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To Borrow or Insure? Long Term Care Costs and the Impact of Housing

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Adam W. Shao*, Hua Chen† and Michael Sherris‡

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Abstract

We consider the impact of housing and the availability of reverse mortgages and long-term care insurance on a retiree's optimal portfolio choice and consumption decisions. Individuals decide how much to borrow against their home equity and how much to insure health care costs with long-term care insurance. We build a multi-period life cycle model that takes into consideration longevity risk, health shocks and house price risk. We use an endogenous grid method along with a regression based approach to improve computational efficiency and avoid the curse of dimensionality. Our results show that borrowing against home equity dominates long-term care insurance reflecting higher consumption in earlier years and inclusion of longevity insurance. Long-term care insurance transfers wealth from healthy states to disabled states but reduces earlier consumption because of the payment of upfront insurance premiums. The highest welfare benefits come from using both reverse mortgages and long-term care insurance because of strong complementary effects between them, highlighting the benefits of innovative products that bundle these two products together.

Keywords: life-cycle model; longevity risk; long-term care insurance; residential house; reverse mortgage

JEL Classifications: G21, G22, I13, I31, J26, R31

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1 Introduction

Longevity risk and health shocks are often intertwined with each other and impose significant challenges on financial budgets for the aged population in retirement. On the one hand, life expectancy at age 60 in advanced economies has increased by an average of up to two years in the past decade (International Monetary Fund, 2012). This increased life expectancy at retirement age increases the longevity risk of outliving their resources on the elderly at older ages. On the other hand, there is a large probability that they will experience deteriorated health and thus incur more medical expenses and/or long-term care (LTC) costs. Health care costs in OECD countries increased steadily over recent decades (Colombo *et al.*, 2011; Shi and Zhang, 2013). Health care costs can be met by buying insurance, or by unlocking home equity using home equity release products, such as a reverse mortgage.

In the U.S., approximately two-thirds of individuals currently aged 65 or above will need some form of LTC, either at home or a LTC facility (Chapman, 2012). Such services have been largely provided by Medicare and Medicaid. However, with funding deficiencies at both the state and federal government levels, there is an increasing need to fund LTC costs by individuals through either out-of-pocket savings or private insurance plans. Therefore, it is important to include private LTC insurance in a retirement product portfolio. We also take into account reverse mortgages, whose origin can be traced back to 1970s when academics and practitioners sought to create mortgage instruments to enable elderly homeowners to raise cash to meet their daily living and medical expenses using their home equity as collateral. Reverse mortgages have become widely available in the U.S. since the Department of Housing and Urban Development (HUD) introduced the Home Equity Conversion Mortgage (HECM) program in 1989.[1] In a reverse mortgage, the provider lends the customer cash and obtains a mortgage charge over the customer's property (or a share of the property). The contract

[1]The HECM program is considered the safest and the most popular program of its kind in the U.S., since it is insured by the U.S. federal government and accounts for 95% of the market share (Ma and Deng, 2006).

is terminated upon the death or permanent move-out of the customer, at which time the property is sold and the proceeds are used to repay the outstanding loan. Typically, a no negative equity guarantee is included in the contract, which stipulates that the customer is not liable if the sale proceeds of the property are insufficient to repay the loan.

We consider home equity and reverse mortgages in our model for several reasons. First, the size and illiquidity of home equity have significant impacts on the optimal demand for LTC insurance. Davidoff (2010) argues that the elderly face two forms of commitment, consumption commitment to their house and asset commitment to home equity, that may explain why demand for LTC insurance is weak, despite the risk of potentially very large LTC costs. Second, reverse mortgage loans provide a new means for individual retirees to supplement their retirement income and pay for LTC costs. If home equity is a substitute for LTC insurance, then home equity extraction using a reverse mortgage, complements LTC insurance as suggested by Ahlstrom *et al.* (2004) and others.

There are only a few papers that jointly consider the roles of reverse mortgages and LTC insurance in retirement planning. Davidoff (2010) investigates welfare gains of the retired by including reverse mortgage loans and LTC insurance in the portfolio in a one-period model. He provides evidence from recent health and retirement study (HRS) data that home equity is a substitute for LTC insurance for a significant portion of the elderly. A two-period model is employed by Hanewald *et al.* (2015) to investigate the demand for two types of home equity release products - reverse mortgage and home reversion, where LTC insurance is also included. Each period in their model spans 19 years. They show that, from a welfare perspective, reverse mortgages are preferred to home reversions. The interaction between reverse mortgages and LTC insurance is not considered.

