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Retirement Eggs and Retirement Baskets

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Abstract

How do people save over their lifetime? Using a dynamic lifecycle model of saving and portfolio choice featuring risky labor income, housing, and safe and risky financial assets inside and outside pension plans with comprehensive choice architecture, we examine the behavior of members of an industry-wide retirement fund to assess how standard saving motives, pension defaults, investment returns, preferences and frictions interact to drive lifetime savings across major asset classes. Our results show considerable heterogeneity in what motivates people how to save. First, we find that financial and housing assets are largely driven by consumption smoothing motives. While these motives also affect plan choices, their role in pension accumulation is more limited due to default switching costs. Removing such costs, on the other hand, encourages pension savings at the expense of financial wealth but not of housing. In fact, we find higher pension assets to drive up housing wealth throughout the lifecycle, as people - anticipating a wealthier retirement and to avoid potentially larger adjustment costs later in life - lock in higher housing investments early on. Second, being luxury goods, bequest motives lead to higher DC take-up and riskier portfolios, but only to a modest mid-life financial savings boost. Third, precautionary savings that insure against wage risks have similar plan effects to bequests, although they do not translate in any wealth dynamic. Finally, removing costless redraws on mortgages leads to higher financial savings, again displacing pension balances considerably more than housing wealth.

Key Words: lifetime savings, portfolio choice, income risk, defaults, method of moments.
JEL Classification: H8, J26, J32.

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1 Introduction

How do people save over their lifetime? First, recall the canonical lifecycle model prediction that wealth accumulation exhibits a familiar hump shape (Modigliani, 1986), with overall wealth built up during working years and consumed in retirement. Three key saving motives were found to shape this pattern - i.e., consumption smoothing, bequests and precautionary saving. Early in one's working life, wealth can be significantly affected by labor income risks that trigger precautionary savings (Gourinchas and Parker, 2002), as well as by volatile transfers such as bequests (De Nardi, 2004). During more mature years, saving is more predominantly driven by consumption smoothing and leaving bequests motives (Gomes et al., 2021), with job tenure and mobility (Brown, 1989; Jung and Kuhn, 2019), investment returns (Benhabib et. al, 2019), preference heterogeneity (Krusell and Smith, 1998) and lifetime earning dynamics more generally (Guvenen and Smith, 2014) further affecting the link between all saving motives and so, wealth accumulation.

Setting aside overall levels, however, there are also significant advantages to accumulating wealth via portfolios with different compositions over the lifecycle (Iacoviello, 2011; Flavin and Yamashita, 2011; Kraft and Munk, 2011; Kraft et al., 2018).[1] Given the role private pensions have come to play in the provision of retirement income (Amaglobeli et al., 2019) and the worldwide shift from defined benefit (DB) to defined contribution (DC) plans that increasingly rely on defaults (OECD, 2019),[2] the quality of people's portfolio decisions will thus bear increasing weight on their old age savings adequacy - a trend further accentuated by the current demographic shifts that continue to challenge the fiscal sustainability of public pensions.

From a policy perspective, designing adequate welfare programs requires therefore adequate understanding of how people accumulate wealth. But there is more. Accumulating different assets for different purposes - particularly the choice of housing capital versus equity or private pensions - has significant implications also for the macroeconomic composition of capital, in turn affecting asset returns, housing prices, rental yields and financial stability (Eckardt et al., 2018). Thus, regardless of the micro or macro perspective one takes when examining savings, understanding

[1]See also Poterba et al. (1995), Ameriks and Zeldes (2004), Benzoni et al. (2007).

[2]Plan defaults assign specific outcomes for key decisions (e.g., participation, contribution rates, investment allocations, benefit type, etc.) when no active choice is made. While theoretically harmless as long as people can easily opt out, research suggests they can significantly affect pension savings (Madrian and Shea, 2001; Dobrescu et al., 2018).

what motivates portfolio composition and its lifecycle dynamics remains key.

So how do standard saving motives and pension choices, investment returns, preferences and frictions interact to drive lifetime savings across the main asset classes? To answer this question, we estimate a structural lifecycle model of optimal consumption and portfolio choice - involving real (housing) and financial wealth in safe and risky assets inside and outside pension plans - in the presence of uninsurable labor income risk and borrowing constraints. In doing so, we use panel administrative data on members of an industry-wide retirement fund matched with nationally representative survey data. The retirement fund we study - Unisuper - is one of the largest in Australia, it covers all higher education and research sector employees and offers both DB and DC plans, each with several reversible and irreversible defaults related to plan type, voluntary contributions and investment choice. Using a multi-step process, we link Unisuper records with Household, Income & Labour Dynamics in Australia (HILDA) data on a wide variety of assets and liabilities, and proceed in two steps. First, we document the relation between the major asset classes (i.e., housing and financial assets inside and outside pension plans) and identify the factors associated with their levels and prevalence. Second, we use our rich structural model to assess whether these empirically motivated elements can explain the data profiles and run counterfactuals to quantify the extent to which they do so. The structural lifecycle model we develop considers that working individuals earning stochastic labor income consume and save via real and financial assets inside and outside pension plans to maximize expected lifetime utility. People in our model can either rent or own a house, with homeowners being allowed to borrow against their real assets by taking a collateralised mortgage. As in our institutional context, pension wealth gets decided in a rich setting that combines automatic plan enrolment with reversible and (time-sensitive) irreversible plan defaults: upon being hired, individuals are automatically enrolled in a DB plan and, within the first period, they have a one-off option to switch to DC. Each period they can also decide to (i) voluntarily contribute and override the default (0%) voluntary contribution rate, and (ii) opt out of the default (balanced) asset allocation, by choosing a different share of pension wealth to be invested in risky assets. Switching out of plan defaults bears the cost of acquiring information, operationally making the change, and/or forgoing the liquidity of saving outside the pension plan. Finally, we include stochastic discount rates and housing preferences to account for preference heterogeneity potentially affecting the wealth distribution (Krusell and Smith, 1998; Guvenen and Smith, 2014; Stachurski and Toda, 2019) and homeownership profiles (Ngai and Sheedy, 2020).

Up to our best knowledge, ours is the first structural study to examine the allocation of wealth across all major liquid and illiquid, risky and safe asset classes in a setup that fully fleshes out pension choices related to plan type, contributions and asset allocations, each with its own default options. The main advantage of this setup is that it allows us to conduct counterfactual simulations that can identify the economic drivers shaping not only the overall wealth profiles but also those of each main asset class separately. We also manage to circumvent a few challenges related to estimating lifecycle models. First, our rich panel data on a sizeable sample of individuals with different ages and job tenures allows us to disentangle the age vs. cohort vs. time effects in wealth accumulation (Ameriks and Zeldes, 2004). Second, we limit the issue of potential measurement errors that can notoriously affect survey data (Kapteyn and Ypma, 2007) by using an extensive matching procedure that links them to comprehensive administrative data (Dobrescu et al., 2018). Third, the usually limited active stock market participation (Bertaut and Starr-McCluer, 2002; Cocco, 2005; Lynch and Tan, 2011) implies that any portfolio insights will be based only on the behavior of a selected group of individuals (i.e., those who engage with such decisions). We bypass this issue by considering the portfolio allocations that all people hold for their (DC-component) pension wealth due to automatic plan enrolment and account for non-choices by carefully modelling plan defaults. Fourth, by modelling pension plan choices jointly with both (i) liquid financial saving, and (ii) homeownership decisions involving precommitments that make housing wealth costly to adjust, we can not only better understand savings patterns but also flesh out the interplay between the savings motives that drive them. Finally, with six endogenous state variables, three discrete choices and frictions, our model is both high-dimensional and non-convex. Solving convex high-dimensional models generally relies on first order conditions (FOCs) to gain computational speed via the endogenous grid method (EGM - see Carroll, 2006). Such conditions, however, are not sufficient without convexity (Iskhakov et al., 2017). To address this issue, we use the novel general sufficient second order conditions (SOCs) for discrete choice dynamic problems by Shanker et al. (2022) that allow us to use EGM via derivatives alone, without adjusting or removing sub-optimal grid points (Fella, 2014; Iskhakov et al., 2017). Additionally, to estimate our parameters using the simulated method of moments, we use the cross-entropy method (De Boer, 2005) - a Monte Carlo, gradient free algorithm shown to perform well on optimisation problems for irregular objective functions (Botev et al., 2011). By doing so, we also advance the understanding of practical scaling in distributed dynamic programming algorithms.

Our reduced form analysis yields several interesting results. For instance, while overall pension wealth is relatively high in our sample, females have lower balances than males but they also invest it slightly more aggressively possibly in an attempt to close this gap. Additionally, we see people becoming homeowners relatively early in their working life and holding higher housing wealth shares as they get older. Our female and less educated subsamples are both more likely to own a home, which is consistent with the former deriving higher utility from housing than males and the latter still using housing as primary vehicle to build up savings. In contrast, those more educated and on higher wages diversify their portfolios more, while net wealth and the wealth share own-home invested appear positively related.

The simulated and empirical patterns are well-matched and show rising age profiles for all types of wealth and voluntary contributions. Plan defaults are highly prevalent for both plan type and asset allocations, with stable risky assets shares over the lifecycle and only slight rebalancing at older ages. Finally, females accumulate less than males in their pension accounts but invest it more aggressively, and they also save slightly more financial wealth outside their pension plan.

Several counterfactuals allow us to better understand these patterns. First, eliminating consumption smoothing motives (by allowing individuals to maintain a uniform income even after retirement - Pashchenko and Porapakkarm, 2020) results in 32.94% less overall savings, with females registering almost double the drop compared to males. We also see an average of 9.47% fewer DC plan opt-ins, an unsurprising finding given their role of offering people a way to directly manage and diversify their portfolio to smooth consumption. Zooming in on wealth while holding plan choices fixed, however, we find consumption smoothing to significantly drive financial wealth accumulation particularly during mature working years (i.e., after the age of 45), with females (males) saving 23.56% (11.44%) more by the end of their working life (Gourinchas and Parker, 2002). Strikingly, the same wealth boost occurs for housing but (i) much earlier due to housing frictions motivating early housing investment (Yang, 2009), and (ii) more prominently for females (30.05%) than males (20.09%). While similar profiles (i.e., from mid-30s onwards) are present also for pensions, the (roughly 26%) balance rise does not seem to exhibit this gender gap.

Second, cancelling the bequest motive generates a significant 28.67% drop in pension balances, but also a modest 1.65% increase in non-pension wealth as the drop in financial wealth (no longer needed for one's heirs) is offset by the boost in housing (due to bequests no longer displacing consumption, including housing consumption - see Kopczuk and Lupton, 2007). Unsurprisingly, this housing boost is 63.05% more prominent for females than for males due to females valuing bequests more in our sample (see also Seguino and Floro, 2003) and thus, facing stronger (housing) consumption displacement effects. Interestingly, however, the bequest effect on pension balances operates almost solely via plan choices: Being a luxury good (Ding, 2013), bequests induce 7-8% more willingness to opt for DC plans in an attempt to build up a larger post-retirement payout (used both as a source of retirement income and bequest). As females value bequests more, they will also adopt roughly 12% riskier portfolios. In fact, ceteris paribus plan choices, bequests only drive-up financial savings after mid-working years, while pension assets are still building up. Since pension wealth (and to some extent, housing) also serves as bequeathable wealth, the increase in non-liquid wealth in later life reduces incentives to store wealth in financial assets (Dynan et al., 2002).

Third, and continuing to hold plan choice fixed, we find precautionary motives related to wage risks not to directly add any extra financial or pension wealth due to mortgages featuring costless redraw options (per our institutional context). Thus paying off mortgages has a dual role - as a form of 'saving' (to avoid paying interest) that also enables early homeownership and as a form of insurance. On the pensions side, wage risks have similar effects to bequests, significantly raising DC uptake (by 20.63%) and thus pension balances (by 23.66%): with DB payouts tightly linked to the full stream of wage shocks up to retirement, switching to DC means people are able to better diversify the impact of wage risks on post-retirement income as once a contribution is made, pension accumulation is driven only by market returns.

Fourth, we turn to pensions architecture and start with a counterfactual that assumes costless opting out of plan defaults. Such flexibility leads to significant wealth effects across the board, due to higher DC plan take-up, higher voluntary contributions prevalence and ultimately higher pension balances. Unsurprisingly, males benefit the most from such changes and end up saving 21.36%

more than females in their pension account. While this pension boost mildly displaces financial savings for everybody, we also find an interesting complementarity between pension and housing wealth - strong enough to generate higher non-pension wealth for males but not for females. This link appears to be generated by higher post-retirement wealth driving up consumption (including housing consumption) close to and after retirement: Since housing levels are locked in during early years due to frictions, the higher housing consumption desired in later years drives up housing wealth, as well as overall wealth throughout the lifecycle. To further confirm our pensions-housing complementarity, we also run a counterfactual that raises risky assets returns, and again rather than displacing non-pension wealth, higher pension balances spur higher housing investments.

Finally, we abstract from our housing market setup (and how it interacts with the precautionary saving motive to influence plan choice) by running a counterfactual that replaces the costless mortgage redraw with a constant amortisation repayment. As expected, we see individuals that can no longer draw down their mortgages accumulate significantly more precautionary financial wealth (i.e., 34.42% males, 25.29% females). This induces a general 29.52% shift away from DC plans as their diversification role becomes less relevant, with the added diversification due to higher financial wealth already reducing the overall exposure of wealth to wage risks. All in all, these effects highlight again the key impact plan architecture has on lifecycle portfolio and its mitigating role within the standard saving motives framework. Carefully designed pension plans are thus central to optimal portfolio allocation and overall financial wellbeing in retirement.

1.1 Related literature

These findings bring together the lifecycle saving literature and the literature examining the behavioural role of pension plan architecture and defaults in a novel methodological setup. The lifecycle savings literature can be traced back to Ando and Modigliani (1963), where household saving decisions were motivated by the desire to smooth consumption through time. Over the next decades, several studies significantly advanced the standard lifecycle hypothesis. Kotlikoff and Summers (1981), for instance, highlighted the importance of bequest motives in driving wealth accumulation. While bequests have initially spurred a debate with Modigliani (see Modigliani, 1988), a substantial body of work has ultimately solidified their role in shaping saving behavior

(Ameriks et al., 2011). Since people do not necessarily have access to complete insurance, precautionary saving to insure against income risk (Aiyagari, 1994), and subsequently also other forms of risk such as medical spending risk (De Nardi et al., 2010; Edwards, 2008; Yogo, 2016) or housing risk (Yao and Zhang, 2005), emerged as important saving motives. Further work by Gourinchas and Parker (2002) decomposed how precautionary saving and consumption smoothing motives operated over one's lifetime, with the former being more crucial during early working years and the latter becoming more central in later life. Finally, recent literature has further considered the impact of (i) transaction costs and frictions on early housing accumulation, at the expense of other consumption and slow wealth decumulation in older age (Cocco, 2005; Yang, 2009; Chetty et al, 2017; Fagereng et. al., 2021), and (ii) housing preferences shifts on housing wealth dynamics (Kaplan et. al., 2019; Ngai and Sheedy, 2020). We advance this literature by additionally considering the impact of time preferences shifts, as well as the role of pension defaults and returns on wealth accumulation, both overall and split between the major asset classes.

In doing so we also contribute to the literature documenting the role of private pension accounts on how people. Depending on their type, pension accounts can be more or less exposed to investment risks and often feature (hard or soft) defaults that 'lock in' people's pension saving behaviour. Defaults affect not only contribution rates (Choi et al., 2004; Beshears et al. 2009) and plan choices (Madrian and Shea, 2001; Caroll et al., 2009; Beshears et al., 2009; Goda and Manchester, 2013),[3] but also portfolio allocation between risky and safe assets and annuities (Choi et al., 2005; Edwards, 2008; Horneff et al., 2009; Inkmann et al., 2011; Koijen et al., 2016; Dahlquist et al., 2018), particularly in the presence of income risk (Polkovnichenko, 2007).

With retirement income increasingly dependent on portfolio allocations (Gomes et al., 2021), it is thus key to examine lifecycle savings in a setup that includes plan choices and considers all main asset classes at once. Doing so also fills the literature gap related to the link between pension and non-pension wealth, particularly housing.[4] We are thus able to not only identify the impact of the main drivers of saving on overall wealth allocation, but also study their lifetime effects based on how decisions on the joint portfolio allocation - inside and outside pension plans - are made.

[3]See also Blake (2000), Mitchell et al. (2009), Cocco and Lopes (2011), and Gerrans and Clark (2013) in the context of DC plan choices.

[4]See Eckardt et al. (2018) for a discussion on the importance of this link in the context of economy-wide effects.

By separately looking at gender-specific lifecycle saving patterns, we also contribute to the understanding of gender heterogeneity in the context of plan enrolment (Handel, 2013; Sunstein, 2013; Chetty et al., 2014). The standard result so far is that females are more risk averse than males (Croson and Gneezy, 2009), they respond differently than males to financial defaults (Dobrescu et al., 2018), and more strongly prefer annuities (Agnew et al., 2008). Recent studies have however reconsidered the presence of a gender gap in risk taking (Filippin and Crosetto, 2014), with Gerrans and Clark-Murphy (2004) and Drupp et al. (2020) showing this gap to fade away among professional working and more informed females, respectively.

Finally, we also contribute methodologically, with our solution method itself being related to two strands of technical literature - i.e., convex geometry/analysis and dynamic programming. In this context and up to our best knowledge, we are the first to deploy a general sufficient first and second order condition for non-convex dynamic programming problems. Non-convex problems have been treated in fields such as convex geometry (Tardella, 2008) and convex analysis (Soland, 1971; Toland, 1978; Ekeland and Turnbull, 1983) primarily for use in irregular machine learning optimisation algorithms. These results do not extend, however, to dynamic economic problems such as lifecycle models. The dynamic programming literature in economics has so far proposed upper envelope methods that remove non-optimal solution points (Fella, 2014; Druedahl, 2017; Iskhakov et al., 2017). In contrast, by exploiting the conditions in Shanker et al. (2022), we use general SOCs to verify optimality, with no additional analytical or computational steps.[5]

The rest of the paper is as follows: Section 2 describes our institutional context. Section 3 discusses the data and reduced form results. Section 4 presents the model, and Section 5 shows the calibration and estimation method. Structural results are presented in Section 6 and counterfactuals in Section 7. Section 8 concludes.

2 Institutional context

We study plan participants of UniSuper, an industry-wide retirement (or superannuation) fund covering all Australians employed in the higher education and research sector. With roughly 460,000

[5]Shanker et al. (2022) key insight is to use an auxiliary quadratic constraint to convert a discrete choice problem into a continuous choice one for which sufficient SOCs can be derived. With this result, we can compute and estimate our rich non-convex dynamic model with no loss of efficiency or computational speed.

members and $85 billion in assets, UniSuper is one of Australia's largest retirement funds. At the time of the study (i.e., 2010-2014), it also exhibited several interesting features: First, it was (and continues to be) one of the few remaining hybrid funds (offering both DB and DC pension plans), with member arrangements dependent on employment type, earnings, and workplace agreements. Second, upon becoming a sector employee, fund enrolment is automatic and membership is compulsory (i.e., one may not elect to have their employer contribute to a fund other than UniSuper). Third, UniSuper DB and DC plans have a system of highly consequential (Dobrescu et al., 2018) reversible and irreversible defaults that specify predetermined outcomes when no choice is made on plan type, contribution rates and investment allocations. Table A.1 summarizes the main UniSuper plan features in 2010-2014 for permanent staff - i.e., staff on continuing, tenured contracts or contracts running for two years or more. Upon being employed, permanent staff were offered a one-off choice of (DB vs. DC) *plan*; this choice was irreversible and had to be made within the first 12 months of employment. They also received employer contributions amounting to 17% of their earnings and are required to contribute a further percentage as *standard contributions*. The default standard contribution rate was 7% of (post-tax) earnings but it could be irreversibly decreased to 0% as follows: (i) DB plan members with standard contributions of at least 4.45% had 3 percentage points of their employer contribution go into the DC component of their plan, while the rest went into their DB component, (ii) DB plan members who reduced their standard contributions below 4.45% had all employer and standard contributions absorbed into the DB component of their plan, retirement and death entitlements reduced proportionally to the standard contribution reduction and were ineligible for extra insurance (see Table A.2),[6] and (iii) DC plan members, regardless of their standard contributions, had all employer and standard contributions go to the DC component of their plan. Besides standard contributions, fund members could also make *voluntary contributions* - regularly or irregularly, from either pre- or post-tax wages and for low income earners, they could attract an annual government co-contribution of up to $1,000. The voluntary contribution rate defaulted to 0% of earnings but when positive, the associated amounts accumulated into the DC component of one's plan. Finally, members could choose to grow their total DC component

[6]Employees received life and total and permanent disability (TPD) insurance coverage by default, but may vary their level of coverage and/or add income insurance.

by selecting from a menu of 15 *investment options* that vary by their asset allocation. Movement between investment options was possible, with the default being a 'balanced' portfolio featuring a 70:30 split between risky and safe assets.[7]

Outside of their pension accounts, people can build up wealth by investing in real and financial (non-pension) assets. The Household, Income & Labour Dynamics in Australia Survey (HILDA)[8] reports considerable amounts in these wealth categories. For instance, the average household net wealth (i.e., overall assets net of debt) was roughly \$740,000 in 2014. About 60% of it was represented by housing, making real assets the largest asset class in one's portfolio. Notably, during the period we analyse (2010-2014), almost all of the (rather weak) growth in housing assets came from price increases rather than quantity changes. Indeed, homeownership rates remained fairly stable at around 66% between 2010 and 2014, and unsurprisingly tightly linked to income, wealth and age (until retirement). Non-housing wealth accounted for about 43% of overall assets in 2014, a 4% increase compared to 2010. In contrast to housing, they increased their average value from \$320,000 in 2010 to almost \$400,000 in 2014. Half of this value was held in deposits (14%), direct equity (15%), business assets (11%), and life insurance and durable goods (e.g., motor vehicles, collectibles). The other half was held in pension accounts, which made pension wealth the second largest asset class in one's portfolio, after housing. Interestingly, most of the 2010-2014 increase in non-housing wealth was due to pension balances: For an average household, their prevalence rose from 80% to 84%, and their value grew by around 4% per annum to \$250,000 in 2014 and was largely invested in risky financial securities.

3 Data and empirical analysis

We use data from UniSuper administrative records and the HILDA survey. Our UniSuper data contains extensive information on all pension choices made by a random subsample of fund members who are permanent employees in the higher education and research sector. Each month, the fund also collects data on member demographics and job characteristics, and uses it to compute pension

[7] Since the time of the study, UniSuper has extended the time window in which one is allowed to change plan type to two years - see here, and added extra investment options - see here.

[8] HILDA is a nationally representative, household-based panel study that collects comprehensive information about economic and personal well-being, labour dynamics and family life in Australia (Ryan and Stone, 2016).

balances. We use two waves of UniSuper data, corresponding to the end of 2010 and 2014 and labelled Wave 10 and Wave 14, respectively. We restrict our sample to non-retirees who were active members in Wave 10, as indicated by whether they (or their employers) made any contributions in the last four months. After merging Waves 10 and 14 of UniSuper data, our sample consists of 9,728 individuals that provide a total of 13,022 observations across waves.

There are four sources of information about pension wealth in the UniSuper data, namely plan type, cumulative pension balance (total and specifically in the DC component), voluntary contribution amounts,[9]and the share of pension wealth invested in risky assets. To capture attitudes towards risk, we use a variable denoting whether one purchased supplementary insurance. To account for decisional inertia, we also include an indicator denoting opting out of the default (balanced) asset allocation. Finally, we use the number of employers contributing, job tenure (in years) and annual wage to account for job characteristics, and age and gender as demographics.

Since UniSuper collects limited background information, we supplement our administrative records with individual data from HILDA Wave 10 and 14, collected simultaneously with UniSuper. Specifically, we use UniSuper-matched[10] HILDA data on (i) non-durable consumption, financial wealth and housing (prevalence, value, and expenses related to the primary residence), and (ii) education, marital status, household size, health and net wealth (i.e., net worth excluding pensions).[11] (We only use (ii) variables in the empirical analysis to shed light on specific data patterns; results are robust to their exclusion.)

[9]We abstract from the decision to make standard contributions and calibrate them directly from the data.

[10]To match HILDA and UniSuper data, we follow Dobrescu et al. (2018): We first select the relevant subsample among HILDA respondents (i.e., higher education and research sector employees), and then use an iterative procedure that first matches UniSuper and HILDA individuals along eight common dimensions: age, gender, quintiles for wage, pension balance and years of contribution, whether the spouse contributes, type of plan selected, and type of employment contract. For the unmatched observations, the procedure then drops one dimension (spouse contributions) and attempts the matching again. Finally, we employ this process two additional times, progressively excluding the plan type and then the type of employment contract. We thus end up matching 82% of our full UniSuper sample.

[11]For both consumption and housing expenses, we compute individual spending using household spending and the imputed individual-to-household spending ratio as predicted by the estimated coefficients of a regression of household consumption on age, gender, marital status, household size, health insurance premium, annual wage, net wealth and net wealth interracted with age (Wachter and Yogo, 2010) - see Tables B.1-B.2. Health is captured by a dummy equal to 1 if self-reported health is excellent or very good. For education, we use two dummies denoting whether individuals have (i) university education (Bachelor degree or above), and (ii) 12 years of education or less, respectively.

3.1 Descriptive statistics

We present relevant sample statistics, both overall and split between those with and without default asset allocations.[12] Opting out of the default allocation might suggest different preferences or a different understanding of available options, which could translate into different choices in other dimensions too. For instance, Panel A in Table 1 shows that while, on average, only 25% of members opt for a non-default (DC) plan, a much lower 6.52% of default allocation members do so. In contrast, 53% of those with non-default allocations are enrolled in DC plans. Similarly, although only 20% (10%) of members contribute voluntarily (buy supplementary insurance), about a third fewer default allocation members do so compared to their non-default peers.

These general differences between default and non-default allocation members are also reflected in their pension and financial wealth. Panel B in Table 1 reports mean and median pension balance (in AU$), contributions and tenure, and selected wealth indicators. As expected, university employees appear to have substantial pension balances, generally one employer contributing and rather lengthy tenures, quite generous wages and high financial wealth.[13] Interestingly, however, default allocation members have slightly lower wages and so, lower pension balances despite contributing for longer (12.38 vs. 11.58 years at the median) and investing more aggressively (70% vs. 53% in risky assets) than non-default members. Finally, in line with the national statistics for this sector, we find a ratio of roughly 2-to-1 in terms of real to financial assets, with the average net wealth around the million dollar mark and median housing expenses of about $1,000 yearly.

Table 2 reports our sample demographics. We note no significant differences between our default and non-default allocation subsamples, with an average UniSuper member being around 46 years old, married, in a 3-person household, and with a Bachelor degree or above.

3.2 Empirical analysis

To study the relation between member choices and demographics, risk and job characteristics and wealth, we estimate linear models that correlate one's pension and non-pension wealth with such factors. Specifically, our outcome variables are indicators related to plan type, voluntary contribu-

[12] Recall that all UniSuper members make investment choices as even DB plans have a (small) DC component.
[13] The 2010 (2014) mean salary for full time jobs was about $69,000 ($80,000) - see ABS (2019).

Table 1. Pension and non-pension wealth characteristics

Panel A.	All		Non-Default Allocation		Default Allocation	
	% of Members	# of Members	% of Members	# of Members	% of Members	# of Members
Plan type:						
DB	74.71	3,287	47.08	837	93.44	2,450
DC	25.30	1,113	52.93	941	6.56	172
Is voluntarily contributing	19.43	855	23.57	419	16.63	436
Has supplementary insurance	10.39	457	12.21	217	9.15	240
Is homeowner	86.80	3,819	85.43	1,519	87.72	2,300

Panel B.	Mean	Median	Mean	Median	Mean	Median
Pension wealth (in \$000)	240.36	146.81	231.15	153.47	246.60	143.73
Number of employers contributing	0.97	1.00	0.98	1.00	0.97	1.00
Number of years contributing	12.69	12.00	12.26	11.58	12.98	12.38
Annual wage (estimated, in \$000)	87.89	81.34	90.23	82.85	86.31	79.06
(DC) share in risky assets	0.63	0.70	0.53	0.52	0.70	0.70
Financial wealth (in \$000)	434.31	326.10	420.67	301.52	443.55	345.12
Housing wealth (in \$000)	840.32	660.00	808.75	645.00	861.73	670.00
Housing share in total wealth	0.46	0.49	0.46	0.49	0.46	0.48
Housing expenses (in \$)	8,994.39	1,000.00	8,640.61	939.96	9,234.30	1,000.00
Total net wealth (in \$000)	1,001.60	803.07	962.97	776.20	1,027.80	848.93

Notes: Panel A presents information on all sample members ("All"), as well as on members in subsamples defined by participation in the default investment allocation ("(Non-) Default Allocation"). Panel B shows mean and median for total amount accummulated in the pension account, number of employers currently contributing, years of contribution, estimated age, share of DC balance invested in risky assets, financial and housing wealth, share of housing in total assets, housing expenses (i.e., repairs, renovations) and total net wealth (i.e., net worth excl. pension wealth). The sample consists of members from UniSuper Wave 10, containing 4,400 permanent employees. Unisuper defaults relate to pension plan type (DB), voluntary contribution rate (0%), asset allocation (70% risky assests) and no supplementary insurance.

Table 2. Demographic characteristics

	All	Non-Default Allocation	Default Allocation
Age	45.89	45.12	46.42
Male (%)	37.11	39.93	35.20
Couple (%)	86.96	86.39	87.34
Household size	3.04	3.04	3.04
Low education (%)	5.91	5.01	6.52
Medium education (%)	11.32	10.35	11.98
High education (%)	82.77	84.65	81.50
Good health (%)	53.75	54.27	53.39

Notes: The table presents averages for all sample members ("All"), as well as for the subsamples defined by participation in the default investment allocation ("(Non-) Default Allocation"). The sample consists of members from the first (2010) UniSuper wave, containing 4,400 permanent employees.

tions, pension wealth, share of risky assets owned, and homeownership prevalence and value. All our ordinary least squares (OLS) and logit models include controls for age, gender, education, marital status, household size, health and net wealth. To tease out the attitude towards risk and defaults, we use two variables denoting whether one bought supplementary insurance and opted for non-default asset allocations. To capture job characteristics, we use tenure, number of employers contributing and estimated annual wage. Finally, we include a wave indicator as pension decisions tend to be persistent and present robust standard errors clustered at individual level.

The marginal effects (m.e.) from the OLS and logit specifications are presented in Table 3A-B. The first three columns in Table 3A presents results from a logit model of one's decision to opt for a DC plan, and two OLS models on log of voluntary contribution amount and log of pension balance. Since the data revealed systematic differences between default and non-default investment allocation members, we also present separate results for each such group next to the overall estimates. We do this for a baseline observation defined as a 46-year-old, married female, living in a 3-person household, with a Bachelor degree or higher, 12 years of contributions, average wage and default asset allocation, and uninsured supplementary.

