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Adequacy, Fairness and Sustainability of Pay-As-You-GO-Pension-Systems: Defined Benefit versus Defined Contribution.

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Adequacy, Fairness and Sustainability of Pay-As-You-Go-Pension-Systems: Defined Benefit versus Defined Contribution

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Abstract

There are three main challenges facing public pension systems. First, pension systems need to provide an adequate income for pensioners in the retirement phase. Second, participants wish a fair level of benefits in relation to the contributions paid. Last but no least, the pension system would need to be financially sustainable in the long run. In this paper, we analyse defined benefit versus defined contribution schemes in terms of adequacy, fairness and sustainability jointly. Also, risk sharing mechanisms, that involve changes in the key variables of the system, are designed to restore the financial sustainability at the same time that we study their consequences on the adequacy and fairness of the system.

JEL: **E62, H55, J26**

Keywords: Pay-as-you-go, Public pensions, Risks, Sustainability.

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1 Introduction

Public pension systems are usually financed on a pay-as-you-go (PAYG) basis where pensions for retirees are paid by the contributions of the working-age population. As a result of the decrease in birth rates and increase in life expectancy combined with the baby boom, a common trend in some European countries has been a wave of parametric or even structural reforms, by changing the formula to calculate the initial pension from a Defined Benefit (DB) to a Defined Contribution (DC), with the aim of reducing the expenditure on pensions (Whitehouse 2012).

DC unfunded pension systems (also called Notional Defined Contribution accounts) has some positive features, such as facing the population ageing more or less automatically or improving the relationship between contributions and pension paid (Palmer 2006). However these schemes do not guarantee sustainability, due to the PAYG nature of the system(Valdés-Prieto 2000; Palmer 2013), or secure an adequate level of benefits at all times. In this line, Auerbach and Lee (2006), Auerbach and Lee (2011) and Auerbach et al. (2013) study numerically the fiscal sustainability of NDCs, their performance in regards of risk-spreading among generations and how economic and demographic shocks are spread among different generations. However, their numerical approach does not study jointly the pension adequacy, the actuarial fairness and the sustainability of the system.

For policymakers, a desirable pension system should be financially sustainable but at the same time should also provide an adequate income for pensioners in the retirement phase (adequacy), a fair level of benefits in relation to the contributions paid (actuarial fairness).[1] There is an obvious trade-off between adequacy and affordability as higher the level of benefits resulting from adequacy will have an impact on sustainability.

With the aim to guarantee the sustainability of the pension system some countries have included some risk-sharing mechanisms. These adjustments, Vidal-Meliá et al. (2009, 2010) can be defined as a set of pre-determined measures established by law to be applied immediately according to an indicator of the financial health of the system. In this sense, D'Addio and Whitehouse (2012), adjustments can be made on benefit levels, revaluation of contribution bases or indexation of pension in payments.

In practice, countries like Sweden, Canada, Germany and Japan, among others, have a combination of risk-sharing adjustments that affect to both contributors and pensioners of state pension systems. In particular, in Sweden and Japan, an asymmetric[2] mechanism is applied to both the contribution bases and indexation of pensions while Canada and Germany adjust adjust both contribution rate and indexation of pensions (Börsch-Supan et al. 2004; Vidal-Meliá et al. 2009). However, as far as we know, the risk sharing between the participants do not have any theoretical basis.

This paper designs, from a theoretical point of view, flexible risk sharing mechanisms, that involve changes in the contribution rate and/or indexation of pensions, to restore the sustainability of the different pension schemes. At the same time we aim to shed some light on the consequences of such mechanisms on the adequacy and fairness of de-

[1] See (Queisser and Whitehouse 2006) for more details on actuarial fairness.

[2] The asymmetric mechanisms are designed to face adverse demographic and economic changes. On the contrary, the symmetric mechanism, Alho et al. (2013), is adjusted for both positive and negative deviation of the financial health of the system.

fined benefit (DB) and defined contribution (DC) schemes. This research will certainly contribute to the debate on pension finance in the sense that sustainability, adequacy and fairness are jointly analysed.

The remainder of the article is structured as follows. Section 2 describes the dynamic overlapping generations model for the population and how sustainability, adequacy and fairness are measured. Section 3 develops a flexible risk-sharing mechanism which restores the sustainability into the pension scheme. Section 4 provides an illustration of the impact of various risk-sharing mechanisms on the adequacy, actuarial fairness and sustainability. Finally Section 5 concludes.

2 A dynamic overlapping generations model

This section describes the demographic-economic structure, the calculation of the sustainability indicator and finally the expressions to compute adequacy and fairness in DB and DC schemes. We also show these expressions for the particular case of the steady state and show the assumptions hold so that DB and DC schemes are equivalent.

2.1 Population and salary dynamics

The demographic-economic structure at any time t is represented as follows:

Age:

$$x = \overbrace{x_0, x_0+1, ...x_r-2, x_r-1}^{\text{Contributors}}, \underbrace{x_r, x_r+1, ..., \omega-2, \omega-1}_{\text{Pensioners}}$$

Population at time t:

$$N_t^x = N_{t-x+x_0}^{x_0} \cdot {}_{x-x_0}p_{x_0}(t-x+x_0), \tag{2.1}$$

where

N_t^x denotes individuals aged x for x =x_0, ..., $\omega-1$ who are alive at time $t>0$ and joined the labour market at time $t-x+x_0$. ω is the last possible lifespan, that is, the population aged ω is equal to zero.

We assume that individuals only enter the system at the age of x_0. In particular, the entries at age x_0 vary at the rate n_t, i.e.:

$$N_t^{x_0} = N_{t-1}^{x_0}(1+n_t) \tag{2.2}$$

${}_{x-x_0}p_{x_0}(t-x+x_0)$ is the survival probability from age x_0 at time $t-x+x_0$ to age x by time t.

Formula 2.1 indicates that the population at time t only depends on the individuals who have survived for the time elapsed between the previous period $t-1$ and t, implying that the population is closed to migration[3].

Individual salaries at time t

$$W_t^x = W_{t-1}^x \left(1+g_t\right), \tag{2.3}$$

where

W_t^x denotes the individual salaries for $x = x_0, ..., x_r - 1$ at time $t > 0$ which are earned by the active population and are assumed to be paid at the beginning of the calendar year.
The retirement age is represented by x_r and is assumed to be constant[4].

Wages are age and time dependent in line with empirical evidence.[5]

g_t is the rate of salary variation from the period $t-1$ to period t.

2.2 Liquidity (Sustainability) indicator

We measure the sustainability of the system in terms of the liquidity indicator that compares the income from contributions, together with financial assets, and pension expenditures in one particular year[6]. Our scheme is sustainable in the long run in the sense that we are restoring the liquidity on an annual basis. Formally, the ratio at time t, LR_t is represented as follows:

$$LR_t = \frac{C_t + F_t^-}{P_t}, \tag{2.4}$$

where

C_t represents the income from contributions at time t.

P_t represents the total pension expenditures at time t.

