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Abstract

In this paper we derive the expression for optimal inheritance tax when agents' preferences are subject to temptation and self control problem. We consider a dynamic stochastic model as in Piketty and Saez (2013) where agents are heterogeneous in terms of bequest motives and labor productivities. In such a setup we show that the optimal inheritance tax rate decreases with the level of temptation, and thus it works as an incentive mechanism that leads to more bequests and makes succumbing to temptation less attractive. In fact, when temptation is acute, a subsidy may be justified at any percentile of bequest received. This holds independent of the variation in the models used in the literature as well as the assumption of labor elasticity. The study also reveals some interesting observations. Though from the point of view of incentives, this result has the same essence as in Krusell et al. (2010) where temptation justifies a subsidy on capital, we show that unlike their other policy prescription, the long run equilibrium does not demand a constant subsidy. Thus, even under temptation and self control issue, the standard Chamley - Judd result which recommends zero capital tax in the long run is still valid. However, in a setup that is comparable to Farhi and Werning (2010), our paper shows that in the presence of temptation and self control, if dynamic efficiency holds, optimality always requires a subsidy independent of whether social welfare function puts zero or positive direct weight on the children. This is in direct contrast to Piketty and Saez (2013). A calibration using the same micro data used by Piketty and Saez (2013) shows that the drop in inheritance tax is significant in the presence of temptation and self control.

Keywords : Inheritance tax, Temptation, Self-control, Wealth mobility

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1 Introduction

In this paper, we revisit the old question: how can we tax inherited wealth optimally? Our paper differs from the present literature since we allow agents' preference reversals as time passes.

Piketty and Saez (2013) bring together various existing results from the tax literature and show that optimal inheritance tax formulas can be expressed in terms of sufficient statistics which are estimable and that they are robust to the underlying primitives of the model. However, their paper along with the other papers existing in this literature (for e.g., Farhi and Werning (2010)), somehow ignore the fact that the agents might suffer from other behavioral issues that can seriously affect the amount of bequest left. A prominent issue among them is when agents suffer from temptation and self control for which the amount of bequest that they leave can be significantly different (compared to a situation when they do not suffer from this problem) because of higher level of consumption at present. We thus explore the idea of inheritance tax after incorporating temptation and self control issues in a model similar to Piketty and Saez (2013). In this paper, not only do we provide useful insights into the role of temptation and self control problem in determining the optimal inheritance tax rate, but also connect our results to some of the prominent studies in the area of inheritance and capital taxation.

In particular we use the Gul and Pesendorfer preferences in order to capture the problem of temptation and self control.¹ Given such preferences, we look at a dynamic stochastic model where agents are heterogeneous in terms of bequest motives and labor productivities. In keeping with Piketty and Saez (2013), we categorize the various models that are frequently observed in the literature into two broad categories. In the first broad category of model which is termed as "bequest in the utility", agents care about the after tax bequest that they leave to their off-springs and the social planner maximizes long-run steady state welfare. Under this category, the analysis has been extended to different environments as well, namely, steady-state welfare maximization that incorporates social discounting and the possibility of leaving accidental bequests. In the second category, we replace the assumption of bequest in the utility function with the standard Barro - Becker dynastic model with altruism.

The present study brings forth many interesting results and observations. First, we clearly derive the expression for the optimal inheritance tax rate when agents are impatient and suffer from the problem of temptation and self control. One particular point that repeatedly arises in the analysis is that the level of temptation and the optimal inheritance tax rate are inversely related, that is, the optimal inheritance tax rate decreases with the level of temptation. In fact we can

¹Following Strotz (1956) and Phelps and Pollak (1968), Gul and Pesendorfer (2004) proposed an alternative class of utility functions that provide a dynamically consistent model for addressing preference reversals created by self-control problems. Preferences are defined over consumption sets instead of consumption sequences. An individual's actual choice is a compromise between the commitment utility (standard utility) and a temptation utility. In other words, individuals face the trade-off between short term temptation and the long term interest. Contrary to time-inconsistent preferences (see Laibson (1997)), the main benefit of the self-control preferences proposed by Gul and Pesendorfer (2004) is that preferences remain perfectly time-consistent and allow commitment. There are many extensions and applications of Gul and Pesendorfer (2004) (see for example Fudenberg (2006), Dekel et al. (2009), Stovall (2010), Dekel and Lipman (2012), DeJong and Ripoll (2007), and Estaban et al. (2007)).

further claim that when temptation is severe, a subsidy can also be recommended at any level of bequest received. This result is robust to the different model specifications mentioned below and holds independent of the assumption of elasticity of labor supply. A calibration exercise using the same micro data from the United States that Piketty and Saez (2013) uses, we show that the effect of temptation can be significant at any percentile of bequests received. This negative relationship between the optimal tax rate and the level of temptation implies that when agents are tempted, lowering the tax rate provides incentive to leave more bequests by making ‘succumbing to temptation less attractive’. Further, in a derived parent child version of the model similar to Farhi and Werning (2010) but with the added feature of temptation and self control, we show that unlike the Piketty and Saez (2013) result, if dynamic efficiency holds, a subsidy is always the optimal.

In the dynastic interpretation of the infinite horizon model of Chamley (1986) and Judd (1985) with no stochastic shocks, the optimal inheritance tax rate is zero. Krusell et al. (2010) however extend Chamley (1986) and Judd (1985) by using the Gul and Pesendorfer preferences and show that the optimal policy is a constant subsidy.² First of all, as opposed to Krusell et al. (2010), our tax formulas are expressed in terms of estimable sufficient statistics. As far as the incentive motive is concerned, our result has a flavor similar to Krusell et al. (2010) since our paper also shows a negative relationship between the optimal tax rate and the level of temptation together with recommending subsidy when temptation is critical. However, unlike Krusell et al. (2010), our findings clearly recommend a zero tax in the long run and therefore the celebrated result of Chamley - Judd still holds even when the preferences are subject to temptation and self control. This result is due to the fact that the elasticity of the present discounted value of the tax base with respect to an increase in tax is infinite in Chamley (1986) - Judd (1985). However as mentioned above, in our analysis, a subsidy is optimal in a parent child version of the model similar to Farhi and Werning (2010) when the economy experiences dynamic efficiency.

The rest of the paper is organized as follows. While section 2 deals with the analysis under the assumption of bequest in the utility function, section 3 presents the analysis under dynastic utility. While a calibration exercise is presented in section 4, section 5 concludes.

2 Bequest in the utility

2.1 The Model

In line with Piketty and Saez (2013), we consider a dynamic economy with a discrete set of generations. Initially we will assume that the economy does not experience any growth. Each generation has a unit mass (of measure 1) of agents who live for one period. In the next period, the present generation is replaced by the next generation. An individual agent ti from dynasty i living in generation t has exogenous pre-tax wage income w_{ti} drawn from a stationary distribution. We assume that every agent has available labor time l_{ti} and therefore the pre-tax wage income is $y_{Lti} = w_{ti}l_{ti}$

²There is also a number of other extensions of Chamley (1986) and Judd (1985) leading to non-zero inheritance tax rates. See Cremer and Pestieau (2006) and Kopczuk (2013) for detailed discussions regarding these extensions.

which they receive at the end of the period. Further, individual ti receives $b_{ti} \geq 0$ amount of bequests from generation $t - 1$ at the beginning of period t . The initial distribution of the bequest, b_{0i} , is assumed to be exogenously given. An exogenous gross rate of return R per generation is received by the agents on the amount of inheritance they receive. At the end of the period, agents allocate their lifetime resources which precisely consists of the net of tax labor income and capitalized bequest received, into consumption c_{ti} and bequest left b_{t+1i} . Both the labor tax and the tax on capitalized bequests are assumed to be linear. Precisely, τ_{Lt} represents the labor tax rate and τ_{Bt} is the tax rate on capitalized bequest in period t . The lump-sum grant that the agents may also receive in period t is represented by E_t . Agents receive utility from consumption, leisure and the net-of-tax capitalized bequest left $\underline{b} = Rb_{t+1i}(1 - \tau_{Bt+1})$. A point to note here is that τ_{Bt} can well be interpreted as a capital tax in our model.

Like w_{ti} , the preferences are also drawn from an arbitrary stationary distribution. Thus, independent of parental taste and ability, agents can draw any productivity and taste. Further, we assume that the agents suffer from temptation and self control problems as in Gul and Pesendorfer (2004). Thus whenever the agents suffer from temptation, they consume more and the risk appears on the amount of bequest left for the next generation. The decision problem of an individual ti can be written as

$$\max_{c_{ti}, b_{t+1i}, l_{ti}} \left\{ V^{ti}(c_{ti}, \underline{b}, 1 - l_{ti}) + \tilde{V}^{ti}(c_{ti}, \underline{b}, 1 - l_{ti}) \right\} - \max_{\tilde{c}_{ti}, \tilde{b}_{t+1i}, \tilde{l}_{ti}} \tilde{V}^{ti}(\tilde{c}_{ti}, \tilde{\underline{b}}, 1 - \tilde{l}_{ti}), \quad (1)$$

where \tilde{c}_{ti} represents the temptation consumption, V^{ti} and \tilde{V}^{ti} represent the commitment utility and temptation utility respectively. For any choice variables c_{ti} , b_{t+1i} , l_{ti} , the cost of disutility from self control is given by

$$\max_{\tilde{c}_{ti}, \tilde{b}_{t+1i}, \tilde{l}_{ti}} \tilde{V}^{ti}(\tilde{c}_{ti}, \tilde{\underline{b}}, 1 - \tilde{l}_{ti}) - \tilde{V}^{ti}(c_{ti}, \underline{b}, 1 - l_{ti}).$$

For simplicity, we assume that $\tilde{V}^{ti}(c_{ti}, \underline{b}, 1 - l_{ti}) = \lambda V^{ti}(c_{ti}, \underline{b}, 1 - l_{ti})$ where $\lambda \geq 0$ is a scale parameter that measures the sensitivity to the temptation alternative. We particularly assume that when the agents succumb to the temptation fully, they leave no bequest at all. Given this simplification, (1) takes the following form

$$\max_{c_{ti}, b_{t+1i}, l_{ti}} (1 + \lambda) V^{ti}(c_{ti}, \underline{b}, 1 - l_{ti}) - \lambda V^{ti}(\tilde{c}_{ti}, \underline{b} = 0, 1 - l_{ti}).$$

It is straightforward to check that our usual no temptation situation can be generated by setting $\lambda = 0$. We denote aggregate consumption, labor income of generation t and aggregate bequest received in t by c_t , y_{Lt} and b_t respectively. Obviously, our focus is on the inheritance tax but a point to note here is that the aggregate bequest flow in this model is the aggregate capital accumulation.

2.2 Optimal inheritance tax under inelastic labor

In this paper we present all the results under the assumption of both elastic and inelastic labor. We start with the case where agents do not value leisure and therefore they supply labor inelastically. This assumption is brought in the model by setting $l_{ti} = 1$ and therefore $y_{Lti} = w_{ti}$. Since under this specification l_{ti} is no more a choice variable, we drop l_{ti} from the expression of V^{ti} but whenever labor is assumed to be elastic, we bring back l_{ti} inside V^{ti} . This notational rule is followed throughout the paper. Given the set up, individual ti has now the following optimization problem

$$\max_{\{c_{ti}, b_{t+1i}\}_{t=0}^{\infty}} (1 + \lambda) V^{ti}(c_{ti}, R(1 - \tau_{Bt+1}) b_{t+1i}) - \lambda V^{ti}(\tilde{c}_{ti}, \underline{b} = 0) \quad (2)$$

subject to

$$\begin{aligned} c_{ti} + b_{t+1i} &= R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) w_{ti} + E_t, \\ \tilde{c}_{ti} &= R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) w_{ti} + E_t. \end{aligned}$$

It is straightforward to verify that the first order condition for bequest left is given by

$$V_c^{ti} = R(1 - \tau_{Bt+1}) V_{\underline{b}}^{ti}. \quad (3)$$

We assume that the economy converges to a unique steady state equilibrium which is independent of the initial distribution of bequests and there exists a steady state equilibrium distribution of bequests and earnings. To derive the optimal tax rate, we assume that the government considers the long run steady state equilibrium of the economy where it chooses the long run economic policy E , τ_L and τ_B that maximizes the steady state social welfare. Social welfare, denoted by SWF , is the weighted sum of individual utilities with Pareto weights $\omega_{ti} \geq 0$, subject to a period-wise budget constraint. Formally,

$$SWF = \max_{\tau_B, \tau_L} \int_i \omega_{ti} \left[\begin{array}{l} (1 + \lambda) V^{ti}(R(1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E - b_{t+1i}, R(1 - \tau_B) b_{t+1i}) \\ - \lambda V^{ti}(R(1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E_t, \underline{b} = 0) \end{array} \right]$$

subject to

$$E = \tau_B R b_t + \tau_L y_{Lt}. \quad (4)$$

We will show that the optimal inheritance tax in this setup will depend on the size of behavioral responses to taxation through their measured elasticities, combination of social preferences and the distribution of bequest and earnings captured by distributional parameters and importantly on the temptation parameter which in this model is represented by λ . In our equilibrium, the social welfare is constant over time.

We now focus on the elasticity parameters that will appear in the expression for the optimal τ_B . The long run elasticities of aggregate bequest flow b_t with respect to the net-of-bequest tax

rate $1 - \tau_B$ given E is represented by e_B . Thus formally,

$$e_B = \frac{db_t}{d(1 - \tau_B)} \frac{1 - \tau_B}{b_t} \Big|_E. \quad (5)$$

The long run elasticity of aggregate labor supply with respect to the net-of-labor-tax rate $1 - \tau_L$, denoted by e_L , is

$$e_L = \frac{dy_{Lt}}{d(1 - \tau_L)} \frac{1 - \tau_L}{y_{Lt}} \Big|_E.$$

As expected, whenever we assume that labor supply l_{ti} is inelastic e_L does not play any role in the determination of the optimal inheritance tax rate. We bring e_L parameter back into our discussion whenever the labor supply is elastic.