Opting for DC plans appears related to all our job characteristics indicators (i.e., tenure, number of employers contributing, wage). A unit increase in log wage, which roughly corresponds to 100% increase in salary relative to the baseline, significantly increases one's chances of choosing a DC plan by 4.80%. This effect is almost double in the non-default investor subsample (9.60%) than in

Table 3A. Estimation results for pension-related decisions and outcomes

Variable	All			Non-Default Allocation			Default Allocation		
	DC opt-in	Ln vol. cont.	Ln balance	DC opt-in	Ln vol. cont.	Ln balance	DC opt-in	Ln vol. cont.	Ln balance
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Age	0.024***	0.185**	0.063***	0.056***	0.187*	0.064***	0.005	0.189*	0.059***
	(0.005)	(0.057)	(0.009)	(0.010)	(0.082)	(0.015)	(0.004)	(0.081)	(0.010)
Male	0.011	0.037	0.159***	0.014	-0.031	0.210***	0.017*	0.108	0.120***
	(0.009)	(0.087)	(0.013)	(0.020)	(0.121)	(0.021)	(0.008)	(0.124)	(0.015)
Low education	0.015	-0.103	-0.037	0.028	-0.250	-0.045	0.002	-0.000	-0.035
	(0.023)	(0.172)	(0.030)	(0.051)	(0.253)	(0.054)	(0.017)	(0.237)	(0.035)
High education	0.018	0.085	0.110***	0.034	0.072	0.145***	0.003	0.097	0.083***
	(0.011)	(0.108)	(0.019)	(0.028)	(0.151)	(0.035)	(0.010)	(0.156)	(0.021)
Couple	-0.011	-0.016	0.129***	-0.013	-0.037	0.198***	-0.006	0.037	0.072***
	(0.011)	(0.105)	(0.020)	(0.025)	(0.139)	(0.035)	(0.010)	(0.163)	(0.022)
Household size	0.006*	-0.049	-0.009*	0.013*	-0.049	-0.013	0.001	-0.054	-0.006
	(0.003)	(0.027)	(0.004)	(0.007)	(0.040)	(0.007)	(0.002)	(0.038)	(0.005)
Good health	0.000	-0.037	-0.028*	0.017	0.033	-0.006	-0.010	-0.111	-0.042**
	(0.007)	(0.069)	(0.011)	(0.017)	(0.099)	(0.019)	(0.006)	(0.098)	(0.013)
Suppl. insurance	0.050***	-0.153	0.048**	0.101***	-0.303*	0.027	0.024	0.004	0.066***
	(0.013)	(0.104)	(0.017)	(0.027)	(0.136)	(0.029)	(0.013)	(0.158)	(0.019)
Years of contribution	0.031***	0.000	0.064***	0.082***	-0.002	0.057***	0.001	0.001	0.069***
	(0.005)	(0.006)	(0.001)	(0.011)	(0.009)	(0.002)	(0.004)	(0.009)	(0.001)
Employers	0.050**	-0.025	-0.021	0.102**	0.036	0.001	0.023	-0.088	-0.032
	(0.015)	(0.152)	(0.023)	(0.035)	(0.202)	(0.041)	(0.014)	(0.238)	(0.026)
Ln annual wage	0.048***	1.031***	1.146***	0.096**	1.090***	1.061***	0.023*	1.020***	1.136***
	(0.013)	(0.159)	(0.021)	(0.031)	(0.154)	(0.036)	(0.011)	(0.178)	(0.023)
Ln net wealth	0.091***	0.483*	0.171***	0.201***	0.462	0.158**	0.026	0.521	0.167***
	(0.018)	(0.238)	(0.030)	(0.040)	(0.346)	(0.051)	(0.014)	(0.336)	(0.035)
Ln net wealth X Age	-0.002***	-0.009	-0.003***	-0.004***	-0.009	-0.003*	-0.000	-0.009	-0.003***
	(0.000)	(0.004)	(0.001)	(0.001)	(0.006)	(0.001)	(0.000)	(0.006)	(0.001)
Non-default allocation	0.388***	-0.457	1.065**						
	(0.009)	(2.183)	(0.371)						
Non-default alloc X Ln wage		0.057	-0.101**						
		(0.191)	(0.032)						
Observations	10548	1648	10548	4304	819	4304	6244	829	6244
Model Fit	0.227	0.251	0.734	0.116	0.254	0.661	0.029	0.241	0.788

Notes: All specifications include a wave indicator and are OLS models, except for (1), (4) and (7), which are logit (marginal effects reported). The Default (Non-Default) Allocation columns present results for the subsamples who opted for (out of) the default investment allocation. Age in specifications (1), (4) and (7) denotes plan enrolment age. Standard errors (robust, clustered by individual id) are in parentheses below estimated parameters. ***p-value<0.01, ** p-value<0.05, * p-value<0.1. Including Age^2 in (3), (6) and (9) leaves results unchanged.

the default one (2.30%). Earlier enrolees,[14] as well as wealthier and (extra) insured members also seem more likely to opt for a DC plan, with the effects mostly coming however from the non-default subsample. Indeed, changing the baseline from a default to a non-default asset allocation increases DC participation by 38.80%, which is unsurprising given that allocation decisions are particularly relevant for DC plan members. These results are consistent with the findings related to those that actively plan for retirement - by opting for a DC plan for instance, which involves more control of one's retirement savings - being older, richer and with higher income (Mitchell et al., 2006; Gerrans, 2012). In contrast, those who rely on defaults seem to do so across decisions, with very little to induce them to take control of their retirement 'pot'.

As expected, making voluntary contributions increases with age both overall and in the two (non-)default investor subsamples, with the m.e. being about 0.2. This is unsurprising given the tendency to more actively build up savings as one nears retirement. Those with more generous wages generally contribute more, with the non-default subsample also exhibiting slightly higher contribution elasticity of wage (m.e. of 1.09) compared to their default peers (m.e. of 1.02). These findings are consistent with Hira et al. (2009) which shows that those who do their own research, review plan information, believe it important to set aside funds regularly and are able to adequately do so (due to higher earnings), are also more likely to maximise their retirement contributions.

Most of the factors driving voluntary contributions also appear to matter for pension wealth. Specifically, we find a positive relation between pension wealth and both age and job tenure, with higher education also having a considerable beneficial effect. Compared to females, males appear to have significantly higher balances (the associated m.e. is 0.16), which can be attributed to both higher wages and longer tenures due to fewer career interruptions (see APH, 2016). Being married definitely makes a positive difference for retirement savings, while the overall beneficial effect of having supplementary insurance comes mostly from the default allocation members subsample. Pension balance elasticity of wages is roughly 1.15 for the full sample, and 1.14 (1.06) for default (non-default) members. These high figures are due to both employers and employees contributing, with an increase in wages affecting pension balance via both these channels. Net wealth matters

[14] The DB vs. DC decision is made upon becoming a plan member, thus the relevant age in *DC opt-in* specifications is the enrolment age.

considerably regardless of the subsample we analyze, with the associated elasticities around 0.17. This is consistent with net wealth levels being quite high in Australia and rich individuals increasingly holding the bulk of their wealth in shareholdings or property (ASFA, 2015). Finally, opting out of a default allocation appears positively and strongly related to pension wealth (m.e.=0.96), via the cumulative effect of being in non-default and being in non-default interacted with log wage.

We now turn to Table 3B to shed further light on asset allocations and homeownership decisions. A quick glance at specification (9) reveals that those with less advanced careers, higher wages and supplementary insurance are also more likely to choose non-default allocations: A one unit increase in log wage is associated with a 47.30% higher chances of opting for non-default portfolios, and compared to those uninsured, those insured are 33.20% more likely to do so. Notably, males are 12.8% more likely than females to opt for non-default allocations, but within this category it is interestingly females who invest more aggressively (see specification (4)), possibly in an attempt to close the pension gap with their male peers. This reversed 'gender investing gap' is likely linked to our sample's high financial literacy (Lusardi and Mitchell, 2007), and to women's higher loss aversion (Schubert et al., 1999) and risk aversion profiles (Dohmen et al., 2010).

As for homeownership, we see people becoming owners relatively early and accumulating a higher share of their assets in housing as they get older (Iacoviello, 2011). Education plays an interesting role, with those highly educated diversifying their portfolios more and committing less to homeownership, and those (very few in our sample) with lower education still using housing as primary vehicle to build up savings (Graham et al., 2009; ASFA, 2015).[15] Being in a couple and part of a large household is unsurprisingly positively associated with homeownership, although the former can also bring down the share of wealth invested in real assets. This is likely due to intra-household risk sharing opportunities or an added need for (possibly job-related) mobility, which makes couples opt for less housing-heavy portfolios, while being part of large families would slant things in the opposite direction. Coming to resources, we see higher wages being associated with higher chances of becoming a homeowner, an effect coming predominantly from the default subsample (m.e.=0.83). Looking at the actual housing assets share, however, we also see those on

[15]This result is also evident in the cross-sectional wealth distribution, where the wealthiest households can afford to diversify away from real-estate but the middle quintiles cannot and thus end up overweighting housing.

Table 3B. Estimation results for investment allocation and home-related decisions

Variable	All			Non-Default Allocation			Default Allocation		All
	Risky assets share	Homeowner	Housing assets share	Risky assets share	Homeowner	Housing assets share	Homeowner	Housing assets share	Non-default alloc
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Age	0.006	-0.512***	0.018***	0.013	-0.473***	0.024***	-0.533***	0.014**	0.012
	(0.003)	(0.083)	(0.003)	(0.007)	(0.143)	(0.005)	(0.099)	(0.005)	(0.024)
Male	-0.019**	-0.166	-0.003	-0.040**	-0.008	0.012	-0.285*	-0.015*	0.128*
	(0.007)	(0.101)	(0.005)	(0.015)	(0.149)	(0.008)	(0.138)	(0.007)	(0.050)
Low education	0.018	2.244***	0.066***	0.054	2.469***	0.102***	2.143***	0.048**	-0.234*
	(0.013)	(0.239)	(0.012)	(0.036)	(0.379)	(0.021)	(0.305)	(0.015)	(0.116)
High education	0.011	0.111	-0.022**	0.025	0.047	-0.024	0.154	-0.020*	-0.106
	(0.008)	(0.151)	(0.007)	(0.020)	(0.243)	(0.012)	(0.194)	(0.009)	(0.066)
Couple	-0.004	0.637***	-0.035***	-0.010	0.439*	-0.050***	0.771***	-0.023*	-0.069
	(0.008)	(0.115)	(0.008)	(0.019)	(0.182)	(0.012)	(0.151)	(0.011)	(0.063)
Household size	0.001	0.492***	0.029***	0.003	0.525***	0.029***	0.477***	0.028***	-0.010
	(0.002)	(0.044)	(0.002)	(0.005)	(0.068)	(0.003)	(0.058)	(0.002)	(0.016)
Good health	0.001	-0.076	-0.021***	0.001	-0.153	-0.022**	-0.025	-0.021***	0.006
	(0.005)	(0.088)	(0.005)	(0.013)	(0.138)	(0.008)	(0.116)	(0.006)	(0.042)
Suppl. insurance	0.000	-0.124	-0.015*	-0.002	-0.167	-0.015	-0.091	-0.015	0.332***
	(0.009)	(0.149)	(0.007)	(0.020)	(0.206)	(0.010)	(0.217)	(0.009)	(0.070)
Years of contribution	0.001	0.031**	-0.002***	0.002	0.021	-0.003***	0.038**	-0.002***	-0.034***
	(0.001)	(0.010)	(0.000)	(0.001)	(0.016)	(0.001)	(0.013)	(0.001)	(0.004)
Employers	0.017	-0.008	-0.001	0.044	0.159	0.004	-0.158	-0.007	0.165
	(0.011)	(0.193)	(0.010)	(0.026)	(0.332)	(0.015)	(0.233)	(0.013)	(0.085)
Ln annual wage	0.005	0.622**	-0.053***	-0.011	0.334	-0.054***	0.826**	-0.046***	0.473***
	(0.006)	(0.195)	(0.008)	(0.023)	(0.295)	(0.011)	(0.261)	(0.009)	(0.077)
Ln net wealth	0.018	-0.433	0.131***	0.046	-0.199	0.163***	-0.573	0.110***	0.054
	(0.011)	(0.276)	(0.013)	(0.027)	(0.469)	(0.019)	(0.333)	(0.018)	(0.090)
Ln net wealth X Age	-0.000	0.044***	-0.002***	-0.001	0.041***	-0.002***	0.045***	-0.001***	-0.001
	(0.000)	(0.007)	(0.000)	(0.001)	(0.012)	(0.000)	(0.008)	(0.000)	(0.002)
Non-default allocation	0.011	-0.038	-0.087						
	(0.198)	(0.086)	(0.134)						
Non-default alloc X Ln wage	-0.017		0.008						
	(0.017)		(0.012)						
Observations	10548	10548	10548	4304	4304	4304	6244	6244	10548
Model Fit	0.105	0.431	0.122	0.003	0.442	0.138	0.425	0.112	0.015

Notes: All specifications include a wave indicator and are OLS models, except for (2), (5), (7) and (9), which are logit (marginal effects reported). The Default (Non-Default) Allocation columns present results for the subsamples who opted for (out of) the default investment allocation. Standard errors (robust, clustered by individual id) are in parentheses below estimated parameters. ***p-value<0.01, ** p-value<0.05, * p-value<0.1.

higher wages holding a lower proportion of wealth 'locked' in housing, which suggest high earners using multiple savings vehicles to plan for retirement (Clark et al., 2012). For net wealth, we find almost the opposite pattern: Generally wealthier members have higher wealth shares invested in housing, an effect considerably more prominent for non-default members (m.e.=0.16) than for default ones (m.e.=0.11) who seem to turn more to other ways to save as they get rich.

4 The model

Having identified the factors associated with the level and prevalence of the main asset classes in one's wealth portfolio, we construct a rich structural model to assess how these empirically motivated elements can explain the data profiles and run counterfactuals to quantify the extent to which they do so. To this effect, consider an individual who plans to retire at age $T_R = 65$, faces a stochastic time of death and lives up to age $T = 100$. For simplicity, assume T_R is exogenous and deterministic, and let t denote adult age and p_t the probability of being alive at time $t+1$ conditional on being alive at time t.

A. Preferences. While alive, the individual derives utility each period from non-durable consumption c_t and housing services S_t, according to

$$u(c_t, S_t) = \frac{\left[(1-\alpha_t)c_t^{\rho} + \alpha_t S_t^{\rho}\right]^{\frac{1-\gamma}{\rho}} - 1}{1-\gamma}, \tag{1}$$

where $1/\gamma$ is the intertemporal elasticity of substitution, and $1/(1-\rho)$ is the elasticity of substitution between consumption and housing services. People can 'consume' housing services by either renting or owning a house (Sommer and Sullivan, 2018), with H_t denoting the stock of housing capital owned at time t. If an individual is not a homeowner (i.e., $H_t = 0$), then housing services must be rented at a rate P_t^S that we assume to be a constant proportion φ^S of the house price P_t. If an individual is a homeowner (i.e., $H_t > 0$), then they can only benefit from housing services brought by their own housing stock, thus $S_t = H_t$. Finally, the term $\alpha_t \in (0,1)$ is the idiosyncratic utility weight on housing services that follows an autoregressive process similar to the one specified by Kaplan et al. (2019). We introduce stochastic housing preferences to allow our estimation to account for the observed patterns in housing stock adjustment, which are driven in part by time-varying but persistent housing preferences (Ngai and Sheedy, 2020). As such, α_t fluctuates around a long-run stationary value $\overline{\alpha}$,

$$\ln \alpha_t = (1-\rho_\alpha)\ln\overline{\alpha} + \rho_\alpha \ln \alpha_{t-1} + \varepsilon_{\alpha t}, \qquad \varepsilon_{\alpha t} \sim N(0, \sigma^2_{\alpha\varepsilon_t}), \tag{2}$$

with ρ_α driving α's convergence, on average, to its long-run value. An initial α value significantly above $\overline{\alpha}$ coupled with a fast convergence (i.e., small ρ_α) is indicative of individuals placing less weight over time on housing services when making present consumption and housing choices.

Upon dying, any remaining wealth is bequeated to one's heirs, with the associated utility being

$$B(a_t^{DB}, a_t^{DC}, A_t) = \theta \frac{\left(a_t^{DB} + a_t^{DC} + A_t + k\right)^{1-\gamma}}{1-\gamma}, \tag{3}$$

where θ is the bequest weight, k determines the curvature of the bequest function (De Nardi et al., 2010), and total bequeathable wealth B_t includes both pension wealth (DB accumulated a_t^{DB}, and DC accumulated a_t^{DC}) and non-pension wealth A_t (whether accumulated via housing or not).

B. Employment. We assume individuals start working at age t_0 and while working they earn income y_t. Following the standard approach in the lifecycle literature on portfolio allocation,[16] we consider y_t as the sum of a deterministic and a stochastic component,

$$\begin{aligned} \ln y_t &= y(t,\tau) + \xi_t, \\ \xi_t &= \phi \xi_{t-1} + u_t, \qquad u_t \sim \mathcal{N}\left(0, \sigma_u^2\right) \end{aligned} \tag{4}$$

where $y(t,\tau)$ depends on age t and experience (job tenure) τ. ξ_t is an autoregressive term (with innovation u_t) that allows us to consider some level of wage persistence among individuals and is approximated by a discrete Markov process with N_ξ discrete state points.

C. Pension structure. Following the UniSuper plan architecture, we assume that individuals choose plan type p, voluntary contribution rates v_t and asset allocations π_t (i.e., the share π invested in risky assets at time t) as follows: At t_0, one is automatically enrolled into the default

[16]See Campbell et al. (2001), Gourinchas and Parker (2002), Cocco et al. (2005), Brown et al. (2012).

DB plan. There is a one-off option to switch to DC that must be exercised within the first period, otherwise DB membership becomes permanent. After choosing the plan, both employer and the employee start making their contributions.[17] The employer mandatory contribution v_E and the standard employee one v_S are set to specific (fixed) shares of y_t. Voluntary contribution rates v_t can be changed every period, with the default being $v_t = 0\%$. Finally, each period, individuals can choose to invest their DC component balance via 15 investment options that differ in risk and expected returns. Absent a direct choice, all corresponding assets are defaulted into a balanced investment allocation with a risky assets share $\pi^d = 0.7$. Finally, switching away from any default options is costly, with switching costs $u_i, i \in \{p, v, \pi\}$ modelled in terms of utility lost as[18]

$$\begin{aligned}
u_p &= \psi + \exp\left(\nu_0^p + \nu_1^p \hat{t} + \nu_2^p \hat{t}^2\right) \\
u_{v_t} &= \psi + \exp\left(\nu_0^v + \nu_2^v (t - \nu_1^v)^2 + \nu_3^v \max\{0, \ln(a_t)\}\right) \\
u_{\pi_t} &= \psi + \exp\left(\nu_0^r + \nu_1^r t + \nu_2^r t^2 + \nu_3^r \max\left\{0, \ln\left(a_t^{DC}\right)\right\} + \nu_4^r u_p\right)
\end{aligned} \tag{5}$$

where ψ is a 'default preference' parameter, while ν terms capture (i) the effort of researching, comparing options and filing forms that varies with age t or with the age $\hat{t}$ at which one is observed in Wave 10 to capture cohort specific factors that may have influenced plan choices, (ii) the liquidity value of savings outside pension plans a_t associated with making voluntary contributions, and (iii) the (DC) pension amount at stake a_t^{DC} in the risky assets share decision.

DB plans pay members a nominal benefit based on a formula related to the individual's age, job tenure (in years), contribution rates and average wage over the last three years of continuous employment. These plans include both a genuine DB component (where standard contributions v_S are accumulated) and a separate DC component (where voluntary contributions v_t will be made). As for employer's contributions, a share αv_E will be made to the DB component, with the remaining amount transferred to the DC one. Hence, DB plan benefits are calculated as[19]

[17] We differentiate between employer and employee contributions as the benefits from these two sources accumulate differently within each type of plan. We assume all contributions are pre-tax and subject to 15% concessional tax rate for the first $25,000 (concessional contribution cap); any exceeding amount is subject to 46.5% tax rate.

[18] Steel (2007), Ebersbach & Wilkening (2007), Agarwal et al. (2009), Besedes et al. (2012), Dobrescu et. al (2018).

[19] See 2012 UniSuper Defined Benefit Division & Accumulation 2 Product Disclosure Statement.

$$
\begin{aligned}
a_t^{DB} &= f_t^{ACF}(v_S) \cdot f^{LSF}(t) \cdot f^{ASF} \cdot \tau \cdot \bar{y}_t, \text{ with} \\
f^{LSF}(t) &= \max\{18,\ \min\{23,\ 23 - 0.2\,(65 - t)\}\}/100 \text{ and} \\
\bar{y}_t &= \frac{1}{3}\left[y_t + g(y_{t-1}) + g(y_{t-2})\right]
\end{aligned} \tag{6}
$$

where f_t^{ACF} is the Average Contribution Factor over the entire tenure span, $f^{LSF}(t)$ is the Lump Sum Factor (with $f^{LSF}(t \leq 40) = 18\%$ and $f^{LSF}(t \geq 65) = 23\%$), and $\bar{y}_t$ is the average wage over the last three years of continuous employment. We assume permanent staff work full-time (i.e., f^{ASF}=100%), and compute y_{t-1} and y_{t-2} as in Appendix C.

In contrast with DB benefits, DC benefits - associated with either the DC component of a DB plan or with a DC plan - are accumulated according to the asset allocation that each individual chooses. As mentioned, allocation options differ in terms of the share of time-t DC component balance invested in risky assets π_t and thus DC benefits are

$$
a_{t+1}^{DC} = \begin{cases} \left[\pi_t R_t^r + (1-\pi_t) R_t^s\right] \cdot \left[a_t^{DC} + (v_t + (1-\alpha)\, v_E)\, y_t\right], & \text{if in a DB plan} \\ \left[\pi_t R_t^r + (1-\pi_t) R_t^s\right] \cdot \left[a_t^{DC} + (v_t + v_S + v_E)\, y_t\right], & \text{if in a DC plan} \end{cases} \tag{7}
$$

with R_t^r (R_t^s) being the return on risky (safe) assets, computed as

$$
r_t^i = \ln R_t^i = r^i + i\varepsilon_t^d, \tag{8}
$$

where $i \in \{r, s\}$ is a scaling factor that can amplify ($r > 1$) or dampen ($s < 1$) asset market shocks, and $\varepsilon_t^d \sim N(0, \sigma^2_{\varepsilon_t^d})$ is the returns shock of the default allocation with constant mean return r^d and $r^s < r^d < r^r$. This specification allows us to account for the intertemporal effects of time-varying financial market returns on asset allocation (Campbell and Viceira, 1999, 2002).

D. Housing. In our model, housing plays a dual role. First, it is a durable consumption good and yields instantaneous utility. For simplicity, we assume individuals receive one unit of housing services for each unit of housing (stock) owned.[20] Additionally, housing is a severely illiquid form of wealth that can only be traded by incurring a transaction cost of $\tau_H P_t H_t$, where $\tau_H \in [0,1)$, P_t is the house price, and $P_t H_t$ is time-t housing value (Yogo, 2016).[21] Once the transaction cost has been paid, housing accumulates as:

$$H_t = (1-\delta)H_{t-1} + h_t, \tag{9}$$

i.e., individuals enter time t with an initial housing stock H_{t-1}, which depreciates at constant rate $\delta \in [0,1)$ and is replenished via housing expenses h_t (with $h_t < 0$ in case of downsizing). Second, housing is an investment good with capital value P_t, which brings a gross return R_t^h (Yogo, 2016)

$$R_t^h = (1-\delta)P_t/P_{t-1}. \tag{10}$$

Individuals can take out collateral debt m_t (mortgage or equity loan), which they must pay back at interest rate r_t^m. We assume r_t^m is linked to the safe assets returns through

$$r_t^m = \beta^m r_t^s + \kappa \varepsilon_t^d, \tag{11}$$

where β^m is the markup of the mean mortgage over the mean safe assets returns, and κ scales the mortgage volatility over that of safe assets returns.[22] Borrowers may choose their next period debt position m_{t+1} at time t, but must satisfy the collateral constraint

$$m_{t+1} \leq (1-\varphi^C)P_t H_t, \tag{12}$$

where $\varphi^C \in (0,1)$ is the homeowner deposit proportion. This condition ensures borrowers have a minimum equity of $\varphi^C P_t H_t$ if they hold a mortgage. We assume mortgages can be costlessly refi-

[20] In our setup, housing stock H_t captures both the size and quality of the dwelling; housing depreciation and investments appear simply as reductions and enlargements of H_t, respectively.

[21] After transaction costs have been paid, housing can be adjusted continuously. However, such costs are non-convex and will impose a friction that results in lumpy investment .

[22] Unlike Sommer and Sullivan (2018) that assumes a fixed markup over the constant safe assets return, we capture Australian mortgages by instead linking r_t^m to the stochastic safe assets returns to allow it to vary while keeping the model state-space computationally feasible.

nanced by those who are adjusting their housing stock. Thus for housing adjusters, only constraint (12) holds. However, for those who are not adjusting their housing stock, we assume costless adjustment of the mortgage position up to a limit ι in each period, thus $m_{t+1} - (1 + r_t^m)m_t \geq \iota$.[23]

E. Budget constraint. Let $\mathbb{1}_{\{h_t \neq 0\}}$ be an indicator function equal to one if time-t housing expenses deviate from zero. Assuming (i) individuals enter period t with some amount of financial wealth a_t, and (ii) there is only one risk-free asset in which to invest a_t that yields constant gross returns R, per-period financial wealth after consumption, renting and housing expenses is

$$a_{t+1} = Ra_t + (1 - v_t - v_S)\, y_t - c_t - P_t^S S_t - P_t h_t - \eta \mathbb{1}_{\{h_t \neq 0\}} P_t H_t + m_{t+1} - R_t^m m_t, \tag{13}$$

with $\ln R_t^m = r_t^m$ and total non-pension wealth (i.e., financial and housing, net of mortgages)

$$A_{t+1} = a_{t+1} + (1 - \delta) P_{t+1} H_t - m_{t+1}. \tag{14}$$

If we further assume that all individuals cash out their pension benefits as a lump-sum upon retiring at age $T_R = 65$, the intertemporal budget constraint becomes

$$A_{t+1} - m_{t+1} =$$

$$= \begin{cases} Ra_t - R^m m_t + (1 - v_t - v_S)\, y_t - c_t - P_t^S S_t - P_t h_t + (R_{t+1}^h - \tau_H \mathbb{1}_{\{h_t \neq 0\}}) P_t H_t, & \text{if } t < T_R \\ Ra_t - R^m m_t + a_t^{DB} + a_t^{DC} - c_t - P_t^S S_t - P_t h_t + (R_{t+1}^h - \tau_H \mathbb{1}_{\{h_t \neq 0\}}) P_t H_t, & \text{if } t = T_R \\ Ra_t - R^m m_t - c_t - P_t^S S_t - P_t h_t + (R_{t+1}^h - \tau_H \mathbb{1}_{\{h_t \neq 0\}}) P_t H_t, & \text{if } t > T_R \end{cases} \tag{15}$$

[23]The assumption of a limit on redraws from mortgage accounts means that some individuals will hold liquid assets and mortgages at the same time. The literature often assumes unlimited redraws in order to reduce the state-space as mortgages and liquid assets can be combined into a single net asset state - see Yang (2009), for example. In contrast, our motivation for costless but limited redraws here is *both* to match our institutional setting and also to enable us to identify the gross mortgage position in the asset portfolio. The Australian mortgage market is quite unique in that most mortgage accounts are accompanied by offset accounts or redraw facilities. In this setup, individuals can make extra payments into their mortgage account which they can later withdraw costlessly and this makes the mortgage balance relatively liquid (Price et al., 2019). To avoid adding the mortgage repayment schedule as an additional state-space, we capture the mortgage balance liquidity by assuming the redraw limit is constant each period at ι.

where R_t^h is computed as $R_t^h = (1 + r_t^h)$ and

$$r_t^h = r^h + \varepsilon_t^h, \text{ with } \varepsilon_t^h \sim N(0, \sigma^2_{\varepsilon_t^h}), \tag{16}$$

As a result, Eq. 10 then determines the housing price P_t dynamics, where the initial (2010) house price level is normalized to $P_0 = 1$. Finally, we assume that there is no borrowing associated with financial wealth and so, $a_{t+1} \geq 0$ in each period t.

F. Timing of events and Bellman equation. The dynamic problem can be viewed as a two stage optimization. At the start of the first period, each individual with financial wealth a_{t_0} and labor income shock ξ_{t_0} irrevocably chooses their plan. Thus, time-t_0 Bellman equation is

$$V_{t_0}(X_{t_0}) = \max_{\{DB,DC\}} \left\{ V_{t_0}(X_{t_0}|DB) + \zeta_{DB}, V_{t_0}(X_{t_0}|DC) - u_p + \zeta_{DC} \right\}, \tag{17}$$

where $X_t = \left(a_t, (1-\delta)H_{t-1}, m_t, P_t, a_t^{DC}, \xi_t, \tau, \{DB,DC\}, \alpha_t, \beta_t\right)$ is the vector of state variables and $\{DB,DC\}$ captures the set of plan types.[24] We further include an unobservable utility component in each option $\zeta_{\{DB,DC\}}$, which follows a type I extreme value distribution with scale parameter σ_p,[25] to allow for unobservable elements that might affect each individual's decision. Thus, the probability of choosing DC is (McFadden, 1974; Rust, 1987)

$$Pr(DC) = \frac{\exp\left[\left(V_{t_0}(X_{t_0}|DC) - u_p\right)/\sigma_p\right]}{\exp\left[V_{t_0}(X_{t_0}|DB)/\sigma_p\right] + \exp\left[\left(V_{t_0}(X_{t_0}|DC) - u_p\right)/\sigma_p\right]}. \tag{18}$$

In each subsequent period, individuals then choose (i) the voluntary contribution v_t (from the set $\{v_i, i = 1, 2...N_v\}$), (ii) the asset allocation π_t (from the set $\{\pi_i, i = 1, 2...N_\pi\}$), and (iii) optimal consumption c_t, housing expenses h_t and housing services S_t, to maximize the discounted present value of life-time utility,

$$\hat{V}_t(X_t, v_t) = V_t(X_t, v_t) + \zeta_{v_t}. \tag{19}$$

[24] Note that the state space includes P_t as it defines the relative price of housing in terms of consumption, which is time dependent. We also retain mortgages m_t as an explicit state variable to allow us to run the counterfactual that removes mortgage frictions (i.e., no costless redraws).