F_t^- represents the value of the (buffer) fund at time t, also called reserve fund, before new contributions and benefits payments are considered. F_t^- can be expressed as follows:

[3] As (Settergren and Mikula 2005; OECD 2015) we are not considering migration in our analysis. However, in practice migration plays an important role in the population dynamics of most European countries (Eurostat 2011, 2012).

[4] In practice, the retirement age tends to be linked to the increase in life expectancy (Knell (2012) and Chłón-Domińczak et al. (2012).)

[5] In some countries there is an evidence of an inverted U-shaped wage path that peaks in middle age and declines smoothly thereafter (Blanchflower and Oswald 1990; Groot et al. 1992; Sessions 1993).

[6] We use the concept of one-period liquidity following from the works of Haberman and Zimbidis (2002); Gannon et al. (2014); Godínez-Olivares et al. (2016); Alonso-García and Devolder (2016).

$$F_t^- = F_{t-1}^+(1+i_t) = F_0^- \prod_{j=1}^{t}(1+i_j) + \sum_{j=0}^{t-1}(C_j - P_j) \prod_{k=j+1}^{t}(1+i_k) \tag{2.5}$$

where

i_t represents the financial rate of return of the fund from period $t-1$ to t.

The value of the fund at time t after contributions and payments is given by

$$F_t^+ = F_t^- + C_t - P_t. \tag{2.6}$$

2.2.1 Income from contributions

The income from contributions received by the pension system at time t, C_t, is represented as follows:

$$C_t = \pi_t \sum_{x=x_0}^{x_r-1} W_t^x N_t^x, \tag{2.7}$$

where

π_t is the contribution rate at time t of the pension system.

Remark 1. The income from contributions (2.7) can be rewritten in terms of mortality and contribution rates as follows:

$$\begin{aligned} C_t &= \pi_t W_t^{x_0} N_t^{x_0} + \pi_t \sum_{x=x_0+1}^{x_r-1} W_{t-1}^{x-1}(1+g_t) N_{t-1}^{x-1} \cdot \underbrace{{}_1p_{x-1}(t-1)}_{1-{}_1q_{x-1}(t-1)} \\ &= \frac{\pi_t}{\pi_{t-1}}(1+g_t) C_{t-1} + \pi_t W_t^{x_0} N_t^{x_0} - \pi_t W_t^{x_r-1} N_{t-1}^{x_r-1} \\ &- \pi_t \sum_{x=x_0+1}^{x_r-1} W_t^x N_{t-1}^x \cdot {}_1q_{x-1}(t-1), \end{aligned}$$

where

${}_1q_{x-1}(t-1)$ is the cohort's probability of dying of individuals aged $x-1$ at time $t-1$ before attaining age x by time t.

The income from contributions at time t increases with the age-independent salaries' increase g_t and with the contributions paid by the new entrants aged x_0. It decreases with the contributions ceased to be paid by individuals who just retired and with the contributions ceased to be paid by individuals who deceased between $t-1$ and t.

2.2.2 Pension expenditures

The pension expenditures paid by the pension system at time t, P_t, is represented as follows:

$$P_t = \sum_{x=x_r}^{\omega-1} P_t^x N_t^x, \tag{2.8}$$

where

P_t^x represents the individual pension paid to retirees aged x at time t and

$$P_t^x = P_{t-1}^{x-1}(1+\lambda_t), x \in [x_r+1, \omega-1], \tag{2.9}$$

where

λ_t is the pension's indexation rate from period $t-1$ to period t. Note that $P_t^{x_r}$, for $t > 0$ corresponds to the pension paid to individuals who have just entered retirement and is calculated according to the pension system design.

Remark 2. The pension expenditures (2.8) can be rewritten in terms of mortality and indexation as follows:

$$\begin{aligned} P_t &= P_t^{x_r} N_t^{x_r} + \sum_{x=x_r+1}^{\omega-1} P_{t-1}^{x-1}(1+\lambda_t) N_{t-1}^{x-1} \underbrace{{}_1p_{x-1}(t-1)}_{1-{}_1q_{x-1}(t-1)} \\ &= P_t^{x_r} N_t^{x_r} + (1+\lambda_t) \sum_{x=x_r+1}^{\omega-1} P_{t-1}^{x-1} N_{t-1}^{x-1} - \sum_{x=x_r+1}^{\omega-1} P_t^x N_{t-1}^{x-1} \cdot {}_1q_{x-1}(t-1) \\ &= (1+\lambda_t) P_{t-1} + P_t^{x_r} N_t^{x_r} - P_t^{\omega} N_{t-1}^{\omega-1} - \sum_{x=x_r+1}^{\omega-1} P_t^x N_{t-1}^{x-1} \cdot {}_1q_{x-1}(t-1). \end{aligned}$$

The pension expenditures at time t increases with the age-independent pension's increase λ_t and with the pensions paid to the new retirees aged x_r. It decreases with the pensions ceased to be paid to retirees who died at age ω and with the pensions ceased to be paid by retirees who deceased between $t-1$ and t.

Obviously, the expression of the initial pension $P_t^{x_r}$ depends on whether the pension's design is DC or DB.

Defined Benefit

DB pension systems are usually based on a percentage K_t, commonly known as replacement rate, of a wage-dependent amount $PS_t^{x_r}$, which we name pensionable salary. Mathematically, the initial pension for a retiring individual at time t is expressed as follows:

$$P_t^{x_r} = K_t \cdot PS_t^{x_r} \tag{2.10}$$

The most common expressions for the pensionable salary are:

$$\text{Mean wage revalorized: } PS_t^{x_r} = \frac{\sum_{x=x_0}^{x_r-1} W_{t-x_r+x}^x \prod_{j=t-x_r+x+1}^t (1+g_j)}{x_r - x_0} \quad (2.11)$$

$$\text{Last wage revalorized: } PS_t^{x_r} = W_{t-1}^{x_r-1}(1+g_t) = W_t^{x_r-1} \quad (2.12)$$

Defined Contribution

In the case of pay-as-you-go DC, also known as notional or non-financial DC, the pension at retirement depends on the notional capital saved throughout the working career $NC_t^{x_r}$ and the annuity factor $a_{x_r}(t)$. The pension capital in DC can be calculated in two ways: with or without the survivor dividend (SD) (also called inheritance gains).

The survivor dividend consists of the contributions made by individuals who do not survive until retirement. The sum of the contributions of those individuals represent the survivor dividend (SD). The government can choose to redistribute the balances within the same cohort, increasing the notional return (Vidal-Meliá et al. 2015) (Vidal-Meliá et al. 2015). In fact, this is the approach considered in Sweden, the only country which redistributes explicitly the balance of the deceased (Chłón-Domińczak et al. 2012)[7]. In this paper, we study as well the effect of distributing the SD within the cohort in terms of adequacy and actuarial fairness[8].

Mathematically the expression for the initial pension for the *individual* approach is expressed as follows:

$$P_t^{x_r} = \frac{NC_t^{x_r}}{a_{x_r}(t)} \quad (2.13)$$

where the notional capital for the *individual* $NC_t^{x_r}$ is expressed as follows:

$$NC_t^{x_r} = \sum_{x=x_0}^{x_r-1} \pi_{t-x_r+x} W_{t-x_r+x}^x \prod_{i=t-x_r+x+1}^{t} (1+nr_i) \quad (2.14)$$

where π_{t-x_r+x} is the contribution rate[9] at time $t-x_r+x$ and nr_i is the notional (virtual) rate of return on the pay-as-you-go contributions for the period $i-1$ to i. This notional rate is usually is set by law and can be set equal to any indicator of the financial health of the system, such as, growth rate of GDP, average wages or total income from contributions.