We now define the distributional parameters that will also appear in the expression for τ_B . The social marginal welfare weight on individual ti is denoted by $g_{ti} = \frac{\omega_{ti} V_c^{ti}}{\int_j \omega_{tj} V_c^{tj}}$ which is normalized to 1. As explained by Piketty and Saez (2013), this g_{ti} measures the social value of increasing consumption of an individual ti by one dollar relative to distributing one dollar equally across all individuals. With this g_{ti} , the distributional parameters are defined as follows

$$\bar{b}^{\text{received}} \equiv \frac{\int_i g_{ti} b_{ti}}{b_t}, \bar{b}^{\text{left}} \equiv \frac{\int_i g_{ti} b_{t+1i}}{b_{t+1}}, \text{ and } \bar{y}_L \equiv \frac{\int_i g_{ti} y_{Lti}}{y_{Lt}},$$

where $b_t = \int_i b_{ti}$. The social marginal weights for ti under temptation is $\tilde{g}_{ti} = \frac{\omega_{ti} V_c^{ti}}{\int_j \omega_{tj} V_c^{tj}}$. When agents are tempted towards consumption, because of higher level of consumption, marginal utility is lower compared to the situation when agents are free from temptation. To capture this, throughout the analysis, we assume that the marginal utility under temptation is lower by the proportion of $\alpha \in (0, 1)$ and it is the same for all ti , that is, $V_c^{ti} = \alpha V_c^{ti}$, $\alpha \in (0, 1)$ for all ti . For example, a low value of α implies that individuals consume a lot under temptation compared to a situation when they are free from these issues. Given this assumption, it is straightforward to verify that $\tilde{g}_{ti} = g_{ti}$ and therefore we guarantee

$$\bar{b}^{\text{received}} = \tilde{b}^{\text{received}}, \bar{b}^{\text{left}} = \tilde{b}^{\text{left}}, \text{ and } \bar{y}_L = \tilde{y}_L,$$

where $\tilde{b}^{\text{received}} \equiv \frac{\int_i \tilde{g}_{ti} b_{ti}}{b_t}$, $\tilde{b}^{\text{left}} \equiv \frac{\int_i \tilde{g}_{ti} b_{t+1i}}{b_{t+1}}$, and $\tilde{y}_L \equiv \frac{\int_i \tilde{g}_{ti} y_{Lti}}{y_{Lt}}$.

Thus in this analysis, the social marginal welfare weight on individual ti is unchanged in the presence of temptation and therefore the distributional parameters too. Note that in this paper, we keep ourselves away from the differential effects of temptation on agents due to varying level of temptation at different level of income or assets. That is, to capture the pure effect of temptation, we do not focus on the additional source of heterogeneity due to temptation. Rather we assume that independent of the level of assets or income, the level of temptation is same for everybody along with the fact that the distributional parameters are unchanged. Note that if the value of the variable is lower for those with higher social marginal weights, all the above ratios are less than

1. Further, $\widehat{e}_B = \widetilde{e}_B$ where \widehat{e}_B is the average of $e_{Bti} = \frac{db_{ti}}{d(1-\tau_B)} \frac{1-\tau_B}{b_{ti}}$ weighted by $g_{ti}b_{ti}$, that is $\widehat{e}_B \equiv \frac{\int_i g_{ti}b_{ti}e_{Bti}}{\int_i g_{ti}b_{ti}}$ and \widetilde{e}_B is the same expression under the temptation, that is $\widetilde{e}_B \equiv \frac{\int_i \widetilde{g}_{ti}b_{ti}e_{Bti}}{\int_i \widetilde{g}_{ti}b_{ti}}$.

To derive the optimal tax rate, we consider a small reform $d\tau_B > 0$. A balanced budget condition $dE = Rb_t d\tau_B + \tau_B Rdb_t + y_{Lt} d\tau_L = 0$ therefore needs $d\tau_L < 0$. Given b_{t+1i} is chosen to maximize the agent's utility and applying the envelope theorem, the effect of reform $d\tau_B$ and $d\tau_L$ on the steady state social welfare is given by

$$dSWF = (1 + \lambda) \int_i \omega_{ti} \left\{ V_c^{ti} \cdot ((1 - \tau_B) Rdb_{ti} - Rb_{ti} d\tau_B - y_{Lti} d\tau_L) - V_b^{ti} \cdot (Rb_{t+1i} d\tau_B) \right\} \\ - \lambda \int_i \omega_{ti} V_c^{ti} \cdot ((1 - \tau_B) Rdb_{ti} - Rb_{ti} d\tau_B - y_{Lti} d\tau_L).$$

At the optimum, $dSWF = 0$ implies that

$$(1 + \lambda) \int_i \omega_{ti} \left\{ V_c^{ti} \cdot ((1 - \tau_B) Rdb_{ti} - Rb_{ti} d\tau_B - y_{Lti} d\tau_L) - V_b^{ti} \cdot (Rb_{t+1i} d\tau_B) \right\} \\ = \lambda \int_i \omega_{ti} V_c^{ti} \cdot ((1 - \tau_B) Rdb_{ti} - Rb_{ti} d\tau_B - y_{Lti} d\tau_L). \quad (6)$$

We now present our first proposition below.

Proposition 1 (a) For any τ_L , the optimum tax rate τ_B^{temp} which maximizes the long run steady state social welfare with period-wise budget balance is given by

$$\tau_B^{temp} = \frac{1 - \left[\frac{\bar{b}^{received}}{\bar{y}_L} (1 + \widehat{e}_B) + \frac{(1 + \lambda) \bar{b}^{left}}{R[1 + \lambda(1 - \alpha)] \bar{y}_L} \right]}{1 + e_B - \frac{\bar{b}^{received}}{\bar{y}_L} (1 + \widehat{e}_B)}. \quad (7)$$

(b) To incentivise leaving bequests, optimal tax rate should decrease with the level of temptation. Further, severe temptation may justify a subsidy at any level of bequest received.

Proof. (a) Note that $dE = Rb_t d\tau_B + \tau_B Rdb_t + y_{Lt} d\tau_L = 0$ implies that

$$-y_{Lt} d\tau_L = \left(1 - \frac{e_B \tau_B}{1 - \tau_B} \right) Rb_t d\tau_B.$$

Given this relationship and (3), dividing (6) by $\int_i \omega_{ti} V_c^{ti}$ yields

$$\begin{aligned} & (1 + \lambda) \frac{\int_i \omega_{ti} V_c^{ti}}{\int_i \omega_{ti} V_c^{ti}} \left[-Rb_{ti} d\tau_B (1 + e_{Bti}) + \left(1 - \frac{e_B \tau_B}{1 - \tau_B} \right) Rb_t d\tau_B \frac{y_{Lti}}{y_{Lt}} - \frac{b_{t+1i}}{1 - \tau_B} d\tau_B \right] \\ &= \lambda \frac{\int_i \omega_{ti} V_c^{ti}}{\int_i \omega_{ti} V_c^{ti}} \left[-Rb_{ti} d\tau_B (1 + e_{Bti}) + \left(1 - \frac{e_B \tau_B}{1 - \tau_B} \right) Rb_t d\tau_B \frac{y_{Lti}}{y_{Lt}} \right] \frac{\int_i \omega_{ti} V_c^{ti}}{\int_i \omega_{ti} V_c^{ti}}. \end{aligned}$$

Further, dividing the above equation by $Rb_t d\tau_B$ and using the relationship $V_c^{ti} = \alpha V_c^{ti}$, $\alpha \in (0, 1)$, we get

$$\begin{aligned} & (1 + \lambda) \left[-\bar{b}^{\text{received}} (1 + \hat{e}_B) + \left(1 - \frac{e_B \tau_B}{1 - \tau_B} \right) \bar{y}_L - \frac{\bar{b}^{\text{left}}}{R(1 - \tau_B)} \right] \\ &= \lambda \alpha \left[-\tilde{b}^{\text{received}} (1 + \tilde{e}_B) + \left(1 - \frac{e_B \tau_B}{1 - \tau_B} \right) \tilde{y}_L \right]. \end{aligned}$$

Since $\bar{b}^{\text{received}} = \tilde{b}^{\text{received}}$, $\tilde{e}_B = \hat{e}_B$, and $\tilde{y}_L = \bar{y}_L$ from the above equation we get

$$\tau_B^{\text{temp}} = \frac{1 - \left[\frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{(1 + \lambda) \bar{b}^{\text{left}}}{R[1 + \lambda(1 - \alpha)] \bar{y}_L} \right]}{1 + e_B - \frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B)}.$$

(b) It is straightforward to verify that $\frac{d\tau_B^{\text{temp}}}{d\lambda} < 0$ and hence the proof. ■

The above result is interesting on its merit. First of all, when agents' preferences are subject to temptation ($\lambda > 0$), τ_B^{temp} differs from τ_B . More precisely, we show that in the presence of temptation, $\tau_B^{\text{temp}} < \tau_B$. Thus when individuals are tempted to consume more and leave less amount of bequests, the optimal inheritance tax rate should be less than the optimal tax rate under no temptation. This implies that if the agents suffer from temptation, a higher tax rate is detrimental. In the presence of temptation, lowering the tax rate generates incentive of leaving higher amount of bequests by making 'succumbing to temptation less attractive'. In fact, acute temptation may also justify a subsidy in our analysis. This result is somewhat in line with the Krusell et al. (2010) where a subsidy on capital encourages the agents to save more when the agents' preferences are subject to temptation and self-control problem. Along this line, there is one more interesting observation when we compare our expression of optimal tax rate with the one derived in Piketty and Saez (2013) which recommends a subsidy at higher percentile of bequest received. Our results confirm that a subsidy can be recommended even for agents in lower percentile in the presence of acute self-control problem. This is also clear from the analysis under section 4 of this

paper where a calibration exercise is presented. Obviously, in the absence of temptation, that is when $\lambda = 0$, it is straightforward from the above expression (7) that τ_B^{temp} coincides with the tax rate derived in Piketty and Saez (2013) with no temptation under the assumption that labor is perfectly inelastic.

A point to note here is that when α is very close to one, the difference between the marginal utilities under the commitment consumption and temptation consumption is very small. This implies that for a given λ , the agents do not leave any bequests and therefore a subsidy can be recommended on the ground of an incentive generating instrument. On the other hand, when α is very small, marginal utility under the tempted consumption is sufficiently lower than the marginal utility from commitment consumption which presents a case where individual consumes a lot under temptation compared to a situation with no temptation. This means that they have already left a large amount of bequests and therefore there is no need for a subsidy. This implies that τ_B^{Temp} is now very close to τ_B . In this situation, an extra incentive to leave higher amount of bequests by lowering the optimal tax is not needed.

2.3 Optimal inheritance tax under elastic labor

In this subsection, we relax the assumption of fixed labor supply. Formally, the optimization problem of an individual now incorporates the fact that the agents can choose the optimal amount of labor supply along with the decision of consumption and the amount of bequests that they leave. Formally the agent's optimization problem can be written as

$$\max_{\{c_{ti}, l_{ti}, b_{t+1i}\}_{t=0}^{\infty}} (1 + \lambda) V^{ti}(c_{ti}, R(1 - \tau_{Bt+1})b_{t+1i}, 1 - l_{ti}) - \lambda V^{ti}(\tilde{c}_{ti}, \underline{b} = 0, 1 - l_{ti}) \quad (8)$$

subject to

$$\begin{aligned} c_{ti} + b_{t+1i} &= R(1 - \tau_{Bt})b_{ti} + (1 - \tau_{Lt})w_{ti}l_{ti} + E_t, \\ \tilde{c}_{ti} &= R(1 - \tau_{Bt})b_{ti} + (1 - \tau_{Lt})w_{ti}l_{ti} + E_t. \end{aligned}$$

While the first order condition for the above problem remains

$$V_c^{ti} = R(1 - \tau_{Bt+1})V_{\underline{b}}^{ti}, \quad (9)$$

the government's long run social welfare can be written as

$$SWF = \max_{\tau_L, \tau_B} \int_i \omega_{ti} \left[\begin{array}{l} (1 + \lambda) V^{ti}(R(1 - \tau_B)b_{ti} + (1 - \tau_L)y_{Lti} + E - b_{t+1i}, R(1 - \tau_B)b_{t+1i}, 1 - l_{ti}) \\ - \lambda V^{ti}(R(1 - \tau_B)b_{ti} + (1 - \tau_L)y_{Lti} + E, \underline{b} = 0, 1 - l_{ti}) \end{array} \right]$$

subject to $E = \tau_B R b_t + \tau_L y_{Lt}$, with initial E as given.

Unlike the previous case where labor supply is fixed, under this representation, e_L appears in the expression of τ_B . Further, dE will now have an additional term equals to $\tau_L dy_{Lt}$ which implies

that $dE = Rb_t d\tau_B + \tau_B Rdb_t + y_{Lt} d\tau_L + \tau_L dy_{Lt}$. Using the elasticities defined above, under the balanced budget condition, we have

$$Rb_t d\tau_B \left(1 - \frac{e_B \tau_B}{1 - \tau_B}\right) + d\tau_L y_{Lt} \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) = 0. \quad (10)$$

Following the same argument explained above under the previous subsection, when we set $dSWF = 0$, and apply envelope theorem we get

$$0 = (1 + \lambda) \int_i \omega_{ti} \left\{ V_c^{ti} \cdot ((1 - \tau_B) Rdb_{ti} - Rb_{ti} d\tau_B - y_{Lti} d\tau_L) - V_b^{ti} \cdot (Rb_{t+1i} d\tau_B) \right\} \quad (11)$$

$$- \lambda \int_i \omega_{ti} V_c^{ti} \cdot ((1 - \tau_B) Rdb_{ti} - Rb_{ti} d\tau_B - y_{Lti} d\tau_L).$$

We now present our next proposition on the optimum inheritance tax rate τ_B^{temp} under the assumption of elastic labor supply.

Proposition 2 (a) For a given τ_L , the optimum tax rate τ_B^{temp} which maximizes the long run steady state social welfare with period-wise budget balance is given by

$$\tau_B^{\text{temp}} = \frac{1 - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \left\{ \frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{(1 + \lambda) \bar{b}^{\text{left}}}{R[1 + \lambda(1 - \alpha)] \bar{y}_L} \right\}}{1 + e_B - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B)}. \quad (12)$$

(b) To incentivise leaving bequests, optimal tax rate should decrease with the level of temptation. Further, severe temptation may justify a subsidy at any level of bequest received.