[25] The variance of the distribution of $\zeta_{\{DB,DC\}}$ is therefore $\frac{\pi^2}{6}\sigma_p^2$, and note that in our framework, is it more convenient to select the scale of the shock than to multiply the value functions by scaling parameters.

Here, ζ_{v_t} is the unobservable utility of the v_t choice, and the deterministic value $\bar{V}_t(X_t, v_t)$ is

$$V_t(X_t, v_t) = E\left\{\max_{\pi_t} \hat{V}_t(X_t, v_t, \pi_t)\right\} - u_v \cdot \mathbb{1}\{v_t \neq 0\}, \tag{20}$$

where $\hat{V}_t(X_t, v_t, \pi_t)$ is the value of a portfolio with π_t invested in risky assets, defined as

$$\hat{V}_t(X_t, v_t, \pi_t) = V_t(X_t, v_t, \pi_t) + \zeta_{\pi_t}. \tag{21}$$

Similarly to ζ_{v_t}, ζ_{π_t} is the unobservable utility component for the π_t choice, with observable part

$$V_t(X_t, v_t, \pi_t) = \max_{c_t, h_t, S_t} u(c_t, S_t) - u_\pi \cdot \mathbb{1}\{\pi_t \neq \pi^d\} + \beta_t E_t\left[p_t V_{t+1}(X_{t+1}) + (1 - p_t) b(B_{t+1})\right], \tag{22}$$

subject to budget constraint (15), collateral constraint (12), $S_t = H_t$ if $H_t > 0$, and

$$V_{t+1}(X_{t+1}) = E\left\{\max_{v_{t+1} \in \{v_i\}_{i=1}^{N_v}} \tilde{V}_{t+1}(X_{t+1}, v_{t+1})\right\}. \tag{23}$$

The term β_t is the time-t discount factor that follows a similarly structured AR(1) process as α_t, fluctuating around a long-run stationary value $\overline{\beta}$ with convergence rate ρ_β (Dobrescu et al., 2012).[26] Similar to α_t, an initial value for β that lies significantly above $\overline{\beta}$ coupled with a fast convergence (a small value of ρ_β) is indicative of an individual placing less weight over time on future consumption and housing when deciding present consumption and housing. Note that individuals know β_t, but they are uncertain about future values of β_s, for $s > t$. Because today's individual controls all future allocations, the issue here is one of uncertain future desires (i.e., the problem involves preference uncertainty, not time inconsistency).

We assume both ζ_{v_t} and ζ_{π_t} follow type I extreme value distributions independently, with scale parameters allowed to differ across plan types, so $\sigma_v^{DB} \neq \sigma_v^{DC}$ and $\sigma_\pi^{DB} \neq \sigma_\pi^{DC}$, and simplify as

[26] Krusell and Smith (1998) also show that time preference heterogeneity is key for generating higher order moments of the wealth distribution. In our case, it allows us to appropriately capture the joint dynamics of the asset distributions we observe in the data.

follows[27]

$$V_t\left(X_t\right)=\sigma_v^j \log \left\{\sum_{v_h \in\left\{v_i\right\}_{i=1}^{N_v}} \exp \left[\frac{V_t\left(X_t, v_h\right)}{\sigma_v^j}\right]\right\} \tag{24}$$

$$V_t\left(X_t, v_t\right)=\sigma_\pi^j \log \left\{\sum_{\pi_h \in\left\{\pi_i\right\}_{i=1}^{N_\pi}} \exp \left[\frac{V_t\left(X_t, v_t, \pi_h\right)}{\sigma_\pi^j}\right]\right\}-u_v \cdot \mathbb{1}\left\{v_t \neq 0\right\} \tag{25}$$

The discrete choice probabilities are thus

$$Pr\left(v_t=v_i\right)=\frac{\exp \left[V_t\left(X_t, v_i\right) / \sigma_v^j\right]}{\sum_{v_h \in\left\{v_i\right\}_{i=1}^{N_v}} \exp \left[V_t\left(X_t, v_h\right) / \sigma_v^j\right]} \tag{26}$$

$$Pr\left(\pi_t=\pi_i\right)=\frac{\exp \left[V_t\left(X_t, v_t, \pi_i\right) / \sigma_\pi^j\right]}{\sum_{\pi_h \in\left\{\pi_i\right\}_{i=1}^{N_\pi}} \exp \left[V_t\left(X_t, v_t, \pi_h\right) / \sigma_\pi^j\right]} \tag{27}$$

where $j \in \{DB,\ DC\}$.[28]

H. Solving the model numerically. Because there is no analytic solution, we solve the problem numerically using backward induction. Note the model features not only non-smooth housing adjustment costs, but also non-convexities arising from (i) the fixed or lumpy component of the housing adjustment cost, (ii) the jump discontinuity in the individual renting decision, and (iii) the discrete pension and risky assets share choices. To solve it, we employ the non-convex EGM in Shanker et al. (2022) that uses SOCs to verify sufficiency of functional Euler equations, thereby avoiding costly numerical root-finding at each iteration. Specifically, the main result in Shanker et al. (2022) writes the non-convex dynamic problem such that the discrete or non-convex choices occur as controls at time t, and then converts the discrete and non-convex controls to continuous ones with a quadratic constraint. The conversion allows us to exploit the time-separability of the dynamic problem to yield sufficiency of the dynamic via FOCs *and* SOCs. Once the FOCs and SOCs have been derived, the results by Shanker et al. (2022) avoid additional computational or analytical steps such as adjusting or removing sub-optimal grid points (Fella, 2014; Iskhakov et al.,

[27] The position parameters of ζ_{v_t} and ζ_{π_t} are assumed to be $-\sigma_v\gamma_E$ and $-\sigma_\pi\gamma_E$, where $\gamma_E = 0.57721$ is the Euler constant. Since voluntary contribution and investment choices are not relevant at ages beyond 65, we estimate the scale parameters directly (instead of normalizing them to 1 and multiplying the corresponding deterministic value functions by $1/\sigma_v$ or $1/\sigma_\pi$, respectively, for ages below 65). This is essentially a nested logit model (Berkovec and Rust, 1985).

[28] Since the state space X_t includes the plan type $\{DB, DC\}$, $V_t\left(X_t\right)$ in Eq. (24) differs across plan types.

2017), allowing EGM to be implemented similarly to a standard model without non-convexity.[29]

Despite using EGM, the high number of states in our model still imply a significant computational cost. To solve for the full non-linear solution, we proceed by distributing the solution algorithm on high performance compute clusters. For a given vector of parameters, we parallelize across CPU nodes the evaluation of the FOCs across the state-spaces for each age. However, for each iteration of age in the backward induction algorithm, the full policy function must be interpolated and distributed to each node. This generates a significant 'serial' component in the algorithm, which implies that the backward induction algorithm cannot be scaled linearly.[30] Thus, to parallelize the solution method, we first note that the model can be solved for males and females, and DB and DC members separately (since after making a plan choice, the alternative plan's policy function does not feature in an individual's optimisation problem). Examining the scaling performance of the algorithm, we found that indeed solving the model separately for each of the four groups on 24 CPU cores yields acceptable performance in terms of balancing memory requirements and marginal speed gained via each extra CPU node with the compute time costs. For details on the numerical method and the mathematical proofs deriving the Euler equations, see Appendix C.

Finally, note that our specification of mortgage redraws is designed to reduce dimensionality and make our estimation feasible, while preserving the features of the institutional context reflected in our data. The discussion below Equation (12) captures the Australian mortgage market setup, where individuals can costlessly withdraw from their mortgage accounts if they are ahead of their repayment schedule (up to the amount of extra payments made). Our model thus assumes that redraws are available up to a fixed limit ι, and we calibrate ι accordingly based on the amounts by which individuals are, on average, ahead of their mortgage repayments (RBA, 2018) - see also Kaplan et al. (2019). This assumption, however, means we do not capture how the marginal benefit of access to redraw options in the future affects the mortgage repayments individuals make. Nonetheless, we calibrate the redraw limit ι such that each period, individuals in our model face

[29]Note that the results in Shanker et al. (2022) are general and can be applied to a variety of non-convex time-separable dynamic optimization problems, including those considered by Fella (2014), Iskhakov et al. (2017) and Kaplan et al. (2019). Shanker et al. (2022) advances, for instance, the method in Iskhakov et al. (2017) that uses the context of retirement choice models to propose an algorithm that removes non-optimal points from a grid generated by necessary Euler equations, without deriving sufficient conditions for Euler equations.

[30]Strong linear scaling refers to the property of an algorithm where an increase in the number of CPU cores results in a proportional decrease in computation time.

the same redraw constraints as the average individual in our data.

5 Calibrations and estimation approach

To ease the computational load of estimating all parameters together, we first estimate or calibrate the parameters related to the state variables, as well as the rates of return and the DB pension parameters. Next, we run our models and use these estimated data generating processes to simulate lifecycle profiles for a large number of hypothetical individuals. Finally, we implement an iterative process to find the set of parameters that matches the simulated profiles with the (real) data ones.

We calibrate the survival probability p_t using the Human Mortality Database corresponding figures averaged across 2010 and 2014, and set enrolment age $t_0 = 17$ to match UniSuper data.[31] Next, we rely on an OLS model that associates $\ln y_t$ to a quartic in age and a quadratic in tenure years, and discretize ξ_t via a discrete Markov process with $N_\xi = 3$ gridpoints. For pension wealth, we set (i) v_S to 2.35% (and f_t^{ACF} to 86.0% as its data mean), (ii) v_E to 17% for both DC and DB members (hence $\alpha = 100\%$ as $v_s = 2.35\%$), and (iii) the interest rate parameters for the risky and safe assets, as well as for the default asset allocation to match the risk and return targets from the UniSuper product disclosure statements (i.e., r^d =2.88%, r^s=1.93%, r^r=4.76%, $\sigma_{\varepsilon_t^d}$=0.064, s=0.54, r=1.68). Financial wealth has a constant gross return of $R = 1.0223$, which captures the average real return for long-term (indexed) Treasury bonds for the period 1982-2014. Based on Eq. 10, the housing return is estimated using the Bank for International Settlements series on real residential property prices at a depreciation rate δ of 1.10% (Fox and Tulip, 2014). The real housing return rate, deflated by CPI for all items, has a mean of 3.2% and a standard deviation of 4.2% from 1982 to 2014; housing returns are thus calibrated with $r^h = 1.032$ and $\sigma_{\varepsilon_t^h} = 0.042$ annually. We set $\tau_H = 0.08$ (Yogo, 2016) and $\varphi^S = 0.06$ to capture the average rental yield in Gitleman and Otto (2012).[32] Following Guest (2005), we also set the mortgage deposit rate φ^C to 20%. Based on the Reserve Bank of Australia lending rates for 1982-2014, we find an average real mortgage rate (using median inflation) of 0.0649, with a 0.021 standard deviation.[33] Thus,

[31] Setting a common enrolment age reduces the state of the model during estimation; experimenting with varying the enrolment age in the simulations did not significantly alter our key results.

[32] This is consistent with Australian estimates of rental yields (Fox and Tulip, 2014; Saunders and Tulip, 2019).

[33] See RBA Statistics Table F5. Indicator Lending Rates.

we set $\beta^m = 3.36$, $\kappa = 0.33$ and limit the costless redraw ι to \$120,000 to reflect the estimated offset accounts average balance for our sample, calculated using (i) the national statistics on the average extra payments made into mortgage acccounts as a proportion of total mortgage debt and the ratio of debt to average income (RBA, 2018), and (ii) the average income in our data. The bequest shifter parameter k is set to the weighted average of its marital status-specific parameters (Ding, 2013). Finally, initial financial wealth is taken from the data,[34] initial pension and housing wealth are both 0, initial housing capital value is set to a negligible \$1, and the initial real house price is set so that the house price an individual of a given age faces in 2010 is normalised to 1.

Using the Simulated Method of Moments (SMM - see McFadden, 1989; Pakes and Pollard, 1989), we estimate

$$\phi = \left\{\gamma, \overline{\alpha}, \rho_\alpha, \sigma_{\alpha\varepsilon_t}, \rho, \theta, \overline{\beta}, \rho_\beta, \sigma_{\beta\varepsilon_t}, \psi, \{v_i^y\}_{i=0}^{3}, \left\{v_i^p\right\}_{i=0}^{2}, \{v_i^r\}_{i=0}^{4}, \sigma_p, \left\{\sigma_v^j, \sigma_r^j\right\}_{j\in\{DB,DC\}}\right\} \in \mathbb{R}^{27}, \quad (28)$$

by matching the real moments related to wealth and plan choices to the corresponding moments of the same variables in the simulated sample. The objective is to find the vector of preferences $\widetilde{\phi}$ that simulates the distributions such that they fit the data best. To this end, we match (i) first order moments related to consumption, pension, financial and housing wealth, voluntary contributions, and risky assets share - overall and above default levels; (ii) second order moments of consumption, financial and housing wealth; (iii) lagged correlations of consumption, financial and housing wealth; (iv) correlations between consumption and housing wealth, voluntarily contributing and opting for DC, and switching to DC and opting for non-default allocations; (v) plan-specific second order moments of pension wealth, risky assets share and voluntary contributions; (vi) plan-specific correlations between pension wealth and voluntary contributions (amount and prevalence), as well as between pension wealth and risky assets share (level and prevalence of opting for riskier-than-default allocations), and between voluntarily contributing and having non-default allocations, and (vii) proportions of individuals opting for a DC plan, voluntarily contributing and opting for non-

[34] Initial financial wealth was predicted in gender-specific subsamples using the 2^{nd} order age polynomial coefficients of an OLS regression on the available financial (non-pension) wealth data. None of the other HILDA-UniSuper matching dimensions were relevant at the initial (plan enrolment) age.

default allocations. We discuss the identification below.

For efficiency reasons, we estimate our models for males and females separately, using our two waves of UniSuper data.[35] For each model, we calculate the age-specific empirical (real) moments by first selecting the appropriate subsample (i.e., males or females). Next, we assign individuals into 5-year age cohorts, with the 1^{st} cohort consisting of individuals with ages below 25 in 2010, the 2^{nd} containing those aged 25-29 in 2010, and so on.[36] We then take cell means by cohort [37] for the balanced panel in each wave to generate the data moments. For the simulated moments, we simulate $N = 10,000$ paths of individual choices,[38] collect the simulated values for each path and then compute N sets of simulated moments, conditional on the initial values of the state variables X_{t_0} and on the parameters $\widetilde{\phi}$. Finally, the SMM estimator $\hat{\phi}_{SMM}$ minimizes the distance between the set of empirical moments m_T and the average of the N sets of simulated moments $\frac{1}{N}\sum_{n=1}^{N} m_n(X_{t_0}, \widetilde{\phi})$,

$$\hat{\phi}_{SMM} = \arg\min_{\widetilde{\phi}} [m_T - \frac{1}{N}\sum_{n=1}^{N} m_n(X_{t_0}, \widetilde{\phi})]' W_T [m_T - \frac{1}{N}\sum_{n=1}^{N} m_n(X_{t_0}, \widetilde{\phi})], \tag{29}$$

where W_T is the weighting or distance matrix that almost surely converges to $W_T = S^{-1}$, where S is the limit, as $NT \to \infty$, constant full-rank matrix of the covariance of the estimation errors. For a given N, as $T \to \infty$, if the weighting matrix is chosen optimally,

$$T(1+1/\mu)[m_T - \frac{1}{N}\sum_{n=1}^{N} m_n(X_{t_0}, \widetilde{\phi})]' W [m_T - \frac{1}{N}\sum_{n=1}^{N} m_n(X_{t_0}, \widetilde{\phi})] \to \chi^2(j-k),$$

where μ is the ratio of the simulated sample size to the empirical one, j is the number of moments and k is the number of estimated parameters. To minimize the objective function (29), we use the cross-entropy method (Botev el al., 2011). We start with a uniform draw of $\bar{T} = 300$ parameter

[35] Note that the UniSuper-HILDA link involves matching UniSuper individual pension information with similar (individual) level HILDA data. While asset decisions might be, at least to some extent, taken at the household level, our data profiles consistently represent individual behavioural patterns.

[36] The last cohort, labelled ”60-64”, also contains a few observations on individuals 65+. Their data (on wealth, consumption, contributions, allocations) is not very different from the ”60-64” cohort data and so, including the 65+ in the last cohort does not significantly alter the empirical moments.

[37] We deal with housing, financial wealth, and voluntary contributions outliers by excluding the 99th percentile of each series.

[38] Using more than 10,000 paths to compute moments did not change results materially.

vectors and evaluate the objective function (29) across each of them. To do so, we create 300 groups of CPU nodes, each with 48 CPU nodes. Each of these groups is further clustered into sets of 24 nodes, each solving the lifecycle model conditional on DB and DC pension choice for one parameter draw. From this initial draw, we select the top 10% performing parameters that achieve the lowest value of the objective function (29). Next, we use this 'top' parameter set to fit a multi-variate distribution over the parameter space, from which we sample the next iteration of $\bar{T}$ parameter vectors and evaluate the objective function (29) across the new parameter samples. Again selecting the top set of performers from this stage, we repeat the process until the covariance matrix of the parameter sample distribution satisfies a predetermined tolerance.

5.1 Identification

To pin down which moments identify our parameters of interest, we start by presenting the intuition behind why each parameter might significantly affect only a subset of moments. Since an analytical proof is not possible, we next validate these intuitions by establishing identification in a local neighborhood of a selected subset of parameters via simulation.[39] Changing one parameter can affect, however, multiple data moments. For instance, risk aversion, bequest weight and discount factor parameters are jointly identified by cohort-specific first order financial wealth moments: A high γ makes individuals save more, higher β's means people are more future oriented and decumulate more slowly, and high θ's (i.e., strong bequest motives) also lead to higher savings. We better identify these parameters by requiring the model to also match the first order moments of pension wealth and housing wealth, by cohort. The rationale is provided by our Euler equation. Ignoring bequests, the Euler equation shapes the savings profiles (i.e., the financial wealth profiles, at least before retiring and cashing out the pension benefits). So, these wealth profiles are largely dictated by a combination of time discounting (β's) and taste for smoothing (γ's). In the case of β's, this equation however identifies the product $\beta_t p_t(1+r)$, not its individual elements. Therefore, lower values of r and/or p_t can lead to higher β_t estimates. To check whether the interest rate can be separately identified, we set its value to the maximum rate observed for the riskiest asset allocation

[39]To do so, we compute the moments and fit the value function at and around estimated parameter values. Next, we check whether the resulting simulated profiles fit the empirical ones as we vary the value of each parameter and verify the fitted function shape in a neighbourhood of the selected parameter value.

(among the 15) and re-estimate the models. We find that the realized returns are on average higher than our benchmark assumption of 2.23% and our β's are accordingly lower. We thus conclude that we can only identify $\beta_t(1+r)$, but not each term separately. Given the autoregressive nature of the underlying β process, however, we acquire additional identification by also requiring the model to match the variance and first order autocorrelation of financial wealth. To further pin down γ's, we also use the correlation involving non-default choices on voluntarily contributing and plan type. Intuitively, this might bias downwards our estimates of risk as we identify them based only on the 'active' sample when presumably default members have certain attitudes towards risk too. Hence, to acquire extra identification we use the proportion of DC wealth invested in riskier (than default) assets across both the DB and DC subsamples. Going back to the Euler equation, we note that bequest motives are related to the total amount of resources that could be passed on as bequeathable wealth. Thus, the bequest weight θ will apply to all of pension, financial and housing wealth, and we additionally identify this parameter via the age-profile of the mean pension and housing wealth. Finally, the parameters α and ρ are identified by noting that the within-period utility function is CES between consumption and housing wealth: α gives the share of resources corresponding to consumption rather than housing, while ρ indicates the within-period substitution between the two. We thus identify the α AR(1) parameters via the mean, variance and lagged correlation of consumption and housing wealth, and use the correlation between these two series to pin down ρ.

Turning to the switching costs, we note that their identification comes from the observations where individuals actually switched away from defaults. Hence, the latent factor ψ is identified via plan-specific correlations involving non-default choices on voluntarily contributions and asset allocations. To identify the parameters of u_p, we match the age-specific proportion of people that switched to DC. For u_v, we match the age-specific proportion of people contributing (to identify the age coefficients) and the mean level of voluntary contributions by age (to identify v_3^v). As for the investment switching costs, the parameter v_3^r of $\ln\left(a_t^{DC}\right)$ is identified by the mean risky assets share, v_4^r by the correlation between opting for non-default allocations and opting for DC, while the age coefficients are once again identified by the proportion of people with non-default investment allocations by age. Finally, to identify the unobservable utility components associated with our three pension choices we proceed as follows: First, we identify the scale parameter σ_p

that determines the variance of ζ_{DB} and ζ_{DC} using the variability in pension wealth by plan type. Second, since people choosing DB or DC might value liquidity differently or have different attitudes toward risk, we allowed the relative weight of ζ_{v_t} and ζ_{r_t} to differ across plan types. As a result, to identify the parameter σ_v we use the plan-specific variance of voluntary contributions and the correlation between pension wealth and voluntary contributions (both amount and prevalence). And similarly, we can identify σ_r by some plan-specific measures of risky assets share variability and by the correlation between pension wealth and risky assets share (both level and prevalence of opting for riskier-than-default allocations).

Finally, we also use the correspondence between the empirical and simulated profiles for homeownership prevalence as an informal over-identification test. While we did not directly fit this variable, our results show that the model manages to endogenously replicate the high rates of homeownership observed in the data very well. We discuss this in more detail in the next section.

6 Results from the structural model

6.1 Parameter estimates

A quick glance at the top panel in Table 4 reveals economically reasonable SMM parameter estimates for both males and females. For instance, the lifecycle literature generally finds relative risk aversion parameters between 1 and 6 (Chetty, 2006). Our estimated γ is roughly 3.62 for males and 3.26 for females. This overall level is in line with Cagetti's (2003) estimates for U.S. college graduates, while the slight gender gradient confirms previous findings on the risk-taking gap gap fading away for highly educated females (Gerrans and Clark-Murphy, 2004; Drupp et al., 2020). This gender differential does not, however, carry over to our time preference estimates. Indeed, the discount factor β is roughly 0.91 on average for both males and females, a value well within the range reported in the literature (Cocco et al., 2005; Dobrescu et al., 2012). Similarly, we find a utility weight of housing versus consumption of 0.51 (0.49) for males (females), slightly higher than in Kaplan et al. (2019) but lower than Yogo's (2016). As for the intra-period substitution between the two, we estimate ρ to be 0.24 for males and 0.32 for females, in line with Ogaki and Reinhart (1998) and Piazzesi et al. (2007), suggesting that females are more willing to substitute housing for consumption. This is also consistent with Andersson's (2001) finding that females in fact invest less in real estate than males, a result we also find in our reduced-form analysis (see Section 3). Combined with risk aversion being lower for females, this suggests that not only different attitudes towards risk but also different willingness to substitute housing and non-housing consumption can explain the heterogeneous savings behaviours across various groups. Turning to the intensity of the bequest motive, we find θ parameters of roughly \$4,316 for men and \$15,522 for females. This gender differential might be due to the stronger intergenerational altruism of females and thus their greater propensity to save for heirs (Seguino and Floro, 2003).

The bottom panel in Table 4 presents the default switching cost estimates. To help with interpretation, we express these costs as the net present value of the additional DC pension balance (at retirement) required to compensate for the associated utility loss. Figure 1 plots these monetary equivalents. First, we note the marked downward sloping profile of all three cost types, suggesting switching away from defaults becomes cheaper over time. More years of service can help individuals realize the importance of retirement savings, and we would thus expect them to take increasingly more control of their wealth accumulation, particularly near the end of their career. As expected, the highest switching costs are related to opting out of the default plan, this irreversible choice costing on average about \$16,732 for males and \$15,961 for females. These amounts represent roughly 18.60% of annual wage, only slightly lower than the those estimated by Luco (2019). Unlike Luco (2019), however, our switching cost structure includes two additional dimensions. In this respect, we see the next highest switching cost being the one related to voluntary contributions. Indeed, switching away from the 0% contribution rate costs females about \$12,319 and males \$11,909. While the gender gradient is inverted compared to default plan switching, this very subtle difference confirms our estimates in Section 3 on males and females being quite similar in their voluntary contributions. In contrast to these patterns, the gender gradient is significantly more marked for the decision to opt out of the default asset allocation. Indeed, it appears to be 35.51% cheaper for males to do so compared to females. Given the high risky assets share of the default allocation and the lower pension wealth of females, it is not surprising that switching away from this default option means considerably more missed high return opportunities for females that can significantly affect their retirement savings.

Overall, while all these costs might seem high, recall that they do not mean that an average individual would not switch for this amount in cash, but that they would not switch for this amount in their DC component upon retirement. An alternative way to understand their impact is to see how wealth would have changed over time had switching been costless. We do so in Section 6.3.

Table 4: Parameter estimates

		Males		Females	
		Estimates	S.E.	Estimates	S.E.
CRRA	γ	3.617	0.098	3.261	0.016
Housing share	$\overline{\alpha}$	0.512	0.013	0.494	.0144
	ρ_α	0.817	0.029	0.797	0.041
	$\sigma_{\alpha\varepsilon_t}$	0.023	0.002	0.023	.001
CES parameter	ρ	0.244	0.023	0.326	0.024
Bequest	$\ln(\theta)$	8.367	0.075	9.652	0.093
Time discount	$\overline{\beta}$	0.918	0.012	0.901	0.019
	ρ_β	0.843	0.021	0.801	0.045
	$\sigma_{\beta\varepsilon_t}$	0.025	0.001	0.034	0.012
Switching costs:					
Voluntary	v_0^v	-0.689	0.011	0.655	0.022
contribution	v_1^v	59.721	0.015	59.249	0.034
	v_2^v	0.096	0.001	0.093	0.002
	v_3^v	-0.360	0.032	-0.564	0.020
	$\sigma_v^{DB} \times 10^3$	1.530	0.001	1.202	.001
	$\sigma_v^{DC} \times 10^5$	0.503	0.001	1.041	0.001
Asset	v_0^r	4.710	0.081	3.271	0.013
allocation	v_1^r	0.244	0.013	-0.101	0.029
	v_2^r	-0.410	0.011	-0.330	0.071
	v_3^r	0.162	0.002	0.360	0.016
	v_4^r	0.081	0.002	0.051	0.021
	$\sigma_r^{DB} \times 10^3$	3.022	0.001	0.621	0.002
	$\sigma_r^{DC} \times 10^3$	1.351	0.001	0.564	0.001
Plan	v_0^p	-0.321	0.089	-0.651	0.075
	v_1^p	-1.893	0.052	-0.124	0.042
	v_2^p	-1.017	0.018	-0.672	0.016
	σ_p	0.557	0.043	0.424	0.055
Default preference	$\ln(\psi)$	-7.234	0.067	1.481	0.057

6.2 Data patterns and model fit

We now turn to our model's ability to recreate the data patterns. Figures 2 - 10 below plot selected moments by cohort. In all figures, the lines labelled *wave10_data* and *wave14_data* correspond to

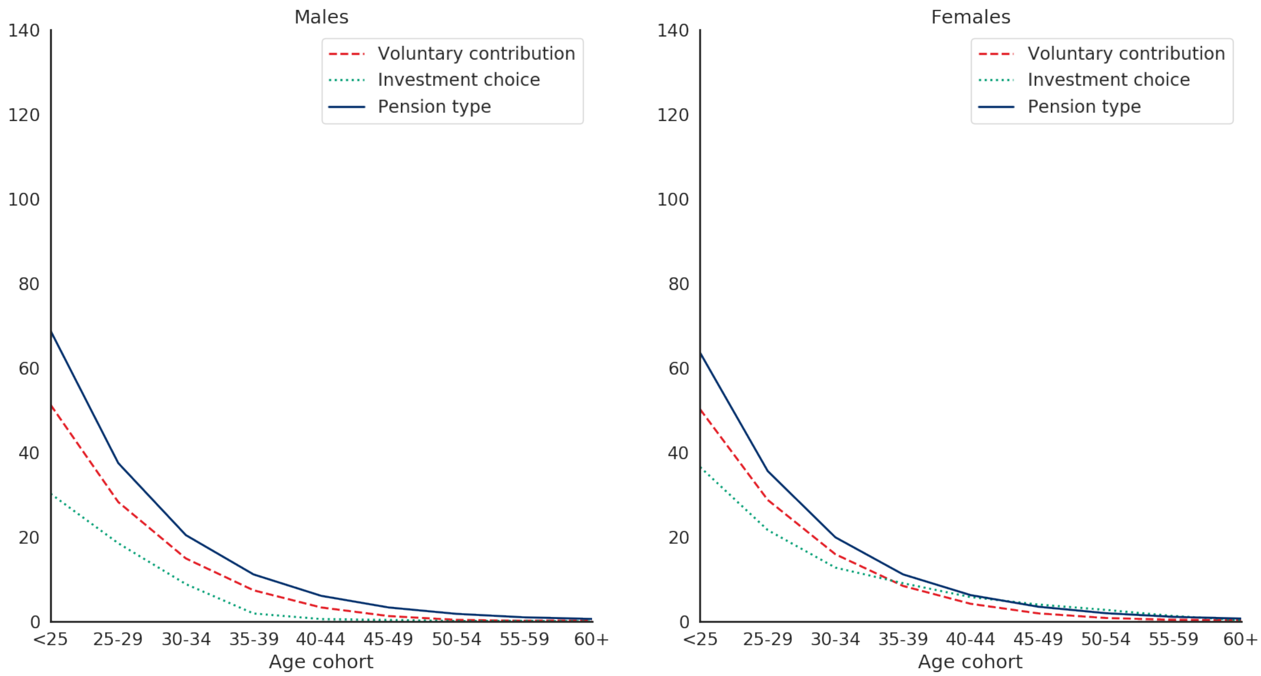

Figure 1: Mean switching costs by cohort (thousands of $)

Waves 10 and 14 data, while *wave10_sim* and *wave14_sim* denote their simulated counterparts.

A quick glance at Figures 2 - 6 reveals that the models fit well overall, for both male and females. In particular, we have successfully replicated (i) the increasing age profiles of wealth for all three types of assets accumulated (Figures 2 - 4), (ii) the gender-specific plan opt-in overall levels (Figure 5), (iii) the relatively stable risky assets share over one's lifecycle with some moderate rebalancing in old age (Figure 6), and (iv) the increasing patterns of voluntary contributions (Figures 7 - 8), that we observe in the data. The goodness of fit between the simulated and the empirical (data) moments is assessed via a χ^2-test (or corresponding $p-value$). In both cases, the model easily passes the χ^2-test of goodness of fit, with χ^2-values well below the 5% critical value. Thus, we cannot reject the null that the simulated and empirical moments are the same at standard significance levels.

