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# Insuring longevity risk and long-term care: Bequest, housing and liquidity

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## Abstract

We study the impact of housing wealth and individual preferences on demand for annuities and long-term care insurance (LTCI). We build a multi-state lifecycle model that includes longevity risk and health shocks. The preference is represented by a recursive utility function that separates risk aversion and elasticity of intertemporal substitution (EIS). When health shocks are considered, a higher level of risk aversion lowers the annuity demand, while a lower level of the EIS has the opposite effect. The impact diminishes with a weaker bequest motive, more liquid wealth, or access to LTCI, all of which increase the demand for annuities. The presence of home equity can enhance annuity demand, but the enhancement is marginal when the LTCI is available. The presence of home equity has a crowding-out effect on LTCI demand, and the effect is strengthened by a lack of bequest motives or a lower degree of risk aversion. The cash poor but asset rich may demand more LTCI coverage than their renter counterparts to preserve bequests. When both life annuities and the LTCI are available, we find that the product demand is robust to changes in risk aversion and the EIS, providing insights into product designs that bundle annuities and LTCI.

*Keywords:* Recursive utility, Housing, Life annuities, Long-term care insurance, Lifecycle model

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# 1 Introduction

The occupation pension funds worldwide have undergone massive transitions from defined benefit to defined contribution (DC) schemes. While DC plans are reaching maturity, the payout phase remains largely under-developed (Rocha et al., 2010). Compared to the accumulation phase, the payout phase features a higher level of heterogeneity. Retirees vary in wealth levels, homeownership status, risk tolerance, bequest motives, among other aspects, making it challenging to design products that meet individual circumstances. The paper helps address the challenge pertinent to life annuities and long-term care insurance (LTCI). We focus on these two products due to their importance in tackling some of the most common retirement risks. Life annuities insure against longevity risk (i.e., the risk of outliving one's financial resources), which is the key consideration in designing the payout phase (Rocha et al., 2011). The LTCI protects against unexpected healthcare costs, which can be the single most severe spending shock for retirees (Brown and Finkelstein, 2011).

We study the impact of housing wealth and individual preferences (including bequest motives and risk aversion) on demand for life annuities and the LTCI. We consider longevity risk together with health shocks in a multi-state lifecycle model that starts at retirement. The preference is represented by an Epstein-Zin-Weil-type utility (Epstein and Zin, 1989; Epstein and Zin, 1991; Weil, 1989) that separately identifies risk aversion and elasticity of intertemporal substitution (EIS), more flexible than the commonly-used power utility. We use alternative parameter values for risk aversion, the EIS, and bequest motives to investigate their impact on product demand. We assume different wealth levels and proportions of net worth in home equity to capture heterogeneous financial profiles. Although we do not intend to explain the low voluntary annuitization rate or the small private LTCI market, our results will help DC pension funds design personalized retirement products and advance the development of the payout phase.

The presence of home equity complicates the decision on life annuities and the LTCI due to its size and illiquidity. This issue is relevant to the majority of older Americans who have high homeownership rates, hovering around 80% over the last few decades (U.S. Census Bureau, 2022). The elderly homeowners have a large fraction of household portfolios held in home equity. The median ratio of home equity to all assets is close to 60% in the U.S. (Davidoff, 2009;

Flavin and Yamashita, 2002). The presence of home equity can pose a liquidity constraint that limits one's capacity to pay for the insurance premiums or to support general consumption. In countries where the homeownership rates are high, their retirees are often asset rich and relatively cash poor (see e.g., Bradbury, 2010; McCarthy et al., 2002).

The impact of illiquid housing wealth on annuity demand is investigated in Pashchenko (2013) who finds that illiquid housing wealth decreases the annuity market participation rates because it reduces the amount of disposal wealth. If housing wealth provides a source of liquidity, Peijnenburg et al. (2017) find a slight increase in the optimal annuitization rate. Both papers do not explicitly consider the interaction between housing wealth and health shocks. In practice, housing wealth is rarely drawn upon to finance non-durable consumption, and selling the house is often associated with losing spouses or moving into a nursing home (Walker, 2004; Venti and Wise, 2004). This means housing wealth can be a significant source of funding for costly long-term care, thus reducing the need to keep a liquid wealth buffer. Precautionary savings for health shocks are known to affect annuity decisions (see e.g., Davidoff et al., 2005; Pang and Warshawsky, 2010; Peijnenburg et al., 2017; Turra and Mitchell, 2008), but how the liquidity released from home equity in the event of health shocks can affect annuity demand remains largely unexplored. The present study fills this gap.

Home equity can substantially reduce demand for the LTCI, provided that home equity is not liquidated unless the homeowner moves to a long-term care facility. This result is proved by Davidoff (2010) in a one-period model and later confirmed by Shao et al. (2019) in a multi-period setting. Housing wealth can also reverse the complementarity between life annuities and the LTCI (Davidoff, 2009). A common feature of these studies is to use a power utility function that imposes an inverse relationship between risk aversion and the EIS. Since empirical experiments find no such correlation (Barsky et al., 1997) and that individuals have relative risk aversion greater than the reciprocal of the EIS (Brown and Kim, 2013), the power utility function is unlikely to capture heterogeneous preferences of retirees.

We use a Markov process to model health state transitions and fit the model to the data collected by the U.S. Health and Retirement Study. The health states of healthy, mildly disabled, and severely disabled are defined based on the number of difficulties in performing activities of daily living. We explicitly consider the link between home equity liquidation and health shocks

by assuming that homeowners will sell housing assets upon becoming severely disabled. This assumption is based on the empirical evidence that home equity is rarely spent before death except for moving into a nursing home, also in line with the assumption made in Davidoff (2009). We solve optimal consumption as well as an optimal portfolio of life annuities and the LTCI. Both products have actuarially fair prices. We abstract from product loading that is often used to explain the thin empirical demand (see e.g., Mitchell et al., 1999; Brown and Finkelstein, 2011), which is beyond the scope of this paper.

We show that housing wealth significantly enhances annuity demand when the LTCI is not available. The presence of home equity can increase the optimal annuitization rate as a fraction of total wealth even though its presence reduces the proportion of total wealth that can be annuitized. The result is stronger than that in Peijnenburg et al. (2017) and contrasts with Pashchenko (2013) due to our assumption of housing wealth liquidation in the event of illness. We confirm the intuitive explanation in Peijnenburg et al. (2017) that home equity lowers liquid wealth buffer for health shocks, thereby increasing the amount of wealth available for annuitization. Moreover, we find that the enhancement effect of home equity is marginal when retirees can access the LTCI. As a result, the optimal annuitization rate as a proportion of total wealth decreases with housing wealth in this case.

We reveal that housing wealth interacts with preferences to affect the LTCI demand, extending the literature that considers the impact of single factors, such as housing wealth (Davidoff, 2010; Shao et al., 2019) or bequest motives (Lockwood, 2018; Pauly, 1990). We find that the crowding-out effect of home equity on the LTCI demand is stronger with a weaker bequest motive or a lower degree of risk aversion. The presence of bequest motives may also reverse the crowding-out effect of home equity. We find that the LTCI helps homeowners in the low wealth groups to preserve their bequests, thereby improving the demand for the LTCI. Furthermore, we find a minimal impact of the EIS on the LTCI demand, robust to changes in homeownership status and amount of housing wealth.

Using the recursive utility allows us to capture a wider degree of heterogeneity in preferences and pinpoint the impact of risk aversion and the EIS on product demand. When the LTCI is not available, we find a higher degree of risk aversion and a lower degree of the EIS drives the optimal annuitization rate in the opposite direction. This suggests the power utility can

confound the impact of risk aversion on annuity demand. When both products are offered in the market, we find the demand for annuities as well as the LTCI is relatively robust to changes in risk aversion and the EIS if we control for the total wealth level and the ratio of home equity to net worth. This result provides a new incentive to bundle life annuities and the LTCI for it can greatly simplify the choice menus of retirement products.

The rest of the paper is organized as follows. Section 2 presents the lifecycle model and the model input in detail. Section 3 discusses how the demand for annuities and the LTCI varies with wealth, homeownership, and individual preferences. Section 4 concludes.

## 2 Lifecycle model in retirement

We set up a discrete-time lifecycle model starting at retirement. The model consists of a series of one-year period that is indexed by  $t \in \{1, 2, \dots, T, T + 1\}$ <sup>1</sup>. The individual retires at  $t = 1$  aged 65, and her maximum attainable age is 100, so  $T = 36$ . All variables are defined in real terms.

### 2.1 Health dynamics and costs

We follow Ameriks et al. (2011) to model the retiree’s health status with states ‘1’ (healthy), ‘2’ (mildly disabled), ‘3’ (severely disabled), and ‘4’ (dead). The categorization of the first three states is based on the number of difficulties in independently performing activities of daily living (ADLs). There are usually a total of six ADLs: dressing, walking, bathing, eating, transferring, and toileting. Mildly disabled state is defined as having difficulties in 1 – 2 ADLs, and severely disabled state is defined as having difficulties in 3 – 6 ADLs. The health state at period  $t$  is denoted as  $s_t$ .

The health state transitions are modeled using a Markov process. Fong et al. (2015) shows a significant proportion of the elderly can recover from the disabled state to the healthy state. On the other hand, severe disability is usually chronic in nature that substantially reduces the possibility of recovery (Ferri and Olivieri, 2000; Olivieri and Pitacco, 2001). We, therefore, allow for transition from the mildly disabled state to the healthy state and do not allow for

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<sup>1</sup>Note that the latest possible consumption occurs at  $t = T$ . The last time index  $T + 1$  is for the purpose of bequest only.

recoveries from the severely disabled state. Figure 1 depicts the health state transitions, where  $\sigma_t(j, k)$  ( $j \in \{1, 2, 3\}, k \in \{1, 2, 3, 4\}$ ) denotes the transition intensity from state  $j$  to state  $k$  at time  $t$ . A more comprehensive approach is to model both care state and health state and allow for all possible transitions (see e.g., Friedberg et al., 2015). We abstract from additional features, and in particular, the recovery from the severely disabled state to focus on the interaction between home equity and healthcare costs as we assume that becoming severely disabled leads to home equity liquidation.

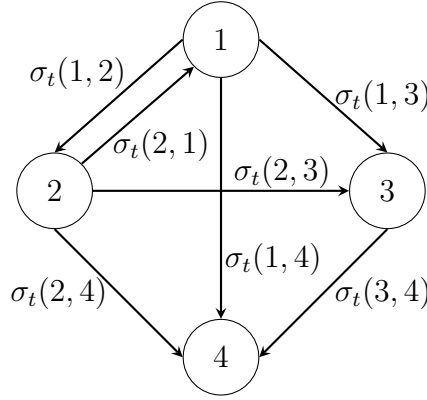


Figure 1. Four-state Markov process that models health state transitions.

Given the transition intensities,  $\sigma_t(j, k)$ , the single-period transition probabilities,  $\pi_t(j, k) \equiv \Pr(s_{t+1} = k | s_t = j)$ , can be solved through Kolmogorov equations. In particular, we assume the transition intensities are constant within an integer age. Then the annual transition probabilities for each period are given by

$$\begin{pmatrix} \pi_t(1,1) & \pi_t(1,2) & \pi_t(1,3) & \pi_t(1,4) \\ \pi_t(2,1) & \pi_t(2,2) & \pi_t(2,3) & \pi_t(2,4) \\ \pi_t(3,1) & \pi_t(3,2) & \pi_t(3,3) & \pi_t(3,4) \\ \pi_t(4,1) & \pi_t(4,2) & \pi_t(4,3) & \pi_t(4,4) \end{pmatrix} = \exp \left[ \begin{pmatrix} \sigma_t(1,1) & \sigma_t(1,2) & \sigma_t(1,3) & \sigma_t(1,4) \\ \sigma_t(2,1) & \sigma_t(2,2) & \sigma_t(2,3) & \sigma_t(2,4) \\ 0 & 0 & -\sigma_t(3,4) & \sigma_t(3,4) \\ 0 & 0 & 0 & 0 \end{pmatrix} \right],$$

where

$$\sigma_t(1,1) = -(\sigma_t(1,2) + \sigma_t(1,3) + \sigma_t(1,4)),$$

$$\sigma_t(2,2) = -(\sigma_t(2,1) + \sigma_t(2,3) + \sigma_t(2,4)),$$

$$\exp(X) = \sum_{k=0}^{\infty} \frac{1}{k!} X^k, \quad X^0 \text{ is the identity matrix with the same dimensions as } X.$$

Given the single-period transition probabilities, the  $n$ -period transition probability,  $\pi_t^n(j, k) \equiv \Pr(s_{t+n} = k | s_t = j)$ , can be obtained through the Chapman-Kolmogorov equations. When  $n = 1$ , it reduces to the single-period transition probability, i.e.,  $\pi_t^1(j, k) = \pi_t(j, k)$ .

We follow Ameriks et al. (2011) to model the out-of-pocket health expenditure as a deterministic process given the health state,  $s_t$ . The deterministic process is preferred over a stochastic model (see e.g., De Nardi et al., 2010) for its simplicity and its ability to capture the characteristics of empirical medical expense risk. Since the healthcare inflation usually exceeds that of the consumer price index (CPI), it is assumed that the relative price of healthcare increases at a rate of  $q$  per annum.

## 2.2 Housing and financial assets

Given that a large majority of retired homeowners have paid off their mortgages, the model assumes the individual lives in a mortgage-free home at retirement. In addition, empirical data shows that housing assets are rarely drawn upon unless the retiree moves to a long-term care facility (see e.g., Venti and Wise, 2004). It is assumed that the retiree will liquidate the house when she becomes severely disabled and subsequently moves to a nursing home. The house has a gross rate of return  $R_{H,t+1}$  from time  $t$  to time  $t + 1$ , where  $\ln(R_{H,t+1})$  follows a normal distribution with mean  $\mu_H$  and variance  $\sigma_H^2$ . The liquid assets earn a constant risk-free return of  $R_f$ . We abstract from the equity market.

## 2.3 Retirement products

At retirement, the individual has access to two types of retirement products, life annuities and the LTCI, both of which are offered by private companies. The retiree decides the proportion ( $\alpha$ ) of liquid assets to annuitize and the percentage coverage ( $\lambda$ ) of LTCI to purchase. The decisions are made at retirement only. The public offering of similar products is not explicitly considered in the model. Nevertheless, the individual's endowment at retirement can be perceived as including the expected present value of public pension paid during retirement, and the out-of-pocket health expenditure can be seen as net of any publicly funded schemes.

The life annuity is of an ordinary type that provides annual level payment for the remaining



lifetime of the annuitant. The payment starts at the beginning of the first period. The annuity is charged at an actuarially fair price. Given an  $\alpha$  proportion of liquid assets annuitized at retirement, the annual income from annuities is given by

$$Y = \frac{\alpha B}{1 + \sum_{t=2}^T R_f^{-(t-1)} \pi_1^{t-1}(s_1, s_t \neq 4)}, \quad (1)$$

where  $B$  denotes the initial endowment of liquid assets,  $\pi_1^{t-1}(s_1, s_t \neq 4)$  denotes the probability that a 65-year-old individual with health state  $s_1$  will survive for the next  $(t - 1)$  years.

The LTCI covers healthcare costs when the policyholder is severely disabled (i.e., health state 3). We assume a lump-sum premium and exclude any loading on the product. The actuarially fair price ( $P$ ) for a full coverage LTCI policy is given by

$$P = \sum_{t=2}^T R_f^{-(t-1)} \pi_1^{t-1}(s_1, s_t = 3) h(s_t = 3, t), \quad (2)$$

where  $h(s_t, t)$  represents the out-of-pocket health expenditure at time  $t$  in health state  $s_t$ .

## 2.4 Budget constraints and wealth dynamics

In the first period, the retiree is endowed with liquid wealth of  $B$  and housing wealth of  $W^H$ , and the retiree is in the healthy state (i.e., health state 1). She then decides the proportion of liquid assets to annuitize and the LTCI coverage to purchase. After that, she receives income from annuities (if any), incurs the healthcare costs, and decides how much to consume. Let  $B_1$  denote the amount of liquid wealth available after purchasing the retirement products. It is given by

$$B_1 = (1 - \alpha)B - \lambda P, \quad B_1 \geq 0. \quad (3)$$

Starting from the second period, the retiree enters the period  $t$  with health state  $s_t$  and wealth  $W_t$ , which consists of housing wealth  $W_t^H$  and liquid wealth  $B_t$ . Note that  $W_t$ ,  $W_t^H$ , and  $B_t$  denote the amount available at the beginning of the period  $t$  (i.e., before any action is taken) except for  $B_1$ , which is specified in Equation (3). The timing of events is as follows.

1. If  $s_t = 4$ , the individual is deceased, so the wealth  $W_t$  is bequeathed.

2. If  $s_t < 4$ , one of the following events will occur.

(a) If  $s_t = 3$  and  $s_{t-1} \in \{1, 2\}$ , the individual will liquidate the home equity and move into a residential care facility.

(b) If  $s_t = 3$  and  $s_{t-1} = 3$ , the individual will remain staying at the residential care.

(c) If  $s_t < 3$ , the individual will remain living at home.

3. If  $s_t < 4$ , the health costs  $h(s_t, t)$  are incurred and a consumption decision ( $C_t$ ) is made.

The remaining liquid assets earn a risk-free return  $R_f$ .

The chosen consumption level must not fall below the consumption floor  $C^f$  to ensure a minimum standard of living. If the individual's budget cannot support the minimum consumption level, we assume the government will provide subsidies to increase the consumption level to  $C^f$ . The liquid wealth in the next period is subsequently set to zero.

The budget constraint for liquid assets  $B$  is given by

$$B_2 = (B_1 + Y - h(s_1, 1) - C_1)^+ R_f;$$

for  $t \in \{2, 3, \dots, T\}$ ,

$$B_{t+1} = \begin{cases} (B_t + Y - h(s_t, t) - C_t)^+ R_f & \text{if } s_t \in \{1, 2\} \\ (B_t + Y + W_t^H \mathbb{1}_{\{s_{t-1} \in \{1, 2\}\}} - (1 - \lambda)h(s_t, t) - C_t)^+ R_f & \text{if } s_t = 3 \end{cases}, \quad (4)$$

where  $(\cdot)^+$  is defined as  $\max(\cdot, 0)$ .

The budget constraint for total wealth  $W$  is given by

$$W_2 = B_2 + W_1^H R_{H,2}, \quad \text{where } W_1^H = W^H;$$

for  $t \in \{2, 3, \dots, T\}$ ,

$$W_{t+1} = \begin{cases} B_{t+1} + W_t^H R_{H,t+1} & \text{if } s_t \in \{1, 2\} \\ B_{t+1} & \text{if } s_t = 3 \end{cases}. \quad (5)$$

## 2.5 Preferences

Individuals in the model are assumed to have Epstein-Zin-Weil-type preferences (Epstein and Zin, 1989; Epstein and Zin, 1991; Weil, 1989) over non-housing consumption and a bequest. Although the housing service consumption is not directly included in the utility function, the housing wealth contributes to the utility through bequests or home equity liquidation that alleviates the budget constraint caused by excessive medical care costs.

The Epstein-Zin model generalizes the power utility model in that it can separately identify the risk aversion and the EIS. The two elements are intrinsically different. Risk aversion describes an individual's willingness to substitute consumption across different states of the world, whereas the EIS describes an individual's willingness to substitute consumption over time. When the individual's EIS is the reciprocal of the coefficient of relative risk aversion, the Epstein-Zin model reduces to the power utility model.

The preferences are specified by

$$\begin{aligned}
 V_t &\equiv V(B_t, W_t^H, s_t, t) \\
 &= \max_{O_t} \left\{ (1 - \beta)C_t^{1-\rho} + \beta \left[ \mathbb{E}_t \left[ \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V(B_{t+1}, W_{t+1}^H, s_{t+1} = k, t+1)^{1-\gamma} \right. \right. \right. \\
 &\quad \left. \left. \left. + \pi_t(s_t, s_{t+1} = 4) b^\gamma W_{t+1}^{1-\gamma} \right] \right]^{\frac{1}{\theta}} \right\}^{\frac{1}{1-\rho}}, \quad \theta = \frac{1-\gamma}{1-\rho}; \tag{6} \\
 O_t &= \begin{cases} \{\lambda, \alpha, C_t\}, & \text{for } t = 1; \\ \{C_t\}, & \text{for } t = 2, \dots, T. \end{cases}
 \end{aligned}$$

The notation  $V_t$  is the indirect utility value at time  $t$ ,  $\beta$  the subjective discount factor,  $\rho$  the inverse of the EIS (i.e.,  $\rho = 1/\psi$ ),  $\mathbb{E}$  the expectation operator,  $\gamma$  the coefficient of relative risk aversion,  $b$  the strength of bequest motive. The subjective discount factor ( $\beta$ ) measures an individual's impatience to defer consumption. It takes values between zero and one, with a lower value representing less willingness to postpone the consumption. The strength of bequest motives ( $b$ ) takes non-negative values, with a higher value meaning a stronger bequest motive.