Proof. Note that as in the proof of Proposition 1, $dE = Rb_t d\tau_B + \tau_B Rdb_t + y_{Lt} d\tau_L + \tau_L dy_{Lt} = 0$ implies that

$$-y_{Lt} d\tau_L = \frac{Rb_t d\tau_B \left(1 - \frac{e_B \tau_B}{1 - \tau_B}\right)}{\left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right)}.$$

Given this relationship and (9), dividing (11) by $\int_i \omega_{ti} V_c^{ti}$ yields

$$(1 + \lambda) \frac{\int_i \omega_{ti} V_c^{ti}}{\int_i \omega_{ti} V_c^{ti}} \left\{ -Rb_{ti} d\tau_B (1 + e_{Bti}) + \frac{\left(1 - \frac{e_B \tau_B}{1 - \tau_B}\right)}{\left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right)} Rb_t d\tau_B \frac{y_{Lti}}{y_{Lt}} - \frac{b_{t+1i}}{1 - \tau_B} d\tau_B \right\}$$

$$= \lambda \frac{\int_i \omega_{ti} V_c^{ti}}{\int_i \omega_{ti} V_c^{ti}} \left\{ -Rb_{ti} d\tau_B (1 + e_{Bti}) + \frac{\left(1 - \frac{e_B \tau_B}{1 - \tau_B}\right)}{\left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right)} Rb_t d\tau_B \frac{y_{Lti}}{y_{Lt}} \right\} \frac{\int_i \omega_{ti} V_c^{ti}}{\int_i \omega_{ti} V_c^{ti}}.$$

We follow the same procedure for the inelastic labor supply. By dividing the above equation by $Rb_t d\tau_B$ and using the relationship $V_c^{ti} = \alpha V_c^{ti}$, $\alpha \in (0, 1)$, we can have

$$\begin{aligned} & (1 + \lambda) \left\{ -\bar{b}^{\text{received}} (1 + \hat{e}_B) + \frac{\left(1 - \frac{e_B \tau_B}{1 - \tau_B}\right)}{\left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right)} \bar{y}_L - \frac{\bar{b}^{\text{left}}}{R(1 - \tau_B)} \right\} \\ &= \alpha \lambda \left\{ -\tilde{b}^{\text{received}} (1 + \tilde{e}_B) + \frac{\left(1 - \frac{e_B \tau_B}{1 - \tau_B}\right)}{\left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right)} \tilde{y}_L \right\} \end{aligned}$$

which, given $\bar{b}^{\text{received}} = \tilde{b}^{\text{received}}$, $\bar{b}^{\text{left}} = \tilde{b}^{\text{left}}$, and $\bar{y}_L = \tilde{y}_L$, guarantees that

$$\tau_B^{\text{temp}} = \frac{1 - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \left[\frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{(1 + \lambda) \bar{b}^{\text{left}}}{R[1 + \lambda(1 - \alpha)] \bar{y}_L} \right]}{1 + e_B - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B)}.$$

(b) It is straightforward to verify that $\frac{d\tau_B^{\text{temp}}}{d\lambda} < 0$ and hence the proof. ■

All our discussions related to Proposition (1) are also valid here. The only added feature is that the optimal tax rate under temptation now contains the elasticity of labor supply.

2.3.1 Growth and wealth loving agents

In this subsection, we extend the analysis to include a labor augmenting economic growth per generation with a rate $G > 1$. We assume that the labor supply is elastic. We present the result under the assumption that we have a steady state where all the variables, including the individual wage rate w_{ti} , grow at the rate of G . This rules out the possibility that labor is affected due to the growth. Further we incorporate “wealth loving” motives which is important when any annuity market is not present or it is imperfect. This supports an important observation that people leave accidental bequest at the time of death. Thus, by assuming wealth loving motive, we also consider the fact that people may leave bequests for other reasons too. In the presence of temptation, this wealth loving motive can play a crucial role. In line with Piketty and Saez (2013), we assume that individuals derive utility from four components: own consumption, after tax bequests, pre tax bequests, and leisure. Formally the function V^{ti} can be written as $V^{ti}(c_{ti}, R(1 - \tau_{Bt+1})b_{t+1i}, b_{t+1i}, 1 - l_{ti})$. When agents do not care about the post-tax bequests, their utility is not affected by the tax rates. However those who receive the inheritance are definitely affected. The relative importance of altruism in bequests motives for individual ti is measured by $\nu_{ti} \equiv \frac{R(1 - \tau_{Bt+1})V_b^{ti}}{V_c^{ti}}$ with population average $\nu \equiv \frac{\int_i g_{ti} b_{t+1i} \nu_{ti}}{\int_i g_{ti} b_{t+1i}}$. We would like to mention here that for the calibration exercise, we use this particular specification of the economy.

The problem of an individual under this setup is given by

$$\max_{\{c_{ti}, l_{ti}, b_{t+1i}\}_{t=0}^{\infty}} (1 + \lambda) V^{ti}(c_{ti}, R(1 - \tau_{Bt+1}) b_{t+1i}, b_{t+1i}, 1 - l_{ti}) - \lambda V^{ti}(\tilde{c}_{ti}, \underline{b} = 0, b_{t+1i} = 0, 1 - l_{ti})$$

subject to

$$\begin{aligned} c_{ti} + b_{t+1i} &= R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t, \\ \tilde{c}_{ti} &= R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t. \end{aligned}$$

The first order condition with respect to b_{t+1i} is given by

$$V_c^{ti} = R(1 - \tau_{Bt+1}) V_{\underline{b}}^{ti} + V_b^{ti}.$$

Therefore the government's long run social welfare can be written as

$$SWF = \max_{\tau_L, \tau_B} \int_i \omega_{ti} \left[\begin{array}{l} (1 + \lambda) V^{ti}(R(1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E - b_{t+1i}, \underline{b}, b_{t+1i}, 1 - l_{ti}) \\ - \lambda V^{ti}(R(1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E, \underline{b} = 0, b_{t+1i} = 0, 1 - l_{ti}) \end{array} \right]$$

subject to $E = \tau_B R b_t + \tau_L y_{Lt}$. We derive

$$\begin{aligned} dSWF &= (1 + \lambda) \int_i \omega_{ti} V_c^{ti} \cdot ((1 - \tau_B) R db_{ti} - R b_{ti} d\tau_B - y_{Lti} d\tau_L) \\ &\quad - (1 + \lambda) \int_i \omega_{ti} V_{\underline{b}}^{ti} \cdot R b_{t+1i} d\tau_B - \lambda \int_i \omega_{ti} V_c^{ti} \cdot (R(1 - \tau_B) db_{ti} - R b_{ti} d\tau_B - y_{Lti} d\tau_L) \end{aligned}$$

and present our next proposition below.

Proposition 3 (a) For a given τ_L , the optimum tax rate τ_B^{temp} which maximizes the long run steady state social welfare with period-wise budget balance is given by

$$\tau_B^{temp} = \frac{1 - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \left\{ \frac{\bar{b}^{received}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{G\nu(1 + \lambda) \bar{b}^{left}}{R[1 + \lambda(1 - \alpha)] \bar{y}_L} \right\}}{1 + e_B - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \frac{\bar{b}^{received}}{\bar{y}_L} (1 + \hat{e}_B)}. \quad (13)$$

(b) To incentivise leaving bequests, optimal tax rate should decrease with the level of temptation. Further, severe temptation may justify a subsidy at any level of bequest received.

Proof. Setting $dSWF = 0$, using envelope theorem, (10), and $V_{\tilde{c}_{ti}}^{ti} = \alpha V_c^{ti}$, and then dividing the equation by $R b_t d\tau_B \int_i \omega_{ti} V_c^{ti}$ give us the following equation

$$0 = - (1 + \lambda(1 - \alpha)) \int_i \frac{g_{ti} b_{ti}}{b_t} (1 + e_{Bti}) + [1 + \lambda(1 - \alpha)] \frac{\left(1 - \frac{e_B \tau_B}{1 - \tau_B}\right)}{\left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right)} \int_i \frac{g_{ti} y_{Lti}}{y_{Lt}} - \frac{1 + \lambda}{R(1 - \tau_B)} \frac{\int_i g_{ti} b_{t+1i} \nu_{ti}}{b_t}$$

Simplifying the above equation and using $b_{t+1} = Gb_t$ we get

$$(1 + \lambda(1 - \alpha)) \bar{b}^{\text{received}} (1 + \hat{e}_B) + \frac{1 + \lambda}{R(1 - \tau_B)} \nu G \bar{b}^{\text{left}} = [1 + \lambda(1 - \alpha)] \bar{y}_L \frac{\left(1 - \frac{e_B \tau_B}{1 - \tau_B}\right)}{\left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right)}$$

from which we derive the desired expression for the optimal tax rate

$$\tau_B^{\text{temp}} = \frac{1 - \left[1 - \frac{e_L \tau_L}{1 - \tau_L}\right] \left[\frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{G \nu (1 + \lambda) \bar{b}^{\text{left}}}{R [1 + \lambda(1 - \alpha)] \bar{y}_L}\right]}{1 + e_B - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B)}.$$

(b) It is straightforward to verify that $\frac{d\tau_B^{\text{temp}}}{d\lambda} < 0$ and hence the proof. ■

Thus the modified version of the tax rate also guarantees that the increase in the level of temptation suggests a lower optimal level of tax rate so that it can incentivise the individuals to leave more bequests. All the discussions regarding the tax rates derived above are also applicable here. The additional variables that appear here are G and ν which have negative effect on the optimal tax rate as expected. Further, when we compare (13) with (12), we see that R has been replaced by R/G since leaving a relative bequest b_{t+1i}/b_{t+1} now requires leaving a bequest G times larger than leaving the same relative bequest b_{t+1i}/b_t and therefore the relative cost of taxation to bequest leavers is multiplied by G . This feature of the model is not affected by the inclusion of temptation parameter λ .

2.4 With Social Discounting

The previous subsection can be considered as a special case where the generational discount rate $\Delta = 1$. In this subsection, we assume that $\Delta \leq 1$ and under this assumption we calculate the optimal policy in the long run (τ_L, τ_B) that maximizes the discounted social welfare across the periods. Further, we assume that the labor supply is elastic. The individual's problem can be written as

$$\max_{\{b_{t+1i}, l_{ti}\}_{t=0}^{\infty}} \left\{ \begin{array}{l} (1 + \lambda) V^{ti} (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t - b_{t+1i}, R(1 - \tau_{Bt+1}) b_{t+1i}, 1 - l_{ti}) \\ -\lambda V^{ti} (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t, \underline{b} = 0, 1 - l_{ti}) \end{array} \right\}$$

and we notice that the form of the first order condition with respect to b_{t+1i} is similar to the previous cases

$$V_c^{ti} = R(1 - \tau_{Bt+1}) V_{\underline{b}}^{ti}.$$

The government's problem under this specification is given by

$$SWF = \max_{\tau_{Bt}, \tau_{Lt}} \left\{ \begin{aligned} & (1 + \lambda) \sum_{t=0}^{\infty} \Delta^t \int_i \omega_{ti} V^{ti} (R(1 - \tau_{Bt})b_{ti} + (1 - \tau_{Lt})y_{Lti} + E_t - b_{t+1i}, R(1 - \tau_{Bt+1})b_{t+1i}, 1 - l_{ti}) \\ & - \lambda \sum_{t=0}^{\infty} \Delta^t \int_i \omega_{ti} V^{ti} (R(1 - \tau_{Bt})b_{ti} + (1 - \tau_{Lt})y_{Lti} + E_t, \underline{b} = 0, 1 - l_{ti}) \end{aligned} \right\}$$

In the long run as all variables converge,

$$dSWF = (1 + \lambda) \left(\sum_{t=T}^{\infty} \Delta^t \int_i \omega_{ti} V_c^{ti} \cdot ((R(1 - \tau_B)db_{ti} - Rb_{ti}d\tau_B - d\tau_{Lt}y_{Lti}) + \sum_{t=T-1}^{\infty} \Delta^t \int_i \omega_{ti} V_b^{ti} \cdot (-Rb_{t+1i}d\tau_B) \right) - \lambda \sum_{t=T}^{\infty} \Delta^t \int_i \omega_{ti} V_{\bar{c}}^{ti} \cdot (R(1 - \tau_B)db_{ti} - Rb_{ti}d\tau_B - d\tau_{Lt}y_{Lti})$$

Assuming period-wise balanced budget holds, we focus on a small reform $d\tau_B$ so that $d\tau_{Bt} = d\tau_B \forall t \geq T$ where T is sufficiently large, keeping $dE_t = 0$. Unlike the steady state maximization, in this case, we have to sum all the effects for $t \geq T$ which are not identical and reform at T also affects those leaving bequests in generation $T - 1$. Before we present the expression for optimal tax rate in this environment, we define three average discounted elasticities as follows:

$$e_B = (1 - \Delta) \sum_{t=T}^{\infty} \Delta^{t-T} e_{Bt} \text{ and}$$

$$\hat{e}_B = (1 - \Delta) \sum_{t=T}^{\infty} \Delta^{t-T} \hat{e}_{Bt}, \text{ where } \hat{e}_{Bt} = \frac{\int_i g_{ti} b_{ti} e_{Bti}}{\int_i g_{ti} b_{ti}}$$

Discounted e_L satisfies

$$\frac{1 - \frac{e_B \tau_B}{1 - \tau_B}}{1 - \frac{e_L \tau_L}{1 - \tau_L}} = (1 - \Delta) \sum_{t=T}^{\infty} \Delta^{t-T} \frac{1 - \frac{e_{Bt} \tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}}$$

With the construction of these, we express the optimal inheritance tax rate under the social discounting in the following proposition.

Proposition 4 (a) *For a given τ_L , the optimum tax rate τ_B^{temp} which maximizes the long run steady state social welfare with period-wise budget balance is given by*

$$\tau_B^{temp} = \frac{1 - \left[1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \left[\frac{\bar{b}^{received}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{1 + \lambda}{1 + \lambda(1 - \alpha)} \frac{\bar{b}^{left}}{\Delta R \bar{y}_L} \right]}{1 + e_B - \left[1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \frac{\bar{b}^{received}}{\bar{y}_L} (1 + \hat{e}_B)}. \quad (14)$$

(b) *To incentivise leaving bequests, optimal tax rate should decrease with the level of temptation.*

Further, severe temptation may justify a subsidy at any level of bequest received.