We start by taking a closer look at wealth. Figures 2 - 4 show the level of pension, financial and housing wealth by cohort, respectively. As expected, all types of wealth increase with age, but females appear to accumulate 25.74% less in their pension account than males. This might be due to females working fewer hours and being more likely to face career interruptions (i.e., maternity

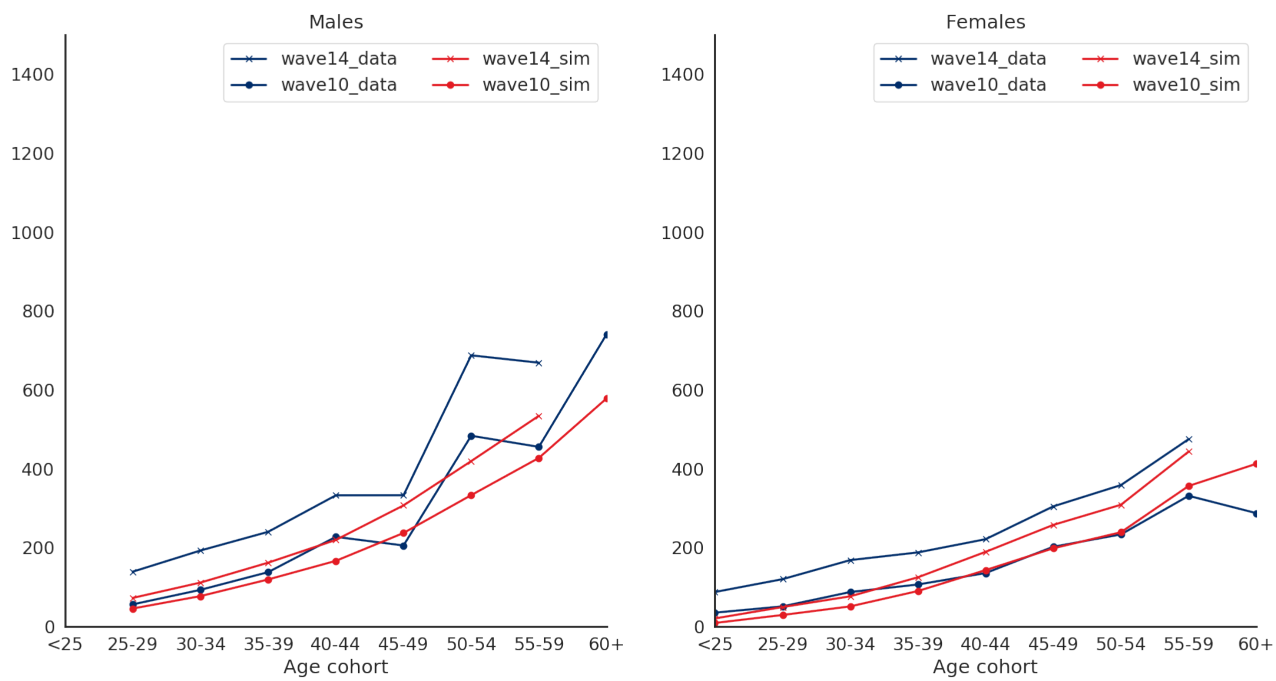

Figure 2: Mean pension wealth (DB+DC) by cohort (thousands of $)

leave, carer responsibilities).[40] With shorter tenures and potentially slower wage growth, they will also have lower pension balances.

Additionally, females are less likely to opt for DC plans, with only 32.62% of females across all cohorts switching out of the default (DB) plan, compared to 35.39% of males (see Figure 5). These differences further deepen the wealth gradient between sexes due to missed opportunities for high return investments. Confirming this conjecture, note that only one in five members in our sample pursue riskier-than-default investment options, with females holding on average 2.13% riskier portfolios. While both males and females show relatively flat age profiles of their risky assets share, for some cohorts there also seems to be a moderate portfolio rebalancing away from risky assets as people age - see Figure 6. In higher education, the decrease in the stock of human capital occurring with age is arguably milder and with a more stable income level and less risk, it is not surprising we see such risky assets profiles (Haliassos et al., 2001; Cocco et al., 2005).

One way to supplement pension wealth is via voluntary contributions. Similar to Beshears et al. (2009), we find rather low overall non-default prevalence when the default contribution rate

[40]See 2013 COAG Reform Council Tracking equity: Comparing outcomes for women and girls across Australia.

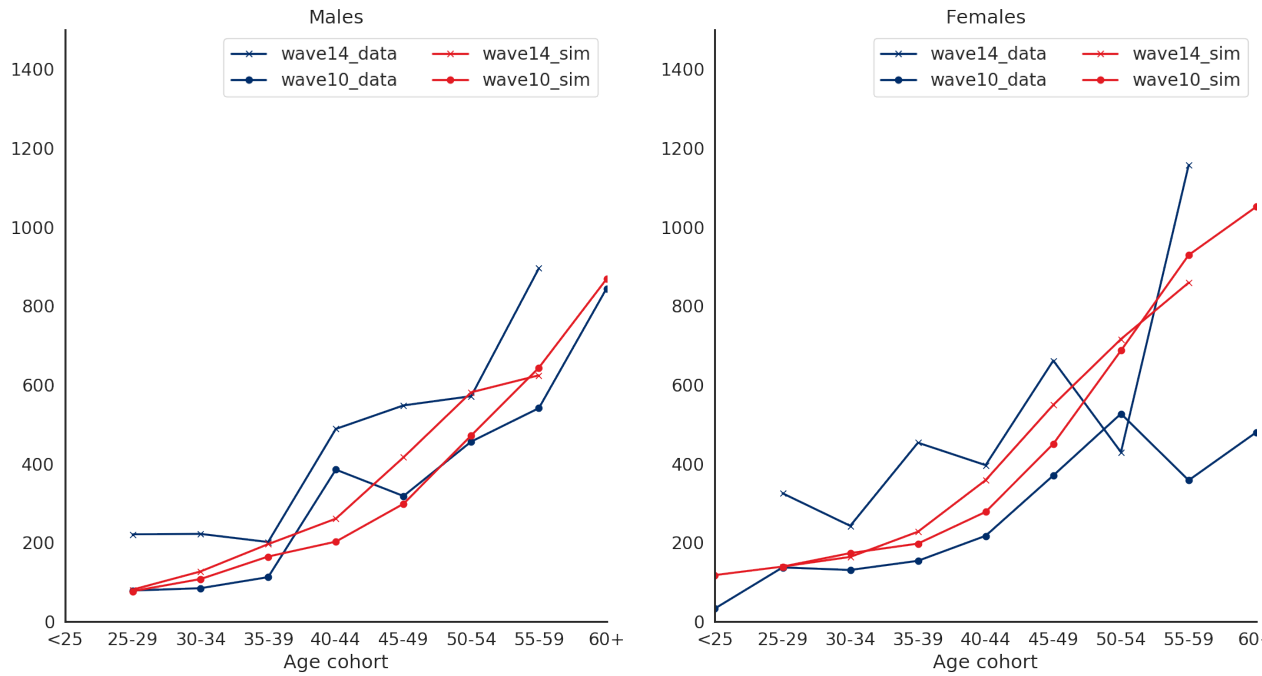

Figure 3: Mean financial wealth by cohort (thousands of $)

is 0%: While towards the end of their career about 40-50% of members contribute, for most of their active life less than 15% make voluntary contributions (see Figure 7). Surprisingly, females rely only slightly more than males on this option to insure their retirement savings against negative labor events and close the pension balance gap with males. Indeed, Figure 8 shows that, across cohorts, females contribute only about 6.14% more than males across all cohorts, which confirms our empirical results in Section 3.

Outside pension wealth, we also find an 7.98% positive difference in financial assets between females and males. This could be due to the higher marginal propensity to accumulate precautionary savings amongst females (Seguino and Floro, 2003), which counteracts their lower wages. As for housing wealth, we find that females accumulate about as much housing wealth as males. This rather high real assets path for females, particularly given also their higher financial wealth, is consistent with females deriving higher utility from housing than males, as discussed in Goldsmith-Pinkham and Shue (2020).

Unsurprisingly, all these wealth patterns generate very reasonable consumption profiles that fit those observed in the data - see Figure 9. As expected, the profiles display the usual slightly

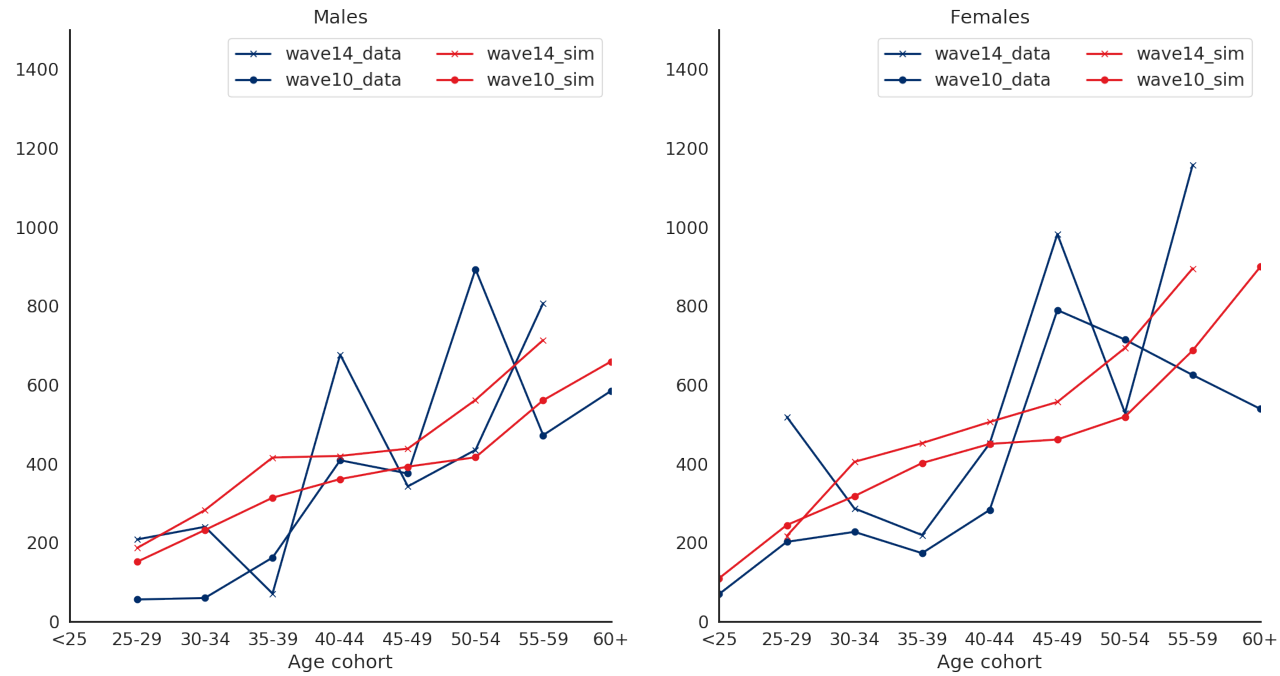

Figure 4: Mean housing wealth by cohort (thousands of $)

increasing, mild hump-shape, with males registering lower profiles than females. The higher gap between the simulated and empirical profiles for males might be due to us under-estimating their real consumption by limiting it to non-durables. In fact, the gap is smaller for females, for whom non-durables (e.g., fuel, power, clothing, toys, personal care, household items) are more prominently budget-featured than for males (Bradbury, 2004; Blow et al., 2004).

6.3 Counterfactual simulations

Several counterfactual experiments evaluate the role of different factors in shaping these data profiles. Specifically, we consider how pension and non-pension wealth changes over a lifetime if individuals (i) maintain a uniform income even after retirement, equal to the total (working-life) expected wages divided by their lifespan, referred to as the "No consumption smoothing" scenario,[41] (ii) do not value leaving a bequest, referred to as the "No bequests" scenario, (iii) do not experience earnings uncertainty, referred to as the "No precautionary savings" scenario, (iv) switch

[41] As in Pashchenko and Porapakkarm (2020), note that individuals are still exposed to wage risk. In calculating mean yearly wage, we assume for simplicity a total lifespan equal to the total number of expected years lived. Experiments with methods that adjust for mortality risk do not alter our results.

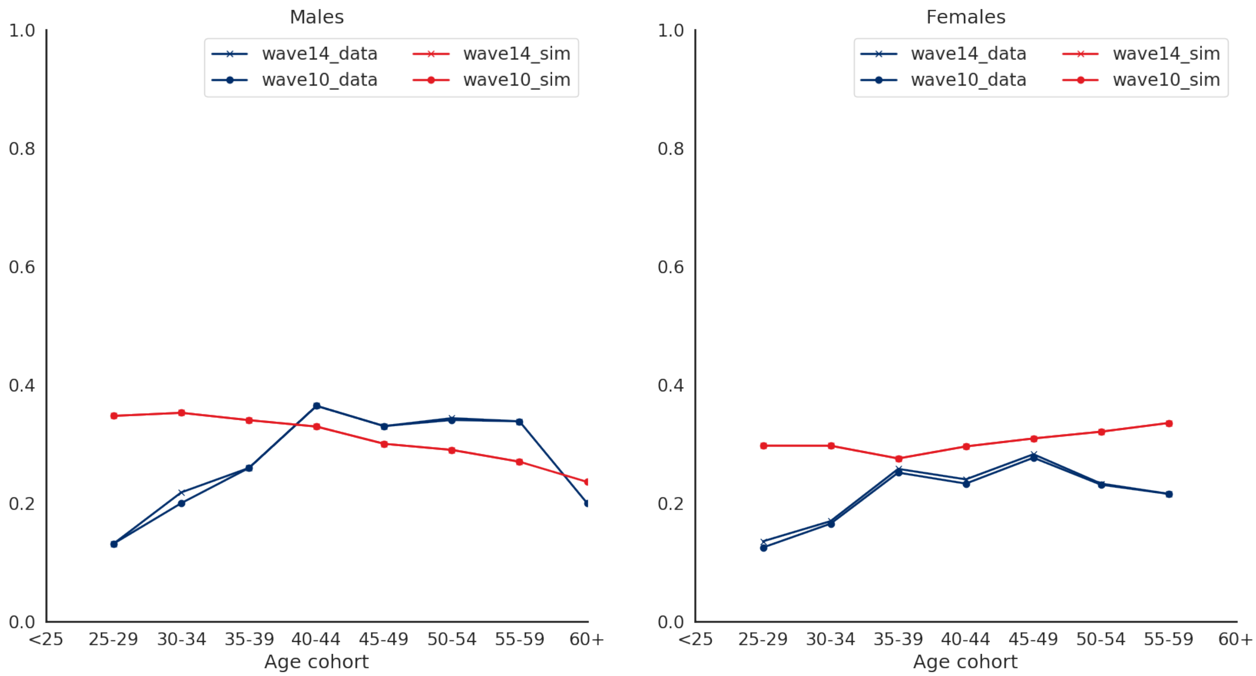

Figure 5: Share of members choosing DC plans by cohort

away from plan defaults (DB plan, 0% voluntary contributions, 70% risky asset allocations) at no cost, captured via the "No switching costs" scenario, (v) earn 4% higher returns for their risky assets (Bansal et al., 2002; Brailsford et al., 2012; Damodaran, 2020), captured by the "Higher risky returns" scenario, and (vi) are unable to make a costless redraws from their mortgage, referred to as the "No redraw" scenario.[42] Every simulation modifies certain parameters associated with a particular scenario, solves the model numerically and generates the corresponding wealth patterns. In Table 5 below, we compare how our counterfactual wealth allocations across all cohorts compares with our baseline levels ("Baseline" scenario).

These exercises reveal several ways in which individuals employ various types of assets to build their savings. Let us start with the scenario that cancels consumption smoothing motives. As predicted by standard lifecycle theory, this scenario leads to a drop in wealth accumulation across all types of assets. Since DC accounts offer people a way to optimally choose their asset portfolio to smooth consumption according to their idiosyncratic preferences, without this motive,

[42]Here we assume individuals repay their mortgage at a constant amortisation rate and can make extra repayments, but not withdrawals from their mortgage. We follow the amortisation formula in Kaplan et al. (2019) and, for tractability, we also assume individuals may refinance their mortgage only by adjusting their housing stock.

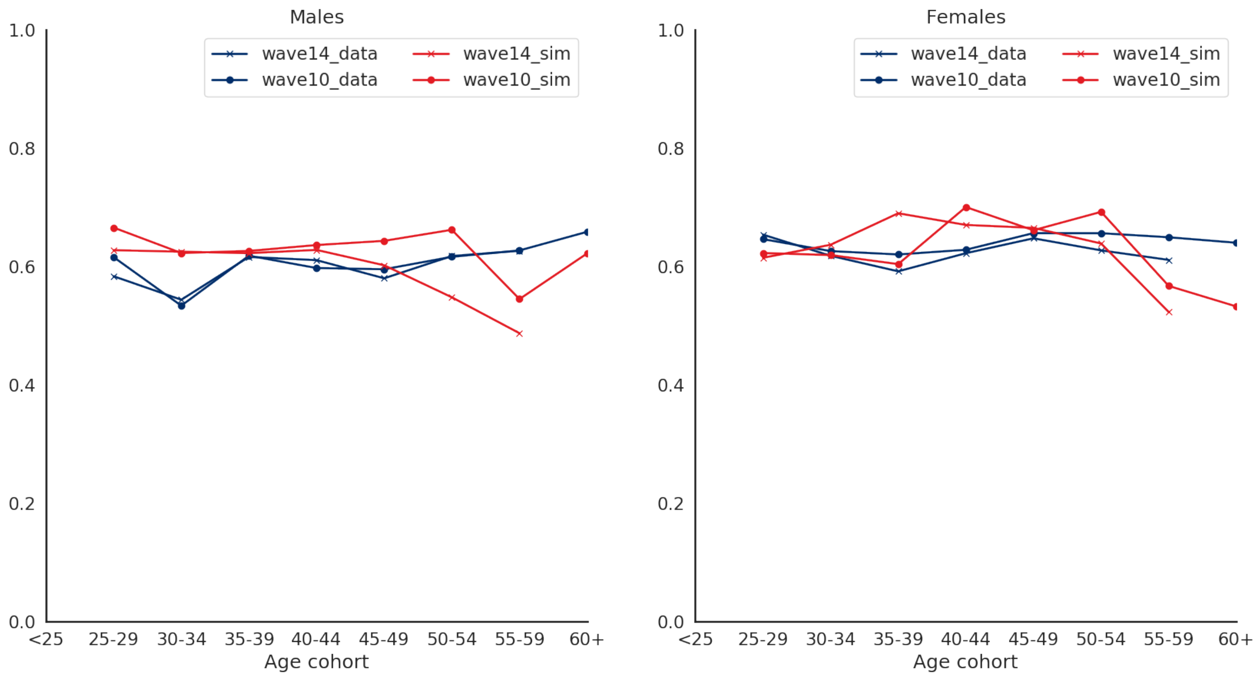

Figure 6: Mean risky assets share by cohort

DC enrolments fall for both genders. While females maintain their contributions to build up their pension savings, males do so slightly less and they also opt for a smaller increase in risky assets share compared to females. Taken together, these changes lead to about 35% less pension wealth for males and 25% less for females. Turning to non-pension wealth, we see the financial wealth of males falling by over 40% without the consumption smoothing motive, although their housing wealth falls only modestly by 6.3%. In contrast, both financial and housing wealth of females fall by over 40%. Housing is thus as important a form of consumption smoothing for females as financial wealth is, while this holds significantly less for males. We return to the role of housing as a mechanism for consumption smoothing when we decompose the saving motives in Section 6.4.

Second, turning off bequest motives seems to have largely the same pension effects as cancelling consumption smoothing motives, namely (i) lower pension wealth and DC plan enrolment across the board, (ii) relatively stable contribution profiles for females and lower ones for males, and (iii) higher risky asset share for males *but* considerably lower ones for females. This (iii) effect arises due to bequests being luxury goods: Since the curvature of the bequest function is less prominent than that of the consumption function, a higher value placed on bequests makes females

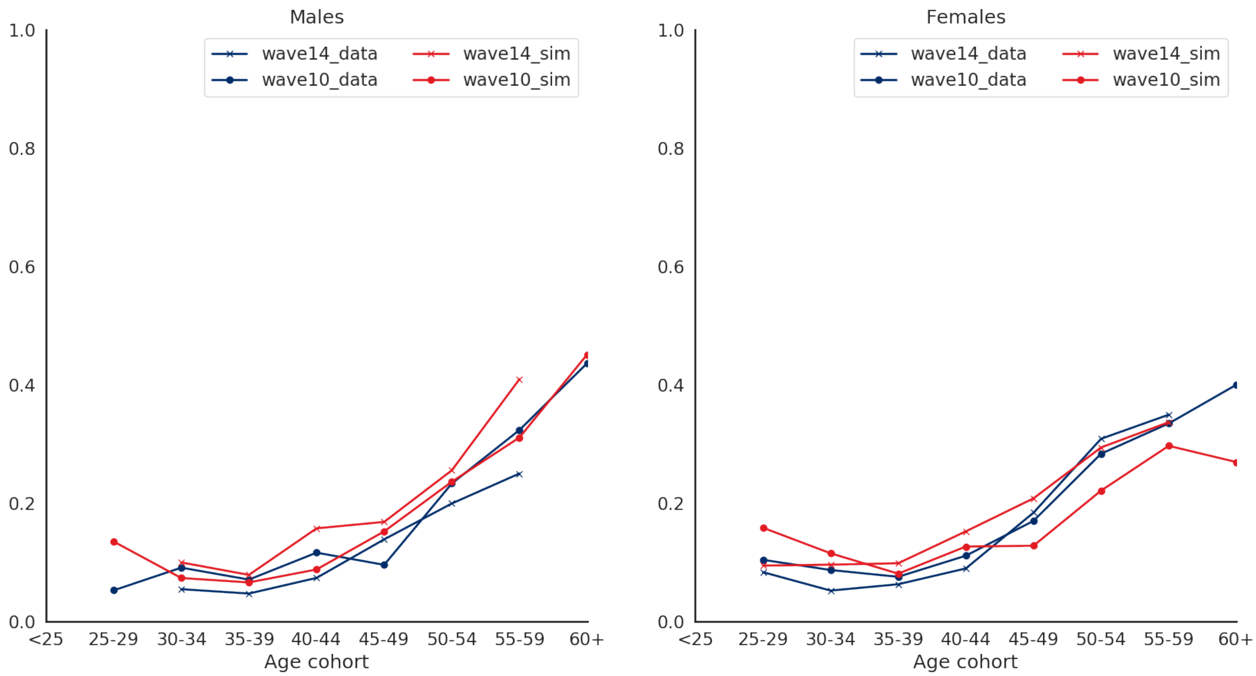

Figure 7: Share of members voluntarily contributing by cohort

less risk averse (Ding, 2013). Without an active bequest motive, the risky assets share falls by 12% for females. Males become more risk averse once they lack bequest motives too, but this effect is dominated by the effect of plan choice on risky assets share: Without bequests, males are more likely (by roughly 7%) to stay in DB, and DB members have on average riskier assets than DC ones, which ultimately dominates the bequest effect on risk taking since males value bequests less than females.[43] While removing bequest motives unsurprisingly also reduces by about one fifth financial wealth as it is no longer needed for one's heirs, housing wealth increases because bequests displace consumption, including housing services consumption (Kopczuk and Lupton, 2007). This rise in housing wealth happens unsurprisingly more for females (26%) than for males (16%) as females value bequests more than males and thus this housing consumption displacement effect of bequests appears stronger for them. Additionally, the transaction cost associated with buying a house means individuals might not view housing as an efficient means to save purely for bequest

[43]Recall DB members are less likely to opt away from the default (70% risky assets) allocation into safer ones, although switching costs for both DB and DC members are similar. This is due to DB members having 'less at stake' by not switching out of the default allocation (as their DC component is less relevant to their post-retirement income than for DC members), and thus being less likely to overcome their switching costs and change to a safer portfolio.

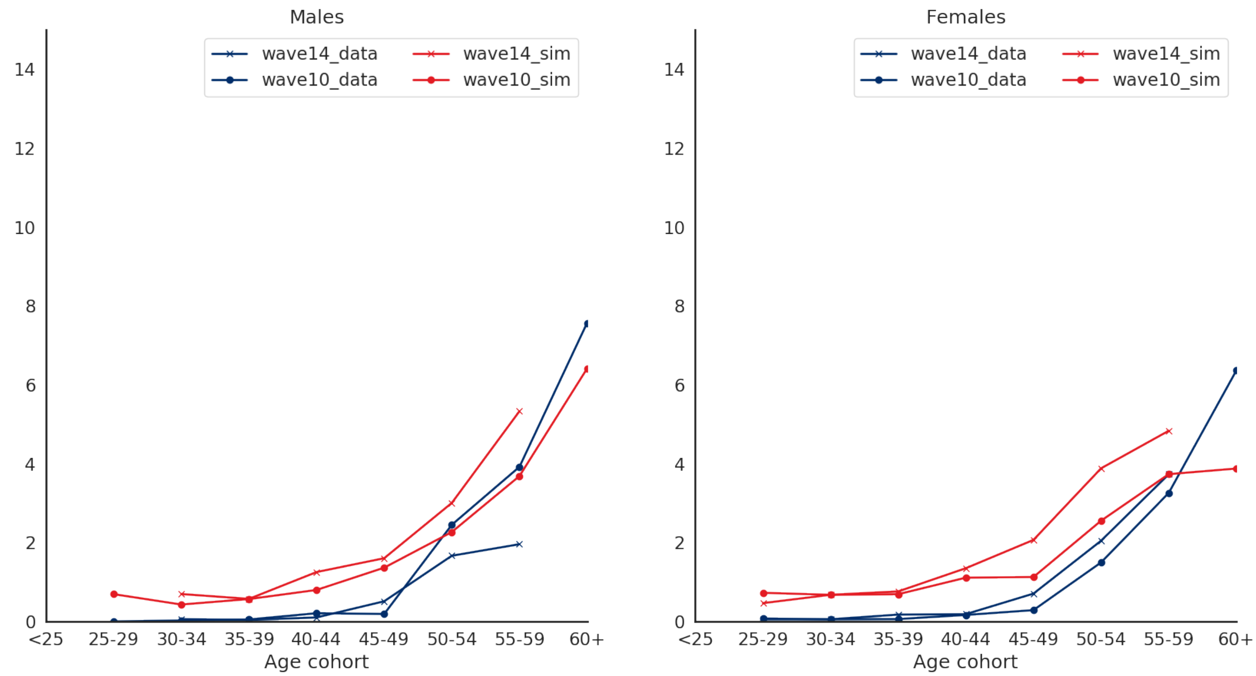

Figure 8: Mean voluntary contributions by cohort (thousands of $)

motives. Thus the marginal effect of no bequest motives does not lead to a high enough drop in housing as a form of bequeathable wealth to offset the housing gain from the non-displacement effect of bequests. From this perspective, financial wealth - and to a considerable extent pension balances - seems to be in our case a more effective means of bequeathing wealth than housing as they do not incur transaction costs.[44] We further examine how bequests affect pension balances when we decompose saving motives in Section 6.4.

Third, we switch off wage uncertainty, which means that individuals are left with no precautionary saving motives related to self-insuring against labour income risks. Without such a motive to save, the reliance on (earnings-based) DB plans increases slightly for both females and males, with (i) everybody investing more aggressively, and (ii) females contributing more to compensate for the earnings differential. As a result, we see an overall drop in pension wealth - once more, more prominent for males (34%) than for females (14%) - but this time also accompanied by the reverse pattern in net non-pension wealth. This means that additional non-pension wealth, and in particular financial wealth is not being accumulated as a form of precautionary saving, a result

[44]Note that our model does not account for taxes, which could affect this result in practice.