## 2.6 Optimization problem and solution method

Individuals optimize over consumption, annuitization rate, and the LTCI coverage to maximize the expected lifetime utility in (6), subject to conditions (1) to (5). We set up grid points on liquid wealth, housing wealth, and current health state to solve the optimization problem. The method of endogenous grid points (Carroll, 2006) is used to set up the grid points for the liquid assets. The grid points on housing wealth are given exogenously. The log-normal distribution of house price growth is discretized by the Gauss-Hermite quadrature. The first-order condition for consumption can be solved analytically to speed up the solution process. The analytical form is derived in Appendix A. The optimization problem is solved backward, starting from the last period. For the points not lying on the grid, a hybrid interpolation method introduced in Ludwig and Schön (2018) is used to find the optimal consumption and the indirect utility value.

The optimal annuitization rate and LTCI coverage are solved in the first period using the following steps. First, we set up the grid points on annuitization rate and LTCI coverage. On each grid point, we solve the optimal consumption and indirect utility levels backward from the last period to the first period. Given the initial liquid wealth and housing wealth, the indirect utility value in the first period for a healthy individual can be found through the hybrid interpolation method. The optimal annuitization rate and LTCI coverage are found by searching for the grid point that gives the highest value of indirect utility.

After solving the optimal decision rules defined on the state space, the time-series profiles of a retiree's optimal consumption can be obtained through simulation. Specifically, we first simulate house price growths and health states, and then use the optimal policy rules to calculate the optimal consumption. The corresponding liquid and total wealth levels can also be obtained. The simulation is run 10,000 times.

## 2.7 Model parameterization

We set the liquid wealth endowment at between \$50K and \$1 million, with an increment of \$50K. When retirees are endowed with home equity, we consider home equity comprises a quarter, a third, or a half of total wealth. The housing wealth proportions are lower than those

reported in Davidoff (2009) and Flavin and Yamashita (2002) because the pre-annuitized wealth is implicitly included in the total wealth. The varieties of liquid wealth levels and home equity proportions allow us to investigate the impact of housing wealth and liquidity on demand for life annuities and the LTCL.

We proceed to discuss the remaining inputs to the lifecycle model: health state transitions in Section 2.7.1, and preference parameters in Section 2.7.2.

### 2.7.1 Health state transitions

The health state transition is estimated using the data from the U.S. Health Retirement Study that surveys a nationally representative sample of Americans over age 50 every two years, starting from 1992. The data before 1998 is removed due to inconsistent question structure. We use the data between 1998 and 2010 and focus on the female experiences since they have longer life expectancy than males, and tend to spend more years in the disabled state (Fong et al., 2015).

We follow the method in Fong et al. (2015) to estimate the health state transitions using a generalized linear model (GLM) with the log link function. The number of transitions at age  $x$  is assumed to follow a Poisson distribution with mean ( $m_x$ ) defined as a polynomial function of age. The mean is given by

$$m_x = e_x \sum_{k=0}^K \eta_k x^k, \quad (7)$$

where  $e_x$  is the central exposure to risk for  $x$ -year-old individuals,  $K$  the degree of the polynomial,  $\eta_k$  the coefficients of the polynomial. We use the Akaike information criterion corrected for sample size (AICc), Bayesian information criterion (BIC), and the likelihood ratio test to select the degree of polynomials. The detailed results are presented in Appendix B. Figure 2 compares fitted transition rates with the crude ones, and shows that the estimation achieves a good fit.

We calculate the survival probability and the probability of being in each health state based on the estimated transition rates. Figure 3 shows that a 65-year-old healthy female has a more than 50% chance of living to the mid-80s, and that the probability of being severely disabled increases substantially after age 85, so the overall risk of requiring long-term care is high. We

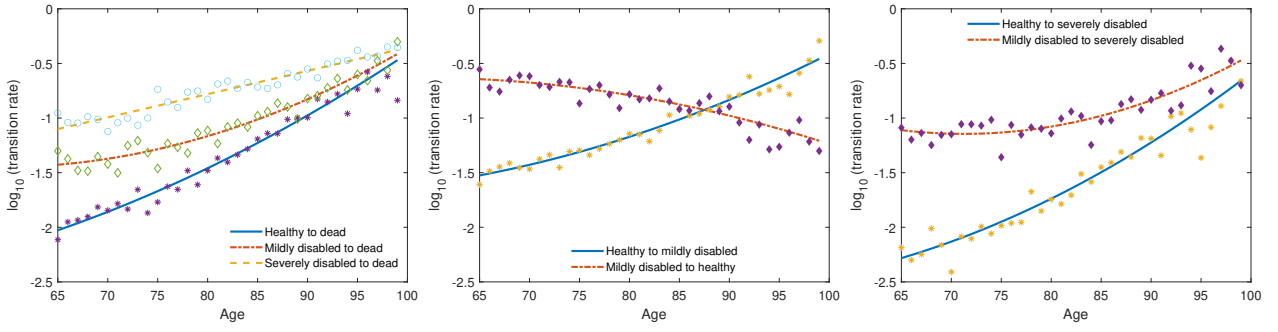


Figure 2. Crude and estimated health transition rates. The scattered points are the crude rates and the curves show the estimated rates.

follow Yogo (2016) to set the risk free rate at 2.5%. As a result, the actuarially fair price of life annuities for a healthy 65-year-old individual is \$14.89 per \$1 of annual income, and that of the LTCI is \$94,752.31 for the full coverage.

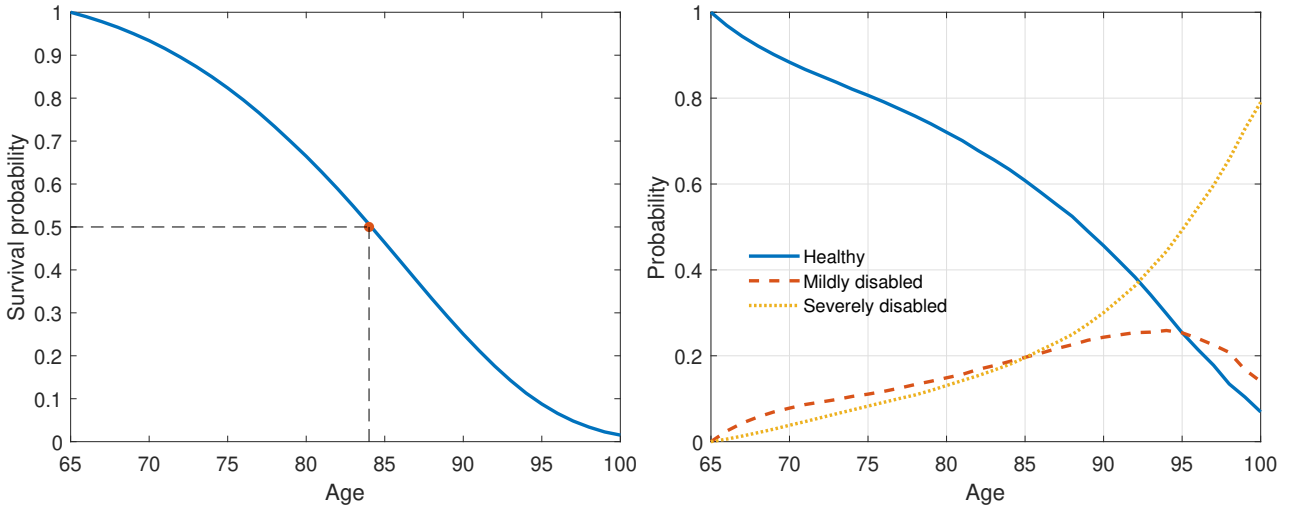


Figure 3. (Left panel) Survival curve and (right panel) probability of being in each health state conditional on being alive for a 65-year-old healthy female.

### 2.7.2 Preference parameters

The preference parameters used in the numerical simulation take the commonly used values in the literature. Their baseline values are displayed in Table 1 along with other parameter values. The sources of the parameters, unless otherwise specified, are listed in the brackets. To study the impact of bequest motives, we consider two cases: no bequest motives ( $b = 0$ ) and a certain bequest motive ( $b = 2$ ). We will separately change the value of  $\gamma$  and  $\psi$  to examine the impact of risk aversion and the EIS. The alternative values of  $\gamma$  are 2 and 8, and the alternative values of  $\psi$  are 0.2 and 0.7.

Table 1. The parameter values used for the base case.

Parameter	Explanation	Value
Preference (Pang and Warshawsky, 2010)		
$b$	Strength of bequest motive	0 and 2
$\beta$	Subjective discount factor	0.96
$\gamma$	Coefficient of relative risk aversion	5
$\psi$	Elasticity of intertemporal substitution (EIS)	0.5
Asset returns (Yogo, 2016)		
$R_f$	Risk free rate	1.025
$\mu_H$	Parameters of the lognormal distribution	0.34%
$\sigma_H^2$	of house price growth	3.5%
Consumption floor (Ameriks et al., 2011)		
$C^f$	Floor for healthy and mildly disabled states	\$4,630
	Floor for severely disabled states	\$5,640
Health expenditure (Ameriks et al., 2011)		
$h(s_1, 1)$	Initial cost for healthy state	\$1,000
$h(s_2, 1)$	Initial cost for mildly disabled state	\$10,000
$h(s_3, 1)$	Initial cost for severely disabled state	\$50,000
$q^\dagger$	Health expenditure inflation in excess of CPI inflation	1.90%

<sup>†</sup> *Source:* Yogo (2016).

### 3 Variations in demand for life annuity and LTCI

The demand for annuities can be affected by the availability of the LTCI (see e.g., Ameriks et al., 2008), and the LTCI payment can be substituted by annuity income (see e.g., Koijen et al., 2016). To isolate the possible interaction between the two products, we consider the following three scenarios: 1) annuities alone are offered (Section 3.1), 2) the LTCI alone is offered (Section 3.2), and 3) both annuities and the LTCI are offered in the market (Section 3.3). Each subsection starts with verification of prior results in the literature before discussing the impact of housing wealth and preferences on product demand.

#### 3.1 Annuities

Our model verifies some well-established results in the literature of optimal annuitization that abstracts from home equity. First and foremost, it is long recognized in the literature that full annuitization is optimal for those who have no bequest motives and face no uncertainty other than their future lifetime (Yaari, 1965; Davidoff et al., 2005). Full annuitization remains optimal in the presence of uncertain healthcare expenditures, provided that they occur later in life (Davidoff et al., 2005; Peijnenburg et al., 2016), while the presence of bequest motives

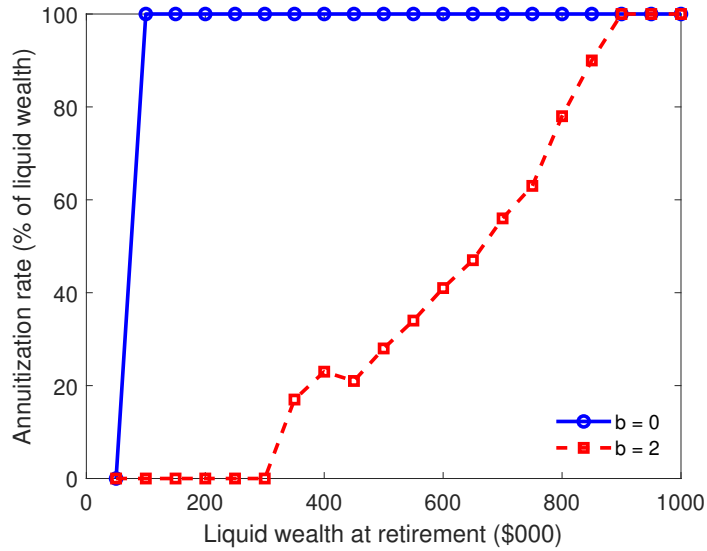


Figure 4. Optimal annuitization rates for retirees endowed with liquid wealth and no housing wealth at retirement. The legend represents the strength of bequest motives. The other preference parameters are  $\gamma = 5, \psi = 0.5$ . The LTCI is not offered in the market.

reduces the annuity demand (Lockwood, 2012). Figure 4 shows that our model reproduces the same set of results. The only exception is for those in the lowest wealth band who purchase no life annuities since they rely heavily on government transfers. In addition, Figure 4 shows that higher wealth can increase the optimal annuitization rate, a result also found in Ai et al. (2017).

That individuals save from annuity income explains the optimality of full annuitization in the presence of uncertain healthcare costs (Peijnenburg et al., 2016). We verify this result by simulating the optimal consumption of a fully annuitized retiree endowed with \$600K liquid wealth and no housing wealth.<sup>2</sup> Figure 5 shows some summary statistics of the simulated consumption paths. The mean and almost all of the quantiles are consistently below the annuity income until late in life, indicating that the annuitants save from annuity income to build up precautionary savings.

### 3.1.1 Housing wealth enhances annuity demand

Having replicated the well-known results in the literature, we extend our model to include housing wealth endowment. Without bequest motives, the optimal annuitization rates are again 100% except for the very poor, so we henceforth focus on the case with bequest motives

<sup>2</sup>The amount of \$600K is chosen for illustrative purposes, and the same result can be found using larger or smaller amount so long as the full annuitization is optimal. We use the total wealth endowment of \$600K in later numerical illustrations as well. The results can be extended to other wealth levels.



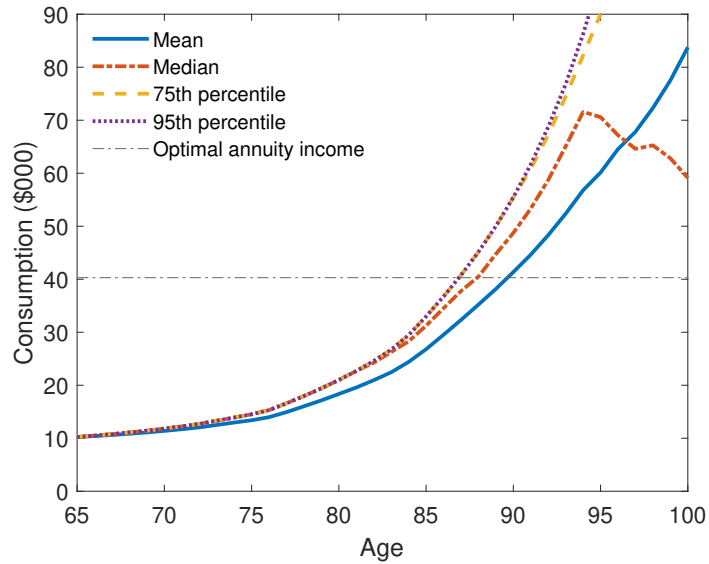


Figure 5. Simulated optimal consumption for retirees endowed \$600K liquid wealth and no housing wealth. The preference parameters are  $b = 0, \gamma = 5, \psi = 0.5$ . The optimal annuitization rate is 100%. The LTCI is not offered in the market.

(i.e.,  $b = 2$ ). Due to the illiquidity of housing wealth, retirees can only annuitize their liquid assets if they are unable to access equity release products (e.g., reverse mortgage). To assess the impact of housing wealth on annuity demand, we investigate how the amount of annuitized wealth as a proportion of liquid wealth varies with housing wealth at retirement. Figure 6 shows that as the ratio between housing and liquid wealth grows, partial annuitization starts at a lower wealth level, and the minimum wealth required for full annuitization is also reduced. Therefore, the presence of housing wealth enhances the annuity demand.

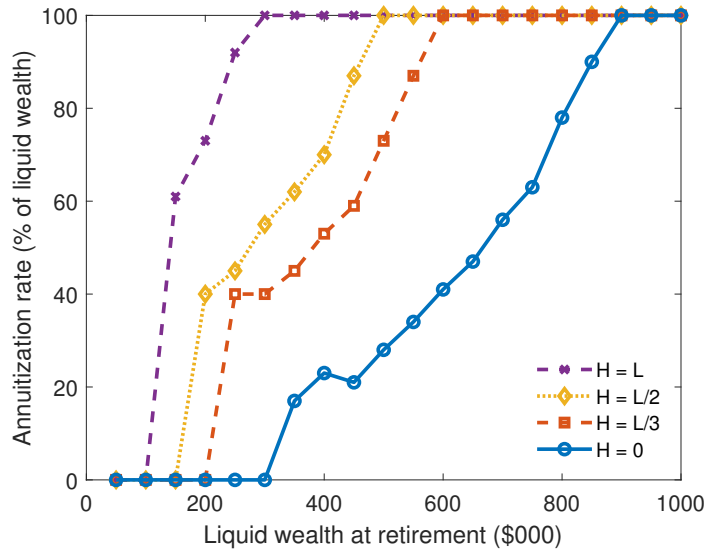


Figure 6. Optimal annuitization rates for retirees endowed with liquid wealth and housing wealth. The legend represents the ratio between housing wealth (H) and liquid wealth (L) at retirement. The preference parameters are  $b = 2, \gamma = 5, \psi = 0.5$ . The LTCI is not offered in the market.

We find that the presence of housing wealth lowers the precautionary savings from liquid wealth, thereby allowing retirees to annuitize a greater proportion of their liquid wealth. To examine the interaction between precautionary savings and housing wealth, we simulate the optimal liquid wealth paths assuming one does not purchase life annuities or the LTCI. Figure 7 plots the average paths in the healthy and mildly disabled states, in which retirees hold precautionary savings. When the retirement endowment has no housing component, the average liquid wealth increases slightly before declining. With a higher proportion of net worth in housing wealth, the curve first flattens and then becomes steeper. This suggests that as housing wealth increases, retirees draw down their liquid wealth at a faster pace and employ less liquid wealth relative to total wealth as precautionary savings.

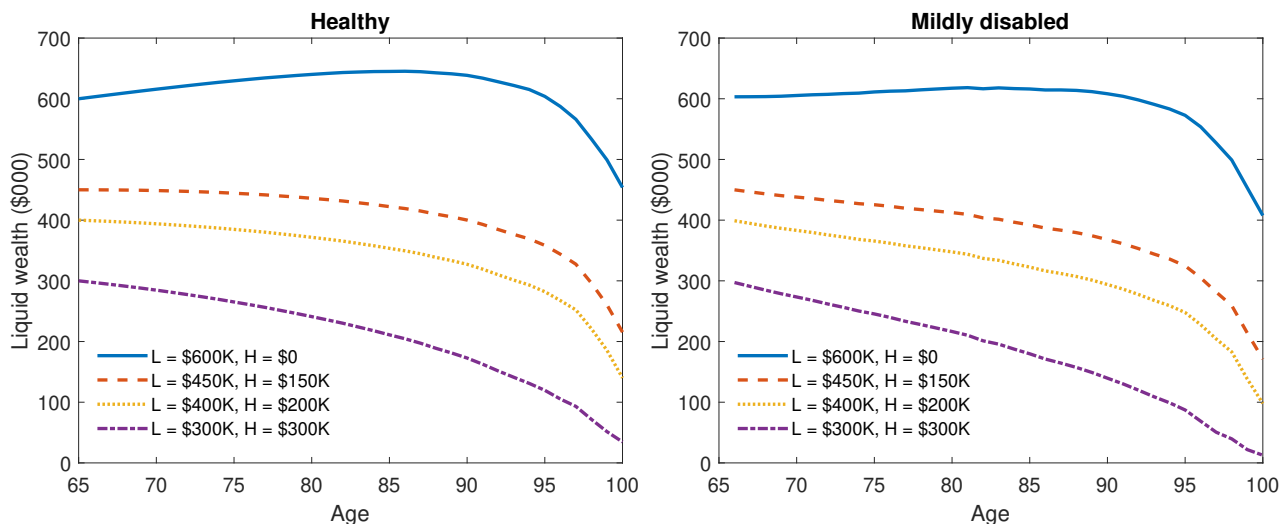


Figure 7. Simulated average optimal liquid wealth paths in (left panel) healthy and (right panel) mildly disabled states. The legend represents the amount of liquid wealth (L) and housing wealth (H) at retirement. The preference parameters are  $b = 2, \gamma = 5, \psi = 0.5$ . Retirees do not purchase life annuities or the LTCI.