Proof. Following our usual process, we get

$$0 = -(1 + \lambda(1 - \alpha)) \sum_{t=T}^{\infty} \Delta^t \left[\int_i \omega_{ti} V_c^{ti} \cdot (Rb_{ti} d\tau_B (1 + e_{Bti})) - \frac{1 - \frac{e_{Bt}\tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt}\tau_L}{1 - \tau_L}} Rb_{ti} d\tau_B \frac{y_{Lti}}{y_{Lt}} \right] \\ - (1 + \lambda) \sum_{t=T-1}^{\infty} \Delta^t \int_i \frac{\omega_{ti} V_c^{ti}}{R(1 - \tau_B)} Rb_{t+1i} d\tau_B.$$

Dividing the above expression by $Rb_t d\tau_B \int_i \omega_{ti} V_c^{ti}$ and using the fact that $g_{ti} = \frac{\omega_{ti} V_c^{ti}}{\int_i \omega_{ti} V_c^{ti}}$, we get

$$0 = -(1 + \lambda(1 - \alpha)) \sum_{t=T}^{\infty} \Delta^t \bar{b}^{\text{received}} (1 + \hat{e}_{Bt}) + (1 + \lambda(1 - \alpha)) \frac{1 - \frac{e_{Bt}\tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt}\tau_L}{1 - \tau_L}} \sum_{t=T}^{\infty} \Delta^t \bar{y}_L - \frac{1 + \lambda}{R(1 - \tau_B)} \sum_{t=T-1}^{\infty} \Delta^t \bar{b}^{\text{left}}.$$

Further simplifying the above gives us

$$\tau_B^{\text{temp}} = \frac{1 - \left[1 - \frac{e_{L}\tau_L}{1 - \tau_L} \right] \left[\frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{1 + \lambda}{1 + \lambda(1 - \alpha)} \frac{\bar{b}^{\text{left}}}{\Delta R \bar{y}_L} \right]}{1 + e_B - \left[1 - \frac{e_{L}\tau_L}{1 - \tau_L} \right] \frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B)}.$$

(b) It is straightforward to verify that $\frac{d\tau_B^{\text{temp}}}{d\lambda} < 0$ and hence the proof. ■

A discussion is now due on the link between this paper and the paper by Farhi and Werning (2010) in the presence of temptation and self control problem. As mentioned by Piketty and Saez (2013), their results on positive inheritance tax crucially depends on the fact that labor income is no more a single source of resources in an individual's life as in Farhi and Werning (2010). There is one more source of inequality and that is inheritance. But now we compare ours with Farhi and Werning (2010) when this flow of inheritance is affected by the presence of temptation and self control behavior. In a two period model of Farhi and Werning (2010) where each dynasty survives for two generations, working parents have no bequests to start with but they have earnings whereas the children receive bequests but never work. While a formal extension of our model in line with Farhi and Werning (2010) could be with preferences $U^{ti}(c, \tilde{c}, \underline{b}, l_{ti}) = (1 + \lambda) V^{ti}(c, \underline{b}, l_{ti}) - \lambda V^{ti}(\tilde{c}, \underline{b} = 0, l_{ti})$ for the parents and $V^{ti}(c)$ for children, we restrict ourselves from that formal analysis. For a general case, Farhi and Werning (2010) focused on a weakly separable utility $V^{ti}(u(c, \underline{b}), l_{ti})$ of parents with nonlinear taxation. By assuming the subutility $u(c, \underline{b})$ homogenous of degree one in line with Piketty and Saez (2013), we can obtain linear tax counterpart of their results. Though our specification of utility suffers from temptation, this assumption is also applicable to our framework. Further, the requirement for the dynamic efficiency condition $\Delta R = 1$ is also unchanged in our model, that is, as

explained by Piketty and Saez (2013), $\Delta R = 1$ is the only situation where the equilibrium survives. Given these, a crucial observation from this analysis is presented in the following proposition.

Proposition 5 *In the parent child version similar to Farhi and Werning (2010) where $V^{ti} = V^{ti}(u(c, \underline{b}), 1 - l_{ti})$ with $u(c, \underline{b})$ homogenous of degree one, independent of whether social welfare function puts zero or positive direct weight on children, if dynamic efficiency holds, $\tau_B < 0$ is always the optimal.*

Proof. The proof is straightforward. Since with $u(c, \underline{b})$ homogenous, bequests decisions are linear in lifetime resources, i.e., $b_{t+1i} = s(1 - \tau_L)y_{Lti}$ which guarantees $E(\omega_{ti}V_c^{ti}b_{t+1i})/b_{t+1} = E(\omega_{ti}V_c^{ti}y_{Lti})/y_{Lt}$, that means $\bar{b}^{\text{left}} = \bar{y}_L$. This is unchanged with the incorporation of λ . Also, since there is inequality of only one dimension, bequest taxes are equivalent to labor taxes on distributional grounds even under temptation. Hence shifting from bequest taxes also has zero net effect on labor supply. Since parents receive nothing in this model, social welfare is only the parents' welfare and $\bar{b}^{\text{received}} = 0$. Tax calculated in (14) given $\bar{b}^{\text{received}} = 0$ and $e_L = 0$ confirms that $\tau_B < 0$ since $(1 + \lambda) / (1 + \lambda(1 - \alpha)) > 1$. If children are also considered in the social welfare function and therefore weights are put on them, $\bar{b}^{\text{received}} > 0$ which along with $\bar{b}^{\text{left}} = \bar{y}_L$ and $e_L = 0$ implies $\tau_B < 0$. Hence the proof. ■

Instead of the cross sectional budget constraint $\tau_B Rb_t + \tau_L y_{Lt} = E_t$, the proof relies on the use of generational government budget constraint $\tau_B b_{t+1} + \tau_L y_{Lt} = E_t$. If the cross sectional one is used and dynamic efficiency does not hold, i.e., $\Delta R > 1$, even assuming $\bar{b}^{\text{received}} = 0$, τ_B^{temp} may not be negative. This is because the formula for τ_B^{temp} will have ΔR , $\tau_B^{\text{temp}} = \frac{1 - \frac{1+\lambda}{1+\lambda(1-\alpha)} \frac{\bar{b}^{\text{left}}}{\Delta R \bar{y}_L}}{1 + e_B}$, while ΔR disappears if the generational one is used. The numerator depends on whether $\frac{1+\lambda}{1+\lambda(1-\alpha)} \frac{1}{\delta R} > 1$ or < 1 .

Interestingly, the above result is in contrast to the Piketty and Saez (2013) results which suggests $\tau_B = 0$ when social welfare considers only parents and $\tau_B < 0$ when children enter into the social welfare function. We show that when temptation is present and parents do not inherit any assets but take the decision of leaving bequests whereas children are the receiver without any work and bequest leaving decision, optimality always recommends a subsidy. This result has another important implication in the literature of capital tax when preferences are subject to temptation and self control. The same essence of Krusell et al. (2010) who recommend a subsidy on capital (discussed in detail in the following section) is restored in the parent child version similar to Farhi and Werning (2010) with temptation and self control problems.

3 Dynastic setup

3.1 The Model

In this setup, instead of enjoying utility directly from the net bequest left, an individual ti derives her utility from the utility of her next generation \mathcal{U}^{t+1i} . This guarantees a recursive structure of the

utility function, specifically, $\mathcal{U}^{ti} = V^{ti}(c_{ti}, 1 - l_{ti}) + \delta \mathcal{U}^{t+1i}$ where $\delta \in (0, 1)$ represents the discount factor. When V^{ti} is assumed to follow Gul-Pesendorfer preferences of the form $(1 + \lambda) u^{ti}(c_{ti}, 1 - l_{ti}) - \lambda u^{ti}(\tilde{c}_{ti}, 1 - l_{ti})$ as discussed above, the utility of an individual ti can be written as

$$\mathcal{U}^{ti} = (1 + \lambda) u^{ti}(c_{ti}, 1 - l_{ti}) - \lambda u^{ti}(\tilde{c}_{ti}, 1 - l_{ti}) + \delta \mathcal{U}^{t+1i}. \quad (15)$$

In this framework too, we restrict ourselves to the same set of tax instruments. Individual maximizes utility as in (15) subject to a budget constraint $c_{ti} + b_{t+1i} = R(1 - \tau_{Bt})b_{ti} + (1 - \tau_{Lt})w_{ti}l_{ti} + E_t$ where $E_t \mathcal{U}^{t+1i}$ is the expected utility of individual $t + 1i$ based on the information available in period t . Thus individual's utility maximization problem is as follows:

$$\max_{\{c_{ti}, l_{ti}, b_{t+1i}\}_{t=0}^{\infty}} (1 + \lambda) \sum_{t=0}^{\infty} \delta^t E_t u^{ti}(c_{ti}, 1 - l_{ti}) - \lambda \sum_{t=0}^{\infty} \delta^t E_t u^{ti}(\tilde{c}_{ti}, 1 - l_{ti})$$

subject to

$$\begin{aligned} c_{ti} + b_{t+1i} &= R(1 - \tau_{Bt})b_{ti} + (1 - \tau_{Lt})y_{Lti} + E_t, \\ \tilde{c}_{ti} &= R(1 - \tau_{Bt})b_{ti} + (1 - \tau_{Lt})y_{Lti} + E_t. \end{aligned}$$

The optimization problem formulated above can be rewritten as

$$\max_{\{c_{ti}, l_{ti}, b_{t+1i}\}_{t=0}^{\infty}} \left\{ \begin{aligned} &(1 + \lambda) \sum_{t=0}^{\infty} \delta^t E_t u^{ti}(R(1 - \tau_{Bt})b_{ti} + (1 - \tau_{Lt})y_{Lti} + E_t - b_{t+1i}, 1 - l_{ti}) \\ &- \lambda \sum_{t=1}^{\infty} \delta^t E_t u^{ti}(R(1 - \tau_{Bt})b_{ti} + (1 - \tau_{Lt})y_{Lti} + E_t, 1 - l_{ti}) \end{aligned} \right\}.$$

The first order condition with respect to b_{t+1i} is therefore given by

$$u_c^{ti}(c_{ti}, 1 - l_{ti}) = \delta R(1 - \tau_{Bt+1}) E_t u_c^{t+1i}(c_{t+1i}, 1 - l_{t+1i}). \quad (16)$$

A point to note here is that since b_{t+1i} is known at the end of t , (16) can be essentially expressed as $\bar{b}_{t+1}^{\text{left}} = \delta R(1 - \tau_{Bt+1}) \bar{b}_{t+1}^{\text{received}}$ where $\bar{b}_t^{\text{received}} = \frac{\int_i \omega_{0i} u_c^{ti}(c_{ti}, 1 - l_{ti}) b_{ti}}{b_t \int_i \omega_{0i} u_c^{ti}(c_{ti}, 1 - l_{ti})}$ and $\bar{b}_{t+1}^{\text{left}} = \frac{\int_i \omega_{0i} u_c^{t+1i}(c_{t+1i}, 1 - l_{t+1i}) b_{t+1i}}{b_{t+1} \int_i \omega_{0i} u_c^{t+1i}(c_{t+1i}, 1 - l_{t+1i})}$, where ω_{0i} is any dynastic Pareto weights.³ Again we focus on the equilibrium where in the long run, the individual outcomes are independent of the initial positions. Along with the standard assumption that when the agents succumb to temptation fully they leave no bequests at all, we now make a crucial assumption as follows.

Assumption 1: The amount of b_{ti} is chosen optimally until period T .

Further, it is equally likely to the government in which period individuals will leave no bequest. So the government chooses τ_B as if everyone inherits bequests in all periods. We will show that under both the steady state dynasty and from period zero perspective, we get the same negative

³In our paper we omit the case for general Pareto weights and focus on utilitarian weights, $\omega_{0i} = 1, \forall i$.

relationship (magnitude are different though) between the tax rate and the level of temptation. However, instead of making this logical assumption that b_{ti} is chosen optimally by the agent ti (Assumption 1), if we assume that b_{ti} is just given for all ti until period T , we might get a positive relationship between the tax rate and the level of temptation (see appendix for a detailed analysis). This change is obviously through the envelope condition because if b_{ti} is chosen optimally until period T , at the time of applying Envelope theorem, we omit the derivative with respect to b_{ti} .

3.2 Optimal Inheritance tax under steady state dynasty

We focus on the steady state with constant tax policy τ_B , τ_L and E such that the government budget constraint $E = \tau_B Rb_t + \tau_L y_{Lt}$ holds in every period. The labor supply is assumed to be elastic here. When we calculate the optimal tax policy at the steady state, the equilibrium constant tax rates that obey the balanced budget constraint of the government maximize the social welfare. As usual, in this analysis too, we consider a small deviation in τ_B so that τ_L changes in such a way so that $dE = 0$. Here we have

$$SWF = \max_{\tau_B} \left\{ \begin{array}{l} (1 + \lambda) \sum_{t=0}^{\infty} \delta^t \int_i u^{ti} (R(1 - \tau_B)b_{ti} + (1 - \tau_L)y_{Lti} + E - b_{t+1i}, 1 - l_{ti}) \\ - \lambda \sum_{t=0}^{\infty} \delta^t \int_i u^{ti} (R(1 - \tau_B)b_{ti} + (1 - \tau_L)y_{Lti} + E, 1 - l_{ti}) \end{array} \right\}$$

subject to period-wise budget constraint. As usual the small reform of the tax rates on the steady state social welfare, given b_{t+1i} and l_{ti} are chosen to maximize the individual utility, is given by

$$dSWF = (1 + \lambda) \left[\int_i u_c^{0i} \cdot (R(1 - \tau_B)db_{0i} - Rb_{0i}d\tau_B) - \sum_{t=0}^{\infty} \delta^{t+1} \int_i R u_c^{t+1i} \cdot b_{t+1i}d\tau_B - \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot y_{Lti}d\tau_L \right] \\ - \lambda \int_i u_c^{0i} \cdot (R(1 - \tau_B)db_{0i} - Rb_{0i}d\tau_B) + \lambda \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot y_{Lti}d\tau_L$$

Here too we show that as the level of temptation increases, optimal inheritance tax rate under the dynastic setup, τ_B^{temp} , in fact decreases.

Proposition 6 (a) For a given τ_L , the optimum tax rate τ_B^{temp} which maximizes the long run steady state social welfare with period-wise budget balance is given by

$$\tau_B^{\text{temp}} = \frac{1 - \left[1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \left[\frac{(1 - \delta)\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{1 + \lambda}{1 + \lambda(1 - \alpha)} \frac{\bar{b}^{\text{left}}}{R\bar{y}_L} \right]}{1 + e_B - \left[1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \left[\frac{(1 - \delta)\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B) \right]}. \quad (17)$$

(b) Optimal tax rate τ_B^{temp} should decrease with the level of temptation. Further, severe temptation may justify a subsidy at any level of bequest received.