Mathematically the expression for the initial pension for the *cohort* approach which includes the SD is expressed as follows:

[7] Arnold et al. (2015) state that the SD could be used to finance unexpected longevity increases instead.

[8] In fact, we show that the DC with SD is actuarially fair on a cohort basis whenever the population is closed, as already shown in Boado-Penas and Vidal-Meliá (2014).

[9] Classical notional DC consider that the contribution rate is constant over time, shifting most of the financial burden on the retirees, see Palmer (2013).

$$P_t^{x_r} = \frac{NC_t^{x_r}}{a_{x_r(t)} N_t^{x_r}} \tag{2.15}$$

where the notional capital for the *cohort* $NC_t^{x_r}$ is expressed as follows:

$$NC_t^{x_r} = \sum_{x=x_0}^{x_r-1} \pi_{t-x_r+x} W_{t-x_r+x}^{x} N_{t-x_r+x}^{x} \prod_{i=t-x_r+x+1}^{t} (1+nr_i) \tag{2.16}$$

The SD are accounted for because the the contributions of everyone in the cohort are taken into account and because the first pension is dependent on the number of individuals retiring the same year as highlighted in formula (2.16).

The annuity factor depends on the indexation and discount rate as well as the life table chosen. Mathematically the annuity for the retiring cohort aged x_r at time t is represented as follows:

$$a_{x_r}(t) = \sum_{x=x_r}^{\omega-1} {}_{x-x_r}p_{x_r}(t-x+x_r) \prod_{j=x_r}^{x} \frac{1+\lambda_j}{1+nr_j} \tag{2.17}$$

For instance when the indexation equals the discounting rate the annuity $a_{x_r}(t)$ is then reduced to the expression of the life expectancy at retirement $e_{x_r;t}$[10].

2.3 Adequacy and actuarial fairness

According to Chomik and Piggott (2016), adequacy can be referred to as poverty alleviation or to the income replacement. Income replacement rates are sometimes represented as the first pension over the last salary. In this paper we define the the replacement rate as the proportion that the benefits represent over the average working age income for the same year. It is thus related to the 'Benefit ratio' commonly present in Aggregate Accounting methods (Roseveare et al. 1996; Boldrin et al. 1999; Dang et al. 2001; Jimeno et al. 2008). Mathematically, this replacement rate RR_t^x for an individual aged x at time t is represented as follows:

$$RR_t^x = \frac{P_t^x}{\sum_{x=x_0}^{x_r-1} W_t^x} (x_r - x_0) \tag{2.18}$$

where

P_t^x represent the pension payments for an individual aged x at time t.

[10]This is the annuity taken into account in Poland and Latvia (Chłón-Domińczak et al. 2012).

The replacement rate does not provide a longitudinal measure of the pension system.[11] A way to solve this problem is to calculate the actuarial fairness for a particular individual that can be defined as the difference between the value of the contributions at retirement and the present value of the benefits paid until the death of the individual (Queisser and Whitehouse 2006). A value equal to zero indicates that the system is actuarially fair, i.e, the cohort receives an equivalent value of benefits to the one they contributed. A value greater than (resp. lower than) 0 indicates that the cohort contributes more (less) than she receives.

Mathematically, the actuarial fairness per cohort and individual can be expressed respectively as follows:

$$\begin{aligned}
\text{Cohort}: & \sum_{x=x_0}^{x_r-1} \pi_{t-x_r+x} W^x_{t-x_r+x} N^x_{t-x_r+x} \prod_{i=t-x_r+x+1}^{t} (1+nr_i) \\
& - \sum_{x=x_r}^{\omega-1} P^x_{t-x_r+x} N^x_{t-x_r+x} \prod_{i=t}^{t-x_r+x+1} \frac{1}{1+nr_i} \\
\text{Individual}: & \sum_{x=x_0}^{x_r-1} {}_{x-x_r}p_{x_r}(t-x+x_r) \pi_{t-x_r+x} W^x_{t-x_r+x} \prod_{i=t-x_r+x+1}^{t} (1+nr_i) \\
& - \sum_{x=x_r}^{\omega-1} {}_{x-x_r}p_{x_r}(t-x+x_r) P^x_{t-x_r+x} \prod_{i=t}^{t-x_r+x+1} \frac{1}{1+nr_i}
\end{aligned}$$

where

nr_i is the notional rate, also known as internal rate of return, of the pension system.

2.4 Particular case: Steady State

Public pension systems are often studied when the economic and population is in the steady state with the aim of deriving elegant conclusions on their design and dynamics. In our context we say that our system is in 'steady state' when the wage, population growth and contribution rate are constant and when the survival probability is time-independent. Mathematically, the population (2.1) and wages (2.3) in steady state are expressed as follows:

$$N^x_t = N^{x_0}_{t-x+x_0} \cdot {}_{x-x_0}p_{x_0}, \tag{2.19}$$

$$N^{x_0}_{t-x+x_0} = N^{x_0}_{t-x+x_0-1} (1+n), \tag{2.20}$$

$$W^x_t = W^x_{t-1} (1+g). \tag{2.21}$$

The evolution of the income from contributions C_t (2.7) is this context is simplified as $C_t = C_{t-1}(1+g)(1+n)$. In a similar manner, the pensionable salary, notional capital and first pension in a DC scheme can be represented as follows:

[11] Also, it is not possible to assess the effect of subsequent applications of the risk-sharing mechanisms (RSM) that restore the sustainability of the system.

DB:

$$PS_t^{x_r} = PS_{t-1}^{x_r}(1+g), \tag{2.22}$$

$$P_t^{x_r} = P_{t-1}^{x_r}(1+g) \tag{2.23}$$

DC with SD:

$$NC_t^{x_r} = \sum_{x=x_0}^{x_r-1} \pi W_{t-x_r+x}^{x} N_{t-x_r+x}^{x} (1+nr)^{x_r-x} = (1+g)(1+n) NC_{t-1}^{x_r}, \tag{2.24}$$

$$P_t^{x_r} = \frac{NC_{t-1}^{x_r}(1+g)(1+n)}{a_{x_r} N_{t-1}^{x_r}(1+n)} = P_{t-1}^{x_r}(1+g), \tag{2.25}$$

DC without SD:

$$NC_t^{x_r} = \sum_{x=x_0}^{x_r-1} \pi W_{t-x_r+x}^{x} (1+nr)^{x_r-x} = (1+g) NC_{t-1}^{x_r}, \tag{2.26}$$

$$P_t^{x_r} = \frac{NC_{t-1}^{x_r}(1+g)}{a_{x_r}} = P_{t-1}^{x_r}(1+g). \tag{2.27}$$