While housing wealth can increase the annuity demand, its presence imposes a liquidity constraint that reduces the proportion of total wealth that can be annuitized. To investigate the net effect, we plot the optimal annuitization rate as a percentage of total wealth in Figure 8. We see that the annuitization rates are capped at the proportion of liquid wealth. Before such constraint becomes binding, the enhancement effect outweighs liquidity constraint and housing wealth increases annuity demand that is measured by the fraction of total wealth.

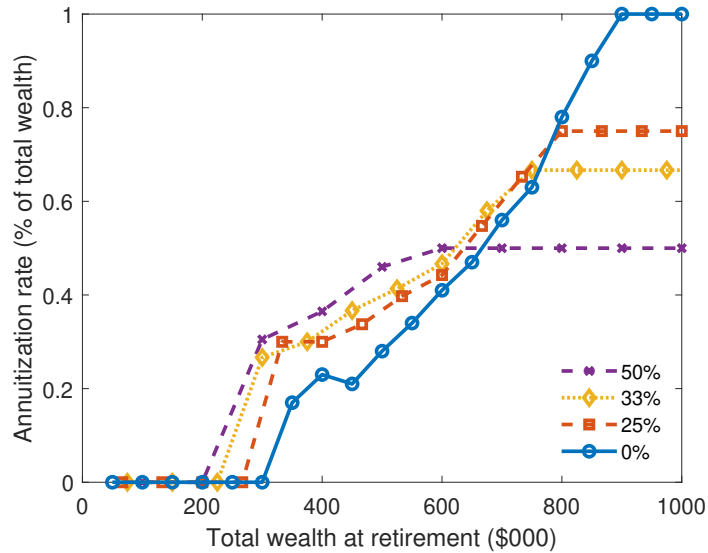


Figure 8. Optimal annuitization rates (as a percentage of total wealth) for retirees with different levels of total wealth. The legend represents the proportion of total wealth in home equity at retirement. The preference parameters are  $b = 2, \gamma = 5, \psi = 0.5$ . The LTCI is not offered in the market.

### 3.1.2 Risk aversion and EIS both affect annuity demand

In addition to housing wealth, we find that both risk aversion and the EIS affect the annuity demand. Figure 9 shows that a higher degree of risk aversion generally reduces the optimal annuitization rate. Individuals with stronger risk aversion are more averse to substituting consumption across different health states, so they set aside more liquid wealth to smooth health shocks. This in turn reduces the optimal annuitization level. Figure 9 also shows that the differences shrink with a higher level of liquid wealth or housing wealth as both factors enhance the annuity demand.

Our finding is in contrast to those in Inkmann et al. (2010) and Pashchenko (2013). Both find that more risk-averse retirees should purchase more annuities. Inkmann et al. (2010) consider a different setting where one can invest in the stock market and has no healthcare costs. More risk-averse individuals invest less in equities and subsequently purchase more annuities. In fact, after removing the component of healthcare costs, we also find that the demand for annuities increases with risk aversion (left panel of Figure 10). Pashchenko (2013) employs a power utility function where a higher degree of risk aversion is tied to a lower degree of the EIS. Our finding does not contradict hers to the extent that we find the demand for annuity generally increases with a smaller EIS as we will discuss next.

Individuals with a higher level of the EIS are known to have higher current consumption

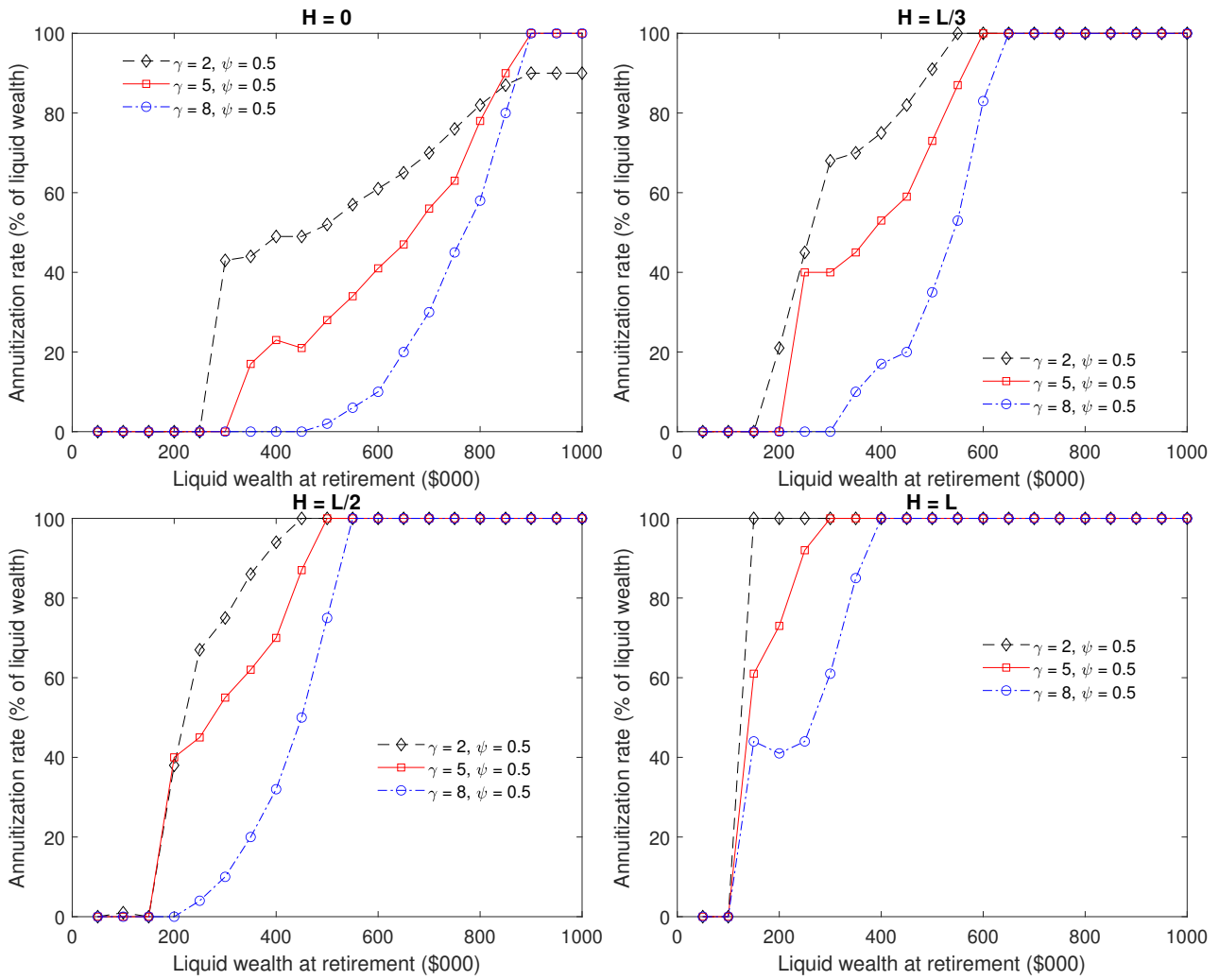


Figure 9. Optimal annuitization rates for retirees with different levels of risk aversion and a certain bequest motive ( $b = 2$ ). The title above each panel denotes the ratio of housing wealth ( $H$ ) to liquid wealth ( $L$ ) at retirement. The LTCI is not offered in the market.

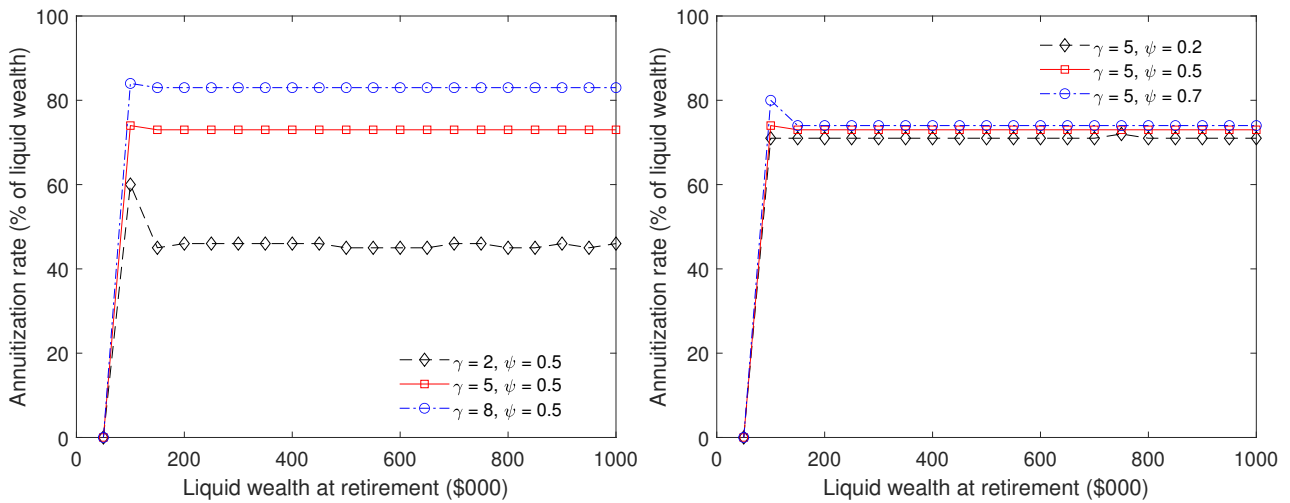


Figure 10. Optimal annuitization rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS in the absence of uncertain healthcare costs. The strength of bequest motives is given by  $b = 2$ . Retirees are endowed with liquid wealth and no housing wealth.

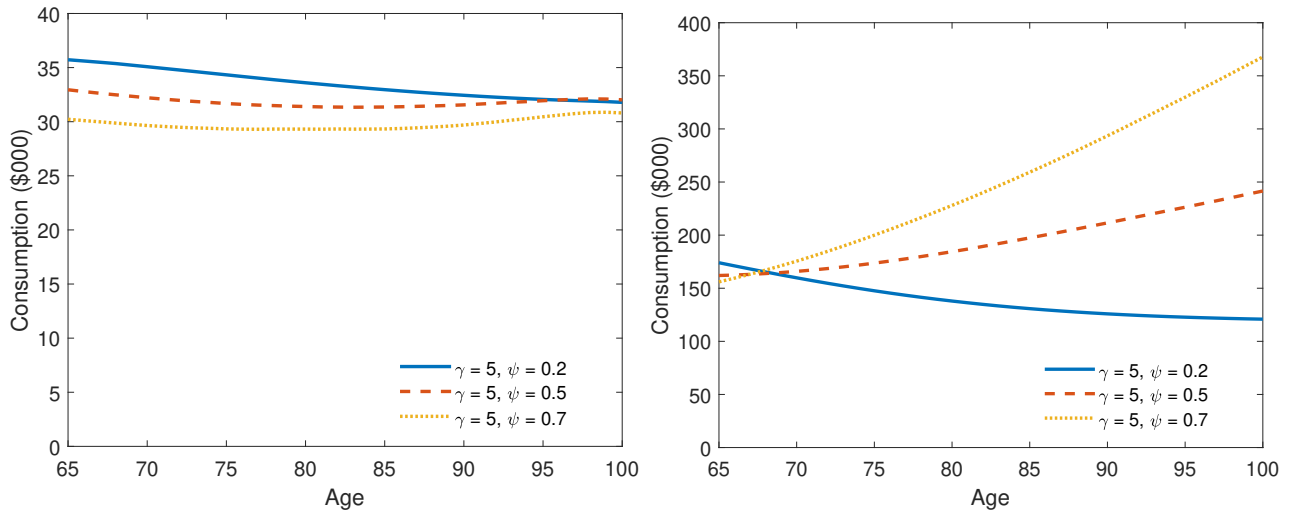


Figure 11. Simulated average (left panel) optimal consumption paths and (right panel) optimal liquid wealth paths for retirees with different levels of the EIS and a certain bequest motive ( $b = 2$ ). Retirees are endowed with \$600K liquid wealth and no housing wealth at retirement. They purchase the optimal amount of annuities at retirement and have no access to the LTCI.

and lower savings if the time-preference-adjusted return on savings is negative (Campbell and Viceira, 1999). We replicate this result using a simplified version of our model that assumes a certain finite lifespan and no healthcare costs. Furthermore, we find that a higher level of the EIS is associated with a larger amount of bequests based on the same set of assumptions. The detailed results are presented in Appendix C.

After incorporating mortality risk back to the model while still abstracting from the uncertain healthcare expenditure, we find that the optimal annuitization rates are similar among retirees with different levels of the EIS (right panel of Figure 10). However, we find noticeable differences in the optimal consumption paths. The left panel of Figure 11 shows that individuals with a higher degree of the EIS tend to have less current consumption and a flatter consumption path. Consequently, they tend to leave larger bequests (right panel of Figure 11), consistent with our prior finding in the case of no mortality risk or healthcare costs.

When facing health shocks, retirees will normally choose to hold precautionary savings if the LTCI is not offered in the market. They can either annuitize less to set aside more liquid wealth upfront, or save from annuity income to build up the buffer. Since our health transition model predicts that the risk of requiring long-term care increases significantly after age 85 (Figure 3), retirees have time to accumulate liquid wealth by spending less than the annuity income during early retirement. For someone without bequest motives, this is a more efficient strategy since

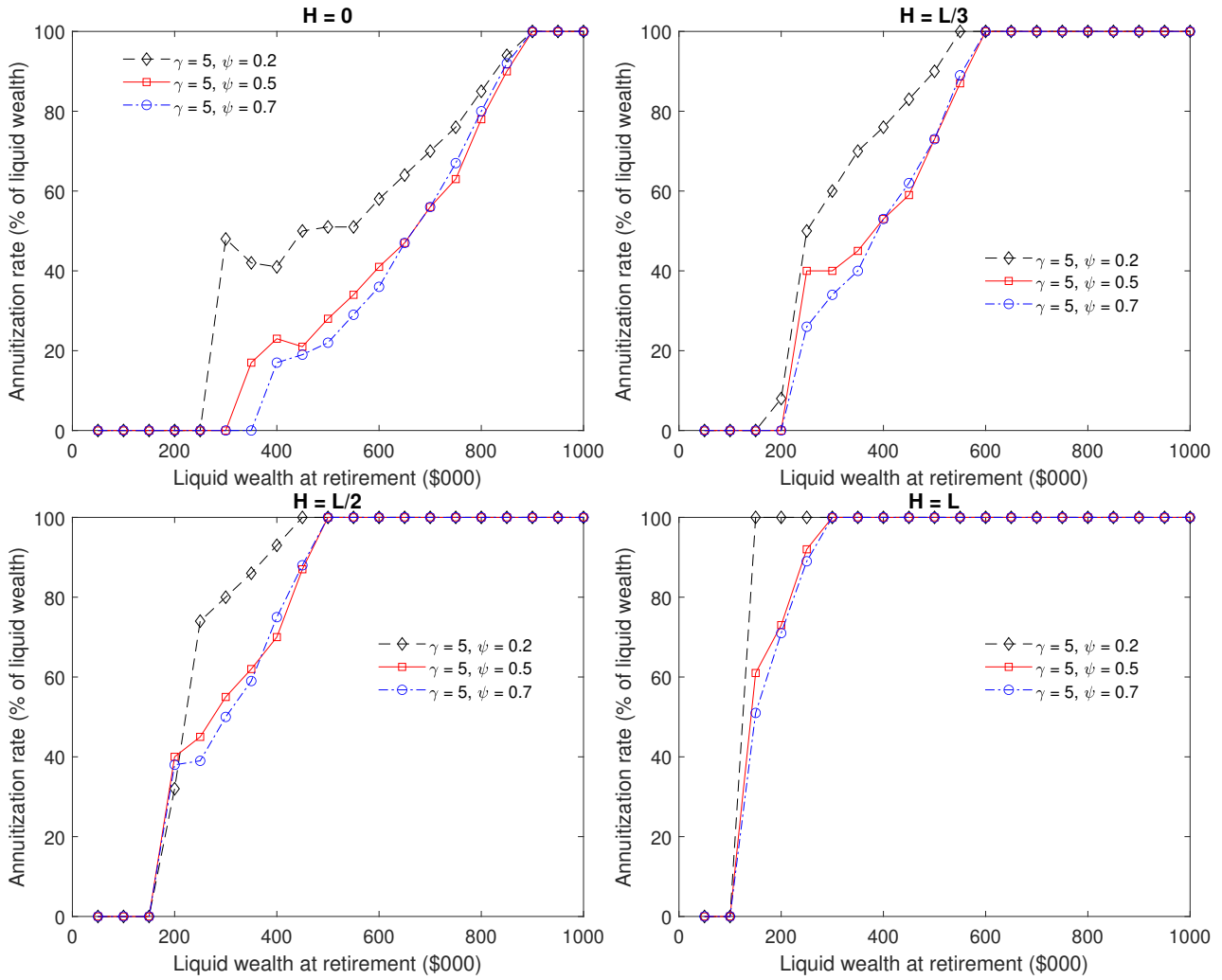


Figure 12. Optimal annuitization rates for retirees with different levels of the EIS and a certain bequest motive ( $b = 2$ ). The title above each panel denotes the ratio of housing wealth ( $H$ ) to liquid wealth ( $L$ ) at retirement. The LTCI is not offered in the market.

wealth, if left unconsumed, generates no utility. For those with bequest motives, using a mixture of upfront savings and annuity income to build a buffer becomes optimal. Their desire to leave bequests lowers the opportunity cost of using liquid wealth as precautionary savings (Lockwood, 2018). We have shown that retirees with a higher level of the EIS are likely to leave a larger amount of bequests, which implies a lower opportunity cost of holding liquid wealth. As a result, Figure 12 shows that retirees with a higher degree of the EIS tend to annuitize less of their wealth. Similar to the case in Figure 9, the variations in the optimal annuitization rate diminish with more liquid or housing wealth.

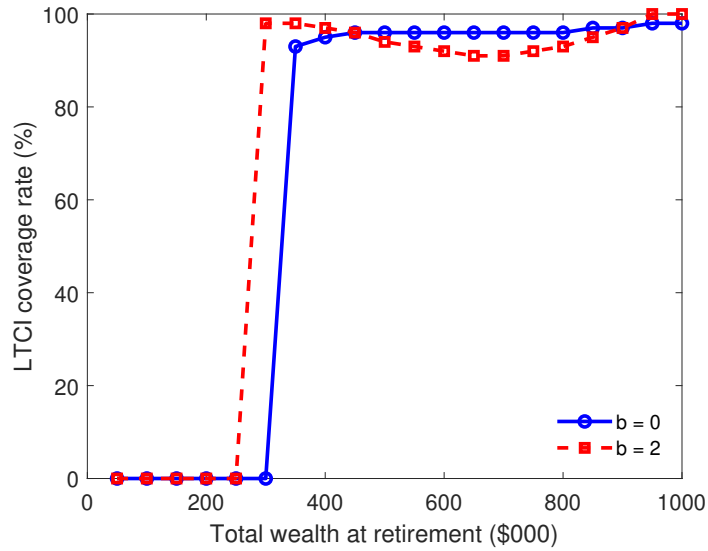


Figure 13. Optimal LTCI coverage rates for retirees endowed with liquid wealth and no housing wealth at retirement. The legend represents the strength of bequest motives. The other preference parameters are  $\gamma = 5$ ,  $\psi = 0.5$ . The life annuity is not offered in the market.

### 3.2 LTCI

The LTCI is an effective instrument in managing the sizable healthcare costs. Figure 13 shows retirees endowed with liquid wealth and no housing wealth demand nearly full LTCI coverage once their wealth levels exceeded a certain threshold. Those who optimally choose to purchase no LTCI coverage rely on government transfers that provide some form of LTCI through the minimum consumption guarantee. The impact of bequest motives is marginal, which is not surprising given the two offsetting effects of bequest motives. On the one hand, the desire to leave bequests can increase the demand for the LTCI since the insurance coverage will add to the bequests left by those who died after becoming severely disabled (Pauly, 1990). On the other hand, bequest motives can lower the opportunity cost of precautionary savings, thereby reducing the demand for the LTCI (Lockwood, 2018).