Proof. (a) Using the first order condition of the individual utility maximization $u_c^{t+1i} \cdot b_{t+1i} = \frac{u_c^{t+i} \cdot b_{t+1i}}{\delta R(1-\tau_B)}$ along with (10), applying envelope theorem, and the assumption $u_c^{ti} = \alpha u_c^{ti}$ we get

$$dSWF = -(1 + \lambda(1 - \alpha)) \int_i u_c^{0i} \cdot b_{0i}(1 + e_{Bi}) R d\tau_B - \frac{1 + \lambda}{1 - \tau_B} \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot b_{t+1i} d\tau_B \\ + (1 + \lambda(1 - \alpha)) R d\tau_B \frac{1 - \frac{e_{B\tau_B}}{1 - \tau_B}}{1 - \frac{e_{L\tau_L}}{1 - \tau_L}} \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot \frac{y_{Lti}}{y_{Lt}} b_t.$$

Further, setting $dSWF = 0$ at the optimum τ_B and then dividing it by $R b_t d\tau_B \int_i u_c^{ti}$, (also note that in the steady state $b_t = b_0$ and $u_c^{ti} = u_c^{0i}$) we get

$$0 = -(1 + \lambda(1 - \alpha)) \frac{\int_i u_c^{0i} \cdot b_{0i}(1 + e_{Bi})}{b_0 \int_i u_c^{0i}} - \frac{1 + \lambda}{1 - \tau_B} \sum_{t=0}^{\infty} \delta^t \frac{\int_i u_c^{ti} \cdot b_{t+1i}}{b_{t+1} \int_i u_c^{ti}} + (1 + \lambda(1 - \alpha)) \frac{1 - \frac{e_{B\tau_B}}{1 - \tau_B}}{1 - \frac{e_{L\tau_L}}{1 - \tau_L}} \sum_{t=0}^{\infty} \delta^t \frac{\int_i u_c^{ti} \cdot y_{Lti}}{y_{Lt} \int_i u_c^{ti}},$$

where $e_{Bi} = \frac{db_{0i}}{d(1 - \tau_B)} \frac{1 - \tau_B}{b_{0i}}$. This implies that

$$0 = -(1 + \lambda(1 - \alpha))(1 - \delta) \bar{b}^{\text{received}}(1 + \hat{e}_B) - \frac{1 + \lambda}{R(1 - \tau_B)} \bar{b}^{\text{left}} + (1 + \lambda(1 - \alpha)) \frac{1 - \frac{e_{B\tau_B}}{1 - \tau_B}}{1 - \frac{e_{L\tau_L}}{1 - \tau_L}} \bar{y}_L.$$

Simplifying it further, we get

$$\tau_B^{\text{temp}} = \frac{1 - \left[1 - \frac{e_{L\tau_L}}{1 - \tau_L}\right] \left[\frac{(1 - \delta) \bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{1 + \lambda}{1 + \lambda(1 - \alpha)} \frac{\bar{b}^{\text{left}}}{R \bar{y}_L}\right]}{1 + e_B - \left[1 - \frac{e_{L\tau_L}}{1 - \tau_L}\right] \left[\frac{(1 - \delta) \bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B)\right]}.$$

(b) It is straightforward to verify that $\frac{d\tau_B^{\text{temp}}}{d\lambda} < 0$ and hence the proof. ■

3.3 Optimal inheritance tax from period zero perspective

In this subsection, we derive the long run optimal inheritance tax rate under temptation from period zero perspective. As mentioned in Piketty and Saez (2013), in this standard model, the bequest behavior changes generations in advance because of anticipated change in the tax rate. This is in contrast to the bequest in utility model presented in the previous section where a future tax change in date T does not affect the generation. Since the anticipated change in the tax rate affects the decision well before the period it is implemented, the optimal tax rate will be different from the computed tax rate presented in the previous section.

Before we figure out the exact expressions for the inheritance tax rate, we focus on some of

the elasticities that will appear in our discussions. As in Piketty and Saez (2013), we divide e_B^{pdv} , the elasticity of the present discounted value of the tax base with respect to an increase in tax in the future into two parts - the usual part measures the post-reform elasticity and the additional part under period zero case measures the anticipated pre-reform behavioral elasticities. Formally, $(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-T} e_{Bt} \equiv e_B^{pdv} = e_B^{post} + e_B^{anticip.}$ with $e_B^{post} = (1 - \delta) \sum_{t=T}^{\infty} \delta^{t-T} e_{Bt}$ and $e_B^{anticip.} = (1 - \delta) \sum_{t=0}^{T-1} \delta^{t-T} e_{Bt}$ as e_B^{post} and $e_B^{anticip.}$ are measured as the discounted average of the elasticities e_{Bt} .

3.3.1 Optimal Inheritance tax under inelastic labor

To have a parity with Piketty and Saez (2013) so that we can compare our results with them, in this section, we assume that the labour supply is inelastic, $\tau_L = 0$. Government's optimization problem in this case can be written as

$$SWF = \max_{\{\tau_{Bt}\}_{t=0}^{\infty}} \left\{ \begin{array}{l} (1 + \lambda) \sum_{t=0}^{\infty} \delta^t \int_i u^{ti} (R(1 - \tau_{Bt})b_{ti} + E_t + y_{Lti} - b_{t+1i}) \\ - \lambda \sum_{t=0}^{\infty} \delta^t \int_i u^{ti} (R(1 - \tau_{Bt})b_{ti} + E_t + y_{Lti}) \end{array} \right\}$$

subject to $\tau_{Bt}Rb_t = E_t$. Again we consider a small reform $d\tau_B$ for all $t \geq T$ where T is sufficiently large so that all the variables converge to the limit. As usual, we can calculate $dSWF$ which in this case is given by

$$dSWF = \begin{aligned} & (1 + \lambda) \left[\sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti} \cdot (-Rb_{ti}d\tau_B + Rb_t d\tau_B) + \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti} \cdot \tau_{Bt} Rdb_t \right] \\ & - \lambda \left[\sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti} \cdot Rb_t d\tau_B + \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti} \cdot \tau_{Bt} Rdb_t \right] - \lambda \delta^T \int_i u_c^{Ti} \cdot (-Rb_{Ti}d\tau_B) \end{aligned} \quad (18)$$

Under the assumption of elastic labor supply, we have the following proposition.

Proposition 7 (a) For a given τ_L , the optimum tax rate τ_B^{temp} which maximizes the long run steady state social welfare with period-wise budget balance is given by

$$\tau_B^{temp} = \frac{1 - \frac{1 + \lambda}{1 + \lambda(1 - \alpha)} \frac{\bar{b}^{left}}{\delta R}}{1 + e_B^{pdv}}.$$

(b) Optimal tax rate τ_B^{temp} decreases with the level of temptation. Further, severe temptation may justify a subsidy at any level of bequest received.

Proof. (a) By setting $dSWF = 0$ at the optimum, applying envelope theorem, using the assumption $u_c^{ti} = \alpha u_c^{ti}$ and then dividing it by $Rd\tau_B$, we have

$$0 = (1 + \lambda) \left[\sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti} \cdot (b_t - b_{ti}) - \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti} \cdot b_t \frac{\tau_B t}{1 - \tau_B t} e_{Bt} \right] - \alpha \lambda \left[\sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti} \cdot b_t - \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti} \cdot b_t \frac{\tau_B t}{1 - \tau_B t} e_{Bt} \right] + \alpha \lambda \delta^T \int_i u_c^{Ti} \cdot b_{Ti}$$

Further, dividing the above by $b_t \int_i u_c^{ti}$ and noting that all the terms converge in the long run, we get

$$0 = (1 + \lambda) \left[\sum_{t=T}^{\infty} \delta^t \left(1 - \frac{\int_i u_c^{ti} \cdot b_{ti}}{b_t \int_i u_c^{ti}} \right) - \frac{\tau_B}{1 - \tau_B} \sum_{t=1}^{\infty} \delta^t e_{Bt} \right] - \alpha \lambda \left[\sum_{t=T}^{\infty} \delta^t - \frac{\tau_B}{1 - \tau_B} \sum_{t=1}^{\infty} \delta^t e_{Bt} \right] + \alpha \lambda \delta^T \frac{\int_i u_c^{Ti} \cdot b_{Ti}}{b_t \int_i u_c^{Ti}}$$

Note that since $0 < \delta < 1$, the term $\alpha \lambda \delta^T \frac{\int_i u_c^{Ti} \cdot b_{Ti}}{b_t \int_i u_c^{Ti}} = \alpha \lambda \delta^T \bar{b}^{\text{received}}$ becomes negligible as T gets very large. Therefore, we can write that

$$0 = (1 + \lambda) \left[\frac{\delta^T}{1 - \delta} (1 - \bar{b}^{\text{received}}) - \frac{\tau_B}{1 - \tau_B} \sum_{t=1}^{\infty} \delta^t e_{Bt} \right] - \alpha \lambda \left[\frac{\delta^T}{1 - \delta} - \frac{\tau_B}{1 - \tau_B} \sum_{t=1}^{\infty} \delta^t e_{Bt} \right]$$

which can further be simplified to

$$0 = 1 + \lambda(1 - \alpha) - (1 + \lambda) \bar{b}^{\text{received}} - (1 + \lambda(1 - \alpha)) \frac{\tau_B}{1 - \tau_B} e_B^{\text{pdv}}.$$

Since the first order condition is unchanged and is given by $\bar{b}^{\text{received}} = \frac{\bar{b}^{\text{left}}}{\delta R(1 - \tau_B)}$, we have

$$0 = (1 + \lambda(1 - \alpha)) \left[1 - \frac{\tau_B}{1 - \tau_B} e_B^{\text{pdv}} \right] - (1 + \lambda) \frac{\bar{b}^{\text{left}}}{\delta R(1 - \tau_B)}$$

which implies that

$$\tau_B^{\text{temp}} = \frac{1 - \frac{1 + \lambda}{1 + \lambda(1 - \alpha)} \frac{\bar{b}^{\text{left}}}{\delta R}}{1 + e_B^{\text{pdv}}}.$$

(b) It is straightforward to verify that $\frac{d\tau_B^{\text{temp}}}{d\lambda} < 0$ and hence the proof. ■

Below we extend our above results to the case of elastic labor supply.

3.3.2 Optimal Inheritance tax under elastic labor

When elasticity is assumed to be elastic, it changes the individual's optimization problem to

$$\begin{aligned} & \max_{\{b_{t+1i}, l_{ti}\}_{t=0}^{\infty}} (1 + \lambda) \sum_{t=0}^{\infty} \delta^t E_t u^{ti} (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t - b_{t+1i}, 1 - l_{ti}) \\ & - \lambda \sum_{t=0}^{\infty} \delta^t E_t u^{ti} (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t, 1 - l_{ti}). \end{aligned}$$

Government's optimization problem in this economy can be written as

$$SWF = \max_{\{\tau_{Bt}, \tau_{Lt}\}_{t=0}^{\infty}} \left\{ \begin{aligned} & (1 + \lambda) \sum_{t=0}^{\infty} \delta^t \int_i u^{ti} (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) w_{ti} l_{ti}) + E_t - b_{t+1i}, 1 - l_{ti} \\ & - \lambda \sum_{t=0}^{\infty} \delta^t \int_i u^{ti} (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) w_{ti} l_{ti}) + E_t, 1 - l_{ti} \end{aligned} \right\}$$

subject to period-wise budget balance, $\tau_{Bt} R b_t + \tau_{Lt} y_{Lt} = E_t$. We assume that in response to an anticipatory change in τ_B , b_t changes and therefore to keep the budget balanced, there is a need for a change in τ_{Lt} . This definitely changes the labor supply decision of individuals before and after tax changes and this is captured in the following equations

$$\begin{aligned} \forall t \geq T, \quad \tau_{Bt} R db_t + R b_t d\tau_B + \tau_{Lt} dy_{Lt} + y_{Lt} d\tau_{Lt} &= 0, \text{ and} \\ \forall t < T, \quad \tau_{Bt} R db_t + \tau_{Lt} dy_{Lt} + y_{Lt} d\tau_{Lt} &= 0. \end{aligned}$$

This generates the following two equations

$$\begin{aligned} \forall t \geq T, \quad d\tau_{Lt} y_{Lt} &= - \frac{1 - \frac{e_{Bt}\tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt}\tau_L}{1 - \tau_L}} R b_t d\tau_B \\ \forall t < T, \quad d\tau_{Lt} y_{Lt} &= \frac{\frac{e_{Bt}\tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt}\tau_L}{1 - \tau_L}} R b_t d\tau_B. \end{aligned}$$

Note that the above relationship holds since we assume a small change in τ_B occurs on or after period T , that is $d\tau_B$ reform starts at T . It can be shown that in this case

$$\begin{aligned} dSWF = & (1 + \lambda) \left[- \sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti} \cdot R b_{ti} d\tau_B - \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti} \cdot y_{Lti} d\tau_{Lt} \right] \\ & - \lambda \left[- \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti} \cdot y_{Lti} d\tau_{Lt} \right] - \lambda \delta^T \int_i u_c^{Ti} \cdot R b_{Ti} d\tau_B. \end{aligned}$$

The expression for τ_B^{temp} is presented in the following proposition.

Proposition 8 (a) For a given τ_L , the optimum tax rate τ_B^{temp} which maximizes the long run steady state social welfare with period-wise budget balance is given by

$$\tau_B^{temp} = \frac{1 - \left[1 - \frac{e_L^{pdv} \tau_L}{1 - \tau_L} \right] \frac{1 + \lambda}{1 + \lambda(1 - \alpha)} \bar{b}^{left}}{1 + e_B^{pdv}}.$$

(b) Optimal tax rate τ_B^{temp} decreases with the level of temptation. Further, severe temptation may justify a subsidy at any level of bequest received.

Proof. Using the usual process as in the previous proofs, we get

$$0 = (1 + \lambda) \left[- \sum_{t=T}^{\infty} \delta^t \frac{\int_i u_c^{ti} \cdot b_{ti}}{b_t \int_i u_c^{ti}} + \sum_{t=T}^{\infty} \delta^t \frac{\int_i u_c^{ti} \cdot y_{Lti}}{y_{Lt} \int_i u_c^{ti}} \frac{1 - \frac{e_{Bt} \tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}} - \sum_{t=1}^{T-1} \delta^t \frac{\int_i u_c^{ti} \cdot y_{Lti}}{y_{Lt} \int_i u_c^{ti}} \frac{\frac{e_{Bt} \tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}} \right] - \alpha \lambda \left[\sum_{t=T}^{\infty} \delta^t \frac{\int_i u_c^{ti} \cdot y_{Lti}}{y_{Lt} \int_i u_c^{ti}} \frac{1 - \frac{e_{Bt} \tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}} - \sum_{t=1}^{T-1} \delta^t \frac{\int_i u_c^{ti} \cdot y_{Lti}}{y_{Lt} \int_i u_c^{ti}} \frac{\frac{e_{Bt} \tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}} \right] - \alpha \lambda \delta^T \frac{\int_i u_c^{Ti} b_{Ti}}{b_T \int_i u_c^{Ti}}.$$

This equation can be simplified further to

$$0 = (1 + \lambda(1 - \alpha)) \left[\bar{y}_L(1 - \delta) \sum_{t=T}^{\infty} \delta^{t-T} \frac{1 - \frac{e_{Bt} \tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}} - \bar{y}_L(1 - \delta) \sum_{t=1}^{T-1} \delta^{t-T} \frac{\frac{e_{Bt} \tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}} \right] - (1 + \lambda) \bar{b}^{received}.$$

With $e_B^{pdv} = e_B^{post} + e_B^{anticip.}$, where $e_B^{post} = (1 - \delta) \sum_{t=T}^{\infty} \delta^{t-T} e_{Bt}$, $e_B^{anticip.} = (1 - \delta) \sum_{t=1}^{T-1} \delta^{t-T} e_{Bt}$, and e_L^{pdv} satisfies

$$\frac{1 - \frac{e_B^{pdv} \tau_B}{1 - \tau_B}}{1 - \frac{e_L^{pdv} \tau_L}{1 - \tau_L}} = (1 - \delta) \sum_{t=T}^{\infty} \delta^{t-T} \frac{1 - \frac{e_{Bt} \tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}} - (1 - \delta) \sum_{t=1}^{T-1} \delta^{t-T} \frac{\frac{e_{Bt} \tau_B}{1 - \tau_B}}{1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}},$$

we can show that the above equation can be expressed as

$$0 = (1 + \lambda(1 - \alpha)) \bar{y}_L \frac{1 - \frac{e_B^{pdv} \tau_B}{1 - \tau_B}}{1 - \frac{e_L^{pdv} \tau_L}{1 - \tau_L}} - (1 + \lambda) \bar{b}^{received}.$$

Further, using the same F.O.C of individual's utility maximization $\bar{b}^{\text{received}} = \frac{\bar{b}^{\text{left}}}{\delta R(1 - \tau_B)}$, we can get

$$0 = (1 + \lambda(1 - \alpha))\bar{y}_L \left[1 - \frac{e_B^{\text{pdv}} \tau_B}{1 - \tau_B} \right] - \left[1 - \frac{e_L^{\text{pdv}} \tau_L}{1 - \tau_L} \right] \frac{(1 + \lambda)\bar{b}^{\text{left}}}{\delta R(1 - \tau_B)}.$$

This guarantees that the optimal tax rate τ_B^{temp} is given by

$$\tau_B^{\text{temp}} = \frac{1 - \left[1 - \frac{e_L^{\text{pdv}} \tau_L}{1 - \tau_L} \right] \frac{1 + \lambda}{1 + \lambda(1 - \alpha)} \frac{\bar{b}^{\text{left}}}{\delta R \bar{y}_L}}{1 + e_B^{\text{pdv}}}.$$

(b) It is straightforward to verify that $\frac{d\tau_B^{\text{temp}}}{d\lambda} < 0$ and hence the proof. ■

The above finding is linked to some of the prominent analyses in the present literature. There are some important differences between the results that we derive here and the results in the existing literature. First of all, when $\bar{b}^{\text{received}} < 1$, a positive tax, as recommended by Piketty and Saez (2013) is not necessarily the outcome here when e_B^{pdv} is finite. The same old essence of negative relationship between the tax rate and the level of temptation still restored here and there is a possibility that subsidy is optimal whenever the commitment consumption is different from the consumption under temptation. Thus the presence of temptation breaks the result that the optimal tax rate is always positive as in Piketty and Saez (2013) when $\bar{b}^{\text{received}} < 1$.