Pension expenditures at time t,P_t (2.8), for a general pension system are then expressed as follows:

$$P_t = \sum_{x=x_r}^{\omega-1} P_{t-1}^{x} N_{t-1}^{x} (1+g)(1+n) = P_{t-1}(1+g)(1+n). \tag{2.28}$$

Remark 3. It is straightforward to note that the fund F_t^+ (2.6), assuming that the fund at inception is zero, i.e. $F_0^- = 0$, is simplified as follows:

$$F_t^+ = \sum_{j=0}^{t} (C_j - P_j)(1+i) = (C_0 - P_0)(1+i)^t \sum_{j=0}^{t} \left(\frac{(1+g)(1+n)}{1+i}\right)^j$$
$$= (C_0 - P_0)(1+i)^t \frac{\left(\frac{(1+g)(1+n)}{1+i}\right)^{t+1} - 1}{\frac{(1+g)(1+n)}{1+i} - 1} \tag{2.29}$$

The abovementioned expression shows that if the initial contribution rate is chosen such that there is a systematic surplus, i.e., $C_0 > P_0$, then the fund will be systematically accumulating funds. Alternatively, if the opposite holds, the fund will be systematically indebted. In particular, when the contribution rate is chosen such that $C_0 = P_0$, the fund will be equal to zero at all times. The expression (2.29) shows that there may be interest of using RSM even when the system is in steady state when the initial equilibrium is not guaranteed. However, in this non-dynamic environment, the RSM only works in one way, either reducing the benefits and increasing the contribution rate for a systematic deficit or increasing the benefits and reducing the contribution rate for a systematic surplus.

Steady state pension systems can be sustainable in the long run whenever the contribution rate is chosen carefully, as shown above. This result holds for the three pension systems separately. In terms of pension adequacy, the following proposition shows that the DB and DC with SD pension system provide the same amount of pension under certain circumstances.

Proposition 1. *The pension for the DB and DC with SD schemes are equal whenever the following assumptions hold:*

- *the pensionable salary of the DB formula evolves with wages,*
- *the notional capital is revalued by means of the canonical notional rate*[12]*,*
- *and the contribution rate is chosen such that the DB scheme is in financial equilibrium, i.e,* $C_0 = P_0$ *.*

In particular, this result holds for any choice of the coefficient K_t *multiplying the pensionable salary* $PS_t^{x_r}$*.*

Proof. Let the initial pension for the DB and the DC with SD be represented by P_t^{DB,x_r} and P_t^{DC,x_r} respectively:

$$P_t^{DB,x_r} = K \cdot PS_t^{x_r} = K \cdot PS_{t-1}^{x_r} (1+g) \tag{2.30}$$

$$P_t^{DC,x_r} = \frac{NC_t^{x_r}}{a_{x_r} N_t^{x_r}} = \frac{\pi \sum_{x=x_0}^{x_r-1} \overbrace{W_{t-x_r+x}^x (1+g)^{x-xr}}^{W_t^x} \overbrace{N_{t-x_r+x}^x (1+n)^{x-xr}}^{N_t^x}}{a_{x_r} N_t^{x_r}} \tag{2.31}$$

Let the contribution rate π equal to the one that makes the DB scheme initially liquid:

$$\pi = \frac{\sum_{x=x_r}^{\omega-1} P_t^{DB,x} N_t^x}{\sum_{x=x_0}^{x_r-1} W_t^x N_t^x}$$

In order to obtain the desired result, we need to rewrite the DB pension as follows:

$$P_t^{DB,x} = P_{t-x+x_r}^{DB,x_r} (1+\lambda)^{x-x_r} = P_t^{DB,x_r} \left(\frac{1+\lambda}{1+g}\right)^{x-x_r}$$

Then the contribution rate can be rewritten as follows:

$$\pi = \frac{P_t^{DB,x_r} \sum_{x=x_r}^{\omega-1} \overbrace{N_t^x (1+n)^{x-x_r}}^{N_{t-x_r+x}^x} \left(\frac{1+\lambda}{(1+n)(1+g)}\right)^{x-x_r}}{\sum_{x=x_0}^{x_r-1} W_t^x N_t^x}$$

[12] This rate is known as the 'natural rate' of the NDC scheme (Valdés-Prieto 2000; Börsch-Supan 2006) or the 'biological rate' of the economy (Samuelson 1958).

Finally, if we replace the contribution rate in (2.31) we observe that the amount of the pension under the DC with SD is equal to the DB pension scheme:

$$
\begin{aligned}
P_t^{DC,x_r} &= P_t^{DB,x_r} \frac{\sum_{x=x_r}^{\omega-1} N_{t-x_r+x}^x \left(\frac{1+\lambda}{(1+n)(1+g)}\right)^{x-x_r}}{\sum_{x=x_0}^{x_r-1} W_t^x N_t^x} \frac{\sum_{x=x_0}^{x_r-1} W_t^x N_t^x}{a_{x_r} N_t^{x_r}} \\
&= P_t^{DB,x_r} \underbrace{\frac{1}{a_{x_r}} \sum_{x=x_r}^{\omega-1} \frac{N_{t-x_r+x}^x}{N_t^{x_r}} \left(\frac{1+\lambda}{(1+n)(1+g)}\right)^{x-x_r}}_{1}
\end{aligned}
$$

□

3 Risk-sharing Mechanism (RSM)

Assuming that the salaries and population dynamics are exogenous, contribution rate and indexation of pension are the only parameters that be adjusted to restore the sustainability.

Let $\beta_t \in [0,1]$ be the time-dependent risk-sharing coefficient between the contributors and the pensioners and $F_t^- = 0$ the fund when the RSMs are put in place. The one-period deficit or surplus D_t (2.6) is then denoted by:

$$D_t = C_t - P_t. \tag{3.1}$$

The government shares the burden between the contributors and pensioners as follows:

- $\beta_t D_t$ is the share of the surplus/deficit borne by the contributors, and
- $(1-\beta_t) D_t$ is the share of the surplus/deficit borne by the pensioners.

Proposition 2 shows how the contribution and indexation rates need to be adjusted to achieve liquidity when the risk-sharing coefficient is given by β_t. First the following notation is needed:

$$C_t^* = \pi_{t-1} \sum_{x=x_0}^{x_r-1} W_t^x N_t^x, \tag{3.2}$$

$$C_t = \pi_t \sum_{x=x_0}^{x_r-1} W_t^x N_t^x, \tag{3.3}$$

$$P_t^* = P_t^{x_r} N_t^{x_r} + \sum_{x=x_r+1}^{\omega-1} P_{t-1}^{x-1} (1+\lambda_t^*) N_t^x, \tag{3.4}$$

$$P_t = P_t^{x_r} N_t^{x_r} + \sum_{x=x_r+1}^{\omega-1} P_{t-1}^{x-1} (1+\lambda_t) N_t^x. \tag{3.5}$$

where

C_t^* and P_t^* are the income from contributions and pension expenditures respectively before the application of the RSMs which ensure one-period liquidity. The income from contributions at time t is calculated taking into account the previous period contribution rate π_{t-1} while pension expenditures are calculated by means of the observed indexation rate λ_t^* in absence of RSMs.