The jump in the optimal LTCI coverage rate shown in Figure 13 is not unusual. Shao et al. (2019) also find strong demand for the LTCI at different wealth levels, although they set the lowest wealth level at \$240K, below which retirees are likely to have a minimal demand due to government transfers. The jump can be explained by the non-linear effect of the LTCI on consumption. Figure 14 shows the average optimal consumption paths in the severely disabled state under different LTCI coverage rates. Regardless of bequest motives, the increment in the optimal consumption grows considerably when the LTCI coverage increases in equal steps

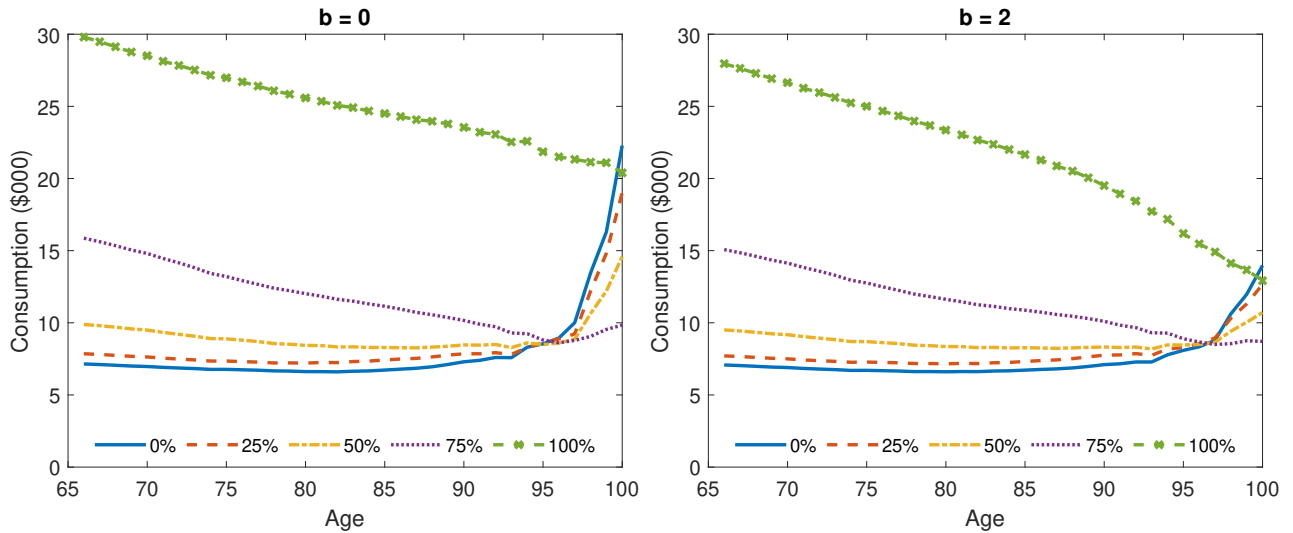


Figure 14. Simulated average optimal consumption paths in the severely disabled state. The legend represents different LTCI coverage rates. Retirees are endowed with \$600K liquid wealth and no housing wealth at retirement. The preference parameters are  $\gamma = 5$ ,  $\psi = 0.5$ . The life annuity is not offered in the market.

from zero to 100%. By contrast, the average consumption in the healthy and mildly disabled states changes more or less evenly with the LTCI coverage rate. The figures are displayed in Appendix D.1. The non-linear effect on consumption implies that, if one does not completely rely on government transfers and purchases some LTCI coverage, the marginal benefit of an extra coverage rate can easily exceed its marginal cost when the coverage is not high. Therefore, the optimal coverage rate for non-homeowners is either at the high end or the low end.

### 3.2.1 Housing wealth interacts with bequest motives

Figure 15 shows that more housing wealth in proportion to total wealth generally lowers the optimal LTCI coverage rate regardless of the desire to leave bequests. This is due to the substitution effect that comes from the overlap between the LTCI payment and housing wealth liquidation. Similar result is also found in Davidoff (2010) and Shao et al. (2019). The comparison between the two panels in Figure 15 shows that the gaps between the curves in the left panel are larger than those in the right panel, suggesting that bequest motives lessen the impact of housing wealth on the optimal LTCI coverage rate. This implies that, between the two offsetting effects of bequest motives, the enhancement effect dominates in the presence of home equity.

The right panel of Figure 15 shows homeowners endowed with less than \$300K total wealth



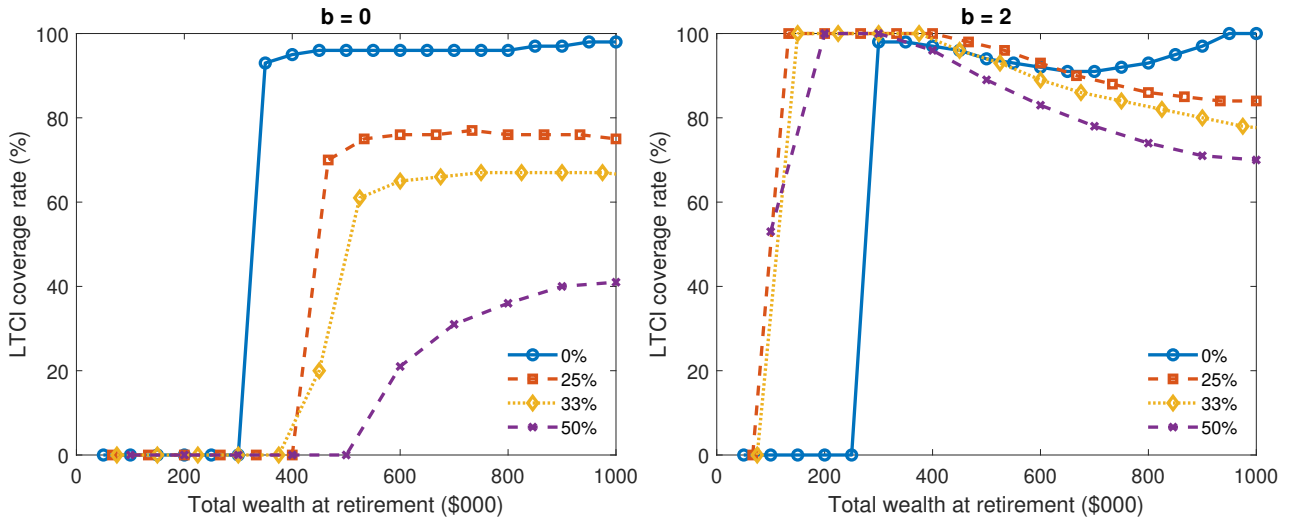


Figure 15. Optimal LTCI coverage rates for retirees endowed with liquid wealth and housing wealth. The legend represents the proportion of total wealth in home equity at retirement. The preference parameters are  $\gamma = 5$ ,  $\psi = 0.5$ . The life annuity is not offered in the market.

demand far more LTCI coverage than non-homeowners endowed with the same amount of total wealth. This is because purchasing the LTCI helps preserve bequests for homeowners more than non-homeowners in the low wealth bands, while the reduction in consumption due to LTCI purchase is limited due to the minimum consumption guarantee. Figure 16 compares the average amount of bequests under no LTCI coverage and full coverage.<sup>3</sup> The left panel shows that the two curves are almost parallel before the solid line falls to zero, suggesting that the LTCI has a limited effect in slowing the wealth drawdown for non-homeowners. By contrast, the middle and right panels of Figure 16 show that increasing LTCI coverage flattens the curve for homeowners. The average amount of bequests under the full coverage almost levels off after age 85. Figure 17 shows the extent of consumption reduction caused by purchasing the LTCI. The difference in annual consumption between no LTCI coverage and full coverage is, on average, around \$1,000 for the first 20 years into retirement. Afterward, the gap closes due to the increased risk of requiring long-term care that incurs substantial costs and triggers the LTCI payment. For homeowners, the right two panels of Figure 17 show that the average consumption with the full LTCI coverage eventually overtakes that of no LTCI coverage.

<sup>3</sup>We select the total wealth endowment amount of \$200K for illustrative purposes. The results can be extended to other wealth levels below \$300K.

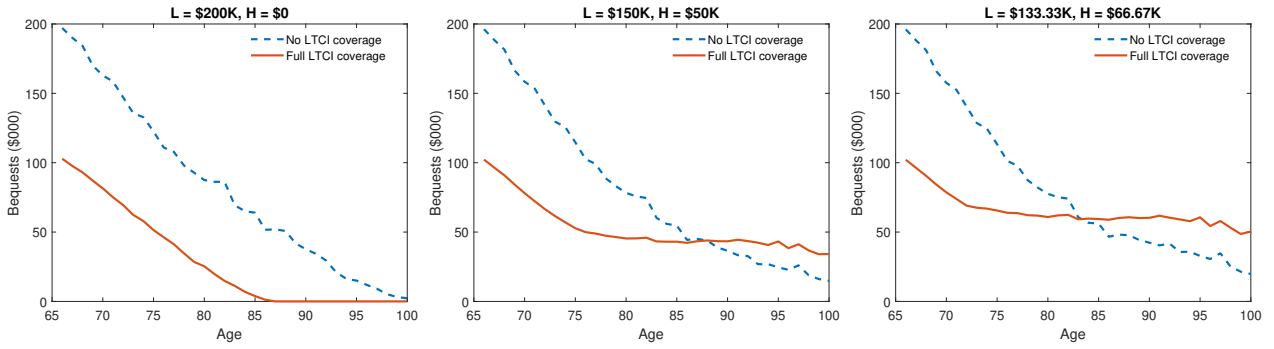


Figure 16. Simulated average bequests for retirees who purchase no LTCI coverage or full LTCI coverage. The title of each panel represents the amount of liquid wealth (L) and housing wealth (H) at retirement. The preference parameters are  $b = 2, \gamma = 5, \psi = 0.5$ . The life annuity is not offered in the market.

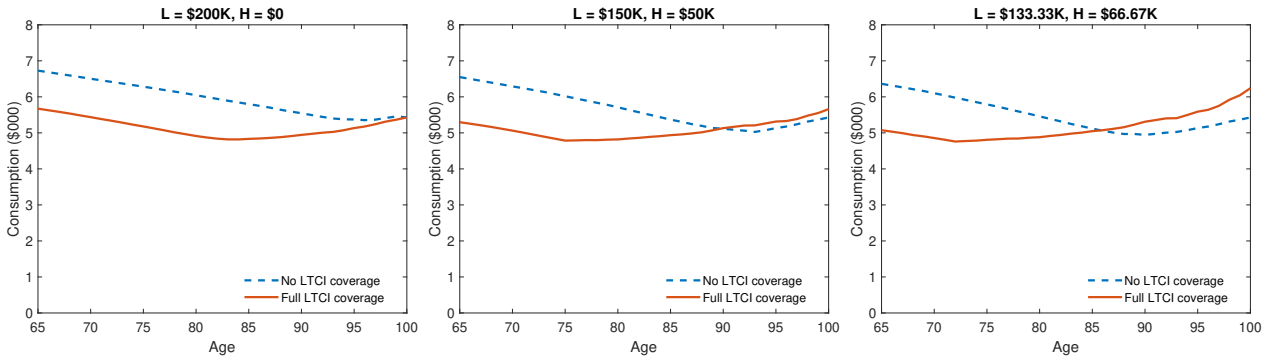


Figure 17. Simulated average optimal consumption paths for retirees who purchase no LTCI coverage or full LTCI coverage. The title of each panel represents the amount of liquid wealth (L) and housing wealth (H) at retirement. The preference parameters are  $b = 2, \gamma = 5, \psi = 0.5$ . The life annuity is not offered in the market.

### 3.2.2 Risk aversion more important than the EIS in affecting the LTCI demand

We find that the EIS has a minimal impact on the optimal LTCI coverage rate for retirees with and without bequest motives alike (Figure 18). The same result holds for homeowners with different levels of housing wealth (see Appendix D.1 for more details). That the EIS has little effect on the demand for the LTCI is intuitive. Unlike life annuities which provide a constant stream of income throughout one's lifetime, the LTCI provides income only when one is severely disabled, limiting its ability to smooth consumption over time. We previously argued that a higher level of the EIS strengthens the role of bequest motives in lowering the opportunity cost of liquid wealth buffers, which can reduce the demand for the LTCI. The effect is offset by the enhancement made to the bequests by a higher LTCI coverage rate.

Figure 19 shows how the demand for the LTCI varies with risk aversion in the absence of housing wealth. Although it appears that a higher risk aversion leads to a lower optimal LTCI

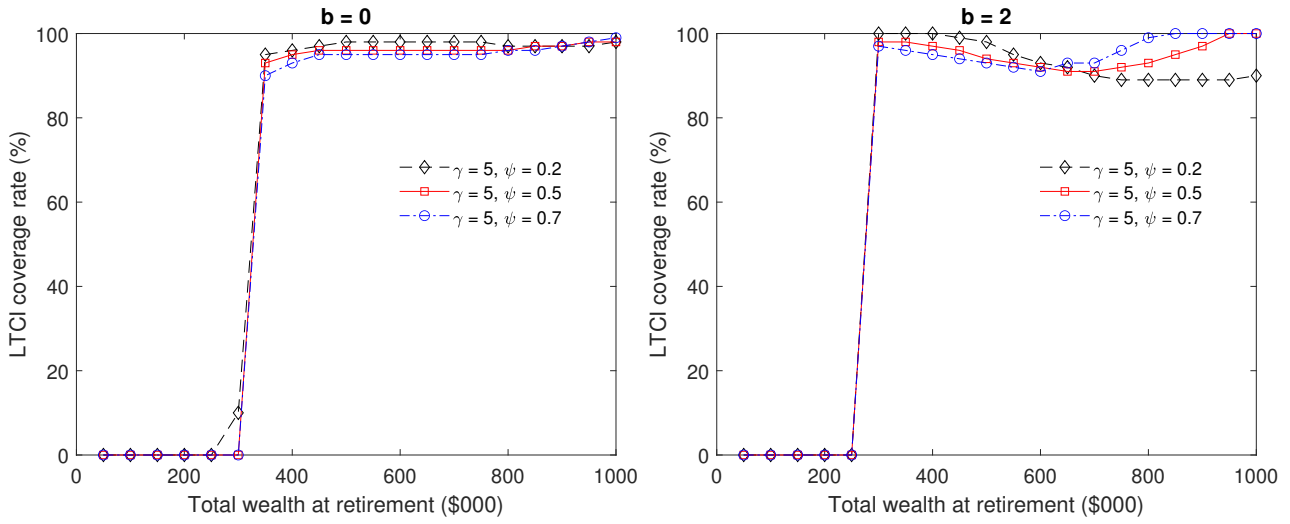


Figure 18. Optimal LTCI coverage rates for retirees with different levels of the EIS. Retirees have no housing wealth. The life annuity is not offered in the market.

coverage rate, it is not necessarily the case for homeowners, which will be discussed later. In addition, the optimal coverage rates (conditional on purchasing the LTCI) in Figure 19 are all close to 100%. Figure 20 shows that the relative difference between the optimal and the full coverage rate, in terms of the objective function, is well below 5%, suggesting that the utility lost from purchasing the full LTCI coverage is minimal.

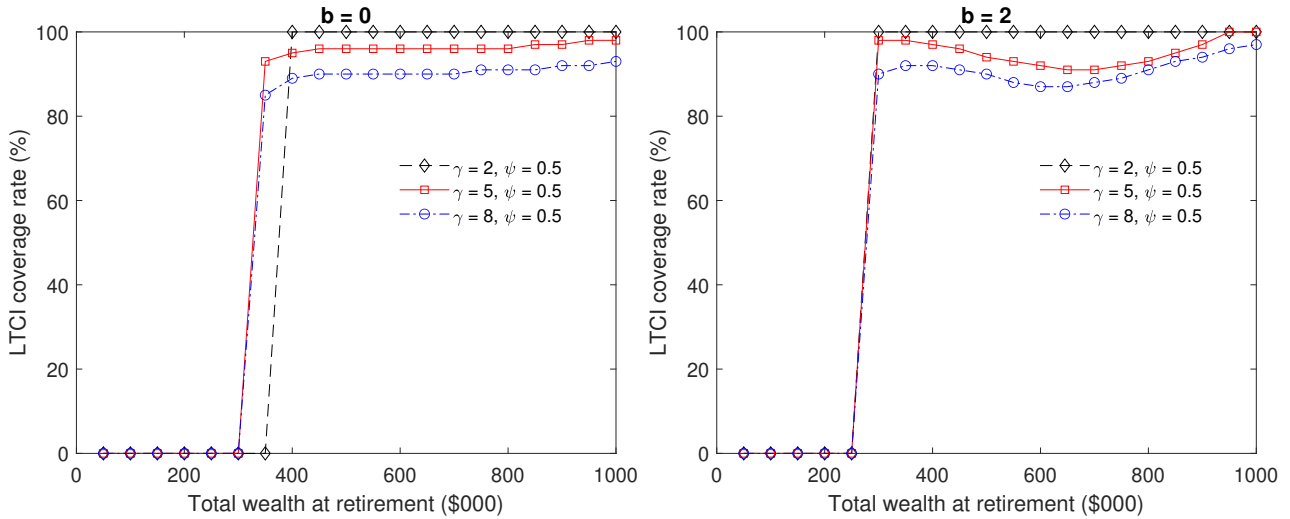


Figure 19. Optimal LTCI coverage rates for retirees with different levels of risk aversion. Retirees have no housing wealth. The life annuity is not offered in the market.

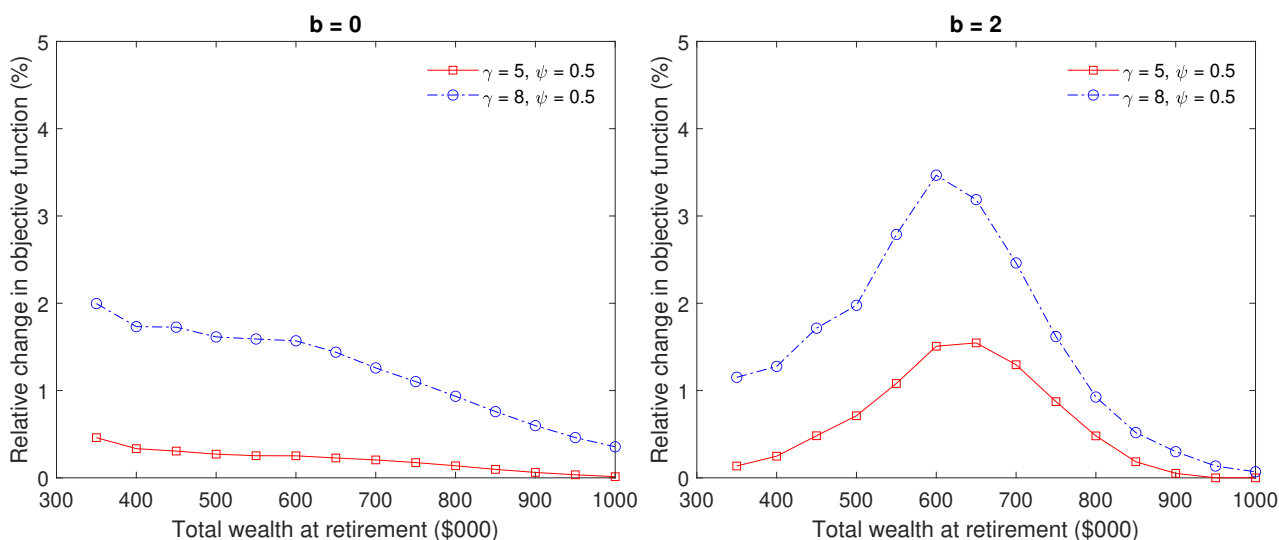


Figure 20. Relative difference in the value of objective function between the full LTCI coverage and the optimal LTCI coverage. Retirees have no housing wealth. The life annuity is not offered in the market.

To further explain the result in Figure 19 that a higher risk aversion drives down the LTCI demand, we plot the simulated average optimal consumption paths in each health state along with the overall average (Figure 21). Deviating away from the optimal LTCI coverage to purchase the full amount widens the gap in consumption between the severely disabled state and other health states. Since more risk-averse individuals prefer a smoother consumption between different health states, retirees with a relatively high level of risk aversion optimally choose to avoid the full LTCI coverage.