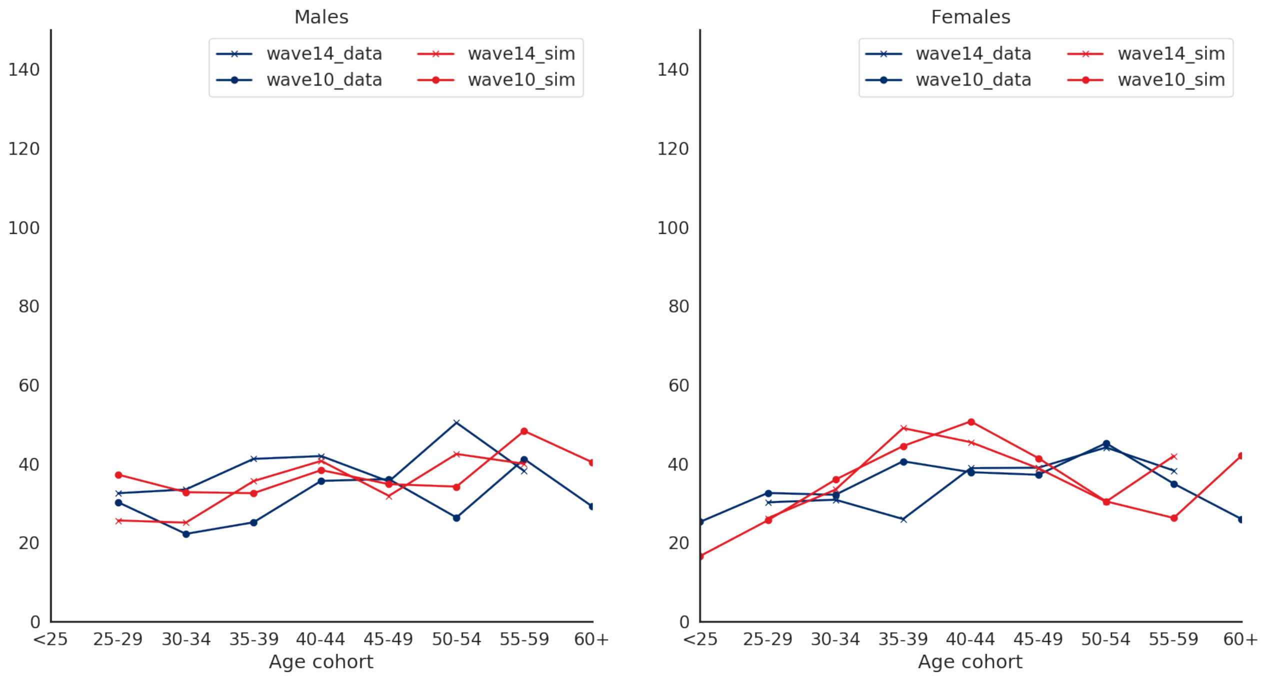

Figure 9: Mean consumption by cohort (thousands of $)

we can understand in terms of our baseline individuals having access to borrowing via mortgage redraws that can be used as insurance. Indeed, paying off mortgages has a dual role - as a form of 'saving' (to avoid paying interest) that also enables early homeownership and as a form of insurance. Hence, even without wage risks, individuals continue to invest in housing and re-pay their mortgages, making the borrowing constraint imposed by the collateral sufficiently generous to not prompt precautionary savings. And Carroll et. al. (2021) points out, it is the possibility of hitting a borrowing constraint due to risk that induces individuals to accumulate precautionary saving. In our case, the borrowing constraint is quite high due to high housing accumulation and somewhat 'fixed' due to housing stock being costly to adjust, and so the addition of wage risks do not lead to a large enough mass of individuals ever facing the prospect of hitting the borrowing constraint. Under a very low probability of hitting the collateral constraint, the extra concavity of the consumption function that would have otherwise led to precautionary saving in the presence of wage risks never arises (see discussion below Proposition 3 by Carroll et. al., 2021), rendering such risks irrelevant to the precautionary saving motive. That being said, also note however the extent to which non-pension wealth increases, especially for males (19%). Without earnings

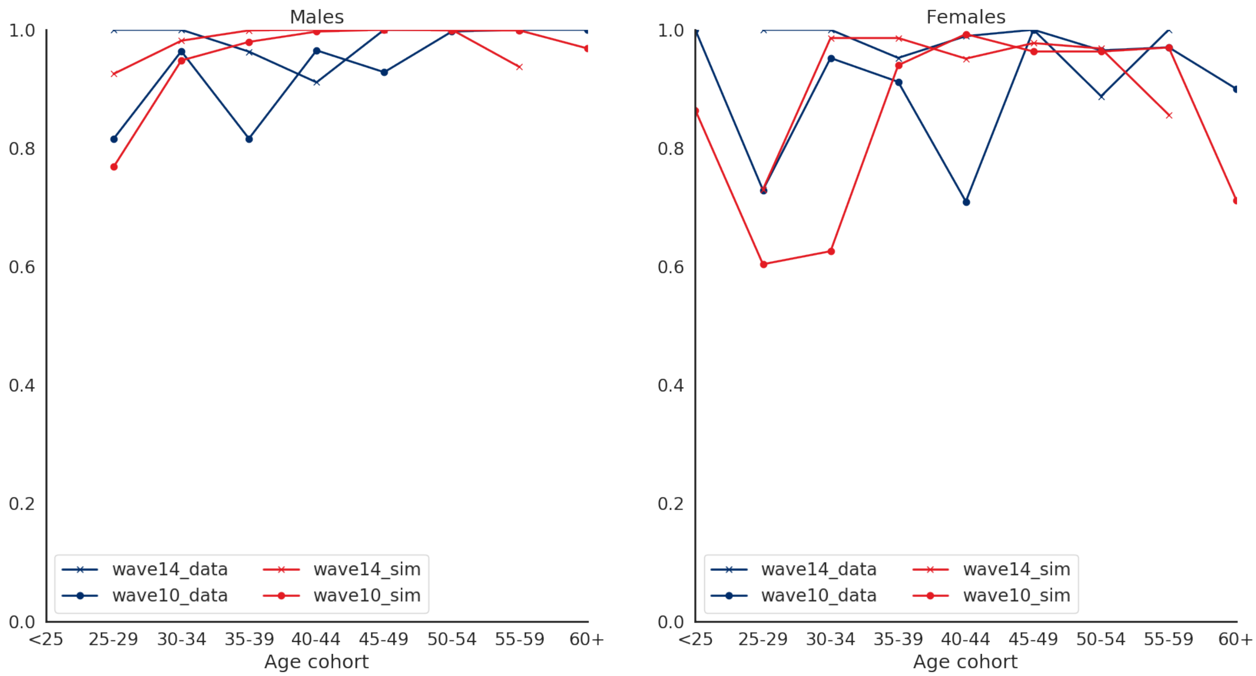

Figure 10: Share of homeowners by cohort

risks, individuals are more likely to invest in housing and take on the risk that holding housing assets brings (which also explains this counterfactual's higher risky assets share, as wage risks drive down risky investment - see Fagereng et al., 2017). However, the key to understanding the non-pension wealth pattern lies in the effect of no earnings risks on plan choice - more individuals remain DB members, thereby accumulating less pension and more non-pension wealth (especially via risky housing assets) as they do not face the risky returns to the same extent as DC members. The general lower DC prevalence in this scenario highlights how DC plans offer an important way to manage wage risk via diversifying the effect of wage risk on post-retirement income: Recall from equation (6) that as a DB member, the post-retirement payout is subject to all the wage shocks up to retirement age (Lindbeck and Persson, 2003; Yang, 2005). DC plans thus offer a way to diversify away from this wage risk exposure since after contributions are made, the accumulated amounts face pension returns risks that are uncorrelated with wage risk. Once earnings uncertainty is removed, the diversification offered by the DC plans becomes less relevant and there is less incentive to switch into them. Rather, people are more likely to stay in DB but ex-post, since they no longer face wage or returns risks, they choose to take on risk by accumulating more housing assets.[45] This counterfactual thus shows how in addition to the standard direct effects of saving motives on wealth accumulation, a quantitatively significant mechanism shaping savings across different asset classes is the indirect effect of saving motives on plan choices. After a saving motive drives people towards certain plan choices, such plans impose a unique set of constraints that, in turn, also shape asset allocation decisions.

Fourth, we directly test the effect of plan defaults by constructing a counterfactual that eliminates switching costs. On the pension wealth side, the results of this scenario confirm lifecycle theory on higher flexibility and opportunities for diversification in default provisions generating a considerable boost (averaging roughly 60%) in pension balances - due to higher DC take-up and likelihood to contribute. As expected, this pension boost directly offsets other financial savings for both males and females. For housing, however, we uncover a complementarity relation, with all individuals accumulating more housing in response to their higher lifetime wealth: As people expect to be wealthier post-retirement, they will wish to smooth future housing consumption and bring some of this consumption forward into their working years.

To further examine this pension-housing complementarity, we conduct a fifth counterfactual that increases risky (pension) returns. Higher risky assets returns unsurprisingly boost pension wealth for both males and females by about one fourth - via higher prevalence of DC plans, voluntarily contributing and holding risky assets, which are all now more rewarding options. However, we also find that overall non-pension wealth, and in particular housing also increases, and more prominently so for males than for females. To understand this, recall that housing adjustment occurs with a fixed cost and individuals accumulate housing early in their working life (see Section 3.2 and Yang, 2009). They thus know when they purchase a house early in their working life that they will prefer not to adjust their stock later in life (due to frictions), meaning housing consumption becomes effectively locked in by decisions taken during early working years. As a result, when individuals purchase a house in their younger years, they will take into consideration not only what they wish to consume immediately, but also what they anticipate consuming in their later life (and even after retirement.) With higher pension returns, younger individuals anticipate a lower

[45] This case is the inverse of the point made by Fagereng et al. (2017) in which wage risks drive down risky investments; without earnings and returns uncertainty, risky housing wealth increases.

marginal utility of consumption after (and close to) retirement and thus increase their housing accumulation in earlier years, as they wish to lock in higher levels of such wealth. Thus, pension and housing wealth, due to the presence of housing frictions and the role of housing as a durable consumption good, behave as complements rather than substitutes.

Finally, we also generalize our results to abstract from our housing setup (and how it interacts with the precautionary saving motive to influence plan choice) by removing the costless redraw option of Australian mortgages. While costless redraws are a standard assumption in the literature (Yang, 2009; Fischer and Stamos, 2013) and fit well our institutional context, we have designed our model to be general enough to accommodate relaxing this assumption. When doing so, we find a substantial drop in the DC uptake and, as a result, a fall in the associated pension wealth - by 11% and 16% for males and females, respectively. The drop in DC opt-ins is due to individuals now being unable to benefit from the risk diversification offered by their DC components and thus preferring to accumulate (safe) financial wealth that fulfils both an insurance and a retirement saving (consumption smoothing) function. Indeed, the third ("No precautionary savings") counterfactual showed that the added diversification offered by the DC plans - that helps smooth consumption and diversify retirement saving when earnings are uncertain - was salient enough to encourage switching into DC. However, saving in safe financial assets also serve as a way to diversify away from wage risks (Fagereng et al., 2017). Cancelling the redraw option will therefore prompt more financial wealth savings, which (i) insure against wage risks in the presence of a salient borrowing constraint, and (ii) represent (safe) pension savings after wage risks dissipate. With the added diversification from higher levels of financial wealth, opting into DC plans to achieve retirement saving diversification becomes less attractive since after plan choice decisions are made, individuals' retirement savings are already less exposed to wage risks. In addition, DC choices do not help with being able to 'commit' to buying a house, do not provide collateral and do not help manage mortgage risks during early working years when liquidity constraints matter. As a result, while financial wealth might increase (by 34% for males and 27% for females), we also find people will hold less housing wealth. To sum up, a reason to opt for DC is to diversify the risk portfolio associated with pension savings in order to mitigate the wage risk effects. Without costless mortgages redraws, precautionary saving motives now drive people to accumulate more (safe) financial wealth and less housing. This higher financial wealth lessens the exposure to wage risks during working years, and also reduces their impact on total post-retirement wealth as the share of such wealth dependent on the DB payout reduces. With this added security offered by financial assets, DC plans become less relevant.

Table 5: Counterfactual scenarios (saving motives)

	Opting into DC plans	Opting to contribute	Risky assets share	Pension wealth	Non-pension wealth:	Financial wealth	Housing wealth
	% of members		%	% change from baseline			
			Panel A. Males				
Baseline	35.392	21.216	59.514	-	-	-	-
No cons. smoothing	32.169	20.009	61.241	-34.764	-19.325	-43.457	-6.339
No bequests	32.798	17.497	65.824	-33.139	0.095	-19.423	16.232
No prec. savings	28.940	21.232	63.671	-33.723	18.730	39.796	7.394
No switching costs	41.185	73.098	48.711	67.946	14.850	-0.046	22.866
Higher R^r	42.644	23.026	61.851	24.477	9.686	9.962	9.537
No redraw	23.967	20.887	60.434	-10.597	35.382	34.423	-1.592
			Panel B. Females				
Baseline	32.619	21.968	61.731	-	-	-	-
No cons. smoothing	29.402	21.871	64.918	-24.542	-44.947	-49.072	-40.873
No bequests	30.040	20.996	54.329	-23.242	2.355	-20.567	26.459
No prec. savings	25.042	25.450	63.690	-13.588	5.017	0.033	9.941
No switching costs	35.447	55.007	52.522	55.989	-0.667	-6.035	4.637
Higher R^r	35.022	23.680	62.538	25.295	1.036	3.082	0.986
No redraw	23.967	19.661	62.233	-15.677	15.276	27.346	-10.789

Notes: Non-pension wealth sums housing and financial wealth. Housing wealth is net of mortgage liabilities.

6.4 Decomposing the saving motives

So far we have focused on studying how various saving motives drive savings decisions, including those related to building up pension balances. However, plan choices have significant second order effects in and of themselves that re-balance people's portfolios. To directly isolate the impact of saving motives on wealth allocation, we now move to examine the saving motives profiles across cohorts and genders, with plan prevalence fixed at their baseline levels. In particular, Figures 11 - 13 show the effect of *adding* a (consumption smoothing, bequest or precautionary) saving motive for financial, gross housing and pension wealth, respectively. For example, the extra precautionary financial savings in Figure 11 is the difference between the baseline financial wealth profile (where all motives are at play) and the counterfactual that eliminates wage risk. This type of calculation allows us to interpret Figure 11 profiles as the marginal effect of each saving motive on each major asset class (see Gourinchas and Parker, 2002; Cagetti, 2003; Pashchenko and Porapakkarm, 2020).

Let us start our discussion of the saving motives decomposition from the financial wealth in Figure 11. A quick glance shows that the consumption smoothing motive induces significantly more financial saving after the age of 40. These patterns are consistent with lifecycle theory on people starting to save for consumption smoothing reasons once middle aged, and increasingly so later in life (Gourinchas and Parker, 2002). Moreover, since males appear more risk averse than females, they prefer investing in safer but also less rewarding (pension) assets and thus end up accumulating more liquid wealth due to this motive than females. What is notable here is that despite access to private pension accounts, males accumulate high financial savings predominantly for consumption smoothing reasons. For females, however, bequest motives also come into play to generate some savings in their middle years, although not as much as for consumption smoothing purposes. This is not surprising given females' higher bequest weight (see also Seguino and Floro, 2003; Dobrescu et al., 2018). Finally, we find precautionary saving motives not to induce any widespread extra liquidity. Note that this results does not imply that financial savings lose their insurance role, but only that when savings due to other motives are present, the added insurance associated with the precautionary motive is negligible.

Now, let us turn to gross housing wealth in Figure 12. Unsurprisingly, on the margin, housing is neither a form of insurance (although housing equity serves as a form of insurance as discussed above) nor a form of bequest. Indeed, housing serves primarily as a consumption smoothing avenue, with all individuals accumulating substantial additional housing assets for this reason by the end of their working life. Females in particular, due to their willingness to take on more risk, accumulate housing wealth to an even larger extent than males for consumption smoothing - roughly twice as much as males from their late 50s onwards. Unlike for financial wealth, however, the consumption smoothing motive starts housing accumulation much earlier in life, for both males and females - a pattern driven by the dual role that housing plays: Recall housing acts not only as a vehicle of wealth that can finance future non-durable consumption, but also directly provides

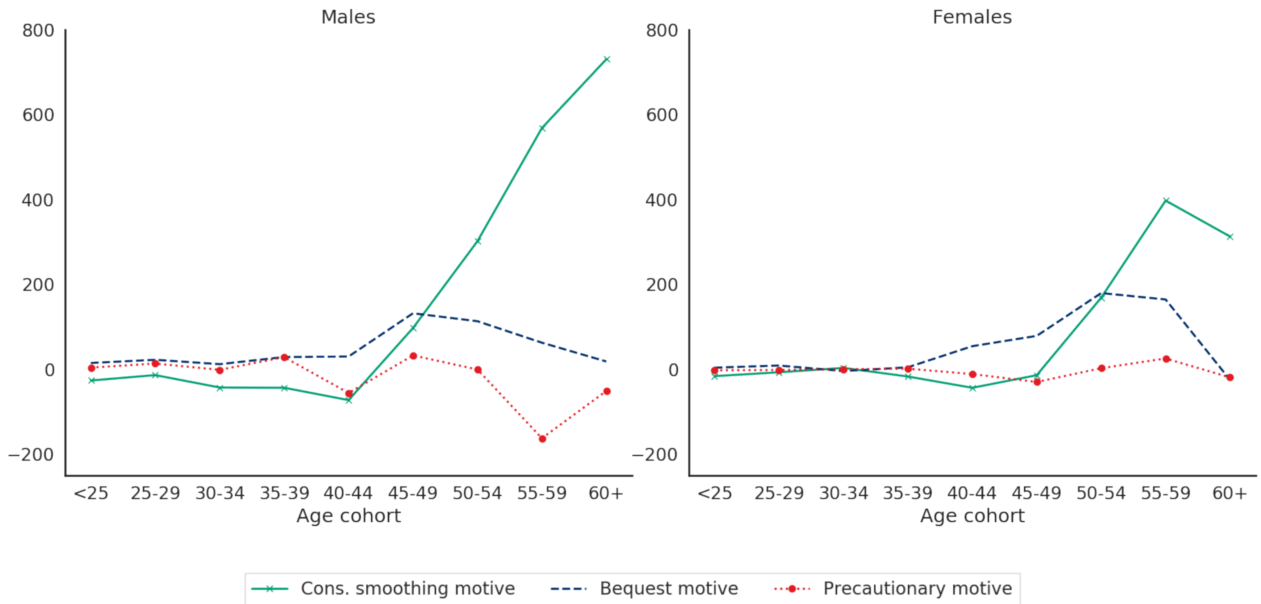

Figure 11: Additional financial wealth by cohort (thousands of $)

durable consumption services. Although housing offers 'necessary' consumption services in addition to real returns, adjusting housing stock is costly and so housing wealth decisions typically occur earlier in life (Yang, 2009). In contrast, the negative role of bequests for housing only starts to appear in middle years and more prominently for males than for females, although it leads to a financial savings boost that drives down housing consumption for all (Kopczuk and Lupton, 2007).

Finally, we see precautionary saving motives in Figure 13 having a negligible effect on pension wealth, as is to be expected since pension wealth cannot be decumulated during work years in response to income shocks. In contrast, consumption smoothing motives seem to affect pension wealth much more, and also much earlier than they affect financial wealth. Despite most people postponing voluntary contributions until much later in their working life, it is the higher returns on pension assets that allow consumption smoothing motives to drive the earlier moderate contributions. As bequests only weakly affect pension balances, we conclude that voluntary contributions during working years are primarily made for consumption smoothing rather than bequest reasons.

One final remark on bequests motives, given the prominence they have had in the lifecycle literature and the fact that they only add moderately to financial and pension wealth accumulation

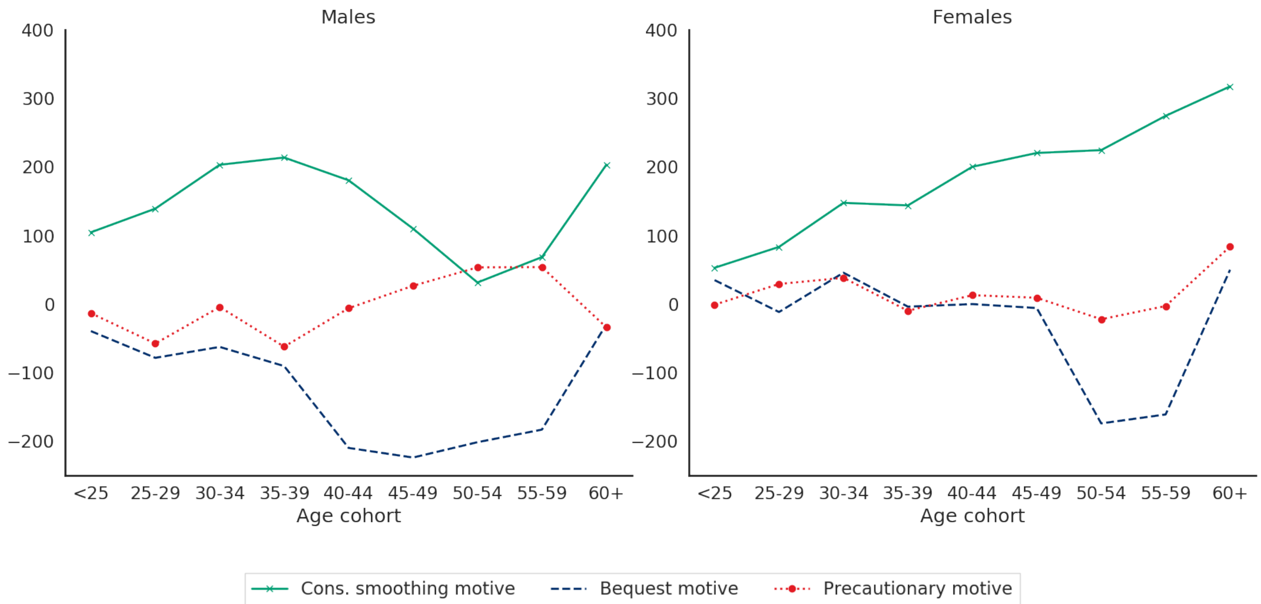

Figure 12: Additional gross housing wealth by cohort (thousands of $)

in our setup: Note how ceteris paribus plan choices, bequest motives boost financial wealth during mid-working years for both and females, but have no wealth effects later on. This finding does not imply bequests are unimportant, but rather that, on the margin, the addition of a bequest motive does not drastically increase savings over and above what is built up for consumption smoothing reasons - a result consistent with Dynan et al. (2002). What our results add to Dynan et al. (2002) is the part on bequest motives boosting financial wealth during mid-years (when retirement savings are not yet built up) to increase savings in case of death before retirement. Once enough retirement savings are accumulated in later years, however, bequest motives do not further alter saving. But, as showed in Section 6.3, they do alter plan decisions, which in turn affects pension balances: Recall bequests make individuals more willing to take on risk, and with an added bequest motive, they will see the exposure to the diversified high risk-high returns allocations offered by DC plans as helping them optimize their capacity to leave bequests. It is indeed this greater willingness to switch to DC that leads to bequest motives indirectly generating the rise in pension wealth accumulation in Table 5, rather than a direct increase in voluntary contributions.

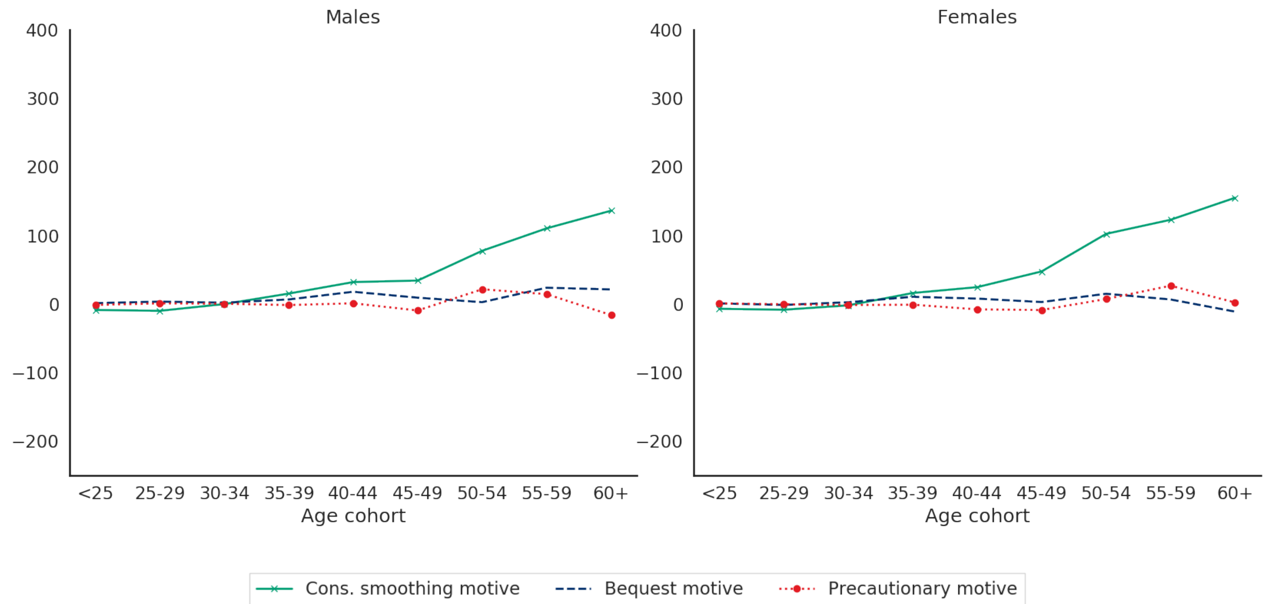

Figure 13: Additional pension wealth (DB+BC) by cohort (thousands of $)

7 Conclusions

How much to save and to diversify over the lifecycle are key decisions faced by all individuals and households, significantly affecting the adequacy of their retirement nest eggs. In aggregate these choices also impact the macroeconomic composition of capital - that in turn affects asset returns, housing prices, rental yields and financial stability (Eckardt et al., 2018), as well as the overall response of the economy to shocks (Kaplan and Violante, 2018). Hence, from a policy perspective, understanding what drives saving decisions during one's lifetime is central both for the design of appropriate public programs and for understanding intergenerational wealth impacts.

This paper disentangles the saving motives behind the lifecycle wealth allocation. In particular, we examine how standard saving motives (i.e., consumption smoothing, leaving bequests and precautionary savings) interact with pension choice architecture, returns, preferences and frictions to drive lifetime saving across all main asset categories. To do so, we structurally estimate a rich dynamic lifecycle model involving real and financial wealth accumulated in safe and risky assets inside and outside pension plans with complex choice architecture, uninsurable labor income risk, housing frictions and borrowing constraints. Our realistic setup, where people (i) can rent or own a house for which they can take a collateralised mortgage, (ii) are automatically enrolled into an employer sponsored plan, (iii) make a wide range of consequential plan decisions (about benefit type, contributions and asset allocations) jointly with homeownership and liquid savings choices, and (iv) incur the possibility that some components of consumption involve precommitments and some choices involve inertia that make them costly to adjust, implies broadly generalizable results. Using panel individual data on members of an industry-wide retirement fund matched with nationally representative survey data, we present two new sets of findings.

First, we empirically identify the various factors that affect the accumulation of various assets. On the pension side, older and wealthier individuals are more likely to opt for DC plans and to voluntarily contribute. Males and non-default allocation members have higher pension balances, while females invest slightly more aggressively possibly in an attempt to close the gap with males. Overall, however, risky assets share profiles appear relatively flat, with only some moderate rebalancing away from risk as people age. Homeownership occurs relatively early in (working) life and, unsurprisingly, housing wealth share increases as people get older. Females with default allocations are more likely to own a home and hold more housing wealth, which given also their slightly higher liquid savings, is consistent with them deriving higher utility from housing than males.

Second, we run counterfactuals to quantify the extent to which various saving motives drive the lifetime accumulation of each of these assets, taking into account the interplay between the standard saving determinants and pension choices, returns, preferences and frictions. These exercises reveal that while all three motives (consumption smoothing, leaving bequests and precautionary savings) have a significant impact on savings, there is considerable heterogeneity in how important they each are for the lifetime dynamics of each asset class. For instance, due to the costless redraw option on mortgages, precautionary saving does not lead to extra financial wealth, rather financial wealth is primarily accumulated as a way to smooth consumption (despite people also having access to pensions). Consistent with lifecycle theory, this rise in liquid savings for consumption smoothing reasons occurs typically after the age of 40, with males reaching the end of their careers with almost double the wealth boost of females. Additionally, we see financial wealth also building up for bequest purposes, with females saving slightly more and definitely earlier than males.

Interestingly, these trends appear somewhat inverted for housing, with consumption smoothing-induced savings occurring earlier, although again more prominently on average for females than for males. So while wage uncertainty does not ultimately matter much for housing wealth, bequests seem to play a negative role starting from middle years and stronger for males than for females (as females value bequests more) due to the financial boost driving down housing consumption for all.

Since pension wealth cannot be decumulated during working years in response to income shocks, we find precautionary motives unable to affect pension balances. The same holds for bequests, leading us to conclude that people voluntarily contribute mostly for consumption smoothing rather than bequest reasons. Indeed, consumption smoothing in isolation significantly drive-up pension wealth for everybody, and when considered jointly with the second order effect of pension choices, we see males additionally benefitting due to higher DC and contribution prevalence.

Finally, flexibility in plan default provisions (related to plan type, voluntary contributions and asset allocation choices) brings substantially higher pension balances, which slightly displaces financial wealth but also considerably increases housing. This pension-housing complementarity is stronger for males than for females, and stems from the higher post-retirement wealth allowing the housing consumption desired in later years to drive up housing wealth throughout the lifecycle. While accessing higher financial returns leads to a milder version of these effects, disposing of costless mortgage redraws has the opposite result - a rise in financial wealth and subsequently, a general shift away from DC plans and lower pension balances. Plan choices thus prove a key secondary channel through which all three saving motives affect wealth, with DC plans serving not only to improve consumption smoothing via retirement income adequacy but also help manage the effects of income risk on pension balances and increase bequests.

As a rising proportion of the population lives longer, grows older and retires relying increasingly on private savings, lifecycle portfolio decisions will soon become crucial in determining post-retirement incomes. Considering the recent rapid shifts towards DC plans with complex choice and default architectures, public policy and financial product design require a strong understanding of portfolio dynamics over the lifecycle. By highlighting the considerable heterogeneity in how and why people save, our results contribute to the ongoing policy debate over approaches to map out adequate welfare programs, while theoretically lending support to modelling saving behavior in a way that allows for different assets to display different lifecycle patterns - some following more and others less the predictions of standard lifecycle theory.

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Appendix

A Retirement Fund Features

Table A1. UniSuper plan features

	Mandatory	Default Option	Alternative Options
Enrolment	✔	-	-
Plan type	-	*DB*	*DC (within 1 yr)*
Employer contributions	*17%*	-	-
*Employee contributions**			
Standard rate	-	*7%*	*(Irreversible) Choice to decrease*
Voluntary rate	-	*0%*	*Choice to increase*
Investment options	-	*Balanced*	*Choice of other 14 options*
Insurance	-	*Life and TPD*	*Choice to change cover*

Notes: The table presents the key features of the retirement fund we study. Bold indicates the choice dimensions that we model. Recall all UniSuper members make investment choices as both DB and DC plans have a DC component *An additional choice dimension (that we do not model here) is that employee contributions can be made pre- or post-tax. TPD denotes total & permanent disability.

Table A2. UniSuper standard contribution schedule

Standard contribution rates %		ACF %	% of employer contribution to DB component
Pre-tax	Post-tax		
8.25	*7.00*	*100.00*	*82.30*
5.25	*4.45*	*100.00*	*100.00*
4.70	*4.00*	*97.40*	*100.00*
3.55	*3.00*	*91.70*	*100.00*
2.35	*2.00*	*86.00*	*100.00*
1.20	*1.00*	*80.20*	*100.00*
0.00	*0.00*	*74.50*	*100.00*

Notes: The table presents the standard contribution rates that an employee can opt for, before- or after-tax, along with the corresponding Average Contribution Factor (ACF) and share of employer contributions to the DB component.