Now we compare our result with the much discussed Chamley - Judd results and also with the result of Krusell et al. (2010) who extended Chamley - Judd and recommended constant subsidy in the long run when the preferences are subject to temptation and self control. They work with the Gul and Pesendorfer preferences and prescribe that in the long run, saving should be subsidized at a constant rate since a subsidy incentivises saving and makes temptation less attractive. Lowering the tax rate to incentivise saving and recommending a subsidy in the presence of acute temptation work in our model too. Yet, e_B^{pdv} plays a crucial role in the analysis. Piketty and Saez (2013) points out that the elasticity e_B^{pdv} is infinite in the Chamley - Judd model with no uncertainty and therefore in the long run, zero tax results can be obtained. It is clear from the above expression of the tax rate that this zero tax result is the outcome in our framework too. This is because e_B^{pdv} is infinite and it is independent of the self-control problem. This zero tax result in the long run thus satisfies the Chamley - Judd result but it is in contrast to Krusell et al. (2010) recommendation, that is, the presence of temptation and self-control does not necessarily demands a subsidy in the long run. This discussion has been summarized and presented below as a corollary.

Corollary 1 *Chamley - Judd result still holds when preferences are subject to temptation and self control problems.*

4 Calibrations

In this section, our aim is to show the impact of various parameters on the optimal tax rate that supports our earlier theoretical results. Our results are explanatory and should not be used for final policy recommendations. The major deviation from the existing literature is that in this paper there is a temptation economy, that is, individuals face temptation and self-control problems. In contrast to Piketty and Saez (2013), we did not provide numerical results for the French economy since the order of magnitudes move in the same directions as in the US economy.

We use the following steady state formula (equation 13) to calculate the optimal tax rates for the US economy:

$$\tau_B^{\text{temp}} = \frac{1 - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \left[\frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B) + \frac{G\nu(1 + \lambda) \bar{b}^{\text{left}}}{R[1 + \lambda(1 - \alpha)] \bar{y}_L} \right]}{1 + e_B - \left(1 - \frac{e_L \tau_L}{1 - \tau_L}\right) \frac{\bar{b}^{\text{received}}}{\bar{y}_L} (1 + \hat{e}_B)}.$$

In the benchmark model, following Piketty and Saez (2013) and Kopczuk and Lupton (2007), we set $e_B = \hat{e}_B = e_L = 0.2$, $\tau_L^{\text{Temp}} = 30\%$, $r - g = 2\%$, $H = 30$ years, and $\nu = 0.7$. Notice that $G/R = 1/e^{(r-g)H} = 0.55$. The values of distributional parameters $\bar{b}^{\text{received}}$, \bar{b}^{left} , and \bar{y}_L are taken from Piketty and Saez (2013).⁴ While there are a number of estimates regarding the value of temptation strength parameter, λ , we do not have any estimates regarding the parameter α .⁵ Hence, we conduct a number of experiments to show the impact of the parameters λ and α on the optimal inheritance tax rates. We also conduct an experiment to explore the interaction between the parameter ν and temptation parameters.

In Figure 1, we explore the implications of the changes in the strength of temptation parameter λ on the optimal inheritance tax rates from the perspective of each percentile p of the distribution of bequest received. We set $\alpha = 0.8$, and vary λ by setting it to 0.4, 0.8, and 1. Since the optimum inheritance tax rate can be a quite large negative number for the high percentiles, we set the lower bound to -20% for the ease of exposition. The optimal linear inheritance tax rate

⁴Piketty and Saez (2013) use the joint micro-level distribution of bequests received (b_{ti}), bequest left (b_{t+1i}), and lifetime labor earnings (y_{Lti}) from the survey data (Survey for Consumer Finances 2010 for the US) to compute the values of distributional parameters $\bar{b}^{\text{received}}$, \bar{b}^{left} , and \bar{y}_L . In order to compute those values, they specify social weights g_{ti} and consider percentile p -weights which concentrate uniformly the weights g_{ti} on percentile p of the distribution of bequests received. As a result, for p weights, $\bar{b}^{\text{received}}$, \bar{b}^{left} , and \bar{y}_L are the the average amount of bequest received, bequest left, and earnings relative to population averages among p th percentile bequest receivers. They compute the aforementioned distributional weights for the individuals who are aged 70 or older. In order to estimate $\bar{b}^{\text{received}}$, retrospective questions about bequest and gift receipts are used. To estimate \bar{b}^{left} , questions about current net wealth is used. Finally, to estimate \bar{y}_L questions regarding wage, self-employment, and retirement incomes used. Married survey participants' wealth is found dividing household wealth by two. When individuals are married, bequest received is calculated by dividing the sum of bequests and gifts received by spouses.

Piketty and Saez (2013) also state the potential problems stemmed from using the survey date. The main problem is reporting bias. Survey participants for various reasons would state incorrect amounts.

⁵Huang et al. (2013) estimate $\lambda = 0.10$ by using National Income and Product Accounts (NIPA) data and estimate $\lambda = 0.24$ by using Consumer Expenditure Survey (CEX) data, assuming agents have self-control preferences in the form of $v(c) = \lambda u(c)$ and the risk aversion parameter is set to the unity.

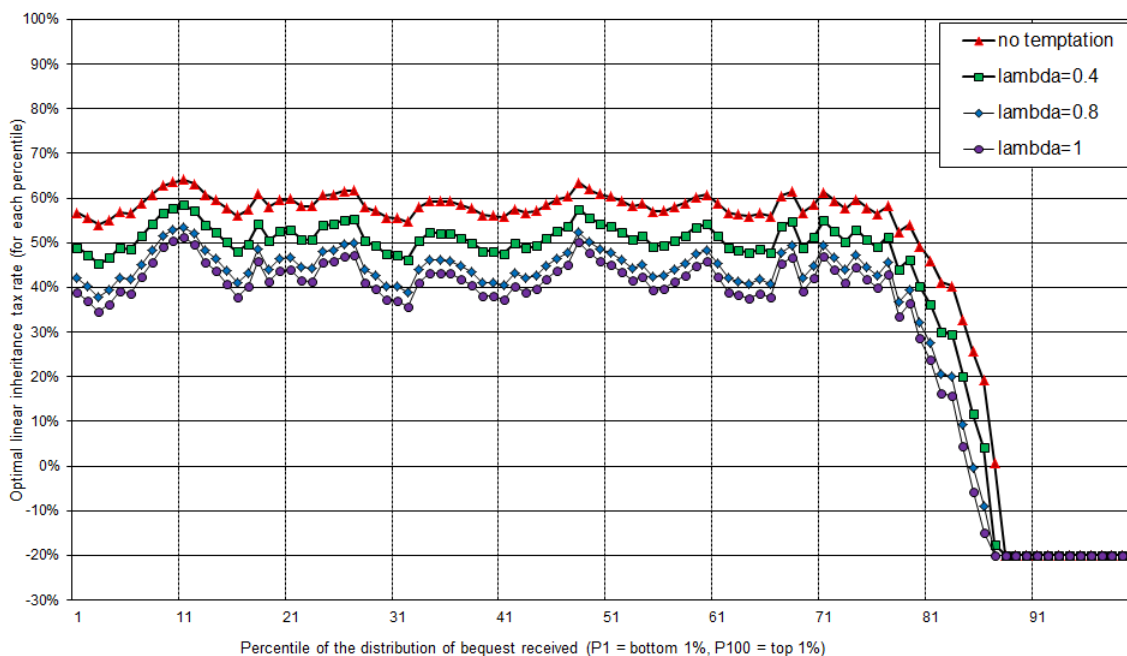


Figure 1: Optimal linear inheritance tax rates, by percentile of $\bar{b}^{\text{received}}$ (temptation parameter λ varies, α is fixed at 0.8)

is around 60% for the bottom 75% of population in no temptation economy which is in line with that of Piketty and Saez (2013).⁶ When individuals face severe temptation as captured by $\lambda = 1$, the optimal tax rate varies between 34% and 51% by clustering around 40% for the bottom 75% of the population. This result clearly shows that the existence of temptation puts a downward pressure on the optimal tax rate. Yet, this pressure, even for the extreme case of temptation, is moderate. When temptation strength is weak, the optimal tax rates deviate minimally from that of the benchmark rates. At around bottom 75% of population, the optimal inheritance tax rate decreases substantially and becomes negative for upper 25% of the population in both temptation and no temptation economies. The optimal bequest tax rate is quite stable across percentiles in the bottom 70%. The reason is as follows.⁷ The inherited wealth is highly concentrated and bottom 70% receive very low amount of bequests ($\bar{b}^{\text{received}}$ is quite close to 0%). Bottom 50% bequest receivers make approximately 90% - 95% of the average earnings \bar{y}_L but they leave substantially less bequest at around 60% - 70% of the average bequest left \bar{b}^{left} . In both the economies, bottom 70% of the population leave some amount of bequests but they prefer higher inheritance tax rates to minimize their burdens on labor tax.

In our model, the strength of temptation is governed by two parameters, α and λ . In this experiment, we fix λ at 0.6 and vary values of the parameter α . For the given value of λ , higher

⁶Piketty and Saez (2013) reported that the optimum rate as about 50% for the bottom 70% of population for the US economy by setting $\nu = 1$. In our benchmark economy, following Kopczuk and Lupton (2007), we set it to 0.70 .

⁷Our explanation follows Piketty and Saez (2013).

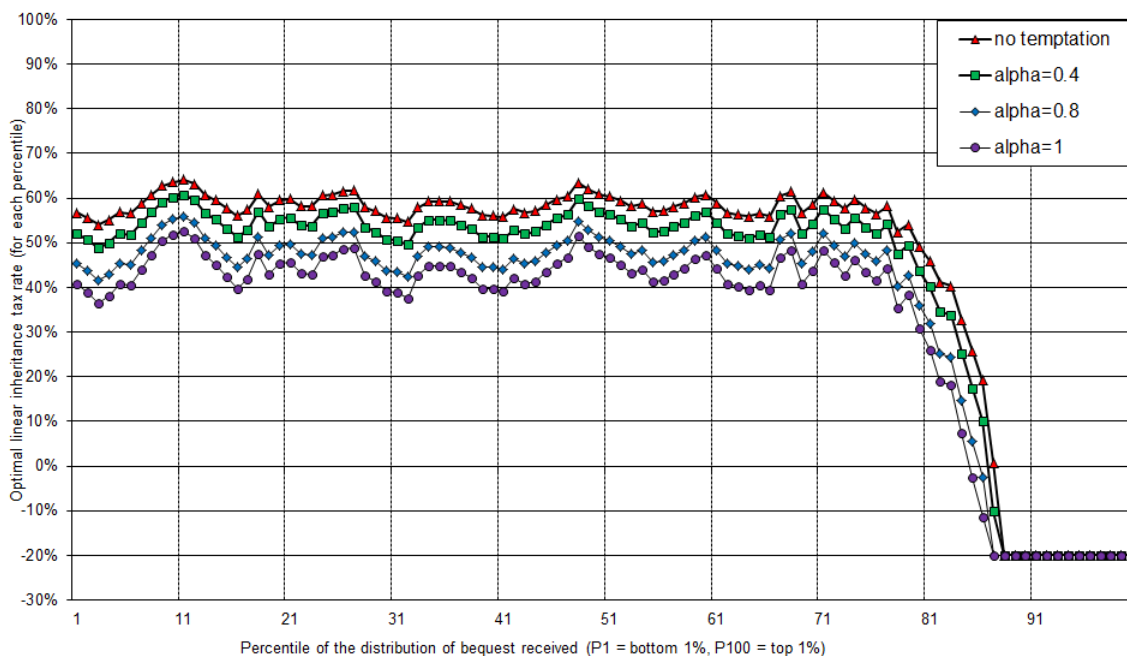


Figure 2: Optimal linear inheritance tax rates, by percentile of $\bar{b}^{\text{received}}$ (α varies, λ is fixed at 0.6)

values of α imply relatively severe temptation. Hence, when $\alpha = 1$, the optimal inheritance tax rate is lower for all income percentiles as in the earlier case. Both exercises support our theoretical findings and show that the existence of temptation problem puts a downward pressure on the optimal inheritance tax rate for all income percentiles. These results have three implications. First, though temptation reduces the optimal tax rates, the role of temptation in terms of reversing the positive inheritance tax result for bottom 80% is somewhat limited. Second, in contrast to Krusell et al. (2010), the negative optimal inheritance tax rate (saving subsidy) apply only to a small group of individuals. Moreover, the negative tax rate (saving subsidy) result is independent of whether an individual faces temptation or not. Third, our tax formulas are expressed in terms of estimable sufficient statistic in contrast to other studies that incorporate behavioral elements to optimal tax studies as in Krusell et al. (2010).