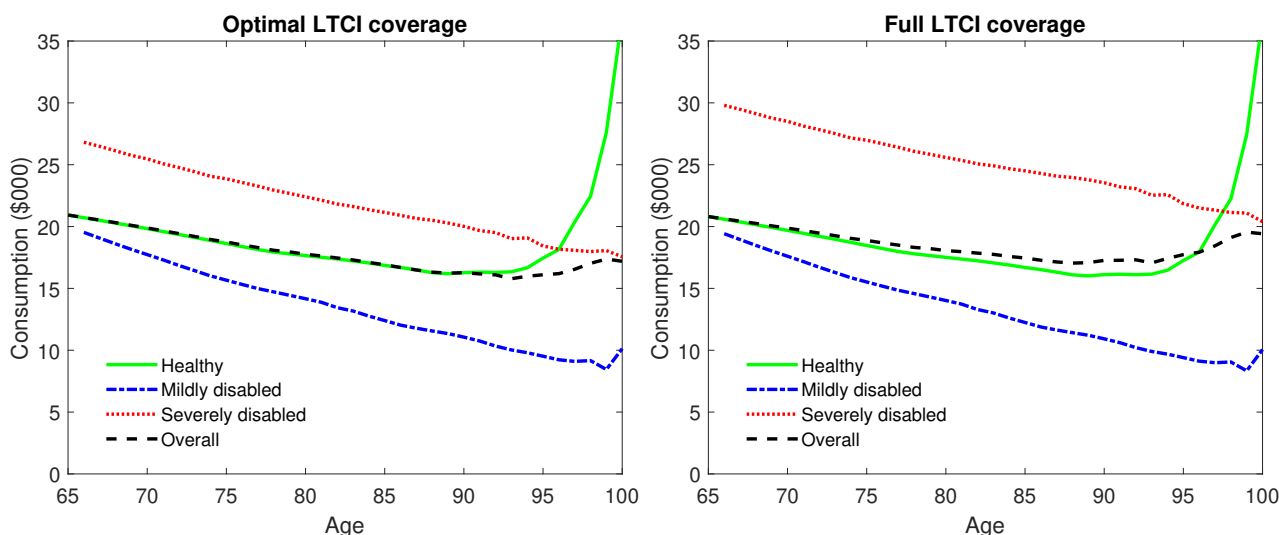


Figure 21. Simulated average optimal consumption paths in each health state and the overall average. Retirees are endowed with \$600K liquid wealth and no housing wealth at retirement. The preference parameters are  $b = 0, \gamma = 5, \psi = 0.5$ . The life annuity is not offered in the market.

Furthermore, we find that risk aversion interacts with housing wealth in affecting the LTCI

demand. Figure 22 and Figure 23 compare the impact of risk aversion on the optimal LTCI coverage among retirees endowed with various levels of housing wealth. As housing wealth grows, the lower the level of risk aversion, the greater the reduction in the optimal LTCI coverage rate. In one case where retirees are endowed with an equal amount of liquid and housing wealth and have no bequest motives (right panel of Figure 22), the optimal LTCI coverage rate increases with risk aversion, reversing the order in Figure 19.

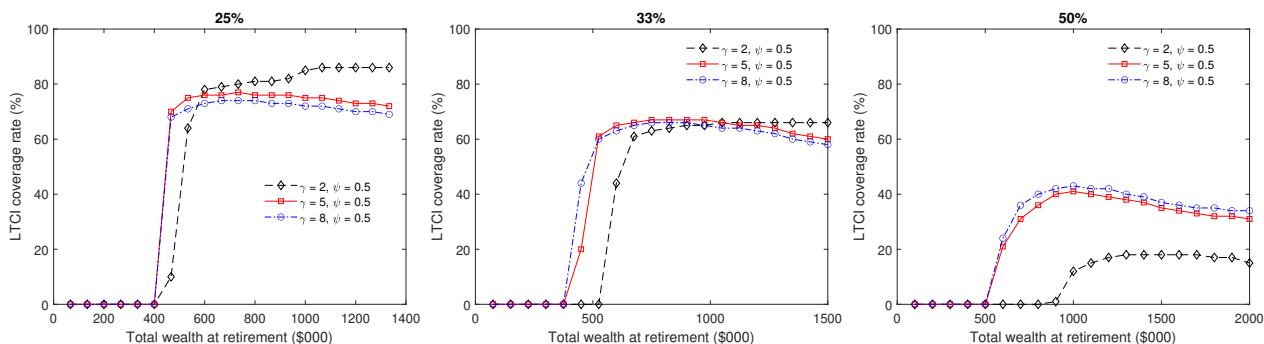


Figure 22. Optimal LTCI coverage rates for retirees with different levels of risk aversion and no bequest motives. The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is not offered in the market.

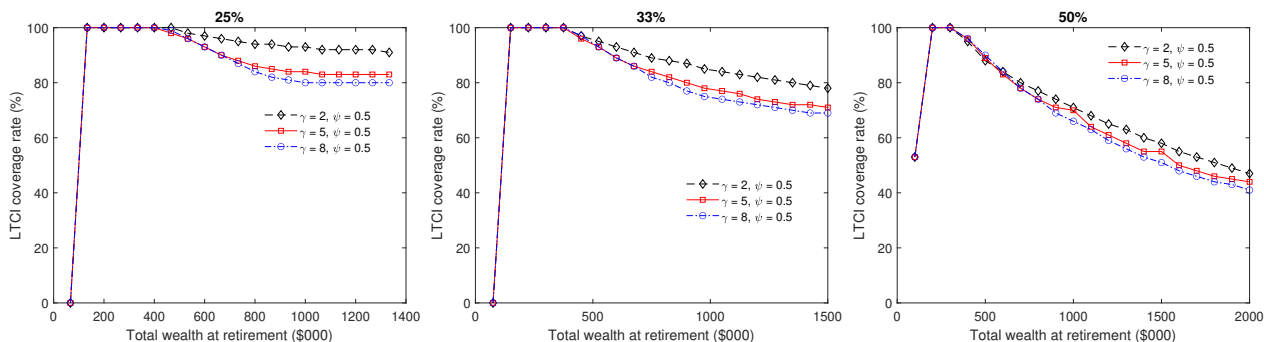


Figure 23. Optimal LTCI coverage rates for retirees with different levels of risk aversion and a certain bequest motive ( $b = 2$ ). The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is not offered in the market.

### 3.3 Both annuities and LTCI

The LTCI is known to enhance the demand for annuities in the absence of housing wealth (see e.g., Ameriks et al., 2008; Wu et al., 2016), and the complementarity between life annuities and the LTCI can be reversed by illiquid housing wealth (Davidoff, 2009). We replicate this pair of results and present the details in Appendix D.2.

When the life annuity or the LTCI alone is offered in the market, we have shown that the product demand is affected by both housing wealth and preferences. When both products are offered, we find that one's wealth level and homeownership status are more important than her risk aversion or the EIS. In addition, the bequest motives remain an important factor in determining the product demand.

#### 3.3.1 Housing wealth and liquidity

We have shown that housing wealth increases annuity demand for retirees with bequest motives when the life annuity alone is offered in the market. For retirees without bequest motives, the improvement is disguised by the optimality of full annuitization in the absence of housing wealth. When the LTCI becomes accessible, retirees without bequest motives do not always find full annuitization optimal. Among those who partially annuitize their wealth, the optimal annuitization rates show slight improvement with housing wealth (left panel of Figure 24). For retirees with bequest motives, homeownership remains an important factor in affecting the annuity demand. The right panel of Figure 24 shows that homeowners tend to have higher optimal annuitization rates than non-homeowners. There is, however, little variation among homeowners endowed with the same level of liquid wealth.

Figure 25 displays the optimal annuitization rate as a proportion of total wealth. The housing wealth almost always reduces the annuity demand when retirees can access the LTCI, in contrast to the case of no LTCI access (Figure 8). That housing wealth enhances annuity demand is unable to offset the liquidity constraint introduced by its presence. The reasons are twofold. Firstly, the presence of the LTCI reduces precautionary savings, thereby narrowing the gap between homeowners and non-homeowners in terms of their optimal annuitization rates as a proportion of liquid wealth (Figure 24). Secondly, that some liquid wealth is allocated to purchase the LTCI further lowers the amount of wealth that can be annuitized.

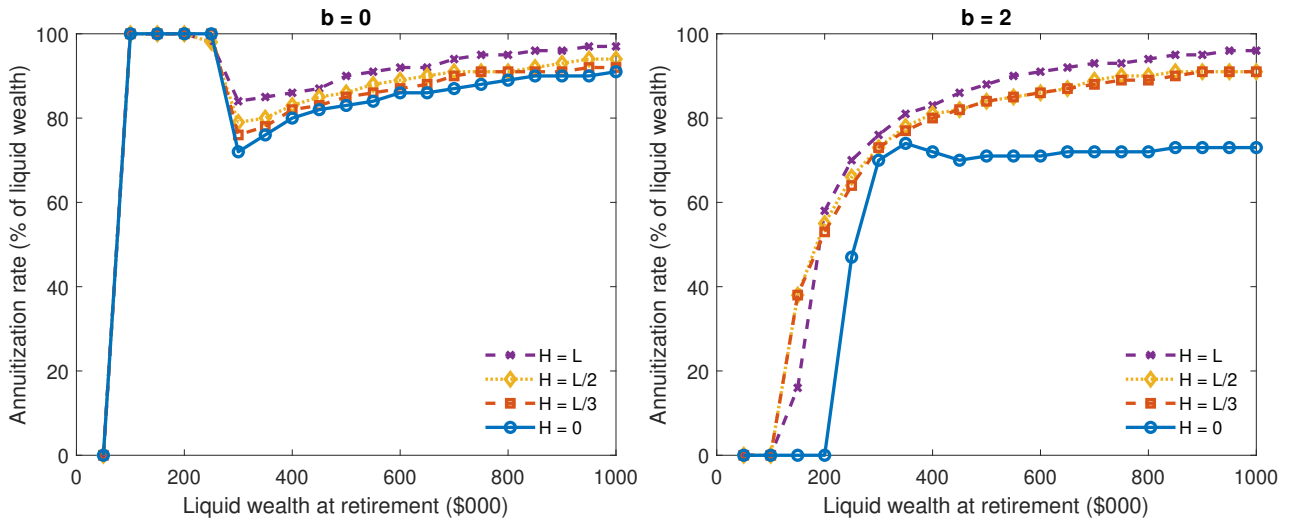


Figure 24. Optimal annuitization rates (as a percentage of liquid wealth) for retirees who have access to the LTCI. The legend represents the ratio between housing wealth (H) and liquid wealth (L) endowment at retirement. The preference parameters are  $\gamma = 5, \psi = 0.5$ .

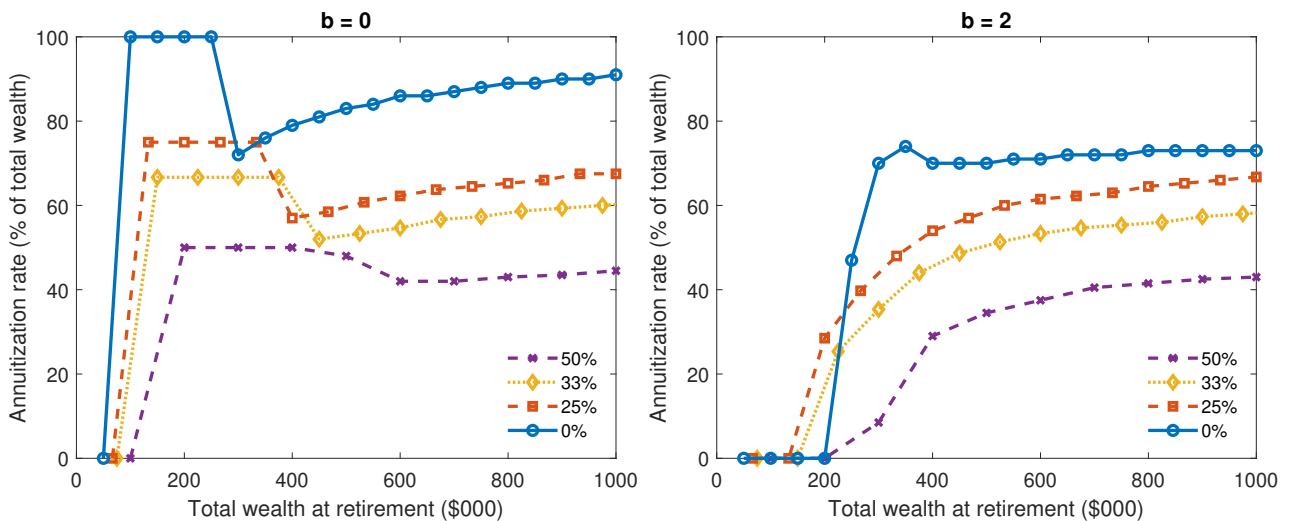


Figure 25. Optimal annuitization rates (as a percentage of total wealth) for retirees who have access to the LTCI. The legend represents the proportion of total wealth in housing at retirement. The preference parameters are  $\gamma = 5, \psi = 0.5$ . The LTCI is offered in the market.

Figure 26 compares the optimal LTCI coverage rate between homeowners and non-homeowners, and among homeowners with different levels of housing wealth. There are noticeable declines in the optimal LTCI coverage rates with higher housing wealth proportions, so the result that housing wealth typically weakens the LTCI demand remains the same regardless of the access to life annuities. The right panel of Figure 26 shows that housing wealth increases the LTCI demand for retirees in the low wealth bands, similar to the case of no access to life annuities.

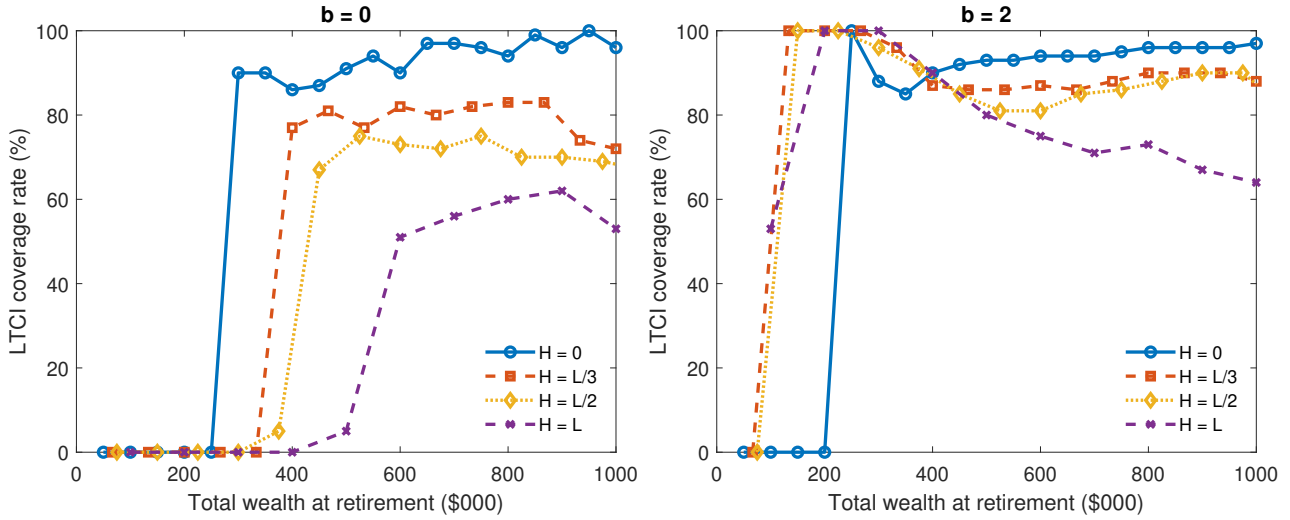


Figure 26. Optimal LTCI coverage rates for retirees who have access to life annuities. The legend represents the ratio between housing wealth ( $H$ ) and liquid wealth ( $L$ ) endowment at retirement. The preference parameters are  $\gamma = 5, \psi = 0.5$ .

Compared to Figure 15 where life annuities are not offered in the market, the curves in Figure 26 move more abruptly with total wealth. This is due to the result that retirees generally use all the liquid wealth to purchase the two products, which we will discuss later. Since a one percentage point increase in the LTCI coverage rate generally requires less liquid wealth than the same percentage point increase in annuitization rate, the capacity to purchase the LTCI is more sensitive to changes in liquid wealth. As the optimal annuitization rate increases steadily with liquid wealth, the optimal LTCI coverage rate might land in a higher or lower position compared to that of the closest wealth band depending on the budget constraint.

Figure 27 and Figure 28 show the allocation of liquid wealth endowment in the absence and presence of bequest motives, respectively. For retirees without bequest motives, both homeowners and non-homeowners usually spend all liquid wealth on life annuities and the LTCI. The proportion allocated to LTCI decreases as housing wealth grows, reflecting a weakening LTCI demand. For retirees with bequest motives, the homeownership status significantly affects the



liquid wealth allocation. Homeowners generally exhaust their liquid wealth on purchasing the two products and leave little cash on hand at the point of retirement. In contrast, the top left panel of Figure 28 shows that non-homeowners usually have some cash on hand after the product purchases. For homeowners with bequest motives, the allocation to LTCI shows an exponential decay with liquid wealth. This is mainly driven by the high LTCI coverage in the low wealth levels among homeowners (right panel of Figure 26).

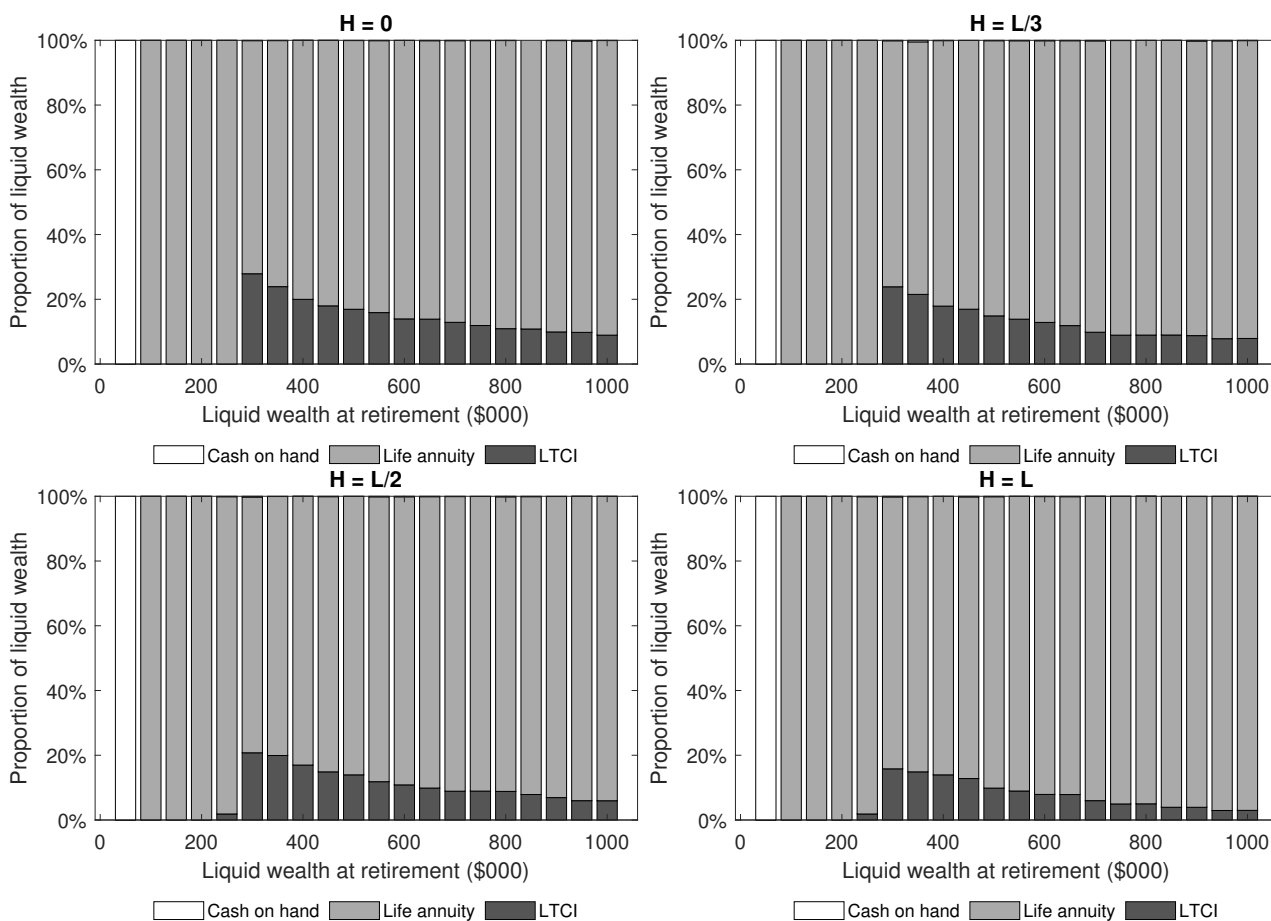


Figure 27. The optimal allocation of liquid wealth endowment in the absence of bequest motives. The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. The preference parameters are  $b = 0, \gamma = 5, \psi = 0.5$ .