C_t and P_t are given by (2.7) and (2.8) and represent respectively the income from contributions and pension expenditures after the application of the RSM. The contribution and indexation rate are represented as follows:

$$\pi_t = \pi_{t-1}\left(1+\alpha_t^{\pi}\right), \tag{3.6}$$

$$\lambda_t = \left(1+\lambda_t^*\right)\left(1+\alpha_t^{\lambda}\right) - 1, \tag{3.7}$$

where

α_t^{π} (resp. α_t^{λ}) is the rate of increase of the contribution rate (resp. indexation rate) after risk-sharing.

Proposition 2 (Risk-sharing)**.** *The RSMs at time t related to the contribution rate, α_t^{π}, and related to the indexation rate, α_t^{λ}, are represented as follows:*

$$\alpha_t^{\pi} = \beta_t\left(\frac{1-LR_t^*}{LR_t^*}\right), \tag{3.8}$$

$$\alpha_t^{\lambda} = \beta_t + (1-\beta_t)\frac{C_t^* - P_t^{x_r}N_t^{x_r}}{P_t^* - P_t^{x_r}N_t^{x_r}} - 1, \tag{3.9}$$

where LR_t^* corresponds to the liquidity ratio in absence of a buffer fund prior to the application of the RSM.

Proof. The expression for α_t^{π} is obtained by forcing the income from contributions C_t after RSM to be equal to the income from contributions before the application of the RSM reduced by the amount of the one-period buffer fund. Mathematically, this is expressed as follows:

$$\begin{aligned} C_t &= C_t^* - \beta_t D_t, \\ (1+\alpha_t^{\pi}) &= \frac{C_t^* - \beta_t D_t}{C_t^*} = 1 - \beta_t\frac{C_t^* - P_t^*}{C_t^*}, \\ \alpha_t^{\pi} &= \beta_t\left(\frac{1-LR_t^*}{LR_t^*}\right). \end{aligned}$$

The expression for α_t^{λ} is obtained in a similar manner by forcing the pension expenditures P_t after RSM to be equal to the pension expenditures before the application of the RSM reduced by the amount of the one-period buffer fund. Mathematically, this is expressed as follows:

$$
\begin{aligned}
P_t &= P_t^* - (1-\beta_t)\, D_t, \\
P_t^{x_r} N_t^{x_r} + \sum_{x=x_r+1}^{\omega-1} P_{t-1}^{x-1}\,(1+\lambda_t)\, N_t^x &= P_t^* - (1-\beta_t)\, D_t, \\
\left(1+\alpha_t^\lambda\right) &= 1 + (1-\beta_t)\,\frac{D_t}{P_t^* - P_t^{x_r} N_t^{x_r}}, \\
\alpha_t^\lambda &= \beta_t + (1-\beta_t)\,\frac{C_t^* - P_t^{x_r} N_t^{x_r}}{P_t^* - P_t^{x_r} N_t^{x_r}}.
\end{aligned}
$$

□

Remark 4. Proposition 2 shows the rate of variation of the contribution and indexation rate needed to restore the liquidity when $\beta_t\%$ of the surplus or deficit is borne by the contributors and the remainder by the pensioners. For instance, when the income from contributions before RSM is greater than the pension expenditures, the liquidity ratio LR_t^* is higher than 1, which indicates that there is a surplus. The parameter α_t^π (3.8) is then negative and the contribution rate is then reduced by $1+\beta_t\left(\frac{1-LR_t^*}{LR_t^*}\right)$. In the same line, the parameter affecting the indexation rate α_t^λ (3.9) is positive and the indexation rate is increased by $1+\beta_t+(1-\beta_t)\frac{C_t^*-P_t^{x_r}N_t^{x_r}}{P_t^*-P_t^{x_r}N_t^{x_r}}$. Note that the variation of the indexation rate is the liquidity ratio corrected by the first pension paid. This occurs because the first pension is not affected by the indexation rate.

Corollary 1 (Particular cases: $\beta_t = 0$ and $\beta_t = 1$)**.** *The risk-sharing coefficients* (3.8) *and* (3.9) *in the extreme cases of $\beta_t = 0$ and $\beta_t = 1$ are:*

- *When the risk-sharing coefficient β_t is equal to* 0*, i.e., when the surplus or deficit is solely borne by the pensioners, the expressions of* (3.8) *and* (3.9) *become:*

$$\alpha_t^\pi = 0, \tag{3.10}$$

$$\alpha_t^\lambda = \frac{C_t^* - P_t^{x_r} N_t^{x_r}}{P_t^* - P_t^{x_r} N_t^{x_r}} - 1. \tag{3.11}$$

- *Alternatively, when the risk-sharing coefficient β_t is equal to* 1*, i.e., when the surplus or deficit is solely borne by the contributors, the expressions of* (3.8) *and* (3.9) *become:*

$$\alpha_t^\pi = \frac{1 - LR_t^*}{LR_t^*}, \tag{3.12}$$

$$\alpha_t^\lambda = 0. \tag{3.13}$$

Remark 5. Corollary 1 shows the extreme cases of our RSM. In the first case, the contribution rate do not change over time and the whole deficit or surplus of the system is borne or benefited by the pensioners through an adjusted indexation. This case relates to the classical notional DC as the contribution rate is by definition constant (Palmer 2013; Chłón-Domińczak et al. 2012). However, it can also be used in DB schemes where the contribution rate is set constant. The second case presents the opposite situation where the deficit or surplus is borne or benefited by the contributors. In particular, this RSM adjusts the contribution rate while keeping the benefits promised by the system. This relates more to a classical DB scheme without carrying out reforms.

4 Numerical illustration

This section presents a numerical example using Belgian data under the generic DB and DC pension systems developed in Section 2. First, the main data and assumptions are presented, secondly the results are discussed under different RSMs as presented in Section 3.