Next, we conduct a sensitivity analysis for temptation (Temp) vis-a-vis no temptation (No temp) economies. Table 1 presents simulations of the optimal inheritance tax rate τ_B using formula (13) for temptation and no-temptation economies. We set labor income tax rate to 30%. For the temptation economy, we set $\alpha = 0.8$ and $\lambda = 0.3$. In each experiment, we display optimal tax rates for $e_B = \hat{e}_B = 0, 0.2, 0.5, \text{ and } 1$. As expected, when e_B approaches to 1, the optimal inheritance tax rates in both economies decrease. In the temptation economy, the tax rate is lower than the benchmark economy.

In the benchmark economy, we set $r - g = 2\%$ and $H = 30$ causing $G/R = 1/e^{(r-g)H} = 0.55$. While decreasing the gap between the rate of return and the growth rate leads to lower optimal rates

Table 1: Optimal Inheritance Tax Rate τ_B Calibrations

	Elasticity $e_B = 0$		Elasticity $e_B = 0.2$		Elasticity $e_B = 0.5$		Elasticity $e_B = 1$	
	Temp	No Temp	Temp	No Temp	Temp	No Temp	Temp	No Temp
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
1. Optimal tax for zero receivers (bottom 50%), $r - g = 2\%$ ($G/R = 0.55$), $\nu = 70\%$, $e_L = 0.2$								
P0-50	64%	70%	53%	59%	43%	47%	32%	35%
2. Optimal linear tax rate for other groups by percentile of bequests received, $r - g = 2\%$ ($G/R = 0.55$), $\nu = 70\%$, $e_L = 0.2$								
P51-70	63%	70%	53%	59%	42%	47%	32%	35%
P71-90	51%	60%	38%	46%	25%	31%	12%	17%
P91-95	-75%	-43%	-111%	-84%	-147%	-126%	-184%	-167%
3. Sensitivity to capitalization factor, $\nu = 70\%$, $e_L = 0.2$								
$r - g = 0\%$ ($G/R = 1$)	34%	46%	28%	38%	23%	31%	17%	23%
$r - g = 3\%$ ($G/R = 0.41$)	73%	78%	61%	65%	49%	52%	37%	39%
4. Sensitivity to bequests motives, $r - g = 2\%$ ($G/R = 0.55$), $e_L = 0.2$								
$\nu = 1$ (100% bequest motives)	48%	58%	40%	48%	32%	38%	24%	29%
$\nu = 0$ (no bequest motives)	100%	100%	83%	83%	67%	67%	50%	50%
5. Sensitivity to labor income elasticity, $r - g = 2\%$ ($G/R = 0.55$), $\nu = 70\%$								
$e_L = 0$	60%	68%	50%	56%	40%	45%	30%	34%
$e_L = 0.5$	69%	75%	57%	62%	46%	50%	34%	37%

This table presents simulations of the optimal inheritance tax rate τ_B using formula (13) for temptation and no-temptation economies. We set labor income tax rate to 30%. Parameters $\bar{b}^{\text{received}}$, \bar{b}^{left} , \bar{y}_L are taken from Piketty and Saez (2013).

in both economies, increasing the gap prescribes higher optimal rates. As in above, the optimal rates in the temptation economy are relatively low.

In our benchmark economy, we set the bequest strength parameter $\nu = 0.7$. When we set it to 1, the optimal rates are relatively lower compared to the benchmark economy. In contrast, when we assume the complete absence of bequest motive (i.e. $\nu = 0$), e_B becomes the only limiting factor for tax rates in both temptation and no temptation economies. Hence, optimal tax rates are higher. This is the only case, in which the existence of temptation does not affect the results.

The changes in the labor supply elasticity has a moderate effect. As it is expected, a higher labor supply elasticity prescribes higher taxes on inheritance, both under the economy with or without temptation. Exactly the opposite happens when it is lower.

In Figure 3, we set $\alpha = 0.8$, $\lambda = 0.6$ and vary the value of the parameter ν to explore the interaction between the parameter ν and temptation parameters. This figure demonstrates that the optimal inheritance tax rates are substantially lower in economies where individuals are more altruistic and/or they have self-control issues. Interestingly, optimal rates in the temptation economy when $\nu = 0.7$ are almost identical to optimal rates in the no temptation economy when $\nu = 1$. This result illustrates the fact that there exists a high degree of substitution between altruism and temptation parameters.

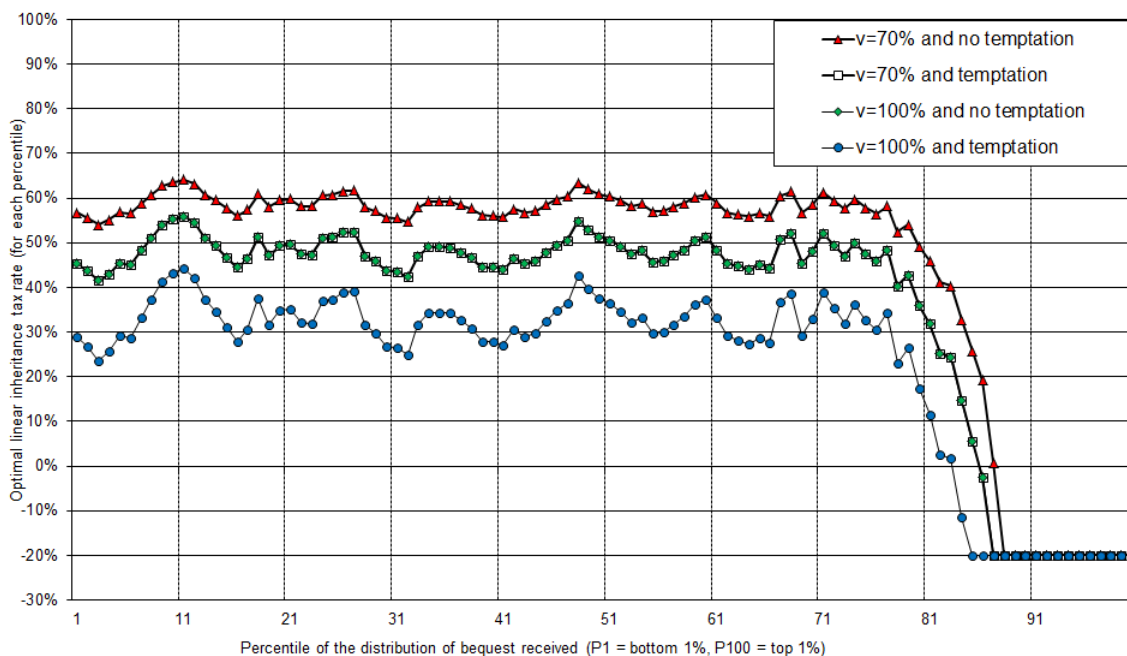


Figure 3: Optimal linear inheritance tax rates, by percentile of $\bar{b}^{\text{received}}$ (temptation is fixed at $\alpha=0.8$ and $\lambda=0.6$; ν varies)

5 Conclusion

Incorporating temptation and self control in the analysis of inheritance tax reveals some interesting results. The added benefit of such a study is that the findings regarding inheritance tax are comparable to some of the important results that exist in the literature of capital taxation. First, we clearly derive the expression for the optimal inheritance tax rate when agents are impatient and suffer from the problem of temptation and self control. We then show that in this framework, there is a negative relationship between the optimal inheritance tax rate and the level of temptation. This feature of the optimal tax in fact leads to a recommendation of a subsidy at any percentile of bequest received when temptation is critical. This result is in line with the existing literature (See Krusell et al. (2010)) on the capital tax which reveals that a subsidy on saving can provide incentive for higher level of saving by making succumbing to temptation less attractive. However, unlike Krusell et al. (2010), our finding recommends a zero tax in the long run and therefore the celebrated result of Chamley - Judd holds even when the preferences are subject to temptation and self control issue. This result is due to the fact that the elasticity of the present discounted value of the tax base with respect to an increase in tax is infinite in Chamley - Judd. However, in the parent child version of the model similar to Farhi and Werning (2010) but with the added feature of temptation and self control, we show that if dynamic efficiency holds, a subsidy is always the optimal which is in contrast to Piketty and Saez (2013).

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Appendix

A.1 Optimal Inheritance tax under steady state dynasty

Here we assume that instead of choosing b_{ti} optimally as assumed in section 3, it is given for all t . However the assumption that when the agents succumbed fully to temptation, they leave no bequest is also valid here. As usual,

$$\begin{aligned} SWF = \max_{\tau_B, \tau_L} (1 + \lambda) \int_i \sum_{t=0}^{\infty} \delta^t u^{ti} (R(1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E - b_{t+1i}, 1 - l_{ti}) \\ - \lambda \int_i \sum_{t=0}^{\infty} \delta^t u^{ti} (R(1 - \tau_B) b_{ti} + (1 - \tau_L) y_{Lti} + E, 1 - l_{ti}) \end{aligned}$$

subject to period-wise budget constraint. Therefore,

$$\begin{aligned} dSWF = (1 + \lambda) \int_i u_c^{0i} \cdot (R(1 - \tau_B) db_{0i} - Rb_{0i} d\tau_B) - (1 + \lambda) \sum_{t=0}^{\infty} \delta^{t+1} \int_i R u_c^{t+1i} \cdot b_{t+1i} d\tau_B \\ - (1 + \lambda) \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot y_{Lti} d\tau_L - \lambda \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot (R(1 - \tau_B) db_{ti} - Rb_{ti} d\tau_B - y_{Lti} d\tau_L). \end{aligned}$$

In this setup, the modified expression for τ_B^{temp} , given τ_L , is given by

$$\tau_B^{\text{temp}} = \frac{1 - \left[1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \left[\frac{(1 - \delta + \lambda(1 - \alpha - \delta)) \bar{b}^{\text{received}} (1 + \hat{e}_B)}{(1 + \lambda(1 - \alpha)) \bar{y}_L} + \frac{(1 + \lambda)}{R(1 + \lambda(1 - \alpha)) \bar{y}_L} \bar{b}^{\text{left}} \right]}{1 + e_B - \left[1 - \frac{e_L \tau_L}{1 - \tau_L} \right] \left[\frac{(1 - \delta + \lambda(1 - \alpha - \delta)) \bar{b}^{\text{received}}}{(1 + \lambda(1 - \alpha)) \bar{y}_L} (1 + \hat{e}_B) \right]}.$$

where τ_B^{temp} increases with the level of temptation.

Let us mention the steps to get the above expression for τ_B^{temp} . Using the first order condition of the individual utility maximization $u_c^{t+1i} b_{t+1i} = \frac{u_c^{t+1i} b_{t+1i}}{\delta R(1 - \tau_B)}$ and $R(1 - \tau_B) db_{ti} - Rb_{ti} d\tau_B =$

$-Rb_{ti}d\tau_B(1 + e_{Bi})$, we get

$$\begin{aligned}
dSWF &= -(1 + \lambda) \int_i u_c^{0i} \cdot b_{0i} (1 + e_{Bi}) R d\tau_B - \frac{1 + \lambda}{1 - \tau_B} \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot b_{t+1i} d\tau_B \\
&\quad + (1 + \lambda) R d\tau_B \frac{1 - \frac{e_{B\tau_B}}{1 - \tau_B}}{1 - \frac{e_{L\tau_L}}{1 - \tau_L}} \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot \frac{y_{Lti}}{y_{Lt}} b_t \\
&\quad + \lambda \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot b_{ti} (1 + e_{Bti}) R d\tau_B - \lambda \frac{1 - \frac{e_{B\tau_B}}{1 - \tau_B}}{1 - \frac{e_{L\tau_L}}{1 - \tau_L}} R d\tau_B \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot \frac{y_{Lti}}{y_{Lt}} b_t,
\end{aligned}$$

where $e_{Bi} = \frac{db_{0i}}{d(1-\tau_B)} \frac{1-\tau_B}{b_{0i}}$ and $e_{Bti} = \frac{db_{ti}}{d(1-\tau_B)} \frac{1-\tau_B}{b_{ti}}$. Setting $dSWF=0$ at the optimum τ_B , we get the following

$$\begin{aligned}
0 &= -(1 + \lambda) \frac{\int_i u_c^{0i} \cdot b_{0i} (1 + e_{Bi})}{b_0 \int_i u_c^{0i}} - \frac{1 + \lambda}{R(1 - \tau_B)} \sum_{t=0}^{\infty} \delta^t \frac{\int_i u_c^{ti} \cdot b_{t+1i}}{b_{t+1} \int_i u_c^{ti}} + (1 + \lambda) \sum_{t=0}^{\infty} \delta^t \frac{1 - \frac{e_{B\tau_B}}{1 - \tau_B}}{1 - \frac{e_{L\tau_L}}{1 - \tau_L}} \frac{\int_i u_c^{ti} \cdot y_{Lti}}{y_{Lt} \int_i u_c^{ti}} \\
&\quad + \lambda \sum_{t=0}^{\infty} \delta^t \frac{\int_i u_c^{ti} \cdot b_{ti} (1 + e_{Bti})}{b_t \int_i u_c^{ti}} \frac{\int_i u_c^{ti}}{\int_i u_c^{ti}} - \lambda \frac{1 - \frac{e_{B\tau_B}}{1 - \tau_B}}{1 - \frac{e_{L\tau_L}}{1 - \tau_L}} \sum_{t=0}^{\infty} \delta^t \frac{\int_i u_c^{ti} \cdot y_{Lti}}{y_{Lt} \int_i u_c^{ti}} \frac{\int_i u_c^{ti}}{\int_i u_c^{ti}}.
\end{aligned}$$

Since $\bar{b}^{\text{left}} = \delta R(1 - \tau_B) \bar{b}^{\text{received}}$, from the above equation we get

$$0 = -(1 - \delta + \lambda(1 - \alpha - \delta)) \bar{b}^{\text{received}} (1 + \hat{e}_B) - \frac{1 + \lambda}{R(1 - \tau_B)} \bar{b}^{\text{left}} + \frac{(1 + \lambda(1 - \alpha)) \left(1 - \frac{e_{B\tau_B}}{1 - \tau_B}\right)}{1 - \frac{e_{L\tau_L}}{1 - \tau_L}} \bar{y}_L$$

which after rearrangement of terms generates the expression for the optimal tax rate

$$\tau_B^{\text{temp}} = \frac{1 - \left[1 - \frac{e_{L\tau_L}}{1 - \tau_L}\right] \left[\frac{1 - \delta + \lambda(1 - \alpha - \delta)}{1 + \lambda(1 - \alpha)} \frac{1 + \hat{e}_B}{\delta} + \frac{1 + \lambda}{1 + \lambda(1 - \alpha)}\right] \bar{b}^{\text{left}}}{1 + e_B R \bar{y}_L}.$$