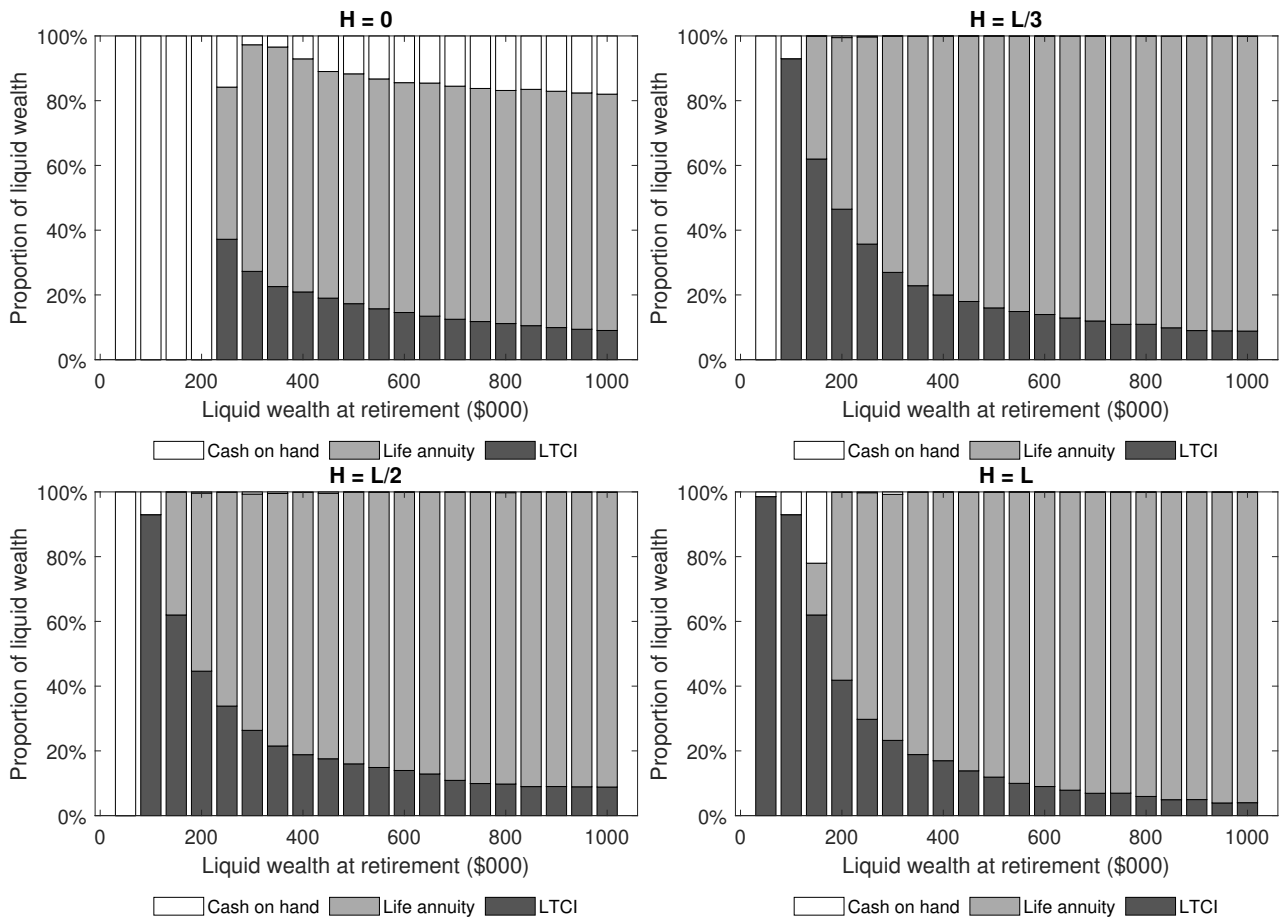


Figure 28. The optimal allocation of liquid wealth endowment in the presence of bequest motives. The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. The preference parameters are  $b = 2$ ,  $\gamma = 5$ ,  $\psi = 0.5$ .

### 3.3.2 Preference

We find that risk aversion and the EIS play a far less important role in determining product demand compared to housing wealth. Figure 29 compares the optimal annuitization rate among different levels of risk aversion and the EIS in the case of no bequest motives. Figure 30 performs the same comparison for retirees with bequest motives. In both figures, the curves almost overlap with each other. The only exception is for retirees who have a relatively low risk aversion and some bequest motives (left panel of Figure 30). They show significantly less annuity demand. The comparison of the optimal LTCI coverage rate shows a similar result. In Figure 31 and Figure 32, the optimal levels vary little with the risk aversion or the EIS. We find similar results for homeowners and that the results are robust to different levels of housing wealth. The figures are displayed in Appendix D.2.

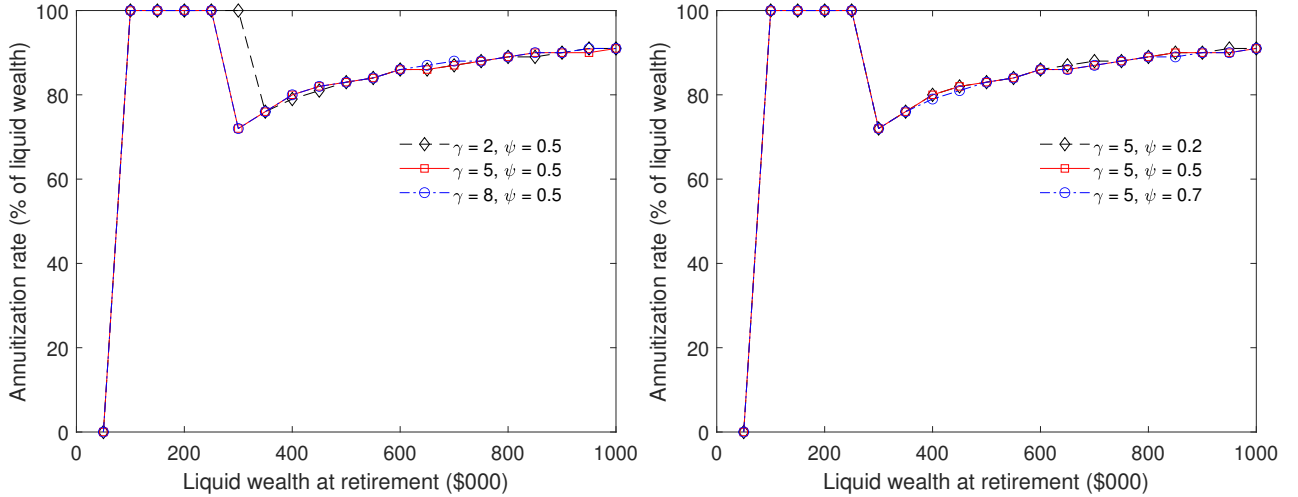


Figure 29. Optimal annuitization rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when the LTCI is offered in the market. Retirees have no housing wealth and no bequest motives.

In the single product case, we have shown the strong impact of bequest motives on annuity demand (Figure 4) and on LTCI demand for homeowners (Figure 15). When both products are available, bequest motives continue to play an important role in product decisions. Figure 24 shows that bequest motives discourage annuity purchase, especially for those in the low wealth bands or do not have housing wealth. Figure 26 shows a similar result to Figure 15 that bequest motives improve the optimal LTCI coverage rate. Moreover, Figures 27 and 28 show that bequest motives affect the liquid wealth spending, especially for non-homeowners. While retirees with no bequest motives tend to spend up their liquid wealth on the two products, non-homeowners with bequest motives leave some cash on hand.

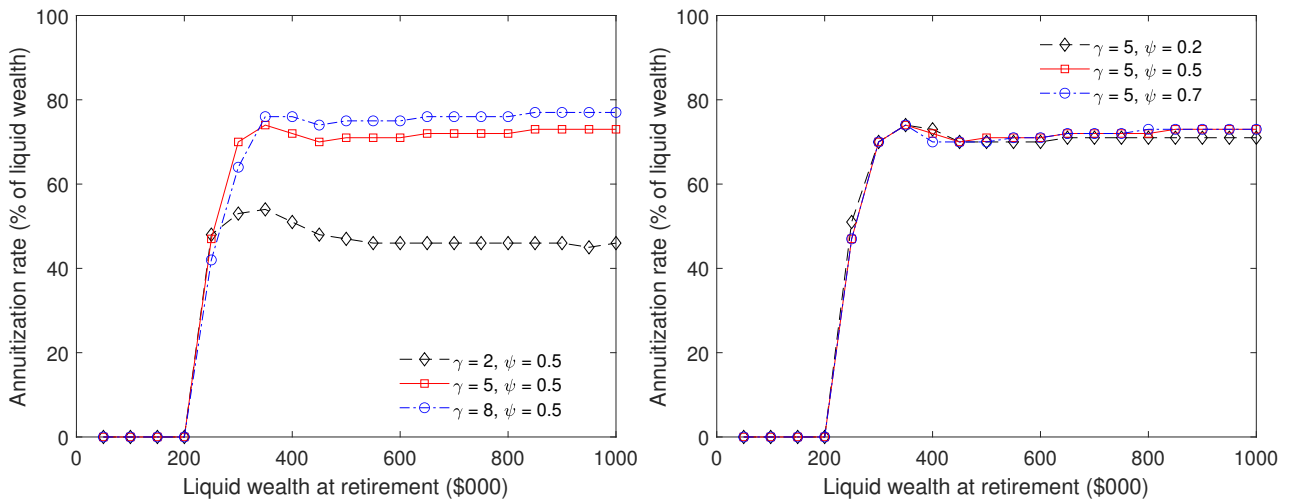


Figure 30. Optimal annuitization rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when the LTCI is offered in the market. The strength of bequest motives is given by  $b = 2$ . Retirees have no housing wealth.

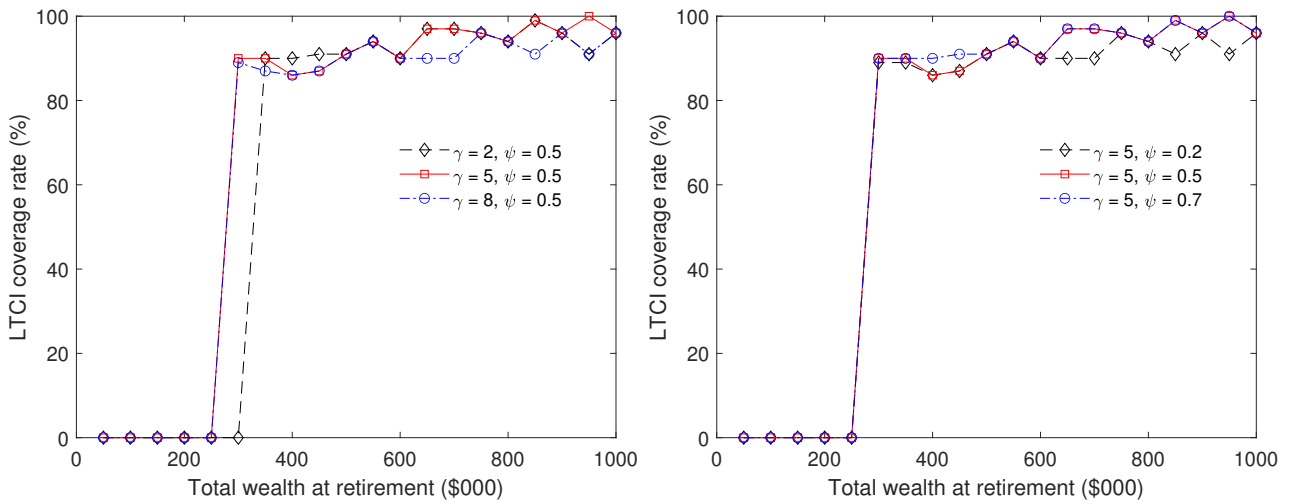


Figure 31. Optimal LTCI coverage rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when life annuities are offered in the market. Retirees have no housing wealth and no bequest motives.

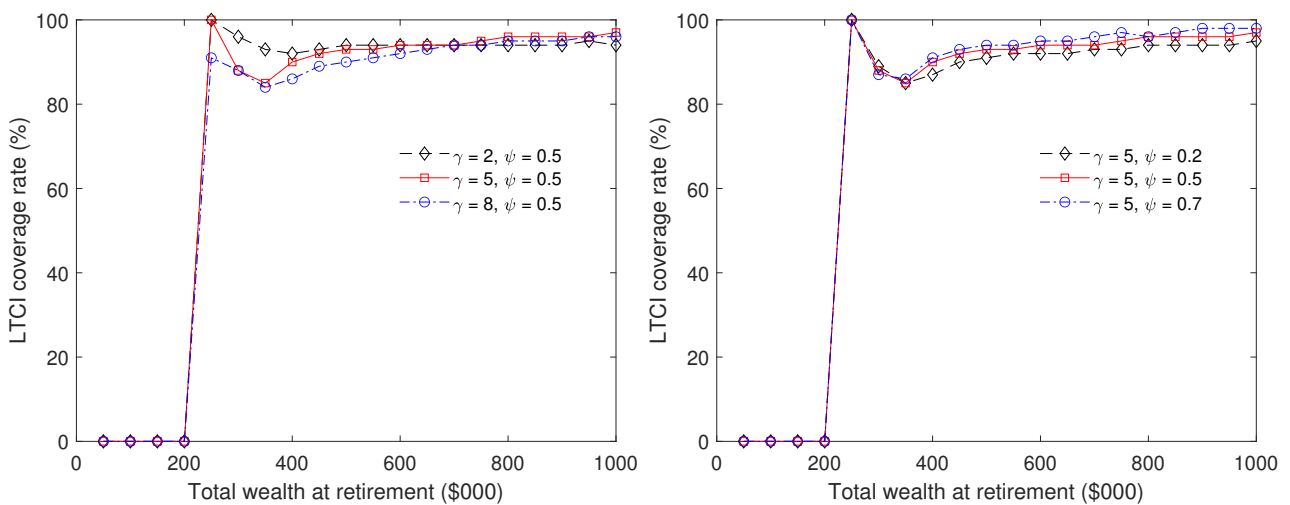


Figure 32. Optimal LTCI coverage rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when life annuities are offered in the market. The strength of bequest motives is given by  $b = 2$ . Retirees have no housing wealth.

## 4 Conclusions

The DC pension funds worldwide are reaching maturity as a growing number of members approach retirement. They need to convert a lump sum into income streams to support their retirement. However, the payout phase remains less developed than the accumulation phase, exposing retirees to longevity risk and health shocks, among other risks during retirement. A major difficulty in developing the payout phase is to design personalized retirement products that meet individual needs and circumstances. Our research offers new insights to help address the challenge.

We study the impact of housing wealth and individual preferences on demand for the products that insure against longevity risk and health shocks, i.e., life annuities and the LTCI. Taking into account housing wealth makes the results relevant to homeowners, who make up the majority of retirees in the U.S. We use Epstein-Zin-Weil-type utility that separates risk aversion from the EIS to capture the preferences of more heterogeneous retirees compared to the commonly-used power utility function.

We find a higher level of risk aversion and a lower level of the EIS has opposite effects on annuity demand, highlighting the need to break their inverse relation imposed by the power utility function. When health shocks are considered, a higher level of risk aversion or a higher level of the EIS decreases annuity demand. The impact diminishes with weaker bequest motives, a higher level of liquid wealth, or access to the LTCI, all of which enhance annuity demand. The presence of home equity enhances annuity demand, albeit to a less extent when retirees can access the LTCI.

Risk aversion and bequest motives interact with housing wealth to affect the LTCI demand, while the impact of the EIS is limited. A lower degree of risk aversion strengthens the crowding-out effect of housing wealth on the LTCI demand. In contrast, the crowding-out effect of housing wealth can be reduced or even reversed by bequest motives. Homeowners with limited wealth may demand higher LTCI coverage than renters endowed with the same amount of total wealth since the LTCI can help preserve the bequests.

We find the demand for life annuities and the LTCI is relatively robust to changes in risk aversion and the EIS when both products are offered simultaneously. Bequest motives, wealth

levels, and homeownership status remain important factors in affecting product demand. Since the information about wealth and homeownership is far easier to obtain than risk aversion or the EIS, the finding implies that bundling life annuities with the LTCI can substantially lower the cost of designing personalized retirement products.

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# Online Appendix

## A First-order condition for consumption

This section derives the first-order condition for consumption given the LTCI coverage and annuitization decisions have been made. The method of solving optimal annuitization rate and LTCI coverage is discussed in Section 2.6. The techniques used below build on the derivations in Chapter 6 of Munk (2013) who solves the optimal consumption problem for an individual with no bequest motive or health risk.

The first-order condition for  $C_t$  implies that

$$(1 - \beta)C_t^{-\rho} = \beta \left\{ \mathbb{E}_t \left[ \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{1-\gamma} + \pi_t(s_t, s_{t+1} = 4) b^\gamma W_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{\theta}-1} \quad (\text{A.1})$$

$$\times R_f \mathbb{E}_t \left[ \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial B_{t+1}} + \pi_t(s_t, s_{t+1} = 4) b^\gamma W_{t+1}^{-\gamma} \right],$$

where  $\partial V_{t+1}/\partial B_{t+1}$  can be derived by taking the derivative on the Equation (6). For the optimal decision, the equation holds without the maximum, that is

$$V_t \equiv V(B_t, W_t^H, s_t, t)$$

$$= \left\{ (1 - \beta) (C_t^*)^{1-\rho} + \beta \left[ \mathbb{E}_t \left[ \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V(B_{t+1}^*, W_{t+1}^H, s_{t+1} = k, t+1)^{1-\gamma} \right. \right. \right. \quad (\text{A.2})$$

$$\left. \left. \left. + \pi_t(s_t, s_{t+1} = 4) b^\gamma (W_{t+1}^*)^{1-\gamma} \right] \right]^{\frac{1}{\theta}} \right\}^{\frac{1}{1-\rho}},$$

where  $C_t^*$  denotes the optimal consumption at time  $t$ ,  $B_{t+1}^*$  and  $W_{t+1}^*$  denotes the next period liquid assets and total wealth, respectively, under the optimal consumption in period  $t$ .

Take the derivative of Equation (A.2) w.r.t.  $B_t$ .

$$\begin{aligned} \frac{\partial V_t}{\partial B_t} = & V_t^\rho \left\{ (1 - \beta)(C_t^*)^{-\rho} \frac{\partial C_t^*}{\partial B_t} \right. \\ & + \beta \left[ \mathbb{E}_t \left( \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{1-\gamma} + \pi_t(s_t, s_{t+1} = 4) b^\gamma (W_{t+1}^*)^{1-\gamma} \right) \right]^{\frac{1}{\theta}-1} \\ & \left. \times \mathbb{E}_t \left[ \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial B_{t+1}^*} \frac{\partial B_{t+1}^*}{\partial B_t} + \pi_t(s_t, s_{t+1} = 4) b^\gamma (W_{t+1}^*)^{-\gamma} \frac{\partial W_{t+1}^*}{\partial B_t} \right] \right\}, \end{aligned} \quad (\text{A.3})$$

where  $\partial B_{t+1}^*/\partial B_t$  and  $\partial W_{t+1}^*/\partial B_t$  can be derived from the budget constraints (4) and (5)

$$\begin{aligned} \frac{\partial B_{t+1}^*}{\partial B_t} &= \left( 1 - \frac{\partial C_t^*}{\partial B_t} \right) R_f, \\ \frac{\partial W_{t+1}^*}{\partial B_t} &= \frac{\partial W_{t+1}^*}{\partial B_{t+1}^*} \frac{\partial B_{t+1}^*}{\partial B_t} = \frac{\partial B_{t+1}^*}{\partial B_t} = \left( 1 - \frac{\partial C_t^*}{\partial B_t} \right) R_f. \end{aligned} \quad (\text{A.4})$$

Substitute the Equation (A.4) into Equation (A.3) and then using the first-order condition (A.1)

$$\begin{aligned} \frac{\partial V_t}{\partial B_t} = & V_t^\rho \beta R_f \left[ \mathbb{E}_t \left( \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{1-\gamma} + \pi_t(s_t, s_{t+1} = 4) b^\gamma (W_{t+1}^*)^{1-\gamma} \right) \right]^{\frac{1}{\theta}-1} \\ & \times \mathbb{E}_t \left[ \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial B_{t+1}^*} + \pi_t(s_t, s_{t+1} = 4) b^\gamma (W_{t+1}^*)^{-\gamma} \right]. \end{aligned} \quad (\text{A.5})$$

Consequently, the first-order condition for  $C_t$  can be re-written as

$$\frac{\partial V_t}{\partial B_t} = (1 - \beta) V_t^\rho C_t^{-\rho}. \quad (\text{A.6})$$

This is the envelope condition for the preferences defined in Equation (6).