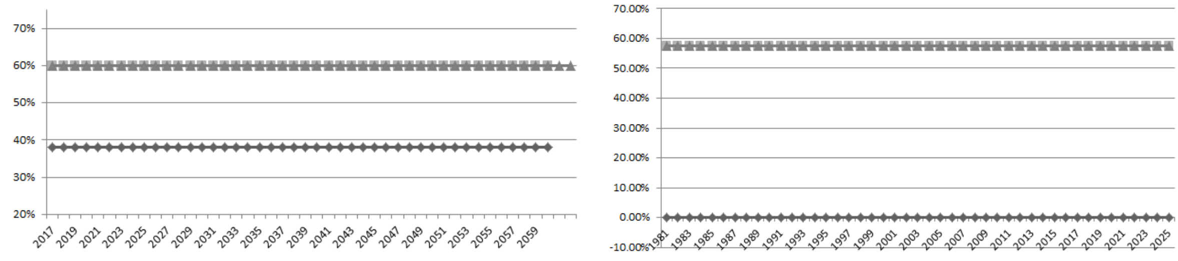
4.1 Data

- We use the demographic structure of the Belgian population from 1935-2016 obtained from Human Mortality Database (2016b). Note that the population before 2016 is open as it represents the total observed population for each age.
- For the forecasted population we use projected values from Eurostat (2013). New entries are assumed to join the system at age 20 and the population after 2016 is closed, that is, exits are only due to death.
- As a base year, Belgian salary structure[13] in 2010 is taken from Eurostat (2010). Past growth salaries are obtained from Statbel (2016) while future salary increases are based on the labour productivity per hour forecast from European Commission (2014).
- Historical data on mortality tables is taken from Human Mortality Database (2016a) and Statbel (2014) while projected values are obtained from Belgian Federal Planning Bureau (2016).
- For the DB pension system, the initial pension is set at 60% of average revalorized salary in line with the current Belgium DB formula.(Federal Pension Service 2016).
- For both DB and DC pension systems, the initial contribution rate in 2016 is the rate that makes the DB system balanced in this particular year, i.e, 19.38%. Note that this contribution rate is higher than the currently used in Belgium, i.e. 16.86%.
- For both schemes, the indexation of pensions is equal to the rate of increase of the income from contributions. Values for the discount rate for the annuity coincide with the indexation of pensions. The value of the annuity is therefore equal to the life expectancy at retirement.
- For both DB and DC pension systems, the retirement age is fixed and equal to 65.
- The buffer fund is assumed to increase at an annual rate of 0%.
- No minimum and maximum pension are considered in our analysis.
- The replacement rate is calculated at age 65 and 85 according to the formula (2.18).

[13]The wage structure is based on the mean annual earnings by sex, age and economic activity including industry, construction and services and excluding public administration, defense and compulsory social security.

Figure 1: No RSM and steady-state. The figure depicts the replacement rate at age 65 and actuarial fairness for an individual under three different pension schemes: DB (light-grey triangle), DC with SD (light-gray square) and DC without SD (dark-grey rhombus)

a) Replacement rate at 65 b) Actuarial fairness



- The actuarial fairness is represent by means of the relative difference between the value of the benefits received and the contributions made at the retirement age x_r.

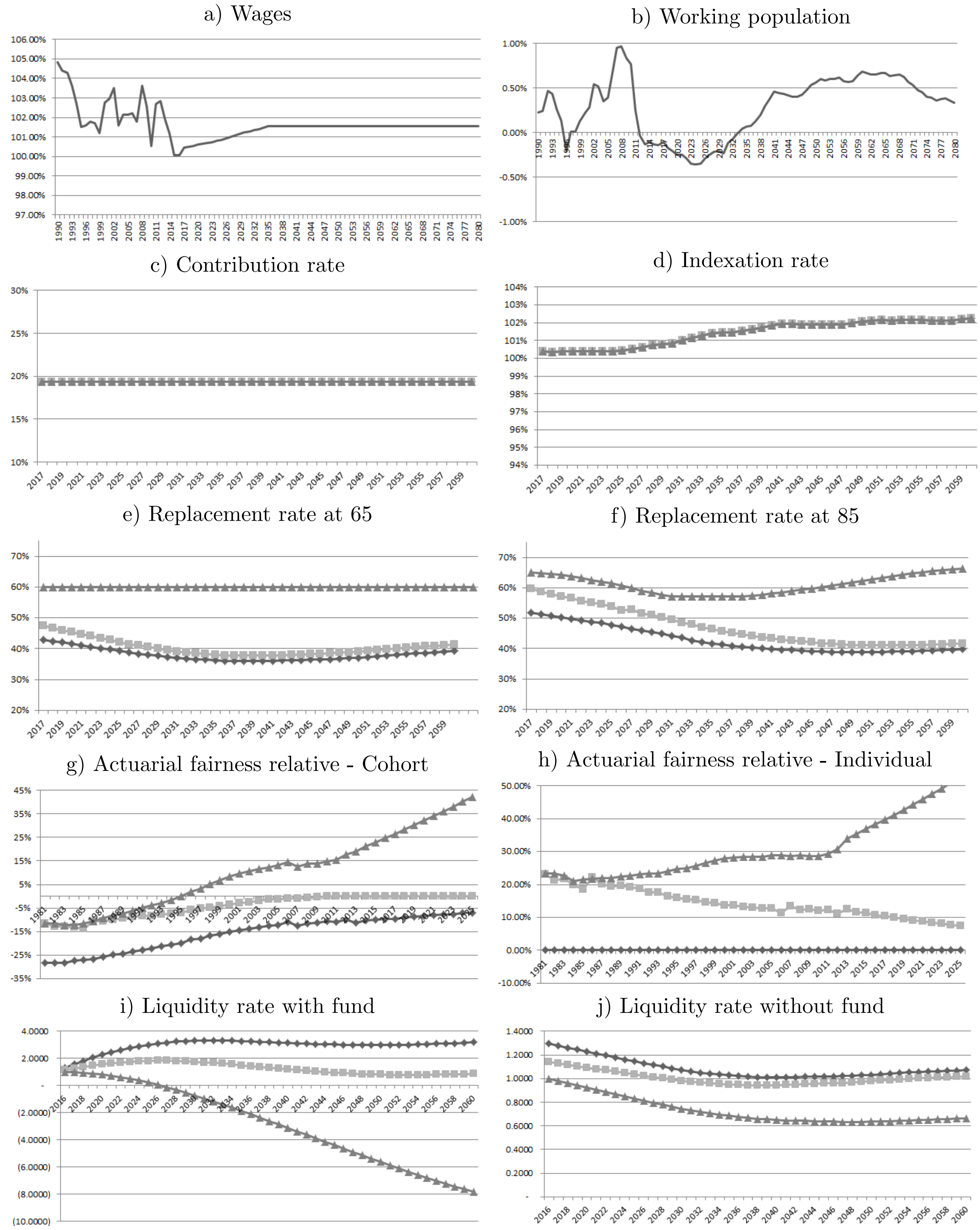
4.2 Base scenario

In an unrealistic steady state[14], Figure 1, both DB and DC scheme with the survivor dividend (SD) provide a replacement rate close to 60% which remains constant over time. As expected, the amount of the initial pension is lower if the survivor dividend is not included and as a consequence the RR is up to 20% lower. Under this particular scenario the system is actuarially fair on an individual basis only if the SD is not taken into account. However, in presence of SD we observe that both DB and DC with SD are not actuarially fair. In fact, individuals in both schemes would have received more benefits than the amount they have been contributing. The contribution and indexation rate would remain constant over time and equal to 15.18% and 0.031% respectively.

Under a dynamic population and economy based on the data presented in Subsection 4.1, we can see, Figure 2, that both the actuarial fairness for the individual as well as the sustainability of the system are compromised for the DB case if no measures are taken. In terms of adequacy the DB pension system provides a replacement rate of 60% just after retirement (Figure 2e). However, this replacement rate changes over time because of various factors. For instance, the generation aged 85 in 2017 which retired in 1997, Figure 2f, has a replacement rate higher than 60% due to the fact that the increase in the working population was positive during most of their retirement period as depicted in Figure 2b. The indexation which depends on both working population and wages was higher than the wages increase. Similarly, the replacement rate at age 85 decreased after 2027 because of the negative evolution of the population, which entailed that pensions were being revalued less than wages. We observe as well that the replacement rate for the DC schemes decreases over time. This is caused by a combination of an increase in life expectancy with a fixed retirement age. However, after 2040 the increase in the working population overcompensates the life expectancy increases and provides a higher initial capital which translates in a higher value of the initial pension. The levels

[14]In this case, the steady state assumes that the growth of the variables involved are constant and equal to the ones in year 2016.