B HILDA Spending Imputation Estimates

Table B1. Share of individual-to-household consumption

	Wave 10				Wave 14
	(1)	(2)	(3)	(4)	(5)
Age	0.397	0.040	0.141*	0.094	-4.540
	(0.978)	(0.966)	(0.063)	(0.915)	(2.583)
Male	-6.432***	-6.331***	-6.124***	-6.076***	-4.849**
	(1.464)	(1.490)	(1.478)	(1.479)	(1.559)
Couple	-1.255	-1.926	-3.067	-3.282	0.470
	(1.720)	(1.885)	(1.915)	(1.989)	(2.132)
Household size		1.295*	1.351*	1.347*	2.191**
		(0.579)	(0.568)	(0.572)	(0.711)
Health insurance premium	0.001*		0.002*	0.001*	0.000
	(0.001)		(0.001)	(0.001)	(0.000)
Ln annual wage	-0.246	-0.130	-0.201	-0.211	0.564
	(0.646)	(0.622)	(0.622)	(0.626)	(0.693)
Ln net wealth	1.723	1.053		0.924	-16.997
	(2.847)	(2.625)		(2.418)	(10.543)
Ln net wealth X Age	-0.021	0.008		0.001	0.355
	(0.072)	(0.071)		(0.067)	(0.188)
Constant	62.467	67.690	81.043***	70.248*	293.585*
	(38.239)	(35.251)	(6.893)	(32.947)	(140.602)
Observations	510	510	510	510	636
AIC	4227.9	4229.1	4220.9	4223.7	5472.6

Notes: All specifications are OLS models. Specifications (4)-(5) are the final ones used for the imputation of individual-to-household consumption share in Wave 10 and Wave 14, respectively. Since net wealth contains negative values, log net wealth is the log of adjusted net wealth, where *Adjusted net wealth= (net wealth - min net wealth) +1*. Robust standard errors are in parenthesis below estimated parameters. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Table B2. Share of individual-to-household housing expenses

	Wave 10				Wave 14
	(1)	(2)	(3)	(4)	(5)
Age	-0.239	-0.016	0.305	-0.016	-14.255
	(2.317)	(2.303)	(0.198)	(2.319)	(7.656)
Male	7.476	7.439	7.265	7.364	2.298
	(5.079)	(5.063)	(5.084)	(5.068)	(5.469)
Couple	3.906	5.258	7.053	6.320	-0.887
	(4.864)	(6.025)	(6.168)	(6.364)	(6.956)
Household size		-1.535	-1.520	-1.590	3.308
		(1.896)	(1.904)	(1.902)	(2.059)
Health insurance premium	-0.001		-0.001	-0.001	-0.002
	(0.001)		(0.001)	(0.001)	(0.001)
Ln annual wage	3.610*	3.558*	3.618*	3.604*	1.899
	(1.453)	(1.432)	(1.439)	(1.444)	(1.619)
Ln net wealth	5.027	5.498		5.675	-62.434*
	(4.634)	(4.594)		(4.626)	(28.886)
Ln net wealth X Age	0.028	0.009		0.011	1.053
	(0.163)	(0.163)		(0.164)	(0.546)
Constant	-18.829	-21.093	43.095*	-23.763	924.231*
	(68.835)	(68.222)	(19.009)	(68.649)	(401.742)
Observations	419	419	419	419	507
AIC	4324.3	4323.8	4326.6	4325.3	5389.9

Notes: All specifications are OLS models. Specifications (4)-(5) are the final ones used for the imputation of individual-to-household consumption share in Wave 10 and Wave 14, respectively. Note that compared to Wave 10, Wave 14 misses (i) holiday and travel costs, and (ii) new vehicles, computers, audio visual equipment, household appliance and furniture. To adjust Wave 14 consumption for these missing categories, we compute the Wave 10 ratio of missing to non-missing consumption categories, where these categories are identified based on whether they appear in Wave 14 or not. We then use the coefficients of specification (4) run on this Wave 10 ratio to impute the value of missing Wave 14 consumption categories and add it to Wave 14 raw consumption to get total consumption. Since net wealth contains negative values, log net wealth is the log of adjusted net wealth, where *Adjusted net wealth= (net wealth - min net wealth) +1*. Robust standard errors are in parenthesis below estimated parameters. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

C Solving the model using non-convex Endogenous Grid Method

The lifecycle model in our paper contains a high dimensional state-space along with discrete choices and non-convexities that arise from housing frictions and renting decisions. Because of the dimensionality, grid search and other brute-force numerical optimization methods to solve for the value function at each t are too time-consuming, in turn making the structural estimation we perform infeasible even with significant computational resources. One way the literature increases computation speeds for high dimensional models is to analytically calculate solutions at each t, using first order Euler equations and deploying the EGM. The EGM was developed by Carroll (2006) in the context of continuous convex problems and allows one to compute solutions to a dynamic problem without costly root-finding, grid-search or other numerical optimization methods (Iskhakov, 2015). The original 'pure' EGM, however, cannot be applied to non-convex and discontinuous problems as standard Euler equations are not sufficient to identify global optima without differentiability and concavity of the value function (Iskhakov et al., 2017).

To take advantage of the computational speeds offered by the EGM and make our structural estimation feasible, we use the results by Shanker et al. (2022) that extend the original EGM to non-convex problems where non-convexity arises from discrete choices. In the sections below, we detail the derivation of the first order sequential Euler equations for the life-cycle model. While the conditions are sequentially sufficient, they will not be recursively sufficient (see Shanker, 2017 for a comparison of sequential and recursive solutions and the role of convexity in relating the two solution concepts). Thus, the computational algorithm imposes additional second order conditions (SOCs) that are derived by converting the discrete choices to continuous ones with an associated quadratic constraint and Lagrange multiplier. Details of the SOCs can be found in Shanker et al. (2022).

C.1 Mathematical preliminaries

We start by stating some mathematical definitions. The Gateaux derivative is a generalisation of derivatives to vector spaces, we follow the terminology used by Penot (2013) - Definition 2.24.

Definition 1 ***(Gateaux differential)*** *Let X be a real topological vector space and Y a normed space. Let W be an open subset of X, let $x \in W$ and let $f\colon W \to Y$. If the following limit exists:*

$$f'_r(x,u) = \lim_{\lambda \to 0} \frac{f(x+\lambda u) - f(x)}{\lambda}$$

then $f'_r(x,u)$ is the radial derivative of f at x in the direction u. If the radial derivative exists for every $u \in X$ and the mapping $f'_g(x)\colon u \mapsto f'_r(x,u)$ is linear and continuous, then f is Gateaux differentiable at x and $f'_g(x)$ is the Gateaux differential of f at x.

Next, let X be a Hilbert space, with $X = \times_i X_i$ where each X_i is space of square integrable random variables on a common separable probability space. The following inner-product defines the dual paring for $x, y \in X$ (Aliprantis and Border, 2006):

$$\begin{aligned} xy = \langle x, y \rangle &= \int \langle x(\omega), y(\omega) \rangle_{\mathbb{R}^i} \mathbb{P}(d\omega) \\ &= \int \sum_i x_i y_i \mathbb{P}(d\omega) \\ &= \sum_i \int x_i y_i \mathbb{P}(d\omega) \end{aligned}$$

Finally, we require a definition of stopping times. Let τ be an arbitrary stopping time relative to the filtration $(\mathscr{F}_t)_{t=0}^T$. For a progressively measurable stochastic process $(l_t)_{t=0}^T$, recall that $(l_t^\tau)_{t=0}^T$ is a *stochastic process stopped at* τ, a random variable measurable with respect to $(\mathscr{F}_t)_{t=0}^T$ such that $l_t^\tau = l_{t \wedge \tau}$. Moreover, the process $(l_t^\tau)_{t=0}^T$ is measurable with respect to the stopped filtration $(\mathscr{F}_t^\tau)_{t=0}^T$, where $\mathscr{F}_t^\tau = \mathscr{F}_{t \wedge \tau}$. Also recall that l_τ is a random variable and is $\mathscr{F}_\tau$ measurable.[46]

C.2 Statement of primitive form lifecycle problem

C.2.1 Model environment

Here we show the sufficiency of generalized sub-derivative Euler equations for the stochastic lifecycle problem. We first write the problem as a sequential primitive form problem with an infinite dimensional state-space. In particular, the sequential problem optimizes over sequences of functions (random variables defined on an underlying probability space) to employ the results from Shanker et al. (2022). While we start with a sequential, infinite-dimensional framing of the problem, the computational algorithm generates a recursive solution. A discussion of the connection between recursive and sequential solutions is given in Shanker (2017).

To begin we start with some formal structure on the uncertainty driving the exogenous shocks. Let $\left(\Omega, \Sigma, (\mathscr{F}_t)_{t=0}^T, \mathbb{P}\right)$ be a filtered probability space on which uncertainty for all individuals will be defined. To simplify the exposition, we will refer to the space $\left(\Omega, \Sigma, (\mathscr{F}_t)_{t=0}^T, \mathbb{P}\right)$ as the underlying probability space throughout this appendix. The sequence of $\sigma-$algebras denoted by $(\mathscr{F}_t)_{t=0}^T$ are a sequence of information sets. Define $e_t = \left\{y_t, R_t^r, R_t^s, P_t, \alpha_t, \beta_t, \{\xi_t^\nu\}_{\nu \in \mathbb{V}}, \{\xi_t^\nu\}_{\nu \in \Pi}\right\}$ as the set of shocks at each t and assume $(e_t)_{t=0}^T$ generates $(\mathscr{F}_t)_{t=0}^T$. Also, let $(\mathscr{F}_j^t)_{j=t}^T$ denote the natural filtration with respect to the sequence $(e_t, \ldots, e_T)$ for any t. For a $\mathscr{F}_k^t$ measurable random variable v, with $k \geq t$, we will use the notation $\mathbb{E}_{j,e}^t v$ to denote the conditional expectation of v with respect to $\mathscr{F}_j^t$, conditioned on a realisation $e = (e_t, \ldots, e_j)$.

Next, we formally define the endogenous states and controls. The sequence of state variables in the model are the progressively measurable sequences of financial assets $(a_t)_{t=0}^T$, housing assets $(H_t)_{t=0}^T$, pension assets $(a_t^{DC})_{t=0}^T$, mortgage liabilities $(m_t)_{t=0}^T$ and risky assets shares $(x_t^\pi)_{t=0}^T$.

[46]Recall that $\mathscr{F}_\tau := \{A \in \Sigma \,|\, A \cap \{\omega \,|\, \tau(\omega) \leq t\} \in \mathscr{F}_t, \forall t \leq T\}$.

Following Shanker et al. (2022), we split the control variables into two groups. The first group includes controls that enter the problem via a convex function conditional on the discrete choices related to renting, adjusting housing stock, voluntary contributing and risky assets share. The convex control variables are the sequences of non-durable consumption $(c_t)_{t=0}^T$, housing services consumed from rented housing services $(hs_t^R)_{t=0}^T$, housing capital investments $(g_t^H)_{t=0}^T$ and mortgage repayments $(g_t^m)_{t=0}^T$. The second group are the discrete choice controls that enter the problem via a non-convex and non-separable function. These are the choices related to renting $(d_t^R)_{t=0}^T$, adjusting the housing capital stock $(d_t^H)_{t=0}^T$, voluntary contributing $(v_t)_{t=0}^T$ and risky assets share $(\pi_t)_{t=0}^T$. Here we have that d_t^R and d_t^H each realize values in $\{0,1\}$, and v_t and π_t realize values in $\mathbb{V}$ and Π. Note that $\mathbb{V}$ and Π are defined as the sets of discrete choices for the voluntary contributions and risky assets share, respectively.

The notation for the states is identical to the main text, except now the states and controls should be formally read as random variables rather than scalars. However, to write the problem as a *primitive* form sequential problem, we introduce additional controls variables to capture housing services $(hs_t^R)_{t=0}^T$, housing capital investments $(g_t^H)_{t=0}^T$ and mortgage repayments $(g_t^m)_{t=0}^T$. In addition to the main text, we capture (i) the choice of renting - that is if $H_{t+1} = 0$ and $S_t > 0$ - by the discrete random variable d_t^R, and (ii) the choice of adjusting - when $H_{t+1} \neq (1-\delta)H_t$ - by the discrete random variable d_t^H. Finally, we add a state variable x_t^π that will transition according to the time t period discrete choice risk share and link the t period risk choice to the $t+1$ period realisation of stochastic return for the DC assets.

We let K denote the number of states, J is the number of convex controls, $\tilde{J}$ is the number of non-convex non-separable controls (the discrete controls), and use J_H to denote the number of constraints. Let $\bar{A}$ (with $\bar{A} \subset \mathbb{R}^K$) be the space where the states take on values and let Z (with $Z \subset \mathbb{R}^J$) be the space where the convex controls z_t realise values. Moreover, let $\tilde{Z} \colon = \{0,1\}^2 \times \mathbb{V} \times \Pi$ be the space where the discrete choices realise values.

To define the state-space and control spaces, denote X as the set of $\mathbb{R}^K$ valued square integrable random variables with finite variance defined on $\left(\Omega, \Sigma, (\mathscr{F}_t)_{t=0}^T, \mathbb{P}\right)$, let Y be the $\mathbb{R}^J$ valued square integrable functions, and let $\tilde{Y}$ be $\tilde{Z}$ valued random variables. The spaces X and Y are Hilbert spaces, and thus reflexive with $X^\star = X$, for instance. We will denote elements of the tuple x_t with $x_t \in X$, using the notation in the main text - $x_t = (a_t, a_t^{DC}, H_t, m_t, x_t^\pi)$. Similarly, $z_t = (c_t, S_t, g_t^H, g_t^m)$ and $\tilde{z}_t = (d_t^R, d_t^H, v_t, \pi_t)$.

The state-spaces of the lifecycle problem will be a sub-set of X, in particular:

$$S_t \colon = \{x \in X \,|x \in \mathrm{m}\mathscr{F}_{t-1}\} \tag{30}$$

where $\mathrm{m}\mathscr{F}_t$ is the sub-space of all $\mathscr{F}_t$ measurable functions. Thus, the states at time t are measurable with-respect to the $(t-1)$ information and are determined at the beginning of period t, before the t period shock is realised. Next, the control spaces will be:

$$Z_t : = \{y \in Y \,| y \in \mathrm{m}\mathscr{F}_t\}, \quad \tilde{Z}_t : = \left\{y \in \tilde{Y} \,| y \in \mathrm{m}\mathscr{F}_t\right\} \tag{31}$$

C.2.2 Constrains, transitions and payoffs

Having described the mathematical environment for the model, we proceed to formally define the constraints, transition functions and pay-offs for the agent.

Constraints. To characterise the constraints, we will define a function $h_t : S_t \times Z_t \times \tilde{Z}_t \to W$ where W is the space of $\mathbb{R}^{J_h}$ valued random variables on the underlying probability space. The mapping h_t maps a random variable to a space of random variables such that the constraints of the lifecycle model hold. The constraints themselves are real-valued functions with real domain, defining the mortgage collateral constraint (mc), the non-negativity constraint for liquid assets (a), the non-negativity constraint for housing assets (H), the non-negativity constraint for mortgages (m) and a rental choice constraint the ensures agents can rent if and only if they own no housing capital (dR). Use the tuple $\mathscr{I}^h = (\mathrm{mc}, \mathrm{a}, \mathrm{m}, \mathrm{H}, \mathrm{dR})$ to index the constraints and define real valued functions φ_l^h for $l \in \mathscr{I}^h$ as follows:

$$\varphi_{mc}^h(m, g^m, H, d^H) = -m + g^m + \phi_t^C((1-\delta)H + d^H g^H)$$

$$\begin{aligned}\varphi_a^h(y, R^s, R^r, P, a, x^\pi, a^{DC}, c, hs^R, g^m, g^H, d^H, d^R) = {} & Ra + (x^\pi R^r + (1-x^\pi)R^r)a^{DC} + (1 - v - v_S)y \\ & - c - P^S hs^R \\ & - Pg^H - d^H \tau_H P\left((1-\delta)H + d_t^H g^H\right) - g^m\end{aligned}$$

$$\varphi_H^h(H, g^H, d^H) = (1-\delta)H + d^H g^H$$

$$\varphi_m^h(m, g^m) = (1 + r^m)m - g^m$$

$$\varphi_m^{dR}(H, g^H, d^R, d^H) = -d^R((1-\delta)H + d^H g^H)$$

The functional constraint function can now be defined as:

$$h_t(x_t, z_t, \tilde{z}_t) = \begin{pmatrix} \varphi_{mc}^h(m_t, g_t^m, H, d_t^H) \\ \varphi_a^h(y_t, R^s, R^r, P_t, a_t, x_t^\pi, \mathbb{1}_{t=R} a_t^{DC}, c_t, hs_t^R, g_t^m, g_t^H, d_t^H) \\ \varphi_H^h(H_t, g_t^H, d_t^H) \\ \varphi_m^h(m_t, g_t^m) \\ \varphi_m^{dR}(H_t, g_t^H, d_t^R, d_t^H) \end{pmatrix} \tag{32}$$

and the feasibility correspondences, $\Gamma_t : S_t \rightrightarrows \times Z_t \times \tilde{Z}_t$, will be:

$$\Gamma_t(x_t) : = \left\{ z_t, \tilde{z}_t \mid h_t(x_t, z_t, \tilde{z}_t) \geq \mathbf{0} \right\} \tag{33}$$

where the inequality in the definition of the feasibility correspondences is assumed to hold $\mathbb{P}$ almost everywhere (a.e.).

Transition equations. We proceed along similar lines to define the transition equation. Use the tuple $\mathscr{I}^f =$ (a,DC,H,m) to collect the indexes for the transition functions for liquid assets (a), pension assets (DC), housing (H), mortgages (m) and risk position (x^π). Define the real valued functions φ_l^f for $l \in \mathscr{I}^f$ as follows:

$$\begin{aligned}
\varphi_a^f(y, R^s, R^r, P, a, x^\pi, a^{DC}, c, hs^R, g^m, g^H, d^H, d^R) = {} & Ra + (x^\pi R^r + (1 - x^\pi) R^r) a^{DC} + (1 - v - v_S) y \\
& - c - P^S hs^R \\
& - P g^H - d^H \tau_H P \left((1 - \delta) H + d_t^H g^H\right) - g^m
\end{aligned}$$

$$\varphi_{DC}^f(R^r, R^s, a^{DC}, v, x^\pi) = (x^\pi R^r + (1 - x^\pi) R^s) a^{DC} + (v + v_S + v_E) y$$

$$\varphi_H^f(H, g^H, d^H) = (1 - \delta) H + d^H g^H$$

$$\varphi_m^f(m, g^m) = (1 + r^m) m - g^m$$

$$\varphi_\pi^f(\pi) = \pi$$

The functional transition function $f_t : \mathrm{Gr}\Gamma_t \to S_{t+1}$, will be:

$$f_t(x_t, z_t, \tilde{z}_t) = \begin{pmatrix} \varphi_a^h(y_t, R^s, R^r, P_t, a_t, x_t^\pi, \mathbb{1}_{t=R} a_t^{DC}, c_t, hs_t^R, g_t^m, g_t^H, d_t^H) \\ \mathbb{1}_{t+1<R} \varphi_{DC}^f(R_t^r, R_t^s, a_t^{DC}, v_t, x_t^\pi) \\ \varphi_H^f(H_t, g_t^H, d_t^H) \\ \varphi_m^f(m_t, g_t^m) \\ \varphi_\pi^f(\pi_t) \end{pmatrix} \tag{34}$$

Payoffs. The payoff function for the functional sequence problem will be defined on a space of random variables which integrates the real-valued payoffs for each realisation on the probability space. The underlying payoffs will be given by a real valued function φ^u, defined as:

$$\varphi^u(s, e, \beta, x, z, \tilde{z}) = -\beta \kappa\big(c, S; \alpha_t\big) + v_v \mathbb{1}_{v \neq 0} + v_\pi \mathbb{1}_{\pi \neq \pi^d} + \xi_v + \xi_\pi - (1 - s) b(B') \tag{35}$$

where $\kappa(c,S;\alpha) = u(c,S,\alpha)$ if $c > 0$ and $s > 0$ or $u(c,S;\alpha) = \infty$ otherwise. Moreover:

$$B' = \varphi_a^h(y, R^s, R^r, P, a, x^\pi, a^{DC}, c, hs^R, g^m, g^H, d^H, d^R) \\ - \varphi_m^f(m, g^m) + P\varphi_H^f(H, g^H, d^H) + \varphi_{DC}^f(R^r, R^s, a^{DC}, v, x^\pi) \quad (36)$$

and

$$S = d^R hs^R + (1-d^R)\left((1-\delta)H + g^H\right) \quad (37)$$

Using the real-valued payoffs, we can define the functional the per-period payoffs, $u_t \colon \mathrm{Gr}\Gamma_t \to \mathbb{R}_\infty$ as:

$$\tilde{u}_t(x_t, z_t, \tilde{z}_t) = \hat{s}_t \mathbb{E}\varphi^u(p_t, \hat{\beta}_t, e_t, x_t, z_t, \tilde{z}_t)$$

where $\hat{\beta}_t = \Pi_{j=0}^t \beta_j$, $\hat{s}_t = \Pi_{j=0}^{t-1} s_j$. By Remark 2.1.1 from Shanker (2017), we will allow the Lebesgue integral above to take on the value $+\infty$.

The fundamentals of the lifecycle model can now be collected as a tuple:

$$\mathscr{E}_{LS} \colon = \left((X, Y, \tilde{Y}), (S_t)_{t=0}^T, (Z_t)_{t=0}^T, (\tilde{Z}_t)_{t=0}^T, (u_t)_{t=0}^T, (f_t)_{t=0}^T, (\Gamma_t)_{t=0}^T\right) \quad (38)$$

C.2.3 Sequential primitive form dynamic optimization problem and sub-problems

We now define the lifecycle problem for which we seek a solution. The lifecycle problem becomes:

$$\min_{(x_t, z_t, \tilde{z}_t)_{t=0}^T} \sum_{t=0}^T \tilde{u}_t(x_t, z_t, \tilde{z}_t) \quad (\mathscr{P}_{LS}(x_0))$$

such that feasibility holds:

$$x_0,\ z_T, \tilde{z}_T \text{ and } x_{T+1} \text{ is given and } z_t, \tilde{z}_t \in \Gamma_t(x_t),\ x_{t+1} = f_t(x_t, z_t, \tilde{z}_t) \text{ for all } t \le T \quad (\mathscr{Y})$$

The sufficiency proofs require us to define a sub-problem of solving the discounted sum of future payoffs given a sequence of discrete choices. In particular, the sub-problem $\mathscr{P}_S(t, e, x, \bar{\tilde{z}}, (\tilde{z}_j)_{j=t+1}^T)$ starts at time t given realised discrete choices $\bar{\tilde{z}}$, exogenous state e and endogenous state x fixed, and given a $(\mathscr{F}_j^{t+1})_{j=t+1}^T$ adapted sequence of discrete choices from t, $(\tilde{z}_j)_{j=t+1}^T$ fixed. Let $V_t^{t,(\bar{\tilde{z}}_j)_{j=t}^T}(e,x)$ be the value function for the problem starting at time t, defined by:

$$V_t^{t,(\bar{\tilde{z}}_j)_{j=t}^T}(y,x) = \min_{(x_j, z_j)_{j=t}^T} \sum_{j=t}^T \tilde{u}_j(x_j, z_j, \bar{\tilde{z}}_j) \quad (\mathscr{P}_{LS}(t, e, x, \bar{\tilde{z}}, (\tilde{z}_j)_{j=t+1}^T))$$

such that x and e are given, $x_{j+1} = f_j(x_j, z_j, \bar{\tilde{z}}_j)$, $z_j, \bar{\tilde{z}}_j \in \Gamma_j^{t+1}(x_j)$ for all $t \leq j \leq T$ and we have:

$$\tilde{u}_j(x_j, z_j, \tilde{z}_j) = \hat{s}_j^t \mathbb{E}_e^t \varphi^u(s_j, \hat{\beta}_j, e_j, x_j, z_j, \tilde{z}_j) \tag{39}$$

where $\hat{\beta}_j^t = \Pi_{l=t}^j \beta_l$, $\hat{s}_j^t = \Pi_{l=t}^{j-1} s_l$.

C.2.4 Recursive primitive form problem

In keeping with the terminology by Sorger (2005) and Shanker (2017), the problem $\mathscr{P}_{LS}$ is a *sequential primitive form finite horizon problem*. The planner selects a sequence of actions and states to maximise a finite horizon discounted sum. Both the actions and staes in the sequential primitive form problem are progressively measurable random variables - thus on a suitable space of random variables, the problem resembles a deterministic one. The sequential primitive form problem furnishes us with a solution that is a sequence of progressively measurable random variables, each dependent on the entire history of shocks up-to time t. By contrast, a *stochastic recursive primitive form problem* is conditioned at each time t and is a problem where the agent selects the *realisations* of the next period states and current period controls, conditioned on a realisation of the current period states. The stochastic recursive problem is characterised by a Stochastic Recursive Bellman Equation (SRBE) and stochastic recursive value function $V_t \colon E \times \bar{A} \to \mathbb{R}$ defined by:

$$V_t(e, x) = \min_{(x_j, z_j, \tilde{z}_j)_{j=t}^T} \sum_{j=t}^T \tilde{u}_j(x_j, z_j, \tilde{z}_j) \tag{$\mathscr{P}_S(t,e,x)$}$$

such that the per-period payoff is given by (39), x and e are given, $x_{j+1} = f_j(x_j, z_j, \tilde{z}_j)$ and $z_j, \tilde{z}_j \in \Gamma_j^{t+1}(x_j)$ for all $t \leq j \leq T$.

If we let $(x_j, z_j, \hat{z}_j)_{j=t}^T$ be a solution sequence for $\mathscr{P}_S(t, e, x)$, by the Bellman Principle of Optimality (BPO), the stochastic recursive value function will satisfy the SRBE:

$$V_t(e, x) = \min_{z, \tilde{z}} \varphi^u(p_t, \beta, \alpha, \xi_v, \xi_\pi, c, S, v, \pi, A') + p_t \beta \mathbb{E}_{t,e}^t V_{t+1}(e_{t+1}, x') \tag{40}$$

such that $x' = \varphi_t^f(x, z, \tilde{z})$ and $\varphi_t^h(x, z, \tilde{z}) \geq 0$. The stochastic recursive problem of solving the SRBE period by period furnishes us with a sequence of measurable policy functions $\sigma_t \colon E \times \bar{A} \to Z \times \tilde{Z}$ such that $\sigma_t(e, x)$ solves the SRBE. Moreover, a stochastic recursive sequence (SRS) generated by $(\sigma_t)_{t=0}^T$, that is, $z_t, \tilde{z}_t = \sigma_t(e_t, x_t)$, with $x_{t+1} = f(x_t, z_t, \tilde{z}_t)$, solves the sequential primitive form problem. This result is standard and we provide a discussion of the BPO in Shanker et al. (2022).

In what follows we will consider a candidate solution of the following form: A sequence of measurable functions $(\sigma_t)_{t=0}^T$ and $\{(\sigma_t^d)_{t=0}^T\}_{d\in\tilde{Z}}$ where $\sigma_j\colon E\times\bar{A}\to Z\times\tilde{Z}$ and $\sigma_t^d:\bar{A}\to Z$ such that

$$\sigma_t=\sum_{d\in\tilde{Z}}\mathbb{1}_{\sigma_{\tilde{z},t}=d}\sigma_t^d \tag{41}$$

In words, σ_t is the policy function for the convex states unconditional on the discrete choice and σ_t^d is the policy function conditioned on a set of discrete choices. We will use $\sigma_{c,t}^d$ (and similarly for all the other controls in the tuple z) to denote the consumption policy function condition on the discrete choice d at time t. And we have used $\sigma_{\tilde{z},t}$ to denote the discrete choice policy function.

Finally, for a sequence of policy function $(\sigma_t)_{t=0}^T$, define the continuation value at each time t:

$$W_t^\sigma(e,x)=\mathbb{E}_e^t\sum_{j=t}^T\tilde{u}_j(x_j,z_j,\tilde{z}_j) \tag{42}$$

where per-period payoff is given by (39), $z_j,\tilde{z}_j=\sigma_j(e_j,x_j)$, with $x_{j+1}=\varphi_t^f(e_j,x_j,z_j,\tilde{z}_j)$ and $x_t=x$.

This completes our formal characterisation of the lifecycle problem and associated concepts. We can now move to characterising the necessary conditions for the lifecycle problem.

C.3 Necessary conditions

Our functional approach to derive Euler equations for the sequence problem utilises the original 18^{th} century line of attack proposed by Euler (see Stokey and Lucas, 1989, Section 3.2), which recognises that the Euler equation is a necessary derivative for a perturbation problem. Typically, the perturbation problem in economic dynamics is implemented as a 'one-shot' deviation problem at time t, maximising over t and $(t+1)$ utility subject to the feasibility of the $(t+2)$ state. However, when the state is not liquid, the $t+2$ state may determine the $(t+1)$ state if no adjustment is being made at $t+1$, thus not providing us with any opportunity for a feasible perturbation to take derivatives. The approach here is to take the perturbation not at $(t+1)$, but at time τ - the next time the illiquid stock is being adjusted.

C.3.1 The τ- shot deviation problem

Now fix t and define the stopping time:

$$\tau\colon=\inf\left\{k\geq t+1\,|\,d_k^H\geq 1\right\}\wedge T \tag{43}$$

Proposition 1 ***(τ - shot Deviation Problem)***

If $(x_t^\star)_{t=0}^{T+1}$ *and* $(z_t^\star, \tilde{z}_t^\star)_{t=0}^{T}$ *solve problem* $\mathscr{P}_S$*, then for each* t*,* $(x_j^{\star,\tau})_{j=t+1}^{T+1}$ *and* $(z_j^{\star,\tau})_{j=t}^{T}$ *solves:*

$$(\mathscr{P}_t^\tau) \qquad \min_{(x_j)_{j=t+1}^{T+1},(z_j)_{j=t}^{T}} \mathbb{E}\varphi^u(c_t^\tau, S_t^\tau, v_t^{\star,\tau}, \pi_t^{\star,\tau}) + \mathbb{E}\sum_{j=t+1}^{\tau} \beta^{j-t}\varphi^u(c_j^\tau, hs_j^\tau, \pi_j^{\star,\tau}, v_j^{\star\tau}) \tag{44}$$

subject to the following constraints for each $j \in \{t, \dots, \tau\}$*:*

1. $x_{j+1} = f(x_j, z_j, \tilde{z}_j^\star)$
2. $(z_j, \tilde{z}_j^\star) \in \Gamma_t(x_j)$
3. $x_t = x_t^\star$
4. $x_{\tau+1} = x_{\tau+1}^\star$

where $S_t = d_t^{R,\star} hs_t^R + (1 - d_t^{R,\star})\big((1-\delta)H_t + g_t^H\big)$.