Substitute the envelope condition (A.6) into Equation (A.1). The first-order condition for  $C_t$

can be re-stated as

$$(1 - \beta)C_t^{-\rho} = \beta \left\{ \mathbb{E}_t \left[ \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{1-\gamma} + \pi_t(s_t, s_{t+1} = 4) b^\gamma W_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{\theta}-1} \\ \times R_f \mathbb{E}_t \left[ (1 - \beta) \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{\rho-\gamma} C_{t+1}^{-\rho} + \pi_t(s_t, s_{t+1} = 4) b^\gamma W_{t+1}^{-\gamma} \right]. \quad (\text{A.7})$$

Therefore, the optimal consumption in period  $t$  is given by

$$C_t^* = \left\{ \beta R_f \left\{ \mathbb{E}_t \left[ \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{1-\gamma} + \pi_t(s_t, s_{t+1} = 4) b^\gamma W_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{\theta}-1} \right. \\ \left. \times \mathbb{E}_t \left[ \sum_{k \neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{\rho-\gamma} C_{t+1}^{-\rho} + \pi_t(s_t, s_{t+1} = 4) \frac{b^\gamma}{1 - \beta} W_{t+1}^{-\gamma} \right] \right\}^{-\frac{1}{\rho}}. \quad (\text{A.8})$$

In the terminal period,  $\pi_T(s_T, s_{T+1} = k) = 0$  for  $k \in \{1, 2, 3\}$  and  $\pi_T(s_T, s_{T+1} = 4) = 1$ , so the optimal consumption in period  $T$  becomes

$$C_T^* = \left( \frac{1}{\beta} - 1 \right)^{\frac{1}{\rho}} \left\{ \mathbb{E}_T [b^\gamma W_{T+1}^{1-\gamma}] \right\}^{\frac{1}{\rho} - \frac{1}{\rho\theta}} \times \left\{ R_f \mathbb{E}_T [b^\gamma W_{T+1}^{-\gamma}] \right\}^{-\frac{1}{\rho}}. \quad (\text{A.9})$$

## B Supplementary results of health state transitions

Tables B.1 and B.2 show the number of transitions and exposure years, respectively, in five-year interval. The results are used to calculate the crude transition rates, which will then be graduated using the generalized linear model. Table B.3 displays the results of selecting the appropriate degree of the polynomial in Equation (7).

Table B.1. Number of transitions between different health states.

	1 → 2	1 → 3	1 → 4	2 → 1	2 → 3	2 → 4	3 → 4
50 – 54	67	21	8	52	13	2	4
55 – 59	280	40	55	212	69	27	16
60 – 64	458	74	114	436	129	37	36
65 – 69	553	112	193	474	147	86	79
70 – 74	575	107	226	441	178	97	86
75 – 79	579	144	257	349	157	116	171
80 – 84	570	162	315	338	190	166	242
85 – 89	445	172	302	235	211	212	312
90 – 94	218	92	160	86	156	172	296
95 – 100	52	24	51	18	76	75	174
Total	3,797	948	1,681	2,641	1,326	990	1,416

*Note:* ‘1’ is healthy state, ‘2’ mildly disabled state, ‘3’ severely disabled state, ‘4’ dead state.

Table B.2. Number of exposure years in healthy, mildly disabled, and severely disabled states.

	Healthy	Mildly disabled	Severely disabled
50 – 54	4,527.18	361.92	121.51
55 – 59	10,816.97	1,136.76	387.61
60 – 64	15,721.89	1,811.16	692.93
65 – 69	16,610.65	2,146.23	802.31
70 – 74	13,975.53	2,079.22	948.19
75 – 79	10,807.98	2,164.77	1,071.76
80 – 84	7,512.86	2,131.81	1,242.44
85 – 89	3,870.87	1,826.11	1,457.01
90 – 94	1,235.42	965.27	1,006.33
95 – 100	235.92	265.35	421.37
Total	85,315.27	14,888.60	8,151.45

Table B.3. Model selection of the Poisson generalised linear models.

K	AICc	BIC	$D_c$	$\Delta D_c$
Disability				
$\sigma_t(1,2)$ : healthy to mildly disabled				
1	334.84	337.96	87.51	
<b>2</b>	304.56	<b>309.05</b>	<b>54.90</b>	<b>32.62***</b>
3	<b>303.87</b>	309.61	51.74	3.16*
$\sigma_t(1,3)$ : healthy to severely disabled				
1	260.49	263.60	64.61	
<b>2</b>	247.74	<b>252.23</b>	<b>49.53</b>	<b>15.08***</b>
3	<b>246.66</b>	252.40	45.99	3.54*
$\sigma_t(2,3)$ : mildly disabled to severely disabled				
1	316.44	319.55	100.70	
<b>2</b>	279.25	<b>283.74</b>	<b>61.17</b>	<b>39.52***</b>
3	<b>279.14</b>	284.88	58.60	2.57
Recovery				
$\sigma_t(2,1)$ : mildly disabled to healthy				
1	301.16	304.27	73.30	
<b>2</b>	<b>292.57</b>	<b>297.06</b>	<b>62.38</b>	<b>10.92***</b>
3	294.97	300.72	62.32	0.06
Mortality				
$\sigma_t(1,4)$ : healthy to dead				
1	272.53	275.64	51.01	
<b>2</b>	<b>265.01</b>	<b>269.50</b>	<b>41.16</b>	<b>9.85***</b>
3	267.02	272.77	40.71	0.45
$\sigma_t(2,4)$ : mildly disabled to dead				
1	246.79	249.90	45.02	
<b>2</b>	<b>243.68</b>	<b>248.18</b>	<b>39.58</b>	<b>5.44**</b>
3	244.11	249.85	37.54	2.04
$\sigma_t(3,4)$ : severely disabled to dead				
<b>1</b>	<b>245.02</b>	<b>248.13</b>	<b>29.59</b>	
2	247.35	251.85	29.58	0.00
3	247.45	253.20	27.22	2.36

*Note:* The chosen degree of polynomial value is in **bold** for each set of nested models.  $D_c$  is the residual deviance statistics.  $\Delta D_c$  denotes the test statistics for the likelihood ratio test. \* is for statistic that is significant at the 10% level, \*\* at the 5% level, \*\*\* at the 1% level.

## C Role of EIS on consumption and savings

Figure C.1 and Figure C.2 replicate the well-established results in the literature using a simplified version of our model that removed mortality risk and healthcare costs. Retirees are assumed to have a certain lifespan and will die upon reaching 100 years old. Figure C.1 compares the simulated average consumption paths for different levels of the EIS. Individuals with a higher level of the EIS tend to have more current consumption and a steeper consumption path over time. Since the time-preference-adjusted return on savings is negative in our model (i.e.,  $R_f - 1 < 1 - \beta$ ), the result is consistent with the finding in Campbell and Viceira (1999).

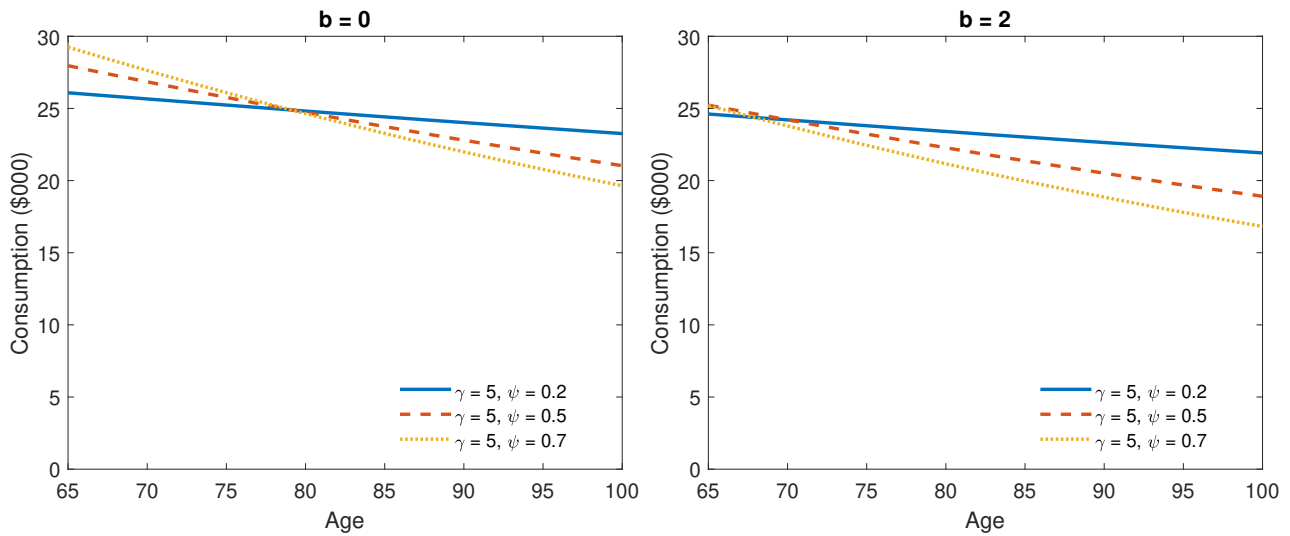


Figure C.1. Simulated average consumption paths for retirees with different levels of the EIS. The 65-year-old retirees have a certain lifespan of 35 years and face no healthcare costs. They are endowed with \$600K liquid wealth and no housing wealth at retirement.

Figure C.2 shows the simulated average liquid wealth paths for different levels of the EIS. The left panel shows that the more the EIS, the lower the curve. In other words, more current consumption leads to less savings in the absence of bequest motives, in line with the result in Campbell and Viceira (1999). The right panel of Figure C.2 shows that this result can be reversed after introducing the bequest motives. Since retirees have certain lifespan, the amount of bequests is the same as the terminal wealth. We find that a higher level of the EIS is associated with greater bequests.



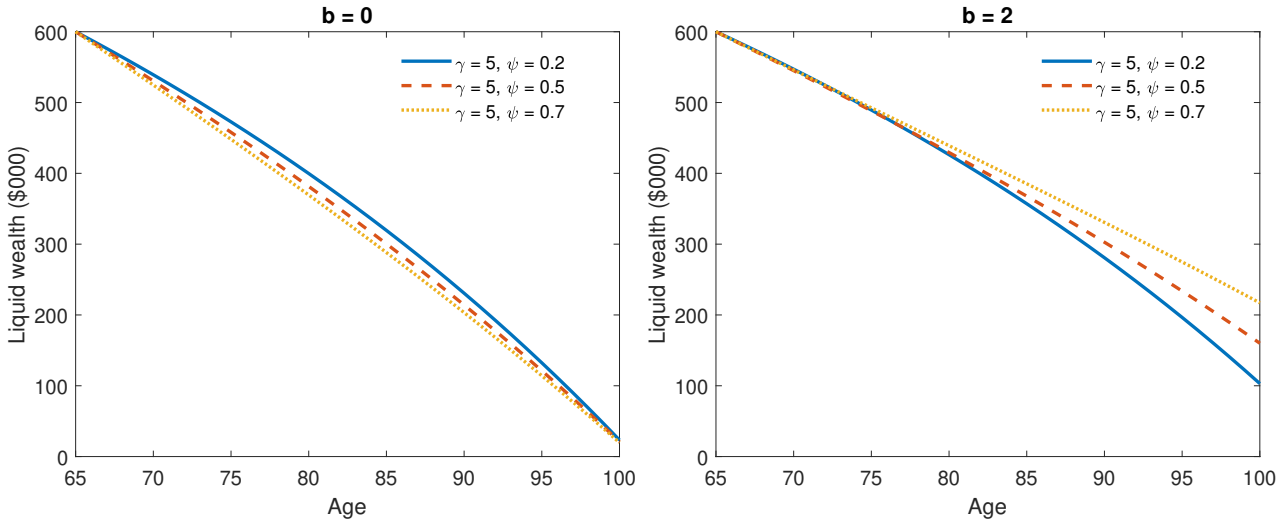


Figure C.2. Simulated average liquid wealth paths for retirees with different levels of the EIS. The 65-year-old retirees have a certain lifespan of 35 years and face no healthcare costs. They are endowed with \$600K liquid wealth and no housing wealth at retirement.

## D Supplementary results of product demand

### D.1 LTCI

Figure D.1 and Figure D.2 show the simulated average consumption paths in the healthy and mildly disabled states, respectively. They contrast with the consumption changes in the severely disabled state shown in Figure 14. As the LTCI coverage increases in equal steps, the optimal consumption in the healthy and mildly disabled states changes more or less evenly, whereas that in the severely disabled state increases at an accelerating pace.

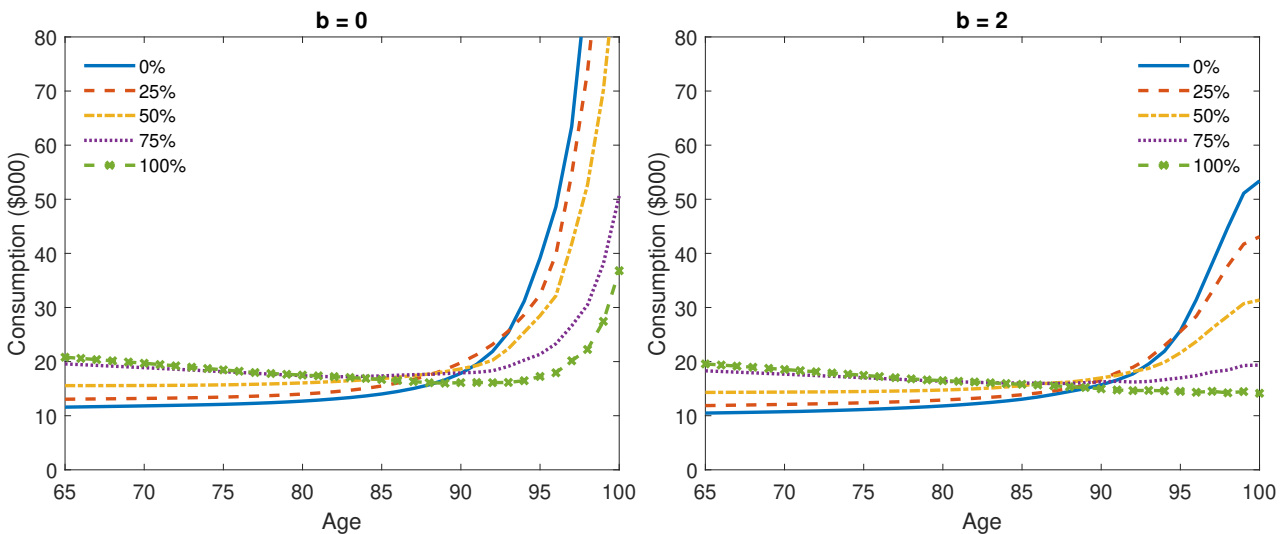


Figure D.1. Simulated average optimal consumption paths in the healthy state. The legend represents different LTCI coverage rates. Retirees are endowed with \$600K liquid wealth and no housing wealth at retirement. The preference parameters are  $\gamma = 5, \psi = 0.5$ . The life annuity is not offered in the market.

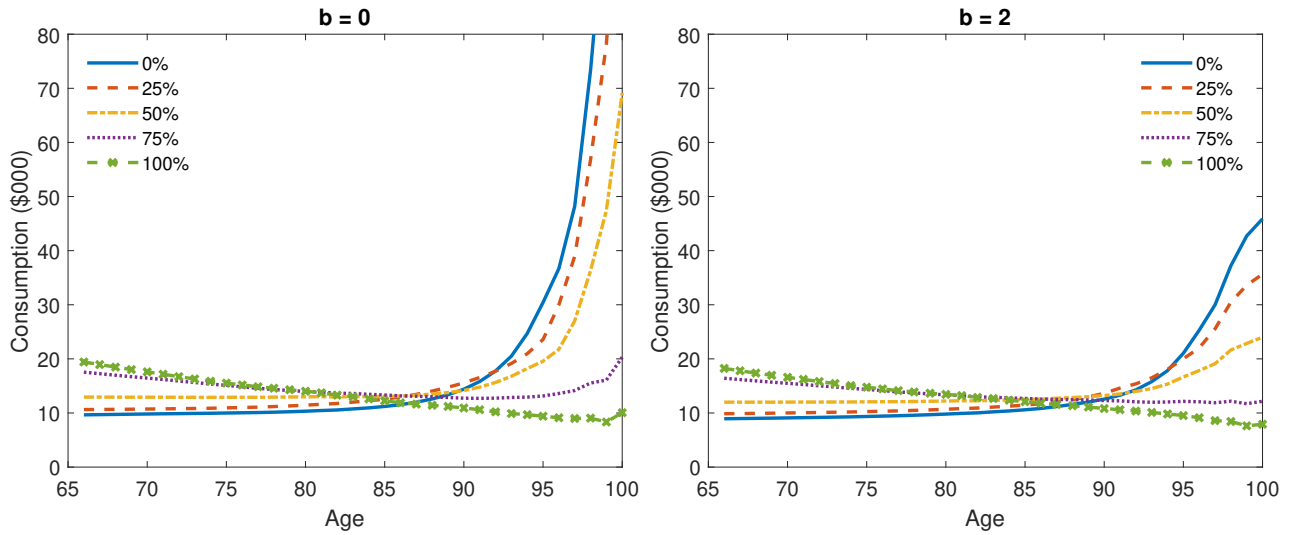


Figure D.2. Simulated average different consumption paths in the mildly disabled state. The legend represents different LTCI coverage rates. Retirees are endowed with \$600K liquid wealth and no housing wealth at retirement. The preference parameters are  $\gamma = 5, \psi = 0.5$ . The life annuity is not offered in the market.

Figure D.3 and Figure D.4 supplement Figure 18 which shows that the EIS has a minimal impact on the optimal LTCI coverage rate for non-homeowners. Figure D.3 displays the optimal LTCI coverage rates with different levels of the EIS for homeowners who do not have bequest motives, each panel representing a different housing wealth proportion. Figure D.4 shows the same comparison for homeowners with bequest motives. Overall, the EIS has a minimal impact on the optimal LTCI coverage rate for homeowners with and without bequest motives alike, and this result is robust to varying levels of housing wealth.

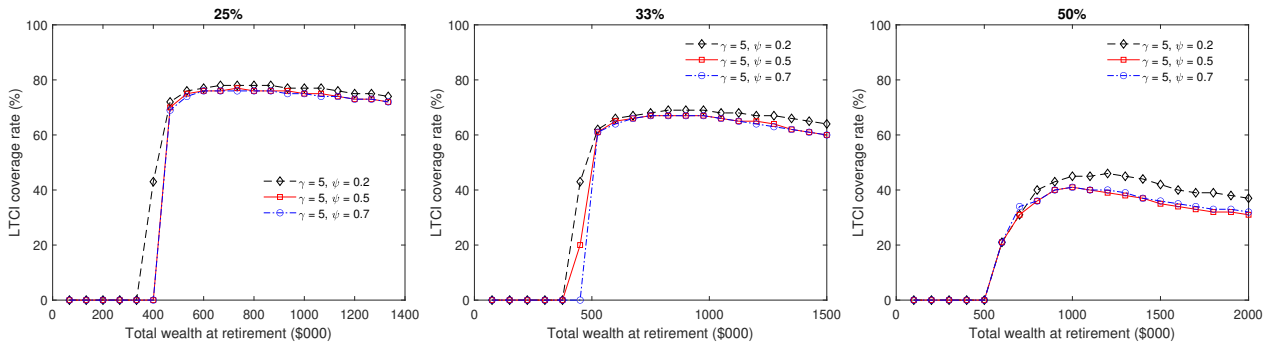


Figure D.3. Optimal LTCI coverage rates for retirees with different levels of the EIS and no bequest motives. The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is not offered in the market.