Figure 2: No RSM: The figure depicts the contribution rate, indexation rate, replacement rate at different ages (65,75,85) and the actuarial fairness for three different pension schemes: DB (light-grey triangle), DC with SD (light-gray square) and DC without SD (dark-grey rhombus)



of adequacy provided by the DB scheme compromise the liquidity of the pension system as depicted in Figure 2i and Figure 2j. The DC schemes (Figure 2i) over-accumulate funds very early, even with a interest rate equal to 0% in line with the work of Auerbach and Lee (2006) and Alonso-García et al. (2015). However, the DB enters into deficit after 2027 and achieves a level of indebtedness of -8, that is, the deficit comprised by the contributions plus the negative fund are 8 times bigger than the pension expenditures. This level of deficit is caused by a level of contributions not being sufficient to cover the pension expenditures as shown in Figure 2j.

In terms of actuarial fairness, we observe that these values change depending on whether we account for the whole cohort or the individual. On a cohort basis, Figure 2g, we observe that the generations retiring after 1995 are receiving much more than they have contributed within a DB scheme. For instance in 2080 the cohort would have received 45% more than they had been contributing, mainly due to the non-contributory pension formula and the fixed retirement age combined with life expectancy increases. The DC with SD is actuarially fair on a cohort basis from 2016, the moment from which the population is closed. On an individual basis, Figure 2h, the interpretation differs. The DC without SD is the only one which is actuarially fair under all circumstances. The other two schemes are not actuarially fair, as individuals receive more than they had been contributing but the trends vary. While the DB scheme is providing more and more benefits compared to the contributions for the reasons above mentioned, the DC scheme with SD is giving less and less additional benefits. The latter is due to the fact that mortality improvements affect all cohorts, which reduce the amount of SD distributed within the cohort.

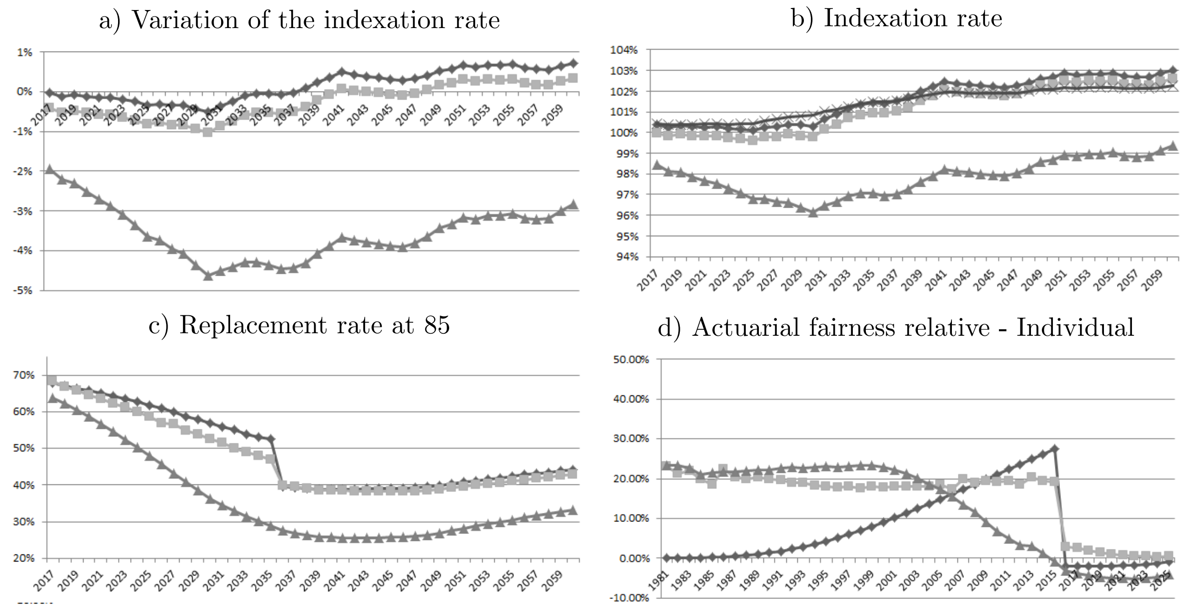
4.3 Risk-sharing mechanisms

This section designs different mechanisms in order to guarantee the financial equilibrium of the system. This mechanism is symmetric, that is, measures are not only taken in bad times, therefore any surplus that might arise would be automatically distributed (Palmer 2013). We also show the consequences on adequacy and fairness after such mechanisms are taken.

Figure 3 shows the path of the indexation of pensions if this was the only variable adjusted to restore the liquidity into the system. As shown in Figure 3a and 3b, the DB scheme needs to keep a much lower negative indexation, compared to the DC schemes, over the whole analysed period to guarantee the financial sustainability of the system. On the contrary the indexation of pension would slightly increase as a result of the system having surplus for the DC scheme after 2038 when the increase in the working population is positive again. However, the replacement rate at age 85, Figure 3c, for the DB scheme reaches a very low value of 26% in 2040 as a result of the negative revaluation of benefits to restore the financial sustainability. This value of the RR is even higher for the DC cases that have not suffered the consequences of the negative revaluation of pensions. In the same line, the actuarial fairness, Figure 3d, drastically improves after the inception of the RSM in 2016 for the DB scheme mainly as a result of a reduction in the indexation of pensions. We observe that once liquidity is enforced via the indexation rate the actuarial fairness also changes proportionally.

When we adjust the contribution rate in order to attain sustainability, Figure 4, we observe that the contribution rate for the DB scheme would have to increase substantially. In fact it would need to increase yearly until achieving a level of 31% by 2050. We observe that the contribution rate should also increase for the DC pension scheme.

Figure 3: RSM on the indexation rate. The figure depicts the variation of the indexation rate, the nominal indexation rate, the replacement rate at age 85 and the actuarial fairness for three different pension schemes: DB (light-grey triangle), DC with SD (light-gray square), DC without SD (dark-grey rhombus) and baseline scenario (dark grey-cross)

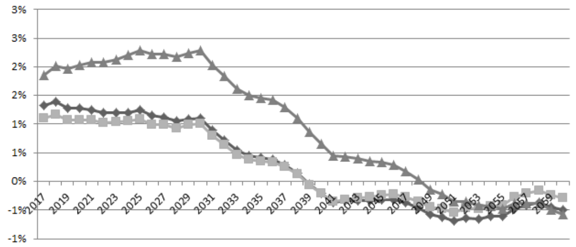


Due to the initial surplus of the system as depicted in Figure 2j, the contribution rate would immediately decrease by 23% to a level of 15% after inception of the RSM. However, given the increase of the old-age population due to the increase in life expectancy the contributions at this low level of contribution rate does not suffice to cover the pension expenditures, therefore increasing every year until 2040 when the contribution rate stops increasing and actually decreases. This evolution is shown in more detail in 4b compared to the level before RSMs were put in place (black line with the cross). In terms of adequacy and fairness, Figure 4c and 4d, there are no significant changes in the trend compared to the cases without RSMs as the indexation of pensions is not affected. However, the contribution rate increase affects the actuarial fairness for both the cohort and the individual, specially in the DB case, as they will contribute more to the system and get the same pension in exchange. For the DC schemes, the actuarial fairness properties remain because an increase in contributions corresponds to an increase in pensions at retirement. However, we are not able to see these effects because of the time horizon studied: the important changes in the amount of contributions will affect cohorts which will retire after 2025, for which we don't have complete life trajectories given that our study stops at 2060.