Proof. Fix $t \in \{1, 2, \dots, T\}$ and suppose for the sake of contradiction that there exist sequences $(x_j)_{j=t+1}^{T+1}$ and $(z_j)_{j=t}^{T}$ that satisfy constraints 1. -5. above such that:

$$\begin{aligned} &\mathbb{E}\varphi^u(c_t^\tau, S_t^\tau, v_t^{\star,\tau}, \pi_t^{\star,\tau}) + \mathbb{E}\sum_{j=t+1}^{\tau} \beta^{j-t}\varphi^u(c_j^\tau, hs_j^\tau, v_j^{\star,\tau}, \pi_j^{\star,\tau}) \\ &< \mathbb{E}\varphi^u(c_t^{\star,\tau}, S_t^{\star,\tau}, v_t^{\star,\tau}, \pi_t^{\star,\tau}) + \mathbb{E}\sum_{j=t+1}^{\tau} \beta^{j-t}\varphi^u(c_j^{\star,\tau}, hs_j^{\star,\tau}, v_j^{\star,\tau}, \pi_j^{\star,\tau}) \end{aligned} \tag{45}$$

Now define the sequence $(x'_k)_{k=0}^{T+1}$ and $(z'_k)_{k=0}^{T}$ as follows:

$$x'_k = x_k^\star \mathbb{1}_{\{k \le t, k > \tau\}} + x_k \mathbb{1}_{\{k > t, k \le \tau\}} \tag{46}$$

and

$$z'_k = z_k^\star \mathbb{1}_{\{k < t, k > \tau\}} + z_k \mathbb{1}_{\{k \ge t, k \le \tau\}} \tag{47}$$

Note, by constraints 3. and 4. above, the sequences $(x'_k)_{k=0}^{T+1}$ and $(z'_k)_{k=0}^{T}$ are feasible. Adding the term $\mathbb{E}\sum_{j=\tau+1}^{T}\beta^{j-t}\varphi^u(c_j^\star, hs_j^\star, v_j^\star, \pi_j^\star)$ to both sides of (45), it follows that:

$$\begin{aligned}
&\mathbb{E}\varphi^u(c_t^\tau, S_t^\tau, v_t^{\star,\tau}, \pi_t^{\star,\tau}) + \mathbb{E}\sum_{j=t+1}^{\tau}\beta^{j-t}\varphi^u(c_j^\tau, hs_j^\tau, v_j^{\star,\tau}, \pi_j^{\star,\tau}) + \mathbb{E}\sum_{j=\tau+1}^{T}\beta^{j-t}\varphi^u(c_j^\star, hs_j^\star, v_j^\star, \pi_j^\star)\\
&\qquad = \mathbb{E}\varphi^u(c_t', S_t', v_t^\star, \pi_t^\star) + \mathbb{E}\sum_{j=t+1}^{T}\beta^{j-t}\varphi^u(c_j', hs_j', v_j^\star, \pi_j^\star)\\
&\qquad < \mathbb{E}\varphi^u(c_t^{\star,\tau}, S_t^{\star,\tau}, v_t^\star, \pi_t^\star) + \mathbb{E}\sum_{j=t+1}^{\tau}\beta^{j-t}\varphi^u(c_j^{\star,\tau}, hs_j^{\star,\tau}, v_j^{\star,\tau}, \pi_j^{\star,\tau})\\
&\qquad\quad + \mathbb{E}\sum_{j=\tau+1}^{T}\beta^{j-t}\varphi^u(c_j^\star, hs_j^\star, v_j^\star, \pi_j^\star)\\
&\qquad = \mathbb{E}\varphi^u(c_t^\star, S_t^\star, v_t^\star, \pi_t^\star) + \mathbb{E}\sum_{j=t+1}^{T}\beta^{j-t}\varphi^u(c_j^\star, hs_j^\star, v_j^\star, \pi_j^\star)
\end{aligned} \tag{48}$$

The first equality follows from the construction of the processes $(x_k')_{k=0}^{T+1}$ and $(z_k')_{k=0}^{T}$ at (46) and (47). The first inequality follows from the contradictory assumption given by Equation (45) and the final equality follows from the definition of a stopped process. To finish the proof, we have:

$$\begin{aligned}
\mathbb{E}\sum_{j=0}^{T}\beta^j\varphi^u(c_j', hs_j', v_j^\star, \pi_j^\star) &= \mathbb{E}\sum_{j=0}^{t-1}\beta^j\varphi^u(c_j^\star, hs_j^\star, v_j^\star, \pi_j^\star) + \beta^t\mathbb{E}\varphi^u(c_t', S_t', v_t^\star, \pi_t^\star)\\
&\quad + \mathbb{E}\sum_{j=t+1}^{T}\beta^{j-t}\varphi^u(c_j', hs_j', v_j^\star, \pi_j^\star)\\
&< \mathbb{E}\sum_{j=0}^{t-1}\beta^j\varphi^u(c_j^\star, hs_j^\star, v_j^\star, \pi_j^\star) + \beta^t\mathbb{E}\varphi^u(c_t^\star, S_t^\star, v_t^\star, \pi_t^\star)\\
&\quad + \mathbb{E}\sum_{j=t+1}^{T}\beta^{j-t}\varphi^u(c_j^\star, hs_j^\star, v_j^\star, \pi_j^\star)\\
&= \mathbb{E}\sum_{j=0}^{T}\beta^j\varphi^u(c_j^\star, hs_j^\star, v_j^\star, \pi_j^\star)
\end{aligned} \tag{49}$$

The first line follows from the construction of the processes $(x_t')_{t=0}^{T+1}$ and $(z_t')_{t=0}^{T}$ at (46) and (46). The inequality follows from (48) and the final equality is obtained by collecting the elements of the summation. However, the processes $(x_t')_{t=0}^{T+1}$ and $(z_t')_{t=0}^{T}$ are feasible, thus, the above yields a contradiction since $(x_t^\star)_{t=0}^{T+1}$, $(z_t^\star)_{t=0}^{T}$ does not solve problem $\mathscr{P}_S$. ■

C.3.2 Heuristic derivation of the Euler equation

Our task now is to derive functional Euler equations based on the necessary condition we stated above. Because our objective is to show how to sufficiently characterise a solution, we start by using the necessary τ shot deviation result to to heuristically derive the Euler equation. We then use Euler equations *with respect to the convex controls* to characterise a candidate sequence and show the Euler equations lead to multipliers that are sufficient for the candidate sequence to be a solution. We give proofs for the sufficiency result here, since our objective is to show that a computed solution is in fact the optimiser. For simplicity, we only give a heuristic derivation of the necessary conditions (see discussion by Shanker et al., 2022).

Setting up the Lagrangian. To derive the Euler Equations using FOCs, let $(x_t^{\star})_{t=0}^{T+1}$ and $(z_t^{\star}, \tilde{z}_t^{\star})_{t=0}^{T}$ solve problem $\mathscr{P}_{LS}$. We must have that for each t, $(x_j^{\star,\tau})_{j=t+1}^{T+1}$ and $(z_j^{\star,\tau})_{j=t}^{T}$ solves:

$$\min_{(x_j)_{j=t+1}^{T},(z_j)_{j=t}^{T}} \mathbb{E}\hat{s}_t\varphi^u(\hat{\beta}^j, \alpha_j, c_t^\tau, S_t^\tau, v_t^\tau, \pi_t^\tau, A_{t+1}^\tau) + \mathbb{E}\sum_{j=t+1}^{\tau} \hat{s}_j\varphi^u(\hat{\beta}^j, \alpha_j, c_j^\tau, S_t^\tau, v_t^\tau, \pi_t^\tau, A_{j+1}^\tau)$$

subject to the constraints 1. - 5. of Proposition 1. The constraints can be written as follows. Start with conditions 2. and 3. - for all periods after t, the feasibility conditions must hold:

$$h_j^l(x_j^\tau, z_j^\tau, \tilde{z}_j^\star) \geq 0, \qquad j \geq t+1, l \in \mathscr{I}^h \tag{50}$$

Moreover, at time t, the feasibility condition must hold such that the time t state is $x_t^\star$:

$$h_t^l(x_t^\star, z_t^\tau, , \tilde{z}_\tau^\star) \geq 0, \qquad l \in \mathscr{I}^h \tag{51}$$

Next, conditions 1. and 4. can be re-written as follows. The transition at time τ, leading to the state at time $\tau+1$ must satisfy:

$$a_{\tau+1}^\star = f^a(x_\tau^\tau, z_\tau^\tau, \tilde{z}_\tau^\star), \quad a_{\tau+1}^{DC,\star} = f^{DC}(x_\tau^\tau, z_j^\tau), \quad m_{\tau+1}^\star = f^m(x_\tau^\tau, z_\tau^\tau) \tag{52}$$

The transition at time t, leading to the state at time $t+1$ must satisfy (given condition 3.):

$$a_{t+1}^\tau = f^a(x_t^\star, z_t^\tau, \tilde{z}_t^\star), \quad a_{t+1}^{DC,\tau} = f^{DC}(x_t^\star, z_t^\tau, , \tilde{z}_t^\star), \quad m_{t+1}^\tau = f^m(x_t^\star, z_t^\tau, \tilde{z}_t^\star) \tag{53}$$

Moreover, for each j after t, the states must satisfy the transition equations:

$$a_{j+1}^\tau = f^a(x_j, z_j^\tau, \tilde{z}_j^\star), \quad a_{j+1}^{DC,\tau} = f^{DC}(x_j, z_t^\tau, \tilde{z}_\tau^\star), \quad m_{t+1}^\tau = f^m(x_j, z_j^\tau, \tilde{z}_\tau^\star), \qquad j \geq t+1 \tag{54}$$

However, note that there is no investment in the housing stock between t and time τ, thus the housing transition equations can be written as:

$$H_\tau = H_t^\star(1-\delta)^{\tau-t} + d_t^{H,\star} g_t^{H,\star}(1-\delta)^{\tau-t+1} \tag{55}$$

$$H^{\star}_{\tau+1} = H_\tau(1-\delta) + g^{H,\star}_\tau \tag{56}$$

Now let Ξ^l_j, with $j \in t,\dots,T$ and $l \in \mathscr{I}^h$ be the multipliers associated with (50) and (51). Next, let Λ^a_j, Λ^{DC}_j, Λ^m_j, with $j \in t,\dots,T$ be the multipliers associated with (52) to (53) and, combine the constraints (55) and (56) to give:

$$H^{\star}_{\tau+1} = \tilde{f}^\tau(H^\star_t, g^H_t, g^H_\tau) :\, = \left(H^\star_t(1-\delta) + d^H_t g^H_t\right)(1-\delta)^{\tau-t} + g^H_\tau \tag{57}$$

let Λ^H be its multiplier. Proceeding heuristically, write the Lagrange for the problem as:

$$\begin{aligned}
\mathscr{L} = {} & \mathbb{E}\hat{s}_t \varphi^u(\hat{\beta}^t, \alpha_t, c^\tau_j, S^\tau_t, v^\tau_t, \pi^\tau_t) + \mathbb{E}\sum_{j=t+1}^{\tau} \hat{s}_j \varphi^u(\hat{\beta}^j, \alpha_j, x^\tau_j, c^\tau_j, d_j) + \sum_{j=t}^{T}\sum_{l\in\mathscr{I}^h} \Xi^l_j h^l_j(x^\tau_j, z^\tau_j, \tilde{z}^\star_j) \\
& + \sum_{j=t}^{T} \Lambda^a_j(f^1(x^\tau_j, z^\tau_j, \tilde{z}^\star_j) - a^{\tau+1}_{j+1}) + \sum_{j=t}^{T} \Lambda^{DC}_j(f^{DC}(x^\tau_j, z^\tau_j, \tilde{z}^\star_j) - a^{DC,\tau+1}_{j+1}) \\
& + \sum_{j=t}^{T} \Lambda^\pi_j(f^\pi(x^\tau_j, z^\tau_j, \tilde{z}^\star_j) - x^{\pi,\tau+1}_{j+1}) + \sum_{j=t}^{T} \Lambda^m_j(f^m(x^\tau_j, z^\tau_j, \tilde{z}^\star_j) - m^{\tau+1}_{j+1}) \\
& + \Lambda^H \tilde{f}^\tau(H^\star_t, g_t, g_\tau)
\end{aligned} \tag{58}$$

The arguments of the Lagrange are $(a^\tau_j, m^\tau_j, a^{\tau,DC}_j)^T_{j=t+1}, (c^\tau_j, hs^{\tau,H}_j, hs^{\tau,R}_j)^T_{j=t}, g^{\tau,H}_t$ and $g^{\tau,H}_\tau$.

FOCs. The multipliers and arguments for the Lagrange are all square integrable random variables defined on the underlying probability space, thus to characterise necessary FOCs formally, we require the use of functional derivatives. However, we proceed heuristically and take the FOCs "as if" each argument was real valued. First, taking the derivative with respect to a_{j+1} gives:

$$(1+r)\mathbb{E}_j(\Lambda^a_{j+1} + \Xi^a_{j+1} + \hat{s}_{j+1}(1-s_{j+1})\hat{\beta}_{j+1} b'(B_{j+2})) - \Lambda^a_j = 0 \tag{a}$$

Next, taking the derivative with respect to a^{DC}_{j+1}, x^π_{j+1} and m_{j+1} gives:

$$\begin{aligned}
\mathbb{E}_j((\Lambda^a_{j+1} + \Xi^a_{j+1})\mathbb{1}_{j+1=R} + \Lambda^{DC}_{j+1}\mathbb{1}_{j+1<R} + \hat{s}_{j+1}(1-s_{j+1})\hat{\beta}_{j+1} b'(B_{j+2})) \\
\cdot (x^\pi_{j+1} R^r_{j+1} + (1-x^\pi_{j+1})R^s_{j+1}) - \Lambda^{DC}_j = 0
\end{aligned} \tag{DC}$$

$$\mathbb{E}_j(\Xi^a_{j+1}\mathbb{1}_{j+1=R} + \Lambda^{DC}_{j+1}\mathbb{1}_{j+1<R})\left(R^r_{j+1} - R^s_{j+1}\right) a^{DC}_{t+1} - \Lambda^\pi_j = 0 \tag{x^π}$$

$$\mathbb{E}_j(1+r^m_t)(\Xi^m_{j+1} + \Lambda^m_{j+1} - \Xi^{mc}_{j+1}) - \Lambda^m_j = 0 \tag{m}$$

We can also take the derivative with respect to each of the controls. Thus, the following hold $m\mathscr{F}_t$ almost everywhere:

$$\hat{s}_j\hat{\beta}_j u_1(c_j,S_j,\alpha_j) - \hat{s}_j(1-s_j)\hat{\beta}_j b'(B_{j+1}) - \Lambda^a_j - \Xi^a_j = 0 \tag{c}$$

$$d^R_t(\hat{s}_j\hat{\beta}_j u_2(c_j,S_j,\alpha_j) - \Lambda^a_j P^S_j - \Xi^a_j P^S_j - P^S_j \hat{s}_j(1-s_j)\hat{\beta}_j b'(B_{j+1})) = 0 \tag{hs^R}$$

$$\begin{aligned} d^H_t P_t(\Lambda^a_t + \Xi^a_t) = {} & \phi^C d^H_t \Xi^{mc}_t + d^H_t \Xi^H_t + \hat{s}_j\hat{\beta}_j d^H_t(1-d^R_t)u_2(c_t,S_t,\alpha_t) \\ & + d^H_t \mathbb{E}_t \sum_{l=t}^{\tau} (1-\delta)^{\tau-t}\hat{\beta}_l P_l \hat{s}_l(1-s_l)b'(B_{l+1}) \\ & + d^H_t \mathbb{E}_t (1-\delta)^{\tau-t}(\Lambda^H + \Xi^H_\tau + \phi^C \Xi^{mc}_\tau + \Xi^H_\tau + (1-d^R_\tau)\hat{s}_\tau\hat{\beta}_\tau u_2(c_\tau,S_\tau,\alpha_\tau)) \\ & - \Xi^{dR}_t d^H_t d^R_t \end{aligned} \tag{g^H_t}$$

$$\begin{aligned} d^H_\tau \Xi^{mc}_\tau \phi^C + d^H_\tau \Xi^H_\tau + d^H_\tau(1-d^R_\tau)u_2(c_\tau,S_\tau,\alpha_\tau) + d^H_\tau \hat{\beta}_\tau P_\tau \hat{s}_\tau(1-s_\tau)b'(B_{\tau+1}) & \\ + \Lambda_H - \Xi^{dR}_\tau d^H_\tau d^R_\tau = d^H_\tau P_\tau(\Lambda^a_\tau + \Xi^a_\tau) & \end{aligned} \tag{g^H_τ}$$

$$\Lambda^m_j + \Xi^{mc}_j - \Xi^m_j - \Xi^a_j - \Lambda^a_j = 0 \tag{g^m_t}$$

Euler equations. We can now combine these FOCs to eliminate most of the multipliers. First, combining (a) and (c) gives us the formulation for the financial assets Euler equation, with the standard interpretation:

$$u_1(c_t,S_t,\alpha_t) - \Xi^a_t = \beta_t p_t \mathbb{E}_t(1+r)u_1(c_{t+1},S_{t+1},\alpha_{t+1}) + (1-p_t)b'(B_{t+1}) \tag{59}$$

Now, using (DC), the Euler equation for DC assets gives us, for period $R-1$:

$$\Lambda^{DC}_{R-1} = \beta_{R-1}\mathbb{E}_{R-1}(\Lambda^a_R + \Xi^a_R + \hat{s}_R(1-s_R)b'(B_{R+1}))\,(x^\pi_R R^r_R + (1-x^\pi_t)R^s_R) \tag{60}$$

and for periods $t < R-1$:

$$\Lambda^{DC}_t = \beta_t \mathbb{E}_t(\Lambda^{DC}_{t+1} + \hat{s}_t(1-s_{t+1})b'(B_{t+1}))\,(x^\pi_t R^r_t + (1-x^\pi_t)R^s_t) \tag{61}$$

Turning to mortgages, combine (m) with (g^m_t) to arrive at:

$$u_1(c_t,S_t,\alpha_t) - \Xi^m_t + \Xi^{mc}_t = \beta_t p_t \mathbb{E}_t(1+r^m_t)(u_1(c_{t+1},S_{t+1},\alpha_{t+1})) - (1-p_t)b'(B_{t+1}) \tag{62}$$

Next, turning to the housing stock, combine (g_t^H) with (g_τ^H) to arrive at:

$$\begin{aligned} d_\tau^H P_t u_1(c_t,S_t,\alpha_t) - d_t^H \Xi_t^H = \mathbb{E}_t\hat{\beta}_t^\tau(1-\delta)^{\tau-t}[&\hat{s}_t^\tau P_\tau u_1(c_\tau,S_\tau,\alpha_\tau) + d_t^H\Xi_t^{mc}\phi^C \\ &+ \hat{s}_t^\tau d_t^H(1-d_t^R)u_2(c_t,S_t,\alpha_t)] \\ &+ \mathbb{E}_t\sum_{l=t}^{\tau}\hat{\beta}_t^l\hat{s}_t^l(1-s_l)b'(B_{l+1})P_l \end{aligned} \tag{63}$$

Finally, we have an intra-temporal constraint for renters:

$$d_t^R\left(u_2(c_j,S_j,\alpha_j)\right) = P_j^S u_1(c_j,S_j,\alpha_j) - (1-p_t)b'(A_{t+1})P_t \tag{64}$$

The above Euler equations (59) - (64), characterise the solution conditional on a sequence of discrete choices. The Euler equations for the liquid capital stock, mortgages and pension assets are standard. However, solving the τ-shot problem allows us to recover an Euler equation for the illiquid stock - equation (63), which would not have been possible following standard approaches (say, by differentiating the Bellman equation). In terms of interpretation, equation (63) tells us that the shadow value (price) of investment (withdrawal) from the illiquid capital stock is given by the discounted expected value of the stock *when the stock is next liquidated*, at random time τ not in the next period as in the standard Euler equation. Thus, subject to the multipliers for the boundary constraints, the agent equates the marginal utility of adjusting the capital stock today with the sum of (i) the marginal utility of the housing capital stock the next time the stock is adjusted, (ii) the utility of housing services if the agent is not renting, (iii) the value of relaxing the collateral constraint, and (iv) the expected marginal value to bequests if the agent dies between period t and τ. We now move to showing how the discrete choices and the above solution sufficiently characterise a solution.

C.4 Sequentially sufficient first order conditions

C.4.1 Constructing candidate sequence of convex controls and multipliers

We now begin the sufficiency results. Consider a sequence of measurable policy functions $(\sigma_t)_{t=0}^T$ and $\left\{(\sigma_t^d)_{t=0}^T\right\}_{d\in\mathbb{D}}$ as described by (41). Suppose the policy functions generate a stochastic recursive sequence $(x_t)_{t=0}^{T+1}$ and $(z_t,\tilde{z}_t)_{t=0}^T$ that satisfies the following conditions $\mathbb{P}$- a.e. The first condition is an Euler equation for liquid assets:

$$u_1(c_t,S_t,\alpha_t) \leq \beta_t p_t(1+r)\mathbb{E}_t u_1(c_{t+1},S_{t+1},\alpha_{t+1}) + (1-p_t)b'(B_{t+1}) \tag{65}$$

with the inequality strict for $B \in \Sigma$, with B having non-zero measure, such that:

$$\varphi_a^h(y_t, P_t, a_t, c_t, hs_t^R, g_t^m, g_t^H, d_t^H)(\omega) = a_{t+1}(\omega) = 0, \qquad \forall \omega \in B \tag{66}$$

Second condition is an Euler equation for mortgages when mortgages are not bounded at the refinance constraint:

$$\begin{aligned} u_1(c_t, S_t, \alpha_t) - (1-p_t)b'(B_{t+1}) & \\ \geq \beta_t p_t \mathbb{E}_t(1+r_t^m)u_1(c_{t+1}, S_{t+1}, \alpha_{t+1}), & \qquad 0 \leq m_{t+1} < \phi^C H_{t+1} \end{aligned} \tag{67}$$

with the inequality strict for $B \in \Sigma$ such that:

$$\varphi_m^h(m_t, g_t^m)(\omega) = m_{t+1}(\omega) = 0, \qquad \forall \omega \in B \tag{68}$$

The third condition is an Euler equation for mortgages when mortgages are bounded at the refinance constraint:

$$\begin{aligned} u_1(c_t, S_t, \alpha_t) - (1-p_t)b'(B_{t+1}) & \\ \leq \beta_t p_t \mathbb{E}_t(1+r_t^m)u_1(c_{t+1}, S_{t+1}, \alpha_{t+1}), & \qquad 0 < m_{t+1} \leq \phi^C H_{t+1} \end{aligned} \tag{69}$$

with the inequality strict for $B \in \Sigma$ such that:

$$\varphi_m^h(m_t, g_t^m)(\omega) = m_{t+1}(\omega) = \phi^C H_{t+1}(\omega), \qquad \forall \omega \in B \tag{70}$$

The fourth condition is an Euler equation for housing if an adjustment is being made at time t:

$$\begin{aligned} P_t u_1(c_t, S_t, \alpha_t) \leq \mathbb{E}_t \hat{s}_t^\tau \hat{\beta}_t^\tau (1-\delta)^{\tau - t} P_\tau u_1(c_t, S_t, \alpha_t) \\ + \Xi_t^{mc} \phi^C + (1-d_t^R) u_2(c_t, S_t, , \alpha_t) \\ + \mathbb{E}_t \sum_{l=t}^{\tau} \hat{\beta}_t^l \hat{s}_t^l (1-s_l) b'(B_{l+1}) P_l \end{aligned} \tag{71}$$

with the inequality strict for $B \in \Sigma$ such that:

$$\varphi_H^h(H_t, g_t^H, d_t^H)(\omega) = H_{t+1}(\omega) = 0, \qquad \forall \omega \in B \tag{72}$$

where we have defined:

$$\Xi_t^{mc} = \min\{0, u_1(c_t, S_t, \alpha_t) - (1-p_t)b'(B_{t+1}) - \beta_t p_t \mathbb{E}_t(u_1(c_{t+1}, hs_{t+1}, \alpha_{t+1}))(1+r_t^m)\} \tag{73}$$

$$\Xi_t^{m} = \min\{0, \beta_t p_t \mathbb{E}_t(u_1(c_{t+1}, S_{t+1}, \alpha_{t+1}))(1+r_t^m) - u_1(c_t, S_t, \alpha_t) + (1-p_t)b'(B_{t+1})\} \tag{74}$$

Finally, if $d_t^R = 1$, then:

$$u_2(c_j, S_j, \alpha_j) = P_j^S u_1(c_t, S_t, \alpha_t) \tag{75}$$

Now define a sequence $(\Lambda_t)_{t=0}^T$ and $(\Xi_t)_{t=0}^T$ as follows. First define:

$$\Xi_t^a = \min\{0, u_1(c_t, S_t, \alpha_t) - \beta_t p_t(1+r)\mathbb{E}_t u_1(c_{t+1}, S_{t+1}, \alpha_{t+1}) + (1-p_t)b'(B_{t+1})\} \tag{76}$$

and then define:

$$\begin{aligned}\Xi_t^H = \min\{0, &P_t u_1(c_t, S_t, \alpha_t) - \beta_t \mathbb{E}_t \hat{\beta}_t^\tau \hat{s}_t^\tau (1-\delta)^{\tau-t} P_\tau u_1(c_t, S_t) \\ &- \Xi_t^{mc}\phi^C - (1-d_t^R)u_2(c_t, S_t, \alpha_t) - \mathbb{E}_t \sum_{l=t}^{\tau} \hat{\beta}_t^l \hat{s}_t^l (1-s_l)b'(B_{l+1})P_l\}\end{aligned} \tag{77}$$

and for the shadow values, we have:

$$\Lambda_t^a = u_1(c_t, S_t, \alpha_t) - \Xi_t^a \tag{78}$$

$$\Lambda_t^m = \Xi_j^a + \Lambda_j^a - \Xi_j^m + \Xi_j^{mc} \tag{79}$$

Now recursively define Λ_t^H as follows. Given Λ_{t+1}^H and Ξ_{t+1}^H, define:

$$\Lambda_t^H = d_t^H \left(P_t u_1(c_t, S_t, \alpha_t) - \Xi_t^H\right) + (1-d_t^H)\beta_t p_t \mathbb{E}_t \left(\Lambda_{t+1}^H + \Xi_{t+1}^H\right) + (1-p_t)b'(B_{t+1}) \tag{80}$$

Note that a sequence $(x_t)_{t=0}^{T+1}$, $(z_t, \hat{z}_t)_{t=0}^T$, $(\Lambda_t)_{t=0}^T$ and $(\Xi_t)_{t=0}^T$ that satisfies (a) - (g_t^m) is equivalent to a sequence that satisfies (65) - (80).

The Euler equations (65) - (80) will characterise a sequence of convex controls and states conditional on a sequence of discrete choices, but we need to impose further conditions to characterise the discrete choices. We turn to these in the next sub-section.

C.4.2 Constructing candidate sequence of discrete choices

Consider again the stochastic recursive sequence described in Section C.4.1 and suppose the discrete choices satisfy:

$$\sigma_{\tilde{z},t}(e,x) = \underset{\tilde{z}\in\tilde{Z},\sigma_t^{\tilde{z}}(x,e,\tilde{z})\in\Gamma_t^t(e,x)}{\arg\min} \quad \varphi_t^u(x,\sigma_t^{\tilde{z}}(x,e,\tilde{z})) + \mathbb{E}^t W_{t+1}^{\sigma}(e_{t+1},\varphi^f(x,\sigma_t^{\tilde{z}}(e,x),\tilde{z})) \tag{81}$$

In words, the discrete choices optimize the sum of the per-period payoff and continuation value under the choice specific policy function. We now turn to showing a candidate sequence that satisfies (65) - (80) and (81) solves problem $\mathscr{P}_{LS}$.

C.4.3 Proof of sufficiency

Consider again the stochastic recursive sequence defined above. Now, to show sufficiency we are going to show that for each t and given time t states x and e and the time t discrete choice $\bar{\tilde{z}}$, the sequence $(z_j)_{j=t}^T$ solves the problem $\mathscr{P}(t,r,x,(\tilde{z}_j)_{j=t}^T)$, where $(\tilde{z}_j)_{j=t}^T$ is the sequence of discrete choice random variables starting at t with value $\bar{\tilde{z}}$.

The j period Hamiltonian, $\mathscr{H}_j$, for the sub-problem can be written as:

$$\begin{aligned}
\mathscr{H}_j(x,z) = \mathbb{E}^t \hat{a}_t^j \varphi_j^u(\hat{\beta}_t^j, \alpha_j, \xi_{v,j}, \xi_{\pi,j}, c_j, S_j, v_j, \pi_j, B_{j+1}) \\
+ \Lambda_j^a \varphi_a^h(e_j, P_j, a_j, c_j, hs_j^R, g_j^m, g_j^H, d_j^H, d_j^R) \\
+ \Lambda_j^{DC} \varphi_{DC}^f(R_j^r, R_j^s, a_j^{DC}, v_j, \pi_j) \\
+ \Lambda_j^H \varphi_H^f(H_j, g_j^H, d_j^H) \\
+ \Lambda_j^m \varphi_m^f(m_j, g_j^m) \\
+ \Lambda_j^\pi \varphi_\pi^f(\pi_j) \\
+ \Xi_j^{mc} \varphi_{mc}^h(m_j, g_j^m, H_j, d_j^H) \\
+ \Xi_j^a \varphi_a^h(y_j, P_j, a_j, c_j, hs_j^R, g_j^m, g_j^H, d_j^H) \\
+ \Xi_j^H \varphi_H^h(H_j, g_j^H, d_j^H) \\
+ \Xi_j^m \varphi_m^h(m_j, g_j^m) \\
+ \Xi_j^{dR} \varphi_{dR}^h(H_j, g_j^H, g_j^R, d_j^H)
\end{aligned} \tag{82}$$

Since the candidate sequence was recursive, we are now viewing the sequence of multipliers as adapted to the filtration $(\mathscr{F}_j^{t+1})_{j=t}^T$ given an initial value d, x and y at time t.

Proposition 2 *Fix t and fix the time t endogenous state x, exogenous state e and discrete choice $\bar{\tilde{z}}$. Let $(x_j)_{t=t+1}^T$ and $(z_j)_{j=t}^T$ be stochastic recursive sequences constructed from a sequence of measurable policy functions $(\sigma_t)_{t=0}^T$ and $\{(\sigma_t^{\tilde{z}})_{t=0}^T\}_{\tilde{z}\in\tilde{Z}}$ that satisfy* (41)*. Moreover, let $(x_j)_{t=t+1}^T$*

and $(z_j)_{j=t}^T$ *satisfy* (65) - (75). *If there exists* $(\Lambda_j)_{j=t}^T$ *and* $(\Xi_j)_{j=t}^T$ *that satisfy* (76) - (80), *then* $(x_j)_{t=t+1}^T$ *and* $(z_j)_{j=t}^T$ *solves problem* $\mathscr{P}(t,e,x,(\tilde{z}_j)_{j=t}^T)$.

Proof. We proceed by verifying the conditions of Proposition 8 by Shanker et al. (2022) applied to the problem $\mathscr{P}(t,e,x,(\tilde{z}_j)_{j=t}^T)$.

First, note that by construction of the multipliers Ξ_t^l for $l \in \mathscr{I}^h$ from equations (73), (74), (76) and (77), we have that $\Xi_t^m \geq 0$. Moreover, in light of conditions (66) - (72), we have that 2. of Proposition 8 is met.