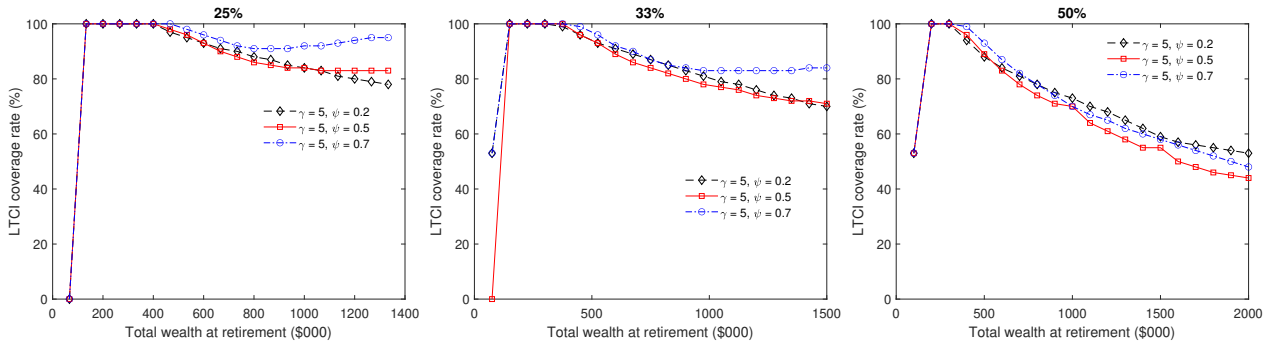


Figure D.4. Optimal LTCI coverage rates for retirees with different levels of the EIS and a certain bequest motive ( $b = 2$ ). The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is not offered in the market.

## D.2 Both annuities and LTCI

The LTCI is known to enhance the demand for annuities in the absence of housing wealth (see e.g., Ameriks et al., 2008; Wu et al., 2016). We find the same result in a wide range of wealth levels. Figure D.5 shows that retirees endowed with between \$200K and \$800K liquid wealth increase their optimal annuitization rates after having access to the LTCI. Retirees in the higher wealth bands, however, optimally cut their annuitization rates to purchase the LTCI.

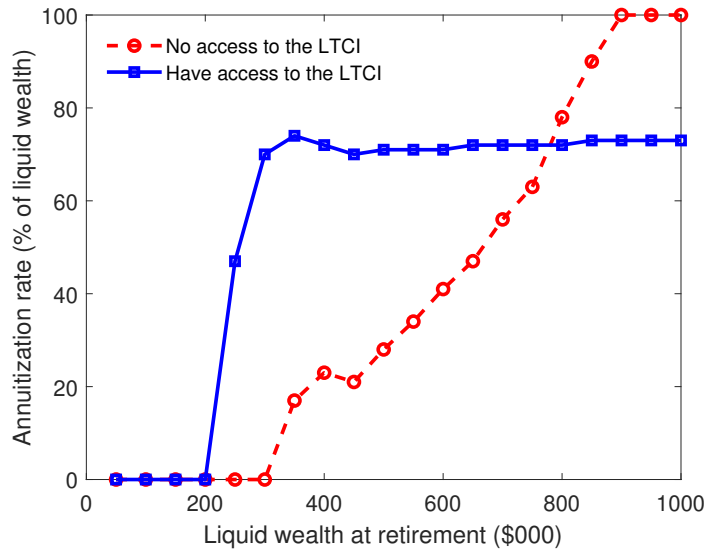


Figure D.5. Optimal annuitization rates with and without access to the LTCI for retirees endowed with liquid wealth and no housing wealth at retirement. The preference parameters are  $b = 2, \gamma = 5, \psi = 0.5$ .

We also verify the result in Davidoff (2009) that the complementarity between life annuities and the LTCI can be reversed by illiquid housing wealth. Figure D.6 compares the interaction between annuity demand and access to the LTCI across different levels of housing wealth. As the ratio between housing wealth and liquid wealth increases, the improvement in the optimal annuitization rate brought about by the LTCI shrinks before it disappears. This shows that

housing wealth undoes the complementarity between the two products.

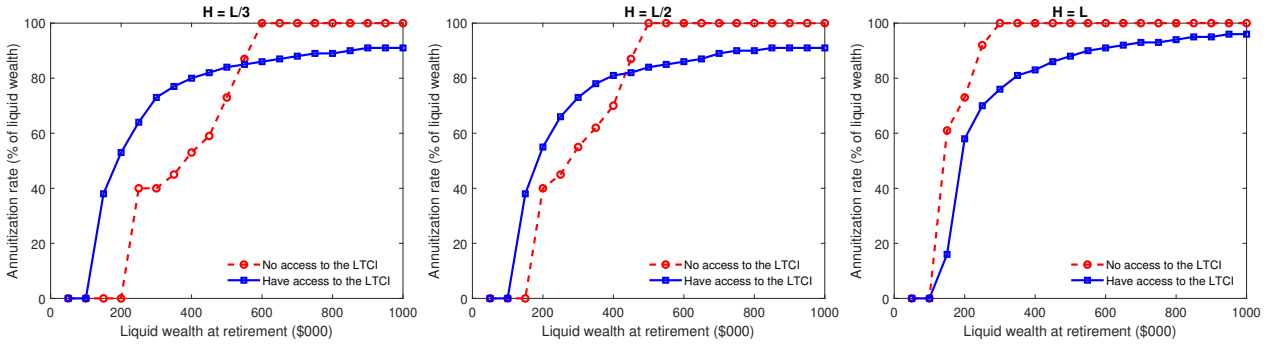


Figure D.6. Optimal annuitization rates for homeowners with and without access to the LTCI. The title above each panel denotes the ratio of housing wealth ( $H$ ) to liquid wealth ( $L$ ) at retirement. The preference parameters are  $b = 2, \gamma = 5, \psi = 0.5$ .

Figure D.7 and Figure D.8 supplement the result shown in Figure 29. They show that for homeowners without bequest motives, the optimal annuitization rates vary little with risk aversion or the EIS.

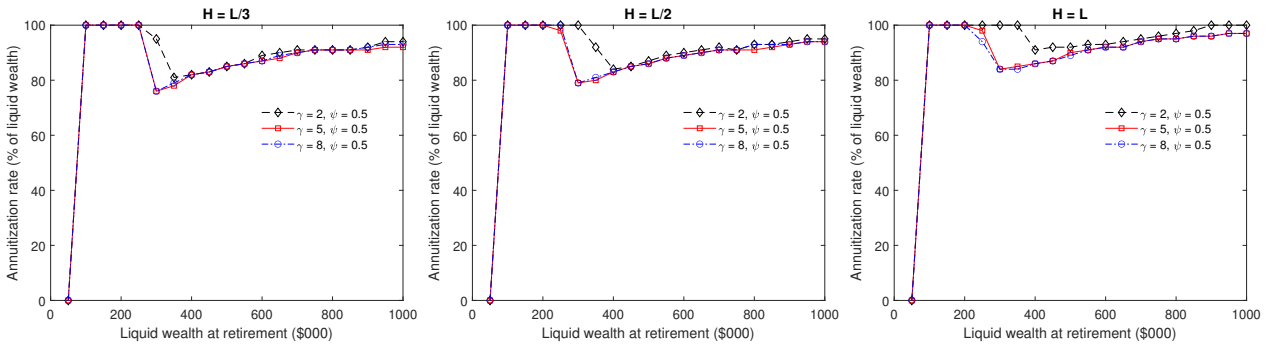


Figure D.7. Optimal annuitization rates for retirees with different levels of risk aversion and no bequest motives. The title above each panel denotes the ratio of housing wealth ( $H$ ) to liquid wealth ( $L$ ) at retirement. The LTCI is offered in the market.

Figure D.9 and Figure D.10 supplement the result shown in Figure 30. They show that for homeowners with bequest motives ( $b = 2$ ), the optimal annuitization rates vary little with risk aversion or the EIS.

Figure D.11 and Figure D.12 supplement the result shown in Figure 31. They show that for homeowners without bequest motives, the optimal LTCI coverage rates vary little with risk aversion or the EIS.

Figure D.13 and Figure D.14 supplement the result shown Figure 32. They show that for homeowners with bequest motives ( $b = 2$ ), the optimal LTCI coverage rates vary little with risk aversion or the EIS.

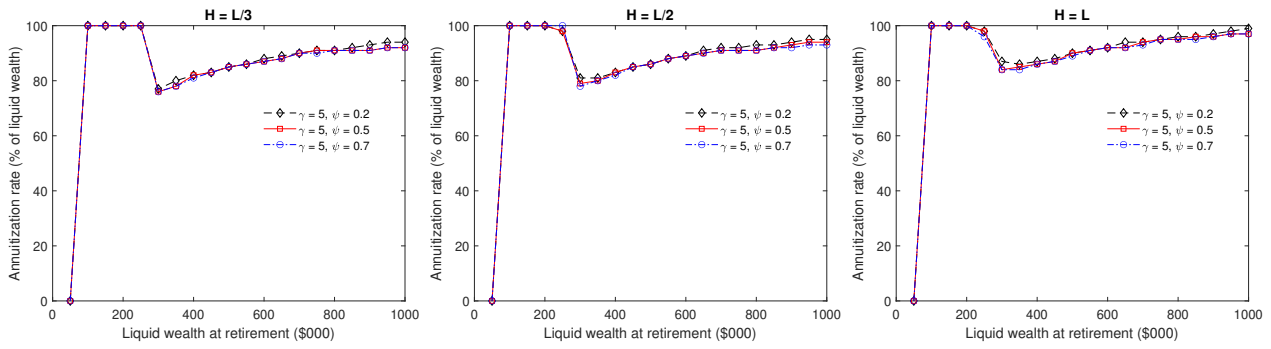


Figure D.8. Optimal annuitization rates for retirees with different levels of the EIS and no bequest motives. The title above each panel denotes the ratio of housing wealth ( $H$ ) to liquid wealth ( $L$ ) at retirement. The LTCI is offered in the market.

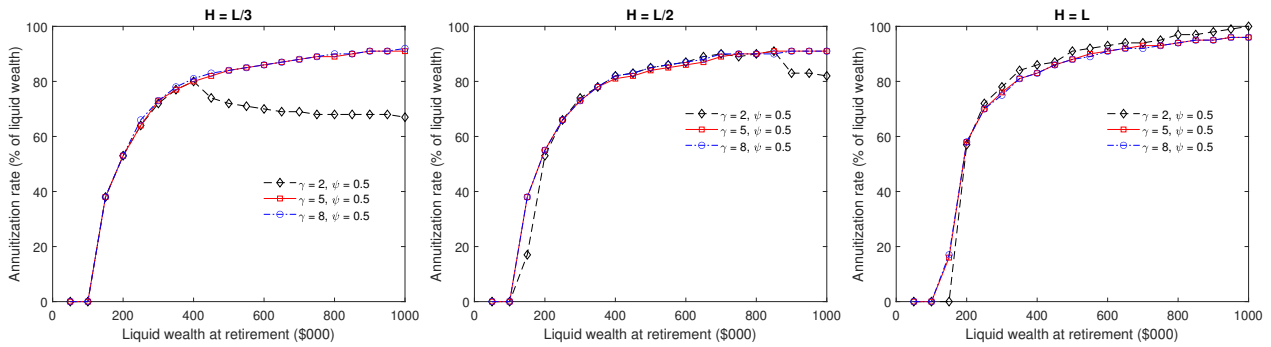


Figure D.9. Optimal annuitization rates for retirees with different levels of risk aversion and a certain bequest motive ( $b = 2$ ). The title above each panel denotes the ratio of housing wealth ( $H$ ) to liquid wealth ( $L$ ) at retirement. The LTCI is offered in the market.

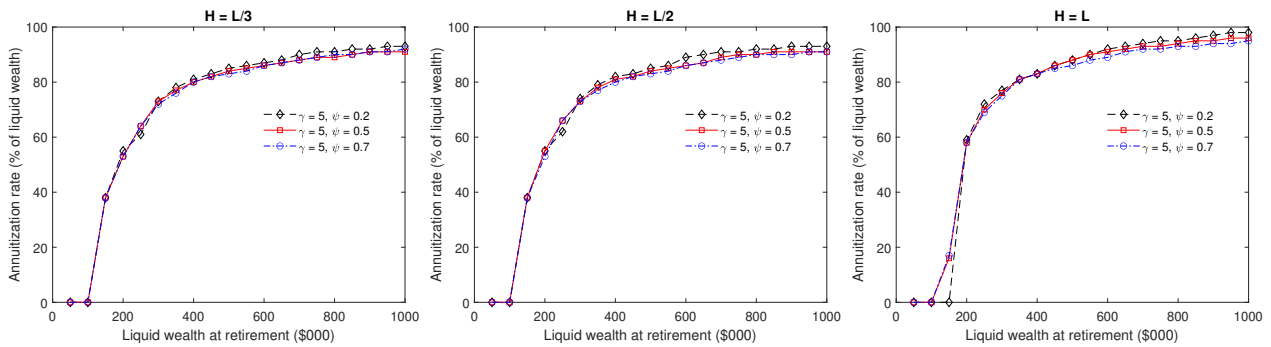


Figure D.10. Optimal annuitization rates for retirees with different levels of the EIS and a certain bequest motive ( $b = 2$ ). The title above each panel denotes the ratio of housing wealth ( $H$ ) to liquid wealth ( $L$ ) at retirement. The LTCI is offered in the market.

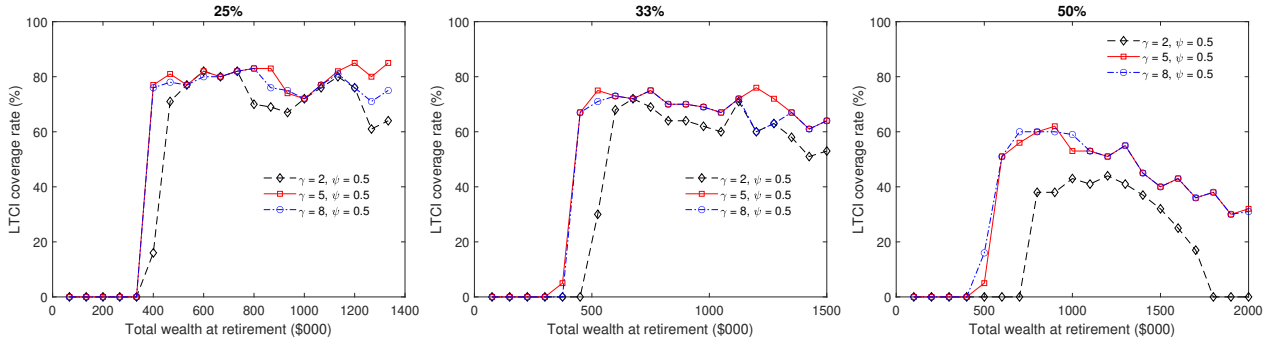


Figure D.11. Optimal LTCI coverage rates for retirees with different levels of risk aversion and no bequest motives. The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is offered in the market.

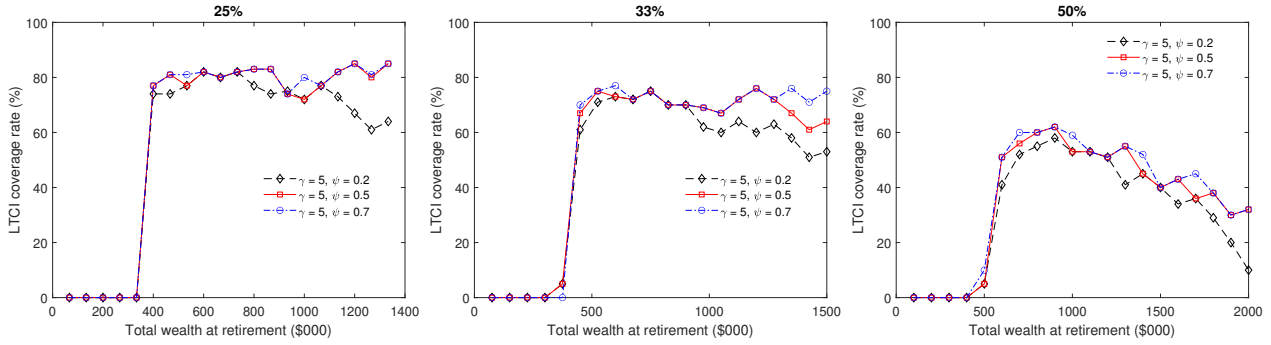


Figure D.12. Optimal LTCI coverage rates for retirees with different levels of the EIS and no bequest motives. The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is offered in the market.

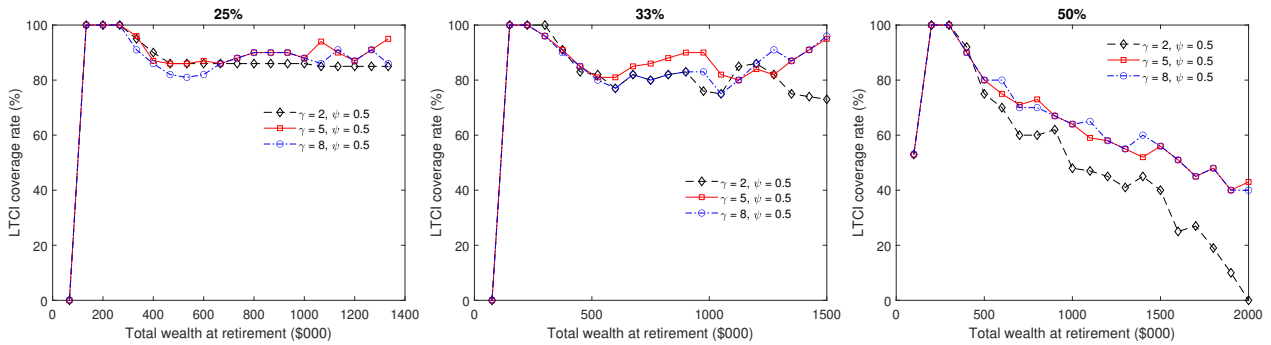


Figure D.13. Optimal LTCI coverage rates for retirees with different levels of risk aversion and a certain bequest motive ( $b = 2$ ). The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is offered in the market.

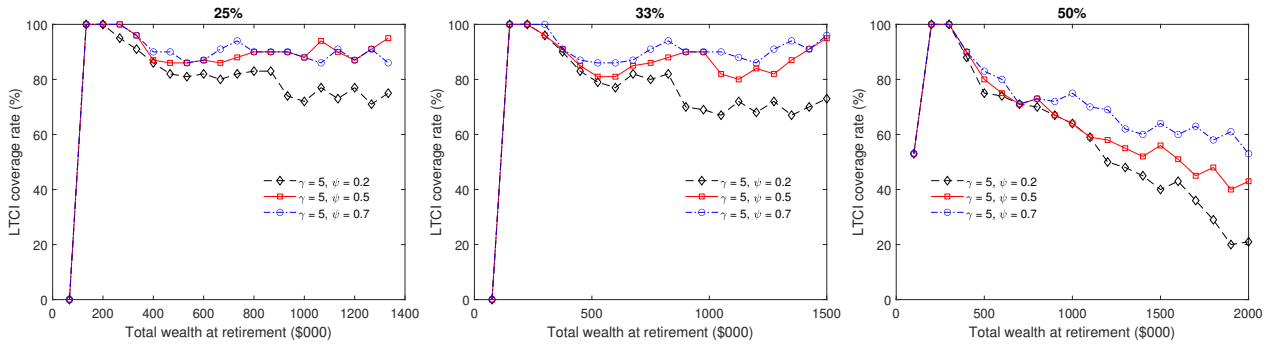


Figure D.14. Optimal LTCI coverage rates for retirees with different levels of the EIS and a certain bequest motive ( $b = 2$ ). The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is offered in the market.