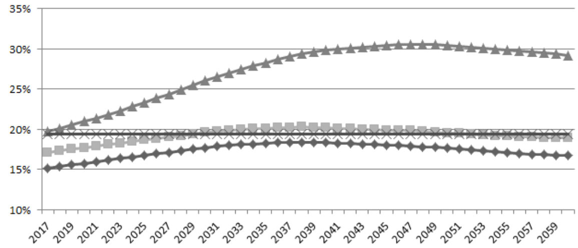
For smoother changes, Figure 5 adapt both the contribution rate and indexation of pension following Formula (3.8) and (3.9) to restore the equilibrium into the system. The risk (deficit or even surplus) is supported equally by both contributors and pensioners in a particular year. For a DB pension system, the contribution rate would need to increase to 26% by 2040 while the indexation of pension would be positive after 2032. The negative indexation for the first year makes that the RR for DB and DC schemes reach similar values although once the indexation reaches a positive value then the RR

Figure 4: RSM on the contribution rate. The figure depicts the variation of the contribution rate, the nominal contribution rate, the replacement rate at age 85 and the actuarial fairness for three different pension schemes: DB (light-grey triangle), DC with SD (light-gray square), DC without SD (dark-grey rhombus) and baseline scenario (dark grey-cross)

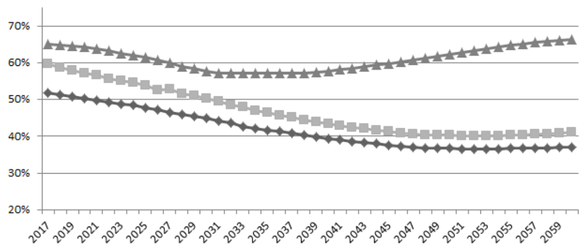
a) Contribution rate variation



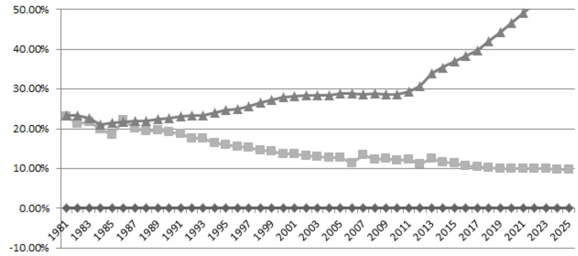
b) Contribution rate



c) Replacement rate at 85



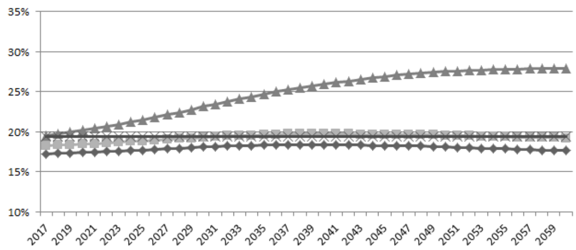
d) Actuarial fairness relative - Individual



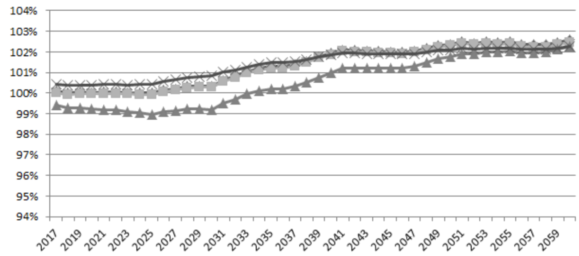
of the DB schemes would reach a higher value.

Figure 5: RSM 50% Contribution Rate and 50% Indexation rate. The figure depicts the contribution rate, indexation rate, replacement rate for different old ages (65,75,85) and the actuarial fairness for three different pension systems: DB (light-grey triangle), DC with SD (light-gray square), DC without SD (dark-grey rhombus) and Base case (dark grey-cross)

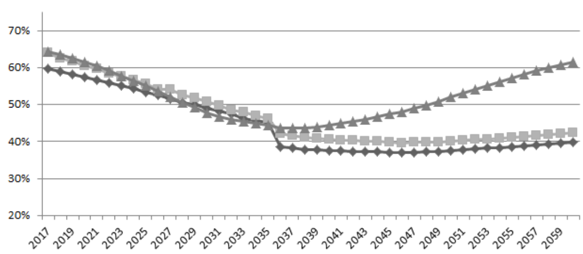
a) Contribution rate



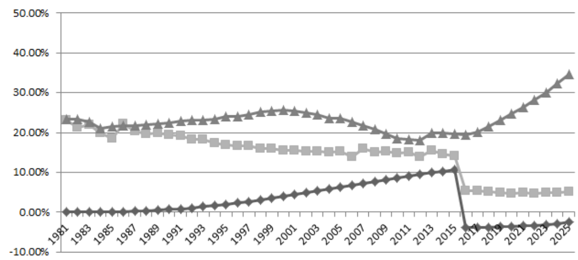
b) Indexation rate



c) Replacement rate at 85



d) Actuarial fairness relative - Individual



5 Conclusion

The challenge of financial sustainability and adequacy are becoming more important in most pay-as-you-go pension systems. Also, structural reforms from DB to DC schemes in countries such as Poland, Latvia, Sweden or Italy open the debate on fairness for participants in the systems.

We show that in the steady state, under certain circumstances, the amount of the pension for DB and DC schemes might be equivalent. Under a dynamic environment, DB and DC schemes reach significantly different values for the initial pension and consequently for sustainability, adequacy and fairness.

This paper design, from a theoretical point of view, flexible RSMs, involving variables such as the contribution rate and/or indexation of pensions, which restore the financial sustainability of pension system in the long run.

We show that, taking into account economic and demographic projections, the sustainability of a generic DB pension scheme (in this case using Belgian data) is seriously compromised unless some mechanisms are implemented immediately. These mechanisms would cause a substantially reduction in the level of pensions for beneficiaries in line with the increase in life expectancy.

As expected, DC schemes show better results for sustainability and fairness and comparable results for adequacy with DB schemes when taking into account mechanisms to restore sustainability.

Our research considers not only the classical DC scheme but also the DC considering the survivor dividend. We show that the dividend involves an increase in the initial pension. Therefore we also study its impact on the sustainability, adequacy and fairness of the system.

Finally, based on the model presented in this paper, at least two important directions for future research can be identified. First, it would be interesting to analyse the consequences of different shocks into the DB and DC schemes. Another direction would be to use different combination of mechanisms involving other different variables such as retirement age and/or replacement rate that guarantee not only the intragenerational but also the intergenerational fairness.

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