Further, it is straightforward that $\frac{d\tau_B^{\text{temp}}}{d\lambda} > 0$ since

$$\begin{aligned}
\frac{d\tau_B^{\text{temp}}}{d\lambda} &= - \frac{\left[1 - \frac{e_{L\tau_L}}{1 - \tau_L}\right]}{1 + e_B} \left[- \frac{\alpha \delta}{[1 + \lambda(1 - \alpha)]^2} \frac{1 + \hat{e}_B}{\delta} + \frac{\alpha}{[1 + \lambda(1 - \alpha)]^2} \right] \frac{\bar{b}^{\text{left}}}{R \bar{y}_L} \\
&= \frac{\left[1 - \frac{e_{L\tau_L}}{1 - \tau_L}\right] \bar{b}^{\text{left}} \alpha \hat{e}_B}{(1 + e_B) R \bar{y}_L [1 + \lambda(1 - \alpha)]^2} > 0.
\end{aligned}$$

A.2.1 Optimal inheritance tax from period zero perspective

Optimal Inheritance tax under inelastic labor

Individual agent's problem under the assumption of inelastic labor supply (where we assume $\tau_L = 0$) is given by

$$\begin{aligned} & \max_{\{b_{t+1i}\}_{t=0}^{\infty}} (1 + \lambda) \sum_{t=0}^{\infty} \delta^t E_t u^{ti} (R(1 - \tau_{Bt}) b_{ti} + y_{Lti} + E_t - b_{t+1i}) \\ & - \lambda \sum_{t=0}^{\infty} \delta^t E_t u^{ti} (R(1 - \tau_{Bt}) b_{ti} + y_{Lti} + E_t). \end{aligned}$$

Further, in this set up,

$$\begin{aligned} SWF &= \max_{\{\tau_{Bt}\}_{t=0}^{\infty}} (1 + \lambda) \sum_{t=0}^{\infty} \delta^t \int_i u^{ti} (R(1 - \tau_{Bt}) b_{ti} + y_{Lti} + E_t - b_{t+1i}) \\ & - \lambda \sum_{t=0}^{\infty} \delta^t \int_i u^{ti} (R(1 - \tau_{Bt}) b_{ti} + y_{Lti} + E_t) \end{aligned}$$

subject to period-wise budget constraint. We follow the same procedure here but because of the changed assumption, new $dSWF$ can be expressed as

$$\begin{aligned} dSWF &= (1 + \lambda) \sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti} \cdot (-Rb_{ti}d\tau_B + Rb_t d\tau_B) + (1 + \lambda) \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti} \cdot \tau_{Bt} Rdb_t \\ & - \lambda \sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot (R(1 - \tau_{Bt}) db_{ti}) - \lambda \sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti} \cdot (-Rb_{ti}d\tau_B + Rb_t d\tau_B) - \lambda \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti} \cdot \tau_{Bt} Rdb_t. \end{aligned}$$

Technically the difference appears because when we assume that b_{ti} s are just given instead of chosen them optimally, the envelope conditions become different. The modified expression for τ_B^{temp} , given τ_L , is then given by

$$\tau_B^{\text{temp}} = \frac{1 - \frac{\bar{b}^{\text{left}}}{\delta R} + \frac{\alpha \lambda \hat{e}_B}{1 + \lambda(1 - \alpha)} \frac{\bar{b}^{\text{left}}}{\delta^{T+1} R}}{1 + e_B^{\text{pdv}}},$$

where the optimal tax rate τ_B^{temp} increases with the level of temptation. By setting $dSWF = 0$ and then dividing it by $Rd\tau_B b_t \int_i u_c^{ti}$ we get

$$0 = (1 + \lambda(1 - \alpha)) \sum_{t=T}^{\infty} \delta^t \left(1 - \bar{b}^{\text{received}}\right) - \frac{\tau_{Bt}}{1 - \tau_{Bt}} (1 + \lambda(1 - \alpha)) \sum_{t=1}^{\infty} \delta^t e_{Bt} + \alpha \lambda \sum_{t=0}^{\infty} \delta^t \bar{b}^{\text{received}} \hat{e}_B$$

since

$$\frac{\int_i u_c^{ti} \cdot b_{ti} e_{Bti}}{b_t \int_i u_c^{ti}} = \frac{\int_i u_c^{ti} \cdot b_{ti} e_{Bti}}{\int_i u_c^{ti} \cdot b_{ti}} \frac{\int_i u_c^{ti} \cdot b_{ti}}{b_t \int_i u_c^{ti}} = \widehat{e}_B \bar{b}^{\text{received}}.$$

This can further be reduced to the following equation

$$0 = (1 + \lambda(1 - \alpha)) \left[1 - \bar{b}^{\text{received}} - \frac{\tau_B}{1 - \tau_B} e_B^{\text{pdv}} \right] + \frac{\alpha \lambda \bar{b}^{\text{received}} \widehat{e}_B}{\delta^T},$$

and using the relationship $\bar{b}^{\text{left}} = \delta R(1 - \tau_B) \bar{b}^{\text{received}}$, we finally get

$$\tau_B^{\text{temp}} = \frac{1 - \frac{\bar{b}^{\text{left}}}{\delta R} + \frac{\alpha \lambda \widehat{e}_B}{1 + \lambda(1 - \alpha)} \frac{\bar{b}^{\text{left}}}{\delta^{T+1} R}}{1 + e_B^{\text{pdv}}},$$

where e_B^{pdv} is defined as in the main text. It is straightforward to verify that $\frac{d\tau_B^{\text{temp}}}{d\lambda} > 0$.

A.2.2 Optimal Inheritance tax under elastic labor

When supply of labor is elastic, individual's optimization problem becomes

$$\begin{aligned} \max_{\{b_{t+1i}, l_{ti}\}_{t=0}^{\infty}} & (1 + \lambda) \sum_{t=0}^{\infty} \delta^t E_t u^{ti} (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t - b_{t+1i}, 1 - l_{ti}) \\ & - \lambda \sum_{t=0}^{\infty} \delta^t E_t u^{ti} (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) y_{Lti} + E_t, 1 - l_{ti}). \end{aligned}$$

Therefore the first order condition with respect to b_{t+1i} is given by

$$u_c^{ti} = \delta R(1 - \tau_{Bt+1}) E_t u_c^{t+1i}.$$

Government's optimization problem in this economy can be written as

$$\begin{aligned} SWF = \max_{\{\tau_{Bt}, \tau_{Lt}\}_{t=0}^{\infty}} & (1 + \lambda) \sum_{t=0}^{\infty} \delta^t \int_i u^{ti} (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) w_{ti} l_{ti} + E_t - b_{t+1i}, 1 - l_{ti}) \\ & - \lambda \sum_{t=0}^{\infty} \delta^t \int_i u^{ti} (R(1 - \tau_{Bt}) b_{ti} + (1 - \tau_{Lt}) w_{ti} l_{ti} + E_t, 1 - l_{ti}) \end{aligned}$$

subject to period-wise budget balance. We derive the expression for modified optimum tax rate

$$\tau_B^{\text{temp}} = \frac{1 - \left[1 - \frac{e_L^{\text{pdv}} \tau_L}{1 - \tau_L} \right] \left[1 - \frac{\alpha \lambda \widehat{e}_B^{\text{pdv}}}{1 + \lambda(1 - \alpha)} \right] \frac{\bar{b}^{\text{left}}}{\delta R \bar{y}_L}}{1 + e_B^{\text{pdv}}}.$$

Here too we observe that τ_B^{temp} increases with the level of temptation λ .

Regarding the proof, we start with the two equations

$$-Rb_t d\tau_B \frac{e_{Bt}\tau_B}{1-\tau_B} = -d\tau_{Lt} y_{Lt} \left(1 - \frac{e_{Lt}\tau_L}{1-\tau_L}\right), t < T$$

and

$$Rb_t d\tau_B \left(1 - \frac{e_{Bt}\tau_B}{1-\tau_B}\right) = -d\tau_{Lt} y_{Lt} \left(1 - \frac{e_{Lt}\tau_L}{1-\tau_L}\right), t \geq T.$$

We derive and set $dSWF=0$ which gives us

$$\begin{aligned} 0 = (1 + \lambda) & \left[-\sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti} \cdot Rb_{ti} d\tau_B - \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti} \cdot y_{Lti} d\tau_{Lt} \right] \\ & - \lambda \left[\sum_{t=0}^{\infty} \delta^t \int_i u_c^{ti} \cdot R(1 - \tau_{Bt}) db_{ti} - \sum_{t=T}^{\infty} \delta^t \int_i u_c^{ti} \cdot Rb_{ti} d\tau_{Bt} - \sum_{t=1}^{\infty} \delta^t \int_i u_c^{ti} \cdot y_{Lti} d\tau_{Lt} \right]. \end{aligned}$$

Dividing the above equation by $\int_i u_c^{ti} \cdot Rb_t d\tau_B$ we get

$$\begin{aligned} 0 = (1 + \lambda) & \left[-\sum_{t=T}^{\infty} \delta^t \frac{\int_i u_c^{ti} b_{ti}}{b_t \int_i u_c^{ti}} + \sum_{t=T}^{\infty} \delta^t \frac{\int_i u_c^{ti} y_{Lti}}{y_{Lt} \int_i u_c^{ti}} \frac{\left(1 - \frac{e_{Bt}\tau_B}{1-\tau_B}\right)}{\left(1 - \frac{e_{Lt}\tau_L}{1-\tau_L}\right)} - \sum_{t=1}^{T-1} \delta^t \frac{\int_i u_c^{ti} y_{Lti}}{y_{Lt} \int_i u_c^{ti}} \frac{\left(\frac{e_{Bt}\tau_B}{1-\tau_B}\right)}{\left(1 - \frac{e_{Lt}\tau_L}{1-\tau_L}\right)} \right] \\ & - \lambda \left[-\sum_{t=0}^{\infty} \delta^t \cdot \frac{\int_i u_c^{ti} b_{ti} e_{Bti}}{\int_i u_c^{ti} b_{ti}} \cdot \frac{\int_i u_c^{ti}}{\int_i u_c^{ti}} - \sum_{t=T}^{\infty} \delta^t \frac{\int_i u_c^{ti} b_{ti}}{b_t \int_i u_c^{ti}} \frac{\int_i u_c^{ti}}{\int_i u_c^{ti}} + \sum_{t=T}^{\infty} \delta^t \frac{\int_i u_c^{ti} y_{Lti}}{\int_i u_c^{ti} y_{Lt}} \frac{\left(1 - \frac{e_{Bt}\tau_B}{1-\tau_B}\right) \int_i u_c^{ti}}{\left(1 - \frac{e_{Lt}\tau_L}{1-\tau_L}\right) \int_i u_c^{ti}} \right. \\ & \left. - \sum_{t=1}^{T-1} \delta^t \frac{\int_i u_c^{ti} y_{Lti}}{\int_i u_c^{ti} y_{Lt}} \frac{\left(\frac{e_{Bt}\tau_B}{1-\tau_B}\right) \int_i u_c^{ti}}{\left(1 - \frac{e_{Lt}\tau_L}{1-\tau_L}\right) \int_i u_c^{ti}} \right], \end{aligned}$$

which can be finally reduced to

$$\begin{aligned} 0 = (1 + \lambda(1 - \alpha)) & \left[-\bar{b}^{\text{received}} + \bar{y}_L \left\{ (1 - \delta) \sum_{t=T}^{\infty} \delta^{t-T} \frac{\left(1 - \frac{e_{Bt}\tau_B}{1-\tau_B}\right)}{\left(1 - \frac{e_{Lt}\tau_L}{1-\tau_L}\right)} - (1 - \delta) \sum_{t=1}^{T-1} \delta^{t-T} \frac{\left(\frac{e_{Bt}\tau_B}{1-\tau_B}\right)}{\left(1 - \frac{e_{Lt}\tau_L}{1-\tau_L}\right)} \right\} \right] \\ & + \alpha \lambda \bar{b}^{\text{received}} (1 - \delta) \sum_{t=0}^{\infty} \delta^{t-T} \hat{e}_{Bt}, \end{aligned}$$

where $\hat{e}_{Bt} = \frac{\int_i u_c^{ti} b_{ti} e_{Bti}}{\int_i u_c^{ti} b_{ti}}$. Since \hat{e}_B^{pdv} , e_B^{pdv} , and e_L^{pdv} satisfy

$$\begin{aligned} e_B^{\text{pdv}} &= e_B^{\text{post}} + e_B^{\text{anticip.}}, \quad e_B^{\text{post}} = (1 - \delta) \sum_{t=T}^{\infty} \delta^{t-T} e_{Bt}, \quad e_B^{\text{anticip.}} = (1 - \delta) \sum_{t=1}^{T-1} \delta^{t-T} e_{Bt} \\ \hat{e}_B^{\text{pdv}} &= (1 - \delta) \sum_{t=0}^{\infty} \delta^{t-T} \hat{e}_{Bt} \end{aligned}$$

$$\frac{1 - \frac{e_B^{\text{pdv}} \tau_B}{1 - \tau_B}}{1 - \frac{e_L^{\text{pdv}} \tau_L}{1 - \tau_L}} = (1 - \delta) \sum_{t=T}^{\infty} \delta^{t-T} \frac{\left(1 - \frac{e_{Bt} \tau_B}{1 - \tau_B}\right)}{\left(1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}\right)} - (1 - \delta) \sum_{t=1}^{T-1} \delta^{t-T} \frac{\left(\frac{e_{Bt} \tau_B}{1 - \tau_B}\right)}{\left(1 - \frac{e_{Lt} \tau_L}{1 - \tau_L}\right)},$$

and given $\bar{b}^{\text{left}} = \delta R(1 - \tau_B) \bar{b}^{\text{received}}$ holds, the above equation can be written as

$$0 = (1 + \lambda(1 - \alpha)) \left[-\bar{b}^{\text{received}} + \bar{y}_L \frac{1 - \frac{e_B^{\text{pdv}} \tau_B}{1 - \tau_B}}{1 - \frac{e_L^{\text{pdv}} \tau_L}{1 - \tau_L}} \right] + \alpha \lambda \bar{b}^{\text{received}} \hat{e}_B^{\text{pdv}}.$$

Rearrangement of the above equation gives us

$$\tau_B^{\text{temp}} = \frac{1 - \left[1 - \frac{e_L^{\text{pdv}} \tau_L}{1 - \tau_L}\right] \left[1 - \frac{\alpha \lambda \hat{e}_B^{\text{pdv}}}{1 + \lambda(1 - \alpha)}\right] \frac{\bar{b}^{\text{left}}}{\delta R \bar{y}_L}}{1 + e_B^{\text{pdv}}}.$$

Further, it can be verified that here too $\frac{d\tau_B^{\text{temp}}}{d\lambda} > 0$.

The above analysis guarantees that the optimal inheritance tax rate can have a positive relationship with λ due to a change in assumption on b_{ti} .