A clear drawback in these studies is that they employ a simple one-period or two-period life cycle model. A more realistic multi-period model is needed in order to better capture the impact of longevity risk, health shocks, and house price risk on a retiree's optimal portfolio choice in both the short run and the long run. Nakajima and Telyukova (2017) use a multi-period life cycle model with biennial transition probabilities calibrated to HRS data to analyze the demand for reverse mortgages. They quantify the welfare gains from reverse mortgages. They also show that bequest motives, uncertainty about health and expenses and loan costs explain the low demand for reverse mortgages. Although their model is richer, LTC insurance is not included.

In this paper, we jointly consider longevity risk, health shocks, house price risk in a multiple-period life cycle model and investigate a retiree's optimal choices of reverse mortgage loans and private LTC insurance in her retirement portfolio. We use an endogenous grid method (EGM) and a regression based approach to improve the computing efficiency and avoid the curse of dimensionality. We also assess the welfare gains from having access to both reverse mortgages and private LTC insurance, supporting the innovation of offering a bundled product from an insurer's perspective.

We build a multi-period life cycle model that takes into account health shocks, idiosyncratic longevity risk, and house price risk. Five health states are defined based on how many Activities of Daily Livings (ADLs) each individual cannot independently perform and whether she resides in a LTC facility or nursing home.[2] A similar health state definition is used in Ameriks *et al.* (2011). However, we use individual level HRS data to estimate the transition probabilities between different health states, which provides more accurate estimations compared to using aggregate data. The house price dynamics are modeled using an ARIMA-GARCH process, reflecting actual U.S. house price evolution. This allows us to take into account the impact of partial predictability of house prices on an individual's optimal portfolio choice. We solve the retiree's optimal choice for consumption, reverse mortgage loans, private LTC insurance, and a risk-free asset. We then quantify the welfare gains from having access to either a reverse mortgage or LTC insurance or both.

[2]The six ADLs that we consider are dressing, walking, bathing, eating, transferring, and toileting.

Because of the incorporation of health states and partial predictability in the house price, computation is demanding and requires the application of new approaches from computational economics. We avoid the curse of dimensionality by using an endogenous grid method (EGM) proposed by Carroll (2005) combined with a regression based method proposed by Brandt *et al.* (2005). Compared to conventional dynamic programming, EGM avoids the time-consuming process of finding roots of non-linear equations by predefining after-consumption wealth. A linear regression method is used to estimate a parametric function for the optimal choice variables with respect to past and current house values, based on simulated house value paths. This further avoids the curse of dimensionality caused by the path-dependent house price model. Using these innovative approaches allows us to extend the simple one-period or two-period model in prior studies to a more realistic multi-period model. We are able to capture both the short-run and long-run effects of different product choices on an individual's wealth, consumption, bequest, and welfare, and provide a richer insight into the trade-off between products and reasons why one product is preferred over another.

The results of this paper have significant implications for individuals considering the use of reverse mortgage loans and private LTC insurance to better manage their retirement risks. We show that private LTC insurance benefits those who have high levels of liquid wealth the most and reverse mortgage benefits those who are asset rich but cash poor the most. The demand for private LTC insurance is stronger when reverse mortgages are also available; having access to private LTC insurance increases the demand for reverse mortgages as well, even for those who are cash rich but asset poor. We quantify the extent to which home equity has crowd-out effects on the demand for private LTC insurance, supporting the theoretical results in Davidoff (2010). We also quantify the extent to which bundling a reverse mortgage and private LTC insurance provides individuals with significantly higher welfare gains with implications for retirement product design. Insurers should consider offering a bundled product of a reverse mortgage and LTC insurance based on these results.

The paper is organized as follows. In Section 2, we develop a multi-period life cycle model and present the analytical framework of the optimization problem for a retiree. In Section 3, we discuss the endogenous grid method and the regression based approach which are used to efficiently solve the multi-dimensional optimization problem. In Section 4, we present numerical results for a baseline case and conduct sensitivity analysis with respect to wealth endowment and other parameter values. We provide concluding remarks and discuss welfare implications and product design in Section 5.

2 Life Cycle Utility Framework

The life cycle utility framework that we use is similar to that in Davidoff (2010) and Nakajima and Telyukova (2017). We only consider homeowners and abstract from the issue of whether an individual should buy or rent. We assume that the utility function of a homeowner has constant relative risk aversion and follows a Cobb-Douglas function with respect to non-housing and housing consumption. It is given in the following equation:

$$U(C_t, H_t) = \frac{\left(C_t^{\eta} H_t^{1-\eta}\right)^{1-\gamma}}{1-\gamma}, \tag{1}$$

where γ is the risk aversion parameter, η is the Cobb-Douglas aggregation parameter of non-housing consumption C_t versus housing consumption H_t. Housing consumption is typically measured by rent (Davidoff, 2010).

We use a bequest function that takes into account the luxury good property of bequests (see Ameriks *et al.*, 2011; De Nardi *et al.*, 2010; Ding *et al.*, 2014).

$$B(W_t) = \begin{