Since the sequence of discrete choices is fixed, the problem is one of minimising over the convex controls only and condition 3. is trivially satisfied. Note that $\mathscr{H}_j$ is Gateaux differentiable (see Shanker et al., 2022). Thus, $\partial\mathscr{H}_j(x,z) = \mathscr{H}'_{j,g}(x,z)$. Where $\mathscr{H}'_{j,g}(x,z)(x',z')$ satisfies the following for any $x',z' \in X_j, Z_j$:

$$\mathscr{H}'_{j,g}(x,z)(x',z') = \begin{pmatrix} \left(\Lambda_j^a(1+r) + \Xi_j^a(1+r) - \Lambda_{j-1}^a\right)a' \\ \left((1-\delta)(\Lambda_j^H + \Xi_j^H + \Xi_j^{mc} + (1-d_j^R)u_2(c_j,S_j,\alpha_j)) - \Lambda_{j-1}^H\right)H' \\ \left((1+r_t^m)(\Lambda_j^m + \Xi_j^m - \Xi_j^{mc}) - \Lambda_{j-1}^m\right)m' \\ \left((\pi_t R_t^r + (1-\pi_t)R_t^s)(\mathbb{1}_{j+1<R}\Lambda_j^{DC}\mathbb{1}_{j+1=R}\Xi_t^a) - \Lambda_{j-1}^{DC}\right)a^{DC,\prime} \\ (u_1(c,S,\alpha) - \Xi_a^1 - \Lambda_j^a)c' \\ d_t^H\left(\Xi_j^{mc}\phi^C + \Xi_j^H + (1-d_j^R)u_2(c_j,S_j,\alpha_j) + \Lambda_t^H - P_t(\Lambda_t^a + \Xi_t^a)\right)g^{H,\prime} \\ \left(\Xi_j^{mc} + \Lambda_j^m - \Xi_j^m - \Xi_j^a - \Lambda_j^a\right)g^{m,\prime} \\ d_j^R\left(u_2(c_j,S_j,\alpha_j) - \Lambda_j^a P_j^S - \Xi_j^a P_j^S\right)hs^{r,\prime} \end{pmatrix}$$

Now, since a', H', m' and $a^{DC,\prime}$ are $\mathscr{F}_{j-1}$ measurable, we have:

$$\mathscr{H}'_{j,g}(x,z)(x',z') = \begin{pmatrix} \left((1+r)\mathbb{E}_{j-1}^t(\Lambda_j^a + \Xi_j^a) - \Lambda_{j-1}^a\right)a' \\ \left(\mathbb{E}_{j-1}^t(1-\delta)(\Lambda_j^H + \Xi_j^H + \Xi_j^{mc} + (1-d_j^R)u_2(c_j,S_j,\alpha_j)) - \Lambda_{j-1}^H\right)H' \\ \left(\mathbb{E}_{j-1}^t(1+r_t^m)(\Lambda_j^m + \Xi_j^m - \Xi_j^{mc}) - \Lambda_{j-1}^m\right)m' \\ \left(\mathbb{E}_{j-1}^t(\pi_t R_t^r + (1-\pi_t)R_t^s)(\mathbb{1}_{j+1<R}\Lambda_j^{DC}\mathbb{1}_{j+1=R}\Xi_t^a) - \Lambda_{j-1}^{DC}\right)a^{DC,\prime} \\ (u_1(c,S,\alpha) - \Xi_a^1 - \Lambda_j^a)c' \\ d_t^H\left(\Xi_j^{mc}\phi^C + \Xi_j^H + (1-d_j^R)u_2(c_j,S_j,\alpha_j) + \Lambda_t^H - P_t(\Lambda_t^a + \Xi_t^a)\right)g^{H,\prime} \\ \left(\Xi_j^{mc} + \Lambda_j^m - \Xi_j^m - \Xi_j^a - \Lambda_j^a\right)g^{m,\prime} \\ d_j^R\left(u_2(c_j,S_j,\alpha_j) - \Lambda_j^a P_j^S - \Xi_j^a P_j^S\right)hs^{r,\prime} \end{pmatrix}$$

Finally, since $(x_j)_{j=t+1}^{T}$ and $(z_t)_{j=t}^{T}$ satisfies (76) - (80), we have that (63) holds and:

$$\Lambda_{j-1}^{H} = \mathbb{E}_{j-1}^{t}(1-\delta)(\Lambda_j^H + \Xi_j^H + \Xi_j^{mc} + (1-d_j^R)u_2(c_j, hs_j)) \tag{83}$$

Thus the first element of the tuple $\mathscr{H}_{j,g}'(x_j, z_j)$ equal $\mathbf{0}$. Moreover, since (a) - (m) also hold, the first, third and forth of $\mathscr{H}_{j,g}'(x_j, z_j)$ are also equal to $\mathbf{0}$. Finally, since (c) - (g_t^m) also hold, the final five elements of $\mathscr{H}_{j,g}'(x_j, z_j)$ are also equal to $\mathbf{0}$. Thus $0 \in \partial\mathscr{H}_j(x_j, z_j)$, completing the verification of condition 4. of Proposition 8 by Shanker et al. (2022).

■

Proposition 3 *Let* $(x_t)_{t=0}^{T}$ *and* $(z_t, \tilde{z}_t)_{t=0}^{T}$ *satisfy* (65) - (75) *and be stochastic recursive sequences constructed from* $(\sigma_t)_{t=0}^{T}$ *and* $\left\{(\sigma_t^d)_{t=0}^{T}\right\}_{d\in\tilde{Z}}$ *starting at time* 0*. If* $(\tilde{z}_t)_{t=0}^{T}$ *satisfies equation* (2) *for each t, then* $(z_t, \tilde{z}_t)_{t=0}^{T}$ *solves* $\mathscr{P}_{LS}(x_0)$*.*

Proof. By Proposition 2, condition 1. of Theorem 19 by Shanker et al. (2022) is satisfied for each t. Since $(\tilde{z}_t)_{t=0}^{T}$ satisfies equation (2) for each t, condition 2. is satisfied. Finally, note that σ_t maximises:

$$V_T(x,y) = \max_{z,d} \varphi_T^u(c,d) = W_T^{\sigma} \tag{84}$$

Thus, we have verified condition 3. of Theorem 19 by Shanker et al. (2022), and $\sigma_t(e,x)$ solves $\mathscr{P}(t,e,x)$ for each t,x,e. Finally, since $\sigma_t(e,x)$ is a measurable sequence, by Claim 16 in Shanker et al. (2022), $(z_j, \tilde{z}_j)_{t=0}^{T}$ solves $\mathscr{P}_S(x_0)$ as was to be shown. ■

From Proposition 3, a pseudo-code becomes apparent. We begin at period T and solve $\mathscr{P}(T,x,y)$ for each x,y and recover a policy function σ_T. Next, at period $(T-1)$, for each $y,x,\tilde{z} \in E\times S\times\tilde{Z}$, we solve the problem $\mathscr{P}_S(t,y,x,\hat{z},(\hat{z}_j)_{j=t+1}^{T})$ for $t = T-1$ to recover $\sigma_{T-1}^{\tilde{z}}$ for $\tilde{z}\in\tilde{Z}$. Since $W_{T-1}^{\sigma} = V_{T-1}$, we then proceed to repeat the process of constructing $\sigma_{T-2}^{\tilde{z}}$ for each $t < T-1$. In solving $\mathscr{P}_S(t,y,x,\hat{z},(\hat{z}_j)_{j=t+1}^{T})$ for $t < T$, we may use sufficient Euler equations. In the next sub-section, we detail the pseudo-code formally.

C.5 Computation

We begin by describing some preliminary details.

Wages. For the average wage over the last three years of continuous employment $\bar{y}_t$ we have that

$$g\left(y_{t-j}\right) = \frac{1}{Pr\left(\xi_t\right)} \sum_{i=1}^{N_\xi} Pr\left(\xi_i\right)\cdot P\left(\xi_i,\xi_t\right)\cdot exp\left[\lambda_0 + \sum_{k=1}^{4}\lambda_k\left(t-j\right)^k + \sum_{k=1}^{2}\lambda_{4+k}\left(\tau-j\right)^k + \xi_i\right].$$

This is derived from the reverse of the Markov process (5) (Chung and Walsh, 1969), with N_ξ discrete state points, distribution $Pr(\cdot)$ and transition matrix $P(\cdot,\cdot)$ and allows us to reduce computational burden and not carry (ξ_{t-2},ξ_{t-1}) in the state space.

Discretizing the state-space. We create state-space grids for each age as follows:

For $t > T_R$, we have

$$\hat{\mathbb{X}}_t = \hat{\mathbb{A}} \times \hat{\mathbb{B}} \times \hat{\mathbb{S}} \times \hat{\mathbb{H}} \times \hat{\mathbb{M}} \times \hat{\mathbb{P}}, \tag{85}$$

while for $t \leq T_R$, we have

$$\hat{\mathbb{X}}_t = \hat{\mathbb{Z}} \times \hat{\mathbb{A}} \times \hat{\mathbb{B}} \times \hat{\mathbb{S}} \times \hat{\mathbb{S}}_{DC} \times \hat{\mathbb{H}} \times \hat{\mathbb{M}} \times \hat{\mathbb{P}} \times \hat{\mathbb{T}}^t \times \{DB, DC\} \tag{86}$$

where $\hat{\mathbb{S}}$ is the financial wealth grid, $\hat{\mathbb{H}}$ is the housing wealth grid, $\hat{\mathbb{P}}$ is the housing price grid, $\hat{\mathbb{S}}_{DC}$ is the DC pension wealth grid, $\hat{\mathbb{M}}$ is the mortgage asset grid, $\hat{\mathbb{Z}}$ is the wages shock grid, $\hat{\mathbb{T}}^t$ is the set of possible tenure levels at age t, $\{DB, DC\}$ is the plan type, $\hat{\mathbb{A}}$ is the grid for housing preferences (α), and $\hat{\mathbb{B}}$ is the grid for time preferences (β). We also let $\hat{\mathbb{V}}$ denote the voluntary contribution grid and $\hat{\Pi}$ denote the risky assets share grid and define an extended grid with the discrete choices as follows:

$$\bar{\mathbb{X}}_t = \hat{\mathbb{Z}} \times \hat{\mathbb{A}} \times \hat{\mathbb{B}} \times \hat{\mathbb{S}} \times \hat{\mathbb{S}}_{DC} \times \hat{\Pi} \times \hat{\mathbb{V}} \times \hat{\mathbb{H}} \times \hat{\mathbb{M}} \times \hat{\mathbb{P}} \times \hat{\mathbb{T}}^t \times \{DB, DC\} \tag{87}$$

We discretize the financial, housing and DC pension wealth space and mortgage assets into 35 grid-points each, and the housing price space into 11 grid-points. The wage term ξ_t is discretized into a 3-state Markov process following Kopecky and Suen (2010). The α and β preference processes are also discretized into 3-point Markov process each, asset returns are discretized to 2 grid-points for safe and risky alternatives, and finally the housing price shock is discretized into a 3 grid-points.

The tenure years state space τ is integer and ranges from 0 to 48 $(= 65 - 17)$. Finally, we consider 5 different levels of voluntary contribution rates (besides the 0% default) and 5 different levels for the share of the DC portfolio invested in risky assets (besides the 70% default). Experiments with the grids fineness suggested that the ones we used produce reasonable approximations.

C.6 Algorithm for non-convex endogenous grid method

Note if the SRS generated by a sequence of measurable policy functions $(\sigma_t)_{t=0}^T$, $(x_t, z_t, \tilde{z}_t, \Xi_t, \Lambda_t)_{t=0}^T$, satisfies ($\mathscr{Y}$) and the conditions of Proposition 3 for each t, then the sequence is a recursive solution by Proposition 3. However, a recursive policy function that solves the Euler equations will not necessarily be a sequential solution, and as discussed in detail by Iskhakov et al. (2017), more than one recursive policy function will solve the Euler equations. The EGM thus proceeds in the standard manner, but utilises the results by Shanker et al. (2022) to impose SOCs that efficiently serve a role analogous to 'eliminating sub-optimal grid points' (Iskhakov et al., 2017). We outline the implementation of the standard EGM steps below and refer readers to Shanker et al. (2022) for the SOC steps. To proceed, note the purpose of the algorithm is to construct interpolants $(l_t)_{t=0}^T$ defined on $(\mathbb{X}_t)_{t=0}^T$ that agree with an approximation of the sequence of policy functions $(\sigma_t)_{t=0}^T$ such that the SRS generated by the policy functions satisfies the Euler equations (65) - (75) and the discrete choices satisfy (2). In particular, the set of interpolants will contain a consumption policy (c), a liquid asset policy (a), a next period housing stock policy (H), a next period mortgage stock policy (m), a decision to rent or not rent (dR) and the decision to adjust or not adjust the housing stock (dH):

$$l_t = (l_t^c, l_t^S, l_t^a, l_t^H, l_t^m, l_t^{\tilde{z}}, l_t^{\Lambda_H}, l_t^W) \tag{88}$$

We will also compute interpolants $l_t^{\Lambda_H}$ and l_t^W denoting interpolants of the housing shadow value and continuation value at each t.

We use the Python and the model consists of a instance of a `LS_model_solver` class. Each `LS_model_solver` is instantized with an instance of a `parameter` class and contains:

1. Numpy grids $\hat{\mathbb{X}}_t$ and $\bar{\mathbb{X}}_t$ for each t
2. Auxiliary grids (see below)
3. Model primitive functions including u, u_1, u_2 and their inverses, the adjustment cost functions and DB pay-out function
4. Auxiliary policy function generators `rent_pol_maker`, `mort_pol_maker`, `no_adj_pol_maker` and `adj_pol_maker` defined below
5. A function `time_iter` that recursively generates the sequence $(l_t)_{t=0}^T$.

We employ EGM, but, in our case, the Euler equations cannot be analytically inverted to avoid all root-find operations (Iskhakov, 2015). Thus, a number of policy functions are recovered via a combination of analytical inversion and single dimensional root-finding. Efficiency is gained since a number of *auxiliary policies* can be defined as *auxiliary grids*, which are smaller than $\hat{\mathbb{X}}_t$. To save notation, we have not stated explicitly the auxiliary grids, but the grids will be the domain of the auxiliary functions defined below.

Before detailing the algorithm, note that we do not explicitly construct the discrete choice as a function of the taste shocks. We also do not condition the policy functions on the taste shocks. Rather we condition the policy functions on a restricted shock vector $\tilde{e}$, the set of shocks other than the taste shocks and an extended state vector $\bar{x}$, the set of endogenous states along with the voluntary contribution choice. The discrete choices themselves are given as a $|\hat{\mathbb{X}}_t| \times |\hat{\Pi} \times \hat{V}|$ probability matrix $\hat{\mathbb{P}}_t$. The rows of $\hat{\mathbb{P}}_t$ index time t states (endogenous and exogenous, excluding the discrete choice risk share at t) and the columns index the discrete choices. It is important to note that the probability matrix $\hat{\mathbb{P}}_t$ corresponds to the probability of making the discrete choice v_t at time t and the discrete state x_t^{π} at time t, *not the choice* π_t *at time* t.

Finally, note that if the matrix $\hat{\mathbb{P}}_{t+1}$ is re-shaped to a matrix $\bar{\hat{\mathbb{P}}}_{t+1}$ with shape $|\hat{Z} \times \hat{A} \times \hat{B}| \times |\bar{\mathbb{X}}_t| \times |\hat{\Pi} \times \hat{V}|$, where $\bar{\hat{\mathbb{X}}}_{t+1}$ is the grid with only non-stochastic states, then $\bar{\hat{\mathbb{P}}}_{t+1}$ can be pre-multiplied with the transition matrix of the exogenous preference, returns and wage shocks to obtain a probability matrix $\hat{\mathbb{E}}_t$ conditioned on time t exogenous stochastic states and time $t+1$ endogenous states.

In generating the auxiliary policy functions below, roots that do not satify the SOCs by Shanker et al. (2022) are not included in the interpolant points.

Evaluate renter auxiliary policy (def `rent_pol_maker`, args = $(l_{t+1}, \hat{\mathbb{E}}_t)$)

- Define the renters' consumption function interpolant $\bar{l}_t^{R^c}(\tilde{e}, hs^R)$ as the root of $c \mapsto -u_1(c, hs^R, \alpha)P^S + u_2(c, hs^R, \alpha)$
- Define the renters' liquid asset policy interpolant $\bar{l}_t^{R^a}(\tilde{e}, a^{DC,\prime}, hs^R, P^S)$ as the root of:

$$a' \mapsto u_1(\bar{l}_t^c(hs^R, P^S, \alpha)) - p_t\beta\hat{\mathbb{E}}_t(1+r)u_1((l_{1,t+1}^C(\bar{x}', \tilde{e}'), l_{1,t+1}^{hs}(\bar{x}', \tilde{e}')) + (1-p_t)b'(A) \quad (89)$$

  where $\bar{x}' = (v', x^{\pi,\prime}, a', \mathbb{1}_{t<R}a^{DC,\prime}, 0, 0)$ and $a^{DC,\prime}$ is the end of period DC assets at time t after contributions have been made but $(t+1)$ returns not realised. Expectations are taken over the random variables e', π and v' and are conditioned on $\tilde{e}$ and x'.

- Define interpolant $\bar{l}_t^{R^{hs}}(\tilde{e}, w, a^{DC,\prime}, P^S)$ as the root to:

$$hs \mapsto w - \bar{l}_t^{R^a}(\tilde{e}, \bar{a}^{DC}, hs, P^S) - P^S \quad (90)$$

Evaluate mortgage auxiliary policy (def `mort_pol_maker`, args = $(l_{t+1}, \hat{\mathbb{E}}_t)$)

- Define mortgage policy interpolant $\bar{l}_t^m(\tilde{e}, a', a^{DC,\prime}, H')$ as the root of:

$$m' \mapsto \hat{\mathbb{E}}_t(1+r_{t+1})u_1((l_{t+1}^C(\bar{x}', \tilde{e}'), l_{t+1}^{hs}(\bar{x}', \tilde{e}')) \\ - \hat{\mathbb{E}}_t(1+r_{t+1}^m)u_1((l_{t+1}^C(\bar{x}', \tilde{e}'), l_{t+1}^{hs}(\bar{x}', \tilde{e}')) \quad (91)$$

  where $\bar{x}' = (v', x^{\pi,\prime}, a', a^{DC,\prime}, 0, 0)$.

Define non-adjusting owner auxiliary (def `no_adj_pol_maker`, args = (l_{t+1},$\hat{\mathbb{E}}_t$))

- Define consumption policy interpolant $\bar{l}_t^{c,nadj}(\tilde{e}, a', a^{DC,\prime}, H, m, P^S)$ as the root of:

$$c \mapsto u_1(c,(1-\delta)) - p_t\beta\mathbb{E}_t(1+r_{t+1})u_1((l_{t+1}^C(\bar{x}',\tilde{e}'), l_{t+1}^{hs}((\bar{x}',\tilde{e}')) + (1-p_t)b'(B')$$

  where

$$\bar{x}' = (v', x^{\pi,\prime}, a', a^{DC,\prime}, (1-\delta)H, \bar{l}_t^m(\tilde{e}, a^{DC,\prime}, (1-\delta)H))$$

- Define non-adjust liquid asset policy interpolant $\bar{l}_t^{a,nadj}(\tilde{e}, x^\pi, v, a, a^{DC}, H, m, P^S)$ as the root of:[47]

$$a' \mapsto a + \mathbb{1}_{t<R}w(1-v_S-v) - \bar{l}_t^{c,adj}(a', a^{DC,\prime}, H, m, P^S) + \bar{l}_t^m(e, a^{DC,\prime}, (1-\delta)H) - (1+r_m)m$$

  where $a^{DC,\prime} = a_{DC}(x^\pi R^r + (1-x^\pi)R^s) + (1+v+v_S)y$.

Define adjusting home-owner auxiliary (def `adj_pol_maker`, args = (l_{t+1},$\hat{\mathbb{E}}_t$))

- Define $\bar{l}_t^{\text{c,adj}}(e, a', a^{DC,\prime}, H', m)$ as the root of:

$$c \mapsto u_1(c,H',\alpha) - \max\{p_t\beta\mathbb{E}_t(1+r_{t+1})u_1((l_{t+1}^C(\bar{x}',\tilde{e}'), l_{t+1}^S(\bar{x}',\tilde{e}')) + (1-p_t)b'(B'), u_1(\bar{c},H')\}$$

  where $\bar{c}$ is the level of consumption if $a' = 0$

- Define liquid asset policy interpolant $\bar{l}_t^{\text{a,adj}}(e, a^{DC,\prime}, H', m)$ as the root of:

$$a' \mapsto \Xi^{mc}\phi^C + \beta\hat{\mathbb{E}}_t l_{t+1}^{\Lambda^H}(\bar{x}',\tilde{e}') + u_2(c,H',\alpha) - u_1(c,H',\alpha)(1+\tau_H P) + (1-p_t)b'(B')P$$

  where

$$c = \bar{l}_t^{\text{c,adj}}(a', a^{DC,\prime}, H', m)$$

$$\Xi^{mc} = \mathbb{1}_{m'=\phi^C\frac{H_{t+1}}{1-\delta}}(u_1(c,hs) - \Lambda^m)$$

$$\Lambda^m = \hat{\mathbb{E}}_t(1+r_{t+1}^m)u_1(l_{t+1}^C(\bar{x}',e'), l_{t+1}^S(\bar{x}',\tilde{e}'))$$

  and

$$m' = \bar{l}_t^m(\tilde{e}, a^{DC,\prime}, H')$$

[47]This step is equivalent to the EGM - see discussion by Shanker et al. (2022).

- Define housing policy function $\bar{l}_t^{H,adj}(\tilde{e}, a, a^{DC}, H, m)$ as the root of:

$$H' \mapsto (1+r)a + \mathbb{1}_{t<R} w(1 - v_S - v) - \bar{l}_t^{c,adj}(a', a^{DC,\prime}, H', m) + \bar{l}_t^{m}(\tilde{e}, a^{DC,\prime}, H') - (1+r_m)m - (1+P_t)(H' - H) - \tau P H'$$

Next, we detail the back-ward induction function.

Time iteration (`time_iter`)

- Initialise l_{T+1} as interpolants filled with zeros defined on $\mathbb{X}^T$
- Solve terminal period problem at time T:
  1. Evaluate auxiliary functions
  2. Interpolate policy functions $\bar{\mathbb{X}}_t$
     - Interpolate renter policy functions:
       * $l_T^{S,R}(x,e) = \bar{l}_T^{R^{hs}}(e, w, 0, P^S)$, where $w = (1+r)a - (1+r)m + (1-\delta)HP$
       * $l_T^{H,R}(x,e) = 0$
       * $l_T^{m,R}(x,e) = 0$
       * $l_T^{c,R}(x,e) = \bar{l}_T^{R^c}(e, \bar{l}_T^{R^{hs}}(\tilde{e}, w, 0, P^S)$, where $w = (1+r)a - (1+r)m + (1-\delta)HP$
     - Interpolate non-adjuster owner functions as:
       * $l^{H,nadj}(x,e) = l^{S,nadj}(x,e) = (1-\delta)H$
       * $l^{m,nadj}(x,e) = \bar{l}_t^{m}(\tilde{e}, 0, (1-\delta)H)$
       * $l^{c,nadj}(x,e) = \bar{l}_T(e, \bar{l}_t^{\tilde{a},nadj}(\tilde{e}, 0, H, m)$
       * $l_t^{a,nadj}(x,e) = \bar{l}_t^{a,nadj}(e, 0, H, m)$
     - Interpolate adjuster owner policy functions as:
       * $l^{H,adj}(x,e) = \bar{l}_t^{H,adj}(e, a, 0, H, m)$
       * $l^{c,adj}(x,e) = \bar{l}_t^{c,adj}(e, \bar{l}_t^{a,adj}(e, 0, l^{H,adj}(x,e), m))$
       * $l^{m,nadj}(x,e) = \bar{l}_t^{m}(e, 0, l^{H,adj}(x,e))$
       * $l_t^{a,adj}(x,e) = \bar{l}_t^{a,adj}(e, 0, l^{H,adj}(x,e), m)$
     - Evaluate discrete choices as follows:
       * For all $x, e \in \bar{\mathbb{X}}_t$ such that $H > 0$ (incumbent owners), evaluate $l_t^{\tilde{z}}$ as:

$$l_t^{\tilde{z}}(e,x) = \underset{\substack{\tilde{z} \in \{\texttt{adj\_rent, adj\_own, nadj\_own}\}, \\ l_t^{\tilde{z}}(x,e,\tilde{z}) \in \Gamma_t^{t}(e,x)}}{\arg\min} \varphi_t^{u}(x, l_t^{\tilde{z}}(x,e,\tilde{z}))$$

* For all $x, e \in \bar{\mathbb{X}}_t$ such that $H > 0$ (incumbent renters), evaluate $l_t^{\tilde{z}}$ as:

$$l_t^{\tilde{z}}(e,x) = \underset{\substack{\tilde{z}\in\{\texttt{adj\_rent},\texttt{adj\_own}\},\\ l_t^{\tilde{z}}(x,e,\tilde{z})\in\Gamma_t^t(e,x)}}{\arg\min} \quad \varphi_t^u(x, l_t^{\tilde{z}}(x,e,\tilde{z}))$$

3. Construct l_T^j, with $j \in \{\texttt{c},\texttt{H},\texttt{a},\texttt{m}\}$ as follows:

$$l_T^j = \sum_{k\in\left\{\substack{\texttt{adj\_rent},\\ \texttt{adj\_own},\\ \texttt{nadj\_own}}\right\}} \mathbb{1}_{k=l_t^{\tilde{z}}(e,x)} l_T^{j,k}$$

4. Define interpolant $l_T^W(e,x) = \varphi_t^u(x, l_T(x,e))$

- While $t > t_0$:

1. Evaluate auxiliary functions
2. Interpolate policy functions $\bar{\mathbb{X}}_t$ as follows:

   - Interpolate renter policy functions on $\bar{\mathbb{X}}_t$:
     * $l_T^{S,R}(\bar{x},\tilde{e}) = \bar{l}_T^{R^{hs}}(\tilde{e}, w, a^{DC,\prime}, P^S)$, where $w = (1+r)a - (1+r)m + (1-\delta)HP + (1 - v_S - v) + \mathbb{1}_{t=R} a^{DC,\prime}$ and $a^{DC,\prime} = a_{DC}(x^\pi R^r + (1-x^\pi)R^s) + (1+v+v_s)y$
     * $l_T^{H,R}(\bar{x},\tilde{e}) = 0$
     * $l_T^{m,R}(\bar{x},\tilde{e}) = 0$
     * $l_T^{c,R}(\bar{x},\tilde{e}) = \bar{l}_T^{R^c}(\tilde{e}, \bar{l}_T^{R^{hs}}(\tilde{e}, w, 0, P^S)$
   - Interpolate non-adjuster owner functions as:
     * $l^{H,nadj}(\bar{x},\tilde{e}) = l^{S,nadj}(x,e) = (1-\delta)H$
     * $l^{m,nadj}(\bar{x},\tilde{e}) = \bar{l}_t^m(\tilde{e}, a^{DC,\prime}, (1-\delta)H)$
     * $l^{c,nadj}(\bar{x},\tilde{e}) = \bar{l}_T(\tilde{e}, \bar{l}_t^{\tilde{a},nadj}(\tilde{e}, a^{DC,\prime}, H, m)$
     * $l_t^{a,nadj}(\bar{x},\tilde{e}) = \bar{l}_t^{a,nadj}(\tilde{e}, a^{DC,\prime}, H, m)$
   - Interpolate adjuster owner policy functions as:
     * $l^{H,adj}(\bar{x},\tilde{e}) = \bar{l}_t^{H,adj}(\tilde{e}, a, a^{DC,\prime}, H, m)$
     * $l^{c,adj}(\bar{x},\tilde{e}) = \bar{l}_t^{c,adj}(\tilde{e}, \bar{l}_t^{a,adj}(\tilde{e}, a^{DC,\prime}, l^{H,adj}(x,\tilde{e}), m))$
     * $l^{m,nadj}(\bar{x},\tilde{e}) = \bar{l}_t^m(\tilde{e}, a^{DC,\prime}, l^{H,adj}(x,\tilde{e}))$
     * $l_t^{a,adj}(\bar{x},\tilde{e}) = \bar{l}_t^{a,adj}(\tilde{e}, a^{DC,\prime}, l^{H,adj}(\bar{x},\tilde{e}), m)$
   - Evaluate discrete choices for housing:
     * For all $\bar{x}, \tilde{e} \in \bar{\mathbb{X}}_t$ such that $H > 0$ (incumbent owners), evaluate $l_t^{\tilde{z}}$ as:

$$l_t^{\tilde{z}}(\bar{x},\tilde{e}) = \underset{\substack{\tilde{z}\in\{\texttt{adj\_rent, adj\_own, nadj\_own}\},\\ l_t^{\tilde{z}}(\bar{x},\tilde{e},\tilde{z})\in\Gamma_t^t(\tilde{e},\bar{x})}}{\arg\min} \quad \varphi_t^u(\bar{x}, l_t^{\tilde{z}}(\bar{x},\tilde{e},\tilde{z})) + \hat{\mathbb{E}}_t l_t^W(\bar{x}',\tilde{e}')$$

where $\bar{x}' = (v', x^{\pi,\prime}, l_t^{a,\tilde{z}}(\bar{x}, e, \tilde{z}), a^{DC,\prime}, l_t^{H,\tilde{z}}(\bar{x}, e, \tilde{z}), l_t^{m,\tilde{z}}(\bar{x}, e, \tilde{z}))$

* For all $\bar{x}, \tilde{e} \in \bar{\mathbb{X}}_t$ such that $H > 0$ (incumbent renters), evaluate $l_t^{\tilde{z}}$ as:

$$l_t^{\tilde{z}}(\bar{x}, \tilde{e}) = \underset{\substack{\tilde{z} \in \{\texttt{adj\_rent}, \texttt{adj\_own}\}, \\ l_t^{\tilde{z}}(\bar{x}, \tilde{e}, \tilde{z}) \in \Gamma_t^t(\tilde{e}, \bar{x})}}{\arg\min} \varphi_t^u(x, l_t^{\tilde{z}}(\bar{x}, \tilde{e}, \tilde{z})) + \hat{\mathbb{E}}_t l_t^W(\bar{x}', \tilde{e}')$$

– Evaluate discrete choice probabilities for pension accounts:

* If $t < R$, then generate the discrete choice probabilities as interpolants on $\hat{\mathbb{X}}_t$ for voluntary contributions and risky assets share choice using equation (26) and (27)
* Generate probability transition matrix $\hat{\mathbb{E}}_{t-1}$

3. Construct l_T^j, with $j \in \{\texttt{c}, \texttt{H}, \texttt{a}, \texttt{m}\}$ as follows:

$$l_t^j(\bar{x}, \tilde{e}) = \sum_{k \in \left\{ \substack{\texttt{adj\_rent}, \\ \texttt{adj\_own}, \\ \texttt{nadj\_own}} \right\}} \mathbb{1}_{k = l_t^{\tilde{z}}(\bar{x}, \tilde{e})} l_t^{j,k}(\bar{x}, \tilde{e})$$

4. Define interpolant $l_T^W(\tilde{e}, \bar{x}) = \varphi_t^u(\bar{x}, l_t(\bar{x}, \tilde{e})) + \hat{\mathbb{E}}_t l_t^W(\bar{x}', \tilde{e}')$, where

$$\bar{x}' = (v', x^{\pi,\prime}, l_t^a(\bar{x}, e), a^{DC,\prime}, l_t^{H,\tilde{z}}(\bar{x}, e), l_t^m(\bar{x}, e, \tilde{z}))$$

5. Set t = -1