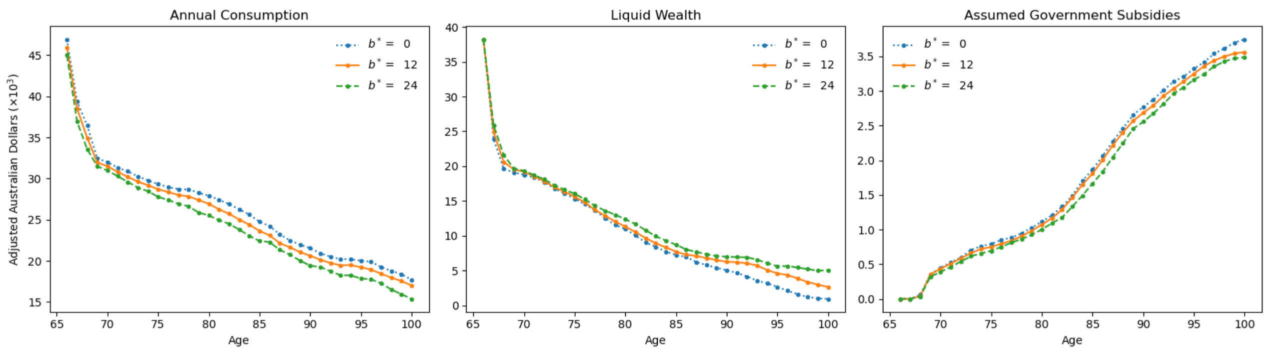


Fig. 13. Simulated average optimal consumption of goods C_t, liquid wealth W_t, and assumed government subsidies of HEAS users with different levels of bequest motive.

The impact of the bequest motive on HEAS users is depicted in Figure 13. It is evident that a stronger bequest motive is associated with a lower level of consumption and a higher level of liquid wealth. Over time, the influence of the bequest motive becomes more pronounced. However, for individuals who are already HEAS users, the effect of the bequest motive is relatively modest. This can be attributed to the fact that the bequest motive can be fulfilled through the remaining value of home equity after the termination of HEAS, mitigating the need for significant adjustments in consumption behaviour.

4.2 Downsizing facilitates immediate gratification

Figure 14 shows different consumption, liquid wealth, and assumed government subsidies for homeowners who downsized at the beginning of retirement. It is presupposed that retirees opt for downsizing at the age of 66. Initially, they are presumed to be in good health. In the baseline case, the ideal size of the new living space post-downsizing is 84.4 square meters, which constitutes approximately 43.75% of the original living area. As such, the initial wealth amounts to AUD 487,450, with the equity value of the new home standing at AUD 385,525. Retirees in good health typically have lower consumption and maintain higher precautionary savings compared to those who are mildly disabled. In the early stages of retirement, those with mild disabilities often spend more than their healthier counterparts. This could be due to their higher risk of mortality, which necessitates a clearer decision between spending for their immediate needs and saving for their inheritance. Initially, the allure of indulging in their retirement outweighs the desire to leave a larger bequest. However, as time progresses, increasing medical costs coupled with regular expenses start depleting their liquid assets. This leads mildly disabled retirees to reduce their consumption, in an effort to save more money for their bequest and to hedge against potential financial and health risks. Consistent with Figure 8, only mildly disabled retirees have a higher level of consumption in the beginning years. On a broader scale, individuals who opt for downsizing at the onset of their retirement typically exhibit a lower consumption level as compared to those utilising the HEAS. In addition, retirees tend to need less financial subsidies compared to HEAS users later in life.

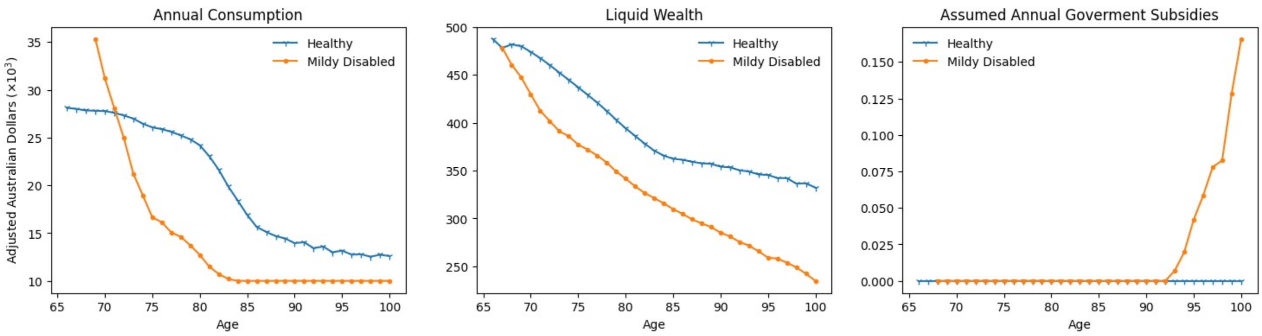
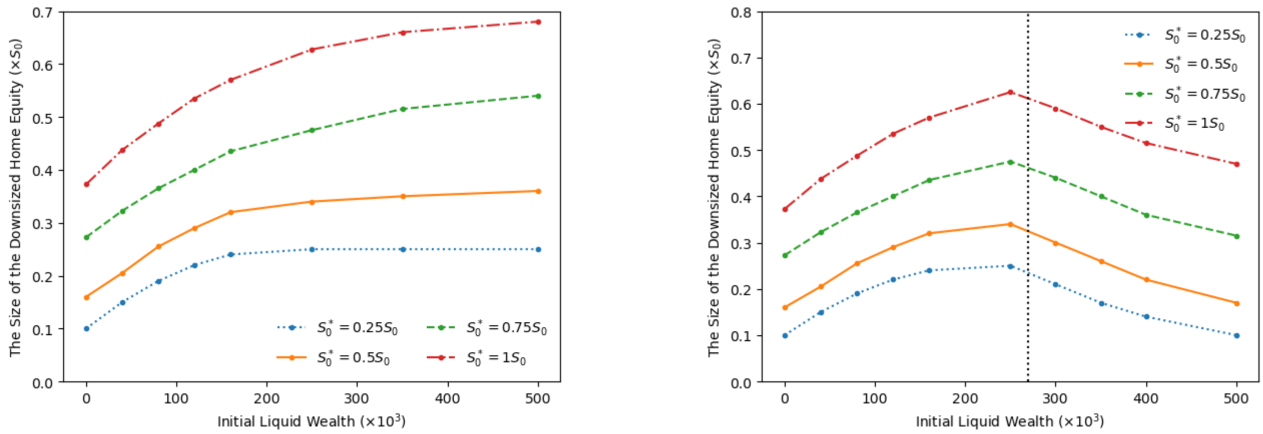


Fig. 14. Simulated average optimal consumption of goods C_t, liquid wealth W_t, and assumed government subsidies for retirees who downsize at age 66. The legend represents two different health states: healthy and mildly disabled

4.2.1 Financial profiles influences downsizing options

An exploration of the factors impacting downsizing choices reveals two main considerations. Firstly, those opting to downsize more significantly are primarily influenced by the financial resources required for their basic needs - both consumption and health-related expenses - and the potential reduction in housing costs that downsizing provides. Conversely, those who prefer to downsize less are often motivated by the utility derived from residing in their current home and the role their home plays as a potential bequest for their heirs. Numerical results reveal that individuals with differing levels of home equity opt for varied optimal downsizing choices based on their initial wealth levels.



(a) Without assets test for pensioners

(b) With assets test for pensioners

Fig. 15. Varying optimal downsizing strategies chosen by healthy retirees at the onset of retirement. The strategies are influenced by the home equity sizes and correspond to distinct initial wealth levels.

Figure 15 illustrates that healthy retirees, depending on their varying levels of home equity, opt for diverse optimal downsizing strategies in response to differing initial wealth levels immediately post-retirement. Figure 15(a) shows that retirees with lower initial liquid wealth levels tend to opt for a greater degree of downsizing. Conversely, retirees with a high initial wealth level are less reliant on extracting wealth from their home equity, leading to a larger size of downsized homes. An interesting pattern emerges when observing homeowners with substantial home equity: these individuals display a predilection towards more pronounced downsizing during the initial stages of retirement when their initial wealth reserves are comparably lower. The implications of this trend suggest a strategic orientation that emphasises the immediate benefits of downsizing. These considerations, under circumstances of lower initial wealth, tend to dominate their decision-making processes. However, as these homeowners' initial wealth levels grow, the difference in their downsizing choices with others begins to diminish. This pattern shows a shift in their weighing of the relative advantages and disadvantages of downsizing, indicating a developing appreciation of their larger residences' utility value and the potential of these assets as bequests. Another noteworthy observation is that homeowners with the lowest initial home equity levels generally avoid downsizing altogether when their initial liquid wealth exceeds AUD 200,000. This implies that the utility derived from residing in their original property often outweighs the potential benefits of downsizing.

Figure 15(b) illustrates the implications of the asset test for pensioners, where the liquid wealth beyond the AUD 270,000 benchmark (with other liquid assets valued at AUD 10,000) triggers a reciprocal decrease in pension income as liquid wealth escalates. As a consequence of this diminished pension income, retirees are driven to withdraw more from their resources to compensate for the income gap. In the following analysis, the asset test is always considered.

4.2.2 Differing preferences affect homeowners

This section explores the impact of three key parameters - the intensity of the bequest motive (b), the expenditure share in non-durable consumption θ, and the intertemporal elasticity of substitution (σ) - on retirees' optimal downsizing choices at retirement. To provide a thorough investigation of the downsizing options, four unique scenarios have been considered and outlined in Table 6. In all these scenarios, retirees share the commonality of having similar total wealth at the time of retirement.

Tab. 6. Different Financial Scenarios

Case	W_0^*/W_0	S_0^*/S_0	H_0^*/W_0^*	$W_0^* + H_0^*$	Y_0^*
1	1	1	22.03	AUD 921,200	AUD 27,664
2	6.5	0.75	2.54	AUD 920,900	AUD 27,664
3	12	0.5	0.92	AUD 921,300	AUD 12,064
4	16.5	0.3	0.40	AUD 924,360	AUD 2,405

† This table depicts various financial scenarios in which retirees all possess similar total wealth upon retirement. The initial calibrated value of home equity, denoted as H_0^*, is contingent upon the size of the initially calibrated home equity, represented as S_0^*. The varied proportions of home equity and liquid wealth influence the yearly pension income, Y_0^*, due to the asset test applied to pensioners.

Figure 16 illustrates the optimal new living spaces chosen as utility function parameters are adjusted. Case 1 and Case 2 exhibit similar trends; in these instances, the total wealth and pension income remain consistent, and the initial ratio of home equity to liquid wealth exceeds 0.791. This value corresponds to the ratio of new home equity to liquid wealth post-downsizing in the baseline scenario. Thus, the downsizing strategies selected by individuals in Cases 1 and 2 tend to align whenever the optimal size of downsized homes falls below $0.75S_0$. Any minor discrepancies may be attributable to factors such as taxation.

As observed in the first subplot in Figure 16. The increase in b corresponds to a rise in the proportion of the size of downsized homes to S_0. This suggests that a stronger bequest motive

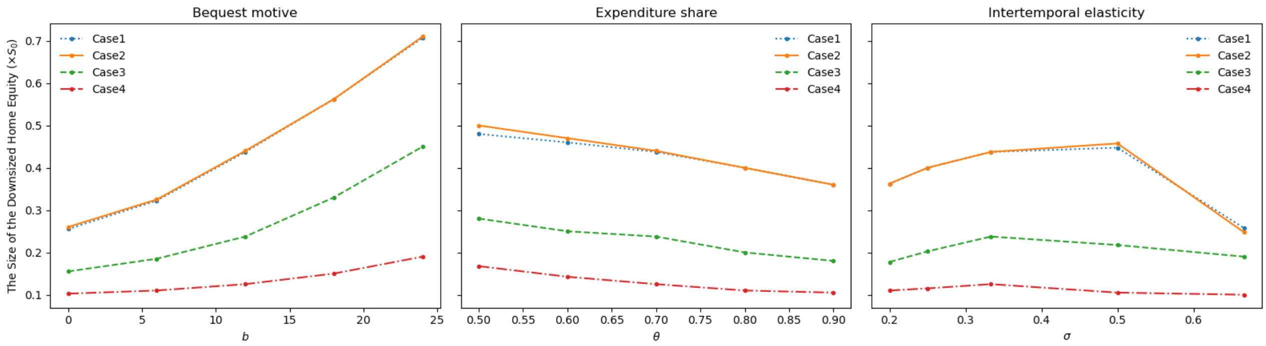


Fig. 16. Varying utilities and financial scenarios influence the downsizing decisions of homeowners. The definitions of different financial scenarios are given in Table 6

prompts retirees to preserve a larger living space, likely viewing a more spacious home as an asset of greater value to leave behind for the next generation. Meanwhile, the second subplot highlights a decrease in the proportion of the size of downsized homes to S_0 with an increasing θ. This indicates that retirees spending more on non-durable goods tend to downsize their living spaces, potentially as a strategy to allocate more funds for consumption. Finally, variations in σ lead to distinct trends across different cases, as shown in the third subplot. For cases 1, 2, and 3, an initial rise in the proportion of the size of downsized homes to S_0 is observed with an increasing σ, followed by a decline. This suggests individuals with a low σ, indicating a high level of risk aversion, prefer to keep a substantial amount of cash available as a hedge against potential risks. Conversely, those with a high σ, reflecting a higher tolerance for risk, seem to opt for greater consumption, particularly in the period immediately post-retirement.

4.3 HEAS and downsizing

This section explores how varying preferences and financial situations influence home release strategies — whether to downsize or opt for HEAS — at age 66. Homeowners will select these optimal strategies at the onset of retirement, based on their expected utility. Table 7 outlines scenarios in which homeowners prefer HEAS over downsizing. In these examples, when one parameter is altered, the remaining two remain constant.

As shown in Table 7, retirees with a stronger bequest motive are less likely to opt for HEAS to unlock home equity. Homeowners are more inclined to choose HEAS when liquid assets make up a smaller percentage of the total assets, as the risk of reaching the outstanding balance limit is lower. When the proportion of initial liquid wealth becomes higher, the possibility of not being able to extract more money from HEAS becomes larger. The deterioration in home equity undermines the bequeathed amount, which is consistent with previous research. As the proportion of initial liquid

Tab. 7. Profiles of HEAS Preferred Retirees Based on Varying Preferences and Financial Scenarios

Scenarios	b			θ			σ		
	b	θ	σ	b	θ	σ	b	θ	σ
Case 1	$0, 6, 12, 18$	0.7	1/3	12	$0.5, 0.6, 0.7, 0.8$	1/3	12	0.7	$1/5, 1/4, 1/3$
Case 2	$0, 6, 12$	0.7	1/3	12	$0.5, 0.6, 0.7, 0.8$	1/3	12	0.7	$1/5, 1/4, 1/3$
Case 3	$0, 6, 12$	0.7	1/3	12	$0.5, 0.6, 0.7$	1/3	12	0.7	$1/4, 1/3$
Case 4	0	0.7	1/3	12	0.5	1/3	12	0.7	-

† Different financial scenarios are defined in Table 6. b values are taken from the set $0, 6, 12, 18, 24$; θ values range within $0.5, 0.6, 0.7, 0.8, 0.9$; and σ is selected from $1/5, 1/4, 1/3, 1/2, 2/3$.

wealth increases, the chances of not being able to withdraw more money from HEAS also rise. This decrease in home equity negatively affects the bequest value.

As the expenditure proportion in non-durable consumption (θ) rises, individuals are more likely to tap into home equity for spending. Therefore, these retirees tend to downsize to satisfy their immediate consumption needs. When initial home equity is limited, only individuals with a high expenditure ratio in home equity consumption (small θ) continue to prefer HEAS, because the benefits they derive from residing in their original home still outweigh potential home equity loss.

Homeowners exhibiting a high level of intertemporal elasticity of substitution prefer to extract more from their home equity for consumption, making them less inclined towards HEAS. In addition, when liquid wealth constitutes a large proportion, highly risk-averse individuals are less likely to utilise HEAS because they might worry about the deterioration of home equity value and lower future income in later life stages. As shown in the table, homeowners represented by Case 4 are not likely to consider HEAS as a home equity release strategy.

To summarise, individuals who have lower motivations to bequeath, derive greater satisfaction from residing in spacious homes, and prioritise long-term benefits over immediate gratification, tend to prefer utilising HEAS. However, when the portion of their total wealth tied up in the initial property value becomes smaller, the interaction between HEAS payments and the assets test diminishes the appeal of HEAS. Consequently, these individuals can be characterised as "asset rich, cash poor".

4.4 Demand analysis at the postcode level

This section presumes that an individual changing residence will select a new location within the confines of the same Greater Capital City Statistical Areas (GCCSAs). The focus of this study is narrowly defined to encompass only suburbs within the Greater Sydney Area. The estimation of home equities size is reliant upon the bedroom count in private dwellings, as provided by the 2021 Census of Population and Housing (Australian Bureau of Statistics, 2022). This estimated home equity size is depicted in table 8. The options `"Not stated"` and `"Not applicable"` from the Census data have been deliberately excluded from the analysis, under the assumption that individuals associated with these responses are not homeowners.

Tab. 8. Sizes of Home Equity for Different Numbers of Bedrooms

Number of Bedrooms	Home Equity (m^2)
0	40
1	65
2	120
3	175
4	230
5	285
6+	340

The Household, Income and Labour Dynamics in Australia (HILDA) Survey provides total wealth information for 80 suburbs in the Greater Sydney Area. The variables that describe the bank accounts, superannuation accounts, credit card liabilities, and other debts of individuals between the ages of 60 and 65 in 2018 are employed to estimate the overall initial cash on hand of retirees upon reaching the age of 66. To enhance the precision of the estimation and provide estimates for all suburbs, the initial cash on hand is characterised by the SA4-level averages, despite the availability of postcode-level data. The average initial cash on hand, life expectancies, and current house price level on the SA4 level are shown in Figure 17. The average home equity size and predicted housing price on the postcode level are shown in Figure 18.

Tab. 9. Table of Variable Names and Descriptions from the HILDA Survey

Variable Name	Variable Description
pwoccdi	Own Credit Card Debt
pwothdi	Other Debt: Car loans/ Investment loans/ Personal loans/ Hire purchase/ Overdue bills
pwobani	Own Bank Accounts
pwsupwi	Superannuation, non-retirees

Finkelstein et al. (2021) discovered a correlation where a single unit rise in average life expectancy coincides with a 0.23 unit enhancement in the causal impact of migration on life expectancy for individuals. However, the advantages of relocating to various areas in the Australian context remain unexplored. Figure 19 illustrates the various suburbs in Sydney where homeowners opt for either the HEAS or downsizing alternatives, given the availability of downsizing options in other suburbs, with ρ (defined in Equation (3)) changing. Contrary to the baseline scenario, where it is presumed that all retirees would opt for HEAS given their utility preferences align with the baseline, a notably small

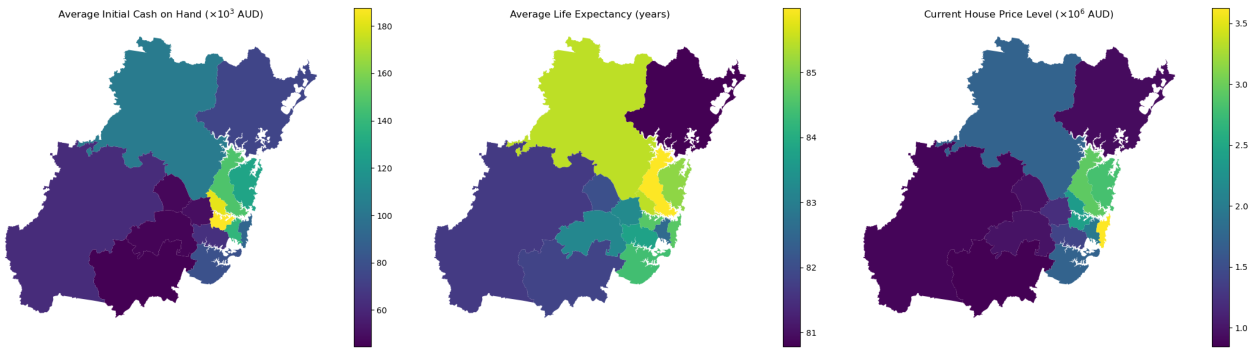


Fig. 17. The Average Initial Cash on Hand, the Mean Life Expectancy over the Past Decade, and the Present Housing Price Median at the SA4 Level. Initial liquid assets are calculated as the mean wealth in each area, excluding short-term debts. The life expectancy values are averages derived from data from the previous decade. The current housing price data is sourced from the Domain House Price Report for the fourth quarter of 2022.

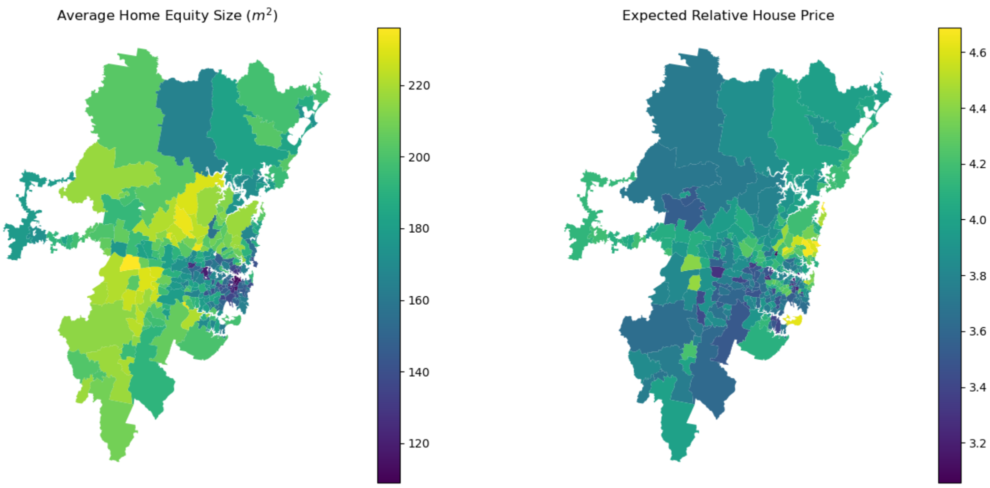


Fig. 18. The Average Size of Home Equity and Predicted Relative House Price at the Postcode Level, Utilising Data From All Postcodes Provided by CoreLogic. The average home equity size is derived from the number of bedrooms in each dwelling. The relative house price is calculated as the ratio of the predicted house price index 35 years later to the current house price index, based on the house price model presented in Section 3.1.

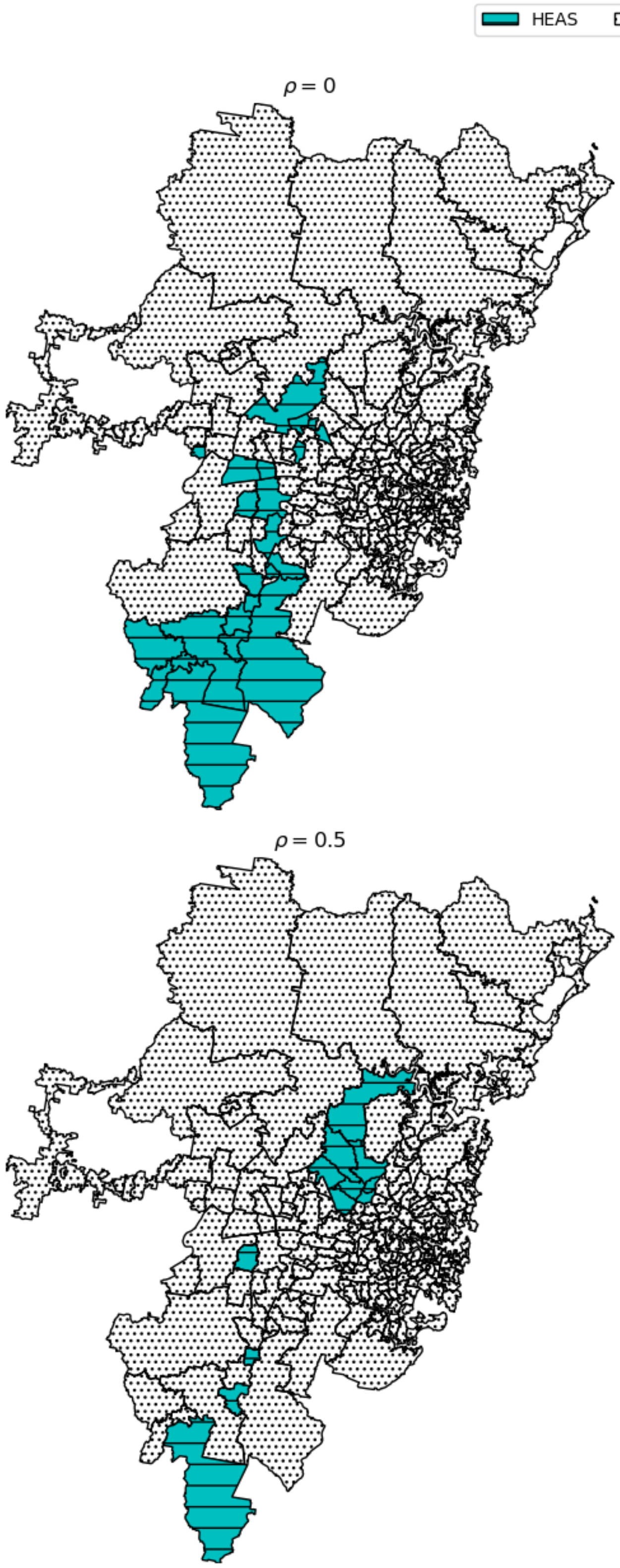


Fig. 19. Comparative Analysis of HEAS vs Downsizing Preferences across Sydney Suburbs. Only a limited number of suburbs appear to favour the HEAS alternative over downsizing, highlighting the significant impact of location on retiree housing choices.

proportion of suburbs are actually favoured for HEAS. This inclination can be attributed to two primary factors: the potential benefits arising from the disparity in house prices and life expectancy across different areas.

In scenarios where increases in lifespan are not taken into account ($\rho = 0$), suburbs advantageous for HEAS tend to exhibit distinct characteristics. These include a lower amount of cash on hand, a substantial quantity of home equity, a moderate current housing price relative to the national average, and a high projected rate of housing price growth over the ensuing 35 years. The significant size of home equity directly enhances the benefits derived from home equity consumption. At the same time, the anticipated surge in housing prices boosts the advantages of retaining ownership of the residence, due to the prospect of high bequest. Moreover, the housing prices in these areas are not exceedingly low, thereby maintaining the likelihood of hitting the HEAS limits at a relatively diminished level. The modest housing prices, in conjunction with the low cash on hand, discourage retirees from considering relocation to other suburbs. Notably, the financial profile, including initial wealth, housing prices, and average home equity size, mirrors the baseline scenario analysed in the preceding sections. As a result, downsizing within the same suburb does not overshadow the benefits of HEAS.

As ρ increases, certain suburbs, with relatively lower life expectancy, transition from being favourable for HEAS to being more suitable for downsizing. This shift in resident preferences can be perceived as a quest for an extended, healthier post-retirement life. The calibrated health transition model suggests that retirees migrating to suburbs with a higher life expectancy not only augment their lifespan but postpone the deterioration in health, thereby reducing the expected health-related expenses. An intriguing observation is the transition of some suburbs from being downsizing-preferred to HEAS-preferred. This phenomenon can be attributed to the elevated risk of medical costs faced by individuals who had opted to downsize and relocate to areas with lower life expectancy when ρ is larger. Broadly, the findings indicate a reduction in the total count of HEAS-preferred suburbs as ρ escalates.

5 Conclusion

This study uses a utility approach to simulate the strategies of homeowners, taking into account realistic factors for retirees releasing locked residential assets. For example, health states, geographically idiosyncratic housing prices and longevity, asset tests for pensioners, and different kinds of expenditures in the Australian context are incorporated into the model, which is an extension of

Nakajima and Telyukova (2017)'s theoretical work.

This study studies the impact of financial profiles and individual preferences on demand for HEAS and downsizing options, using a multi-state lifecycle model that includes longevity risk and health shocks, which is similar to Xu et al. (2023)'s work. Nevertheless, the opportunity for releasing home equity right after retirement is introduced, allowing a more straightforward comparison between HEAS and downsizing. In addition, the downsizing option in this study allows various sizes and among different areas, which is different from Ho (2021), assuming the downsizing amount is fixed and calculated as an actuarial equivalent value of the total withdrawal amount from the reverse mortgage.

The findings of this study reveal that HEAS can enhance consumption levels for both healthy and mildly disabled retirees in certain scenarios, as compared to homeowners opting for downsizing on the national level. HEAS tend to be more appealing to individuals who have a low bequest motive, exhibit a high degree of risk aversion, and stand to gain more from the consumption of home equity. Furthermore, this study confirms a propensity for individuals characterised as "cash-poor, asset-rich" to select HEAS. The elevated percentage of liquid assets can amplify the risk of erosion in home equity for those utilising HEAS, consequently restraining the demand for this scheme.

Furthermore, this study reveals that the spatial disparities in housing prices and life expectancy can limit the appeal for HEAS. Homeowners have the option to relocate to a suburb with more affordable housing, thereby unlocking more funds without a substantial reduction in their home equity size. Moreover, the positive impact of migration on life expectancy can potentially result in a longer and healthier post-retirement life for individuals opting to downsize.

However, it is pertinent to note that in the proposed model, it is assumed that government subsidies are consistently available to address high health costs. This provision inherently enhances the appeal of HEAS, suggesting that the actual demand for HEAS, in reality, can be less pronounced. This gives rise to further inquiries regarding how the eldercare system can be intertwined with other products, such as HEAS, to facilitate healthy aging in the years to come.

Disclosure statement

The authors have examined potential sources of bias and hereby declare that there are no conflicts of interest about this research or its findings.

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A

The numerical techniques used to solve the model are standard. The state space W_t is discretised. The random economic variables are estimated by simulation: a lattice-based grid is constructed, based on the assumption that house price changes are stochastic with two possible values in the next period. The model is solved backward, and the optimal choices for each data are selected according to the first-order conditions. This section derives the first-order condition for consumption given the S_t decisions that have been made. Symbols related to region transfers and those unrelated to calculations are abbreviated in this section. We assume S_t is chosen at the beginning of the simulation. Therefore, S_t is not time-dependent.

The utility function, denoted by $U_1(C_t, S)$, represents the individual's satisfaction from consuming goods (C_t) and home sizes (S). It is defined by the following expression:

$$U_1(C_t, S) = \frac{\left\{\left[\theta^{\frac{1}{\epsilon}} C_t^{\frac{\epsilon-1}{\epsilon}} + (1-\theta)^{\frac{1}{\epsilon}} S^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}\right\}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}.$$

The derivative of the utility function with respect to C_t, denoted by $U_1'(C_t)$, measures the marginal utility or the additional satisfaction gained from a small change in consumption. It is calculated as:

$$U_1'(C_t) = \frac{\partial U_1}{\partial C_t} = \theta^{\frac{1}{\epsilon}} C_t^{-\frac{1}{\epsilon}} \left[\theta^{\frac{1}{\epsilon}} C_t^{\frac{\epsilon-1}{\epsilon}} + (1-\theta)^{\frac{1}{\epsilon}} S^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}\left(1-\frac{1}{\sigma}\right)-1}.$$

The utility function $U_2(W_t)$ represents the individual's satisfaction from the terminal wealth (W_t). It is given by:

$$U_2(W_t) = b\frac{(W_t+\kappa)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}.$$

The derivative of the utility function $U_2(W_t)$ with respect to W_t is denoted by $U_2'(W_t)$ and measures the marginal utility of wealth. It is expressed as:

$$U_2'(W_t) = \frac{\partial U_2}{\partial W_t} = b(W_t+\kappa)^{-\frac{1}{\sigma}}.$$

A.1 $V_{RM}^{renter}(t)$

The value function $V(t)$ represents the maximum expected lifetime utility an individual can attain. It is defined as:

$$V(t) = \max_{C_t}\Big\{U_1(C_t, S) + \beta\mathbb{E}\big[\pi_{x+t}(3,3)V(t+1) + \pi_{x+t}(3,4)U_2\left(W_{t+1}\right)\big]\Big\}.$$

The evolution of wealth, W_{t+1}, depends on current wealth (W_t), consumption (C_t), health expenditure ($HC_t(3)$), rents (RC_t), investment returns (r_t), and labor income (Y_t). It can be expressed as:

$$W_{t+1} = \left(W_t - C_t - HC_t(3) - RC_t\right)(1 + r_t) + Y_t,$$

where $HC_t, RC_t,$ and r_t are stochastic state variables, which can be simulated according to a VAR model.

To find the optimal expression for C_t, we need to solve the maximization problem in the equation for $V(t)$. To do this, we can use the first-order condition by taking the derivative of $V(t)$ with respect to C_t and setting it to zero. We can also use the fact that W_{t+1} is a function of C_t and substitute it in the expression for the first-order condition.

Differentiate the expression inside the curly braces in the equation for $V(t)$ with respect to C_t:

$$\frac{\partial V(t)}{\partial C_t} = U_1'\left(C_t\right) + \beta\mathbb{E}\left[\frac{\partial \pi_{x+t}(3,3)V(t+1)}{\partial C_t} + \frac{\partial \pi_{x+t}(3,4)U_2\left(W_{t+1}\right)}{\partial C_t}\right].$$

Apply the chain rule for the derivative of $U_2(W_{t+1})$:

$$\frac{\partial U_2\left(W_{t+1}\right)}{\partial C_t} = U_2'\left(W_{t+1}\right)\frac{\partial W_{t+1}}{\partial C_t}.$$

Differentiate the expression for W_{t+1} with respect to C_t:

$$\frac{\partial W_{t+1}}{\partial C_t} = \frac{\partial\left(\left(W_t - C_t - HC_t(3) - RC_t\right)(1 + r_t) + Y_t\right)}{\partial C_t} - \left(1 + r_t\right).$$

Substitute the derivatives in the expression for the first-order condition:

$$\frac{\partial V(t)}{\partial C_t} = U_1'\left(C_t\right) + \beta\mathbb{E}\big[\pi_{x+t}(3,3)\frac{\partial V(t+1)}{\partial C_t} - \pi_{x+t}(3,4)U_2'\left(W_{t+1}\right)\left(1 + r_t\right)\big] = 0.$$

Since a complex maximization problem with stochastic variables cannot be provided with an explicit solution for C_t. Instead, the dynamic programming technique is used to solve the problem backward, starting from the beginning of the last period. $V(t+1)$ is a function of W_{t+1}, thus is a function of C_t. The expectation is estimated by considering the two probabilities. The optimal C_t for each time period can be calculated.

A.2 $V_{RM}^{HO}(t)$

For retirees in different health states (1 or 2), the optimization problems are solved simultaneously from the last period. The first-order conditions are:

$$U_1'(C_t) + \beta\mathbb{E}\Big[\pi_{x+t}(1,1)\frac{\partial \overbrace{V_{RM}^{HO}}^{\text{state 1}}(t+1)}{\partial C_t} + \pi_{x+t}(1,2)\frac{\partial \overbrace{V_{RM}^{HO}}^{\text{state 2}}(t+1)}{\partial C_t} +$$
$$\pi_{x+t}(1,3)\frac{\partial V_{RM}^{Renter}(t+1)}{\partial C_t} - \pi_{x+t}(1,4)U_2'(W_{t+1})(1+r_t)\Big] = 0$$

$$U_1'(C_t) + \beta\mathbb{E}\Big[\pi_{x+t}(2,1)\frac{\partial \overbrace{V_{RM}^{HO}}^{\text{state 1}}(t+1)}{\partial C_t} + \pi_{x+t}(2,2)\frac{\partial \overbrace{V_{RM}^{HO}}^{\text{state 2}}(t+1)}{\partial C_t} +$$
$$\pi_{x+t}(2,3)\frac{\partial V_{RM}^{Renter}(t+1)}{\partial C_t} - \pi_{x+t}(2,4)U_2'(W_{t+1})(1+r_t)\Big] = 0$$

A.3 $V^{dHO}(t)$

The corresponding first-order conditions are:

$$U_1'(C_t) + \beta\mathbb{E}\Big[\pi_{x+t}(1,1)\frac{\partial \overbrace{V(t+1)}^{\text{state 1}}}{\partial C_t} + \pi_{x+t}(1,2)\frac{\partial \overbrace{V(t+1)}^{\text{state 2}}}{\partial C_t}$$
$$+\pi_{x+t}(1,3)\frac{\partial \overbrace{V(t+1)}^{\text{state 3}}}{\partial C_t} - \pi_{x+t}(1,4)U_2'(W_{t+1})(1+r_t)\Big] = 0$$

$$U_1'(C_t) + \beta\mathbb{E}\Big[\pi_{x+t}(2,1)\frac{\partial \overbrace{V(t+1)}^{\text{state 1}}}{\partial C_t} + \pi_{x+t}(2,2)\frac{\partial \overbrace{V(t+1)}^{\text{state 2}}}{\partial C_t}$$
$$+\pi_{x+t}(2,3)\frac{\partial \overbrace{V(t+1)}^{\text{state 3}}}{\partial C_t} - \pi_{x+t}(2,4)U_2'(W_{t+1})(1+r_t)\Big] = 0$$

$$U_1'(C_t) + \beta\mathbb{E}\Big[\pi_{x+t}(3,3)\frac{\partial \overbrace{V(t+1)}^{\text{state 3}}}{\partial C_t} - \pi_{x+t}(3,4)U_2'(W_{t+1})(1+r_t)\Big] = 0$$

A.4 Steps after FOC

This subsection primarily focuses on introducing finding the optimal consumption level, denoted as C_t, for a given period t.

Aside from being dependent on the health state, the value functions at the start of each period can be viewed as a function of the wealth at the beginning of the next period. As mentioned earlier, each current node representing the current macroeconomic state is associated with two distinct nodes for the next period, which represent changes in macroeconomic variables based on house prices. The simulation spans 50 years, from age 66 to age 105, although only the results for the subsequent 35 years are presented. If the number of grids in each period is set as 500, the dimensions of each grid would be 500×50. During the t^{th} period, t distinct macroeconomic situations arise, arranged according to housing prices. The "1-1-...-1" trajectory symbolises a steady decrease in home values for the upcoming 50 years, whereas the "1-2-...-50" trajectory implies a uniform rise in housing prices throughout the same duration.

During the t^{th} period, W_t is provided on the grid, establishing the value functions at period $t+1$ as contingent on C_t. Given the value functions for various health states and macroeconomic situations in the ensuing $t+1$ period, the optimal C_t can be computed based on the corresponding first-order conditions. Consequently, this problem necessitates a backward solution. In the final phase, which corresponds to the retiree reaching 105 years of age, it is presupposed that they are unequivocally deceased at the close of this period. Consequently, for each W_t on the grid, the associated C_t and V_t can be identified, facilitating preparations for computations in the preceding period $t-1$. The wealth that is not positioned on the grid will undergo cubic interpolation or linear extrapolation.

Establishing the connection between wealth, consumption, health status, and value functions permits the modelling of individuals with varying health state trajectories. It is presumed that all retirees are in a healthy state at the onset. During each period, a person selects the best consumption option based on their current health status and available liquid wealth.

B

Tab. 10. Age Component Amount (ACP_x)

Age	**Amount**	**Age**	**Amount**
60	\$2,080	75	\$3,750
61	\$2,160	76	\$3,900
62	\$2,250	77	\$4,050
63	\$2,340	78	\$4,210
64	\$2,430	79	\$4,380
65	\$2,530	80	\$4,560
66	\$2,630	81	\$4,740
67	\$2,740	82	\$4,930
68	\$2,850	83	\$5,130
69	\$2,960	84	\$5,330
70	\$3,080	85	\$5,550
71	\$3,200	86	\$5,770
72	\$3,330	87	\$6,000
73	\$3,460	88	\$6,240
74	\$3,600	89	\$6,490
		90+	\$6,750

C

Tab. 11. The number of transitions between and the number of exposure years in different health states (Xu et al., 2023)

	$1 \to 2$	$1 \to 3$	$1 \to 4$	$2 \to 1$	$2 \to 3$	$2 \to 4$	$3 \to 4$	1	2	3
$50 - 54$	67	21	8	52	13	2	4	$4,527.18$	361.92	121.51
$55 - 59$	280	40	55	212	69	27	16	$10,816.97$	$1,136.76$	387.61
$60 - 64$	458	74	114	436	129	37	36	$15,721.89$	$1,811.16$	692.93
$65 - 69$	553	112	193	474	147	86	79	$16,610.65$	$2,146.23$	802.31
$70 - 74$	575	107	226	441	178	97	86	$13,975.53$	$2,079.22$	948.19
$75 - 79$	579	144	257	349	157	116	171	$10,807.98$	$2,164.77$	$1,071.76$
$80 - 84$	570	162	315	338	190	166	242	$7,512.86$	$2,131.81$	$1,242.44$
$85 - 89$	445	172	302	235	211	212	312	$3,870.87$	$1,826.11$	$1,457.01$
$90 - 94$	218	92	160	86	156	172	296	$1,235.42$	965.27	$1,006.33$
$95 - 100$	52	24	51	18	76	75	174	235.92	265.35	421.37
Total	3,797	948	1,681	2,641	1,326	990	1,416	$85,315.27$	$14,888.60$	$8,151.45$

Tab. 12. Model selection of the Poisson generalised linear models

K	AICc	BIC	D_c	ΔD_c
	Disability			
$q_x(1,2)$: healthy to mildly disabled				
1	137.09	37.79	56.22	
2	**104.02**	**2.73**	**18.85**	**37.36*****
3	109.03	4.05	17.86	0.99
$q_x(1,3)$: healthy to severely disabled				
1	111.81	25.93	44.35	
2	85.47	−2.39	**13.73**	**30.63*****
3	**85.44**	**-6.12**	7.70	6.03*
$q_x(2,3)$: mildly disabled to severely disabled				
1	116.10	26.69	18.49	
2	97.41	6.02	11.00	**22.98*****
3	**87.37**	**-7.72**	**10.64**	16.03***
	Recovery			
$q_x(2,1)$: mildly disabled to healthy				
1	152.24	57.59	76.01	
2	**97.08**	0.45	**16.56**	**59.45*****
3	98.06	**-2.27**	11.55	5.01*
	Mortality			
$q_x(1,4)$: healthy to dead				
1	90.14	0.07	45.12	
2	**86.94**	**-5.12**	**22.13**	**7.49****
3	92.58	−3.17	6.10	0.36
$q_x(2,4)$: mildly disabled to dead				
1	82.18	−1.75	16.67	
2	**81.52**	**-4.40**	**11.72**	**4.95***
3	85.35	−4.27	9.55	2.17
$q_x(3,4)$: severely disabled to dead				
1	**82.28**	**-4.09**	**14.33**	
2	85.82	−2.53	13.59	0.75
3	89.76	−2.29	11.53	2.06

D

Tab. 13. Summary of Important Notations - Variables

Variable	Description	Category
x	Age	
t	Elapsed time from age x	
τ_x	Time of death for individual aged x	General
o	Origin location index	
j	Destination location index	
$\bar{m}^{@j}(x)$	Region-specific mortality rate	
$\bar{\vartheta}_j$	Average health capital stock	Mortality
γ_j	Place effect	
s_x	Health status at age x	
$q_x(s_x, s_{x+1})$	Health transition intensity	Health Transition
$\pi_x^{@j}(s_x, s_{x+1})$	Calibrated health transition probability	
C_t	Consumption of nondurable goods	
W_{τ_x}	Bequeathed wealth	Utility
$S_t^{@j}$	Size of home equity	
$HC_t(s_{x+t})$	Annual health costs	
$H_t^{@j}$	Value of home equity	
$A_t^{@j}$	Max annual HEAS payments	Financial
$HE_t^{@j}$	Annual housing expenditures	
$RC_t^{@j}$	Annual rents	
h_t	National house price growth	
$h_{j,t}$	Region-specific house price growth	
$\mathrm{HPI}_{j,t}$	Region-specific House Price Index	Economic
$\boldsymbol{f}_t$	Exogenous risk factors, including cpr, ir, and principal components at time t in Table 3	

† Notations in the table with superscripts $@j$ can have equivalent forms $@o$ and oj to indicate changes in residential status. For example, $S^{@o}$ represents the size of a home sold by someone moving, while S^{oj} denotes their new home's size. An individual can transition within the same area ($o = j$). For non-movers, this size is consistently represented by $S^{@j}$.

Tab. 14. Summary of Important Notations - Parameters

Parameter	Description	Category
ρ	Migration impact degree	Mortality
$\zeta(\cdot,\cdot)$	Coefficients of GLM	Health Transition
$\varsigma_{x,t}^{oj}$	Calibration term of transition probability	
θ	Non-durable cons exp. share	Utility
ϵ	Intratemporal elasticity of subs.	
σ	Intertemporal elasticity of subs.	
κ	Bequest motive curvature	
b	Bequest motive intensity	
β	Discount factor	
ψ_t	Nominal HEAS rate	Financial
λ_1	Transaction cost rate: seller	
λ_2	Transaction cost rate: buyer	
C_f	Lowest consumption level	
$\boldsymbol{\phi}_t$	Coefficients of house prices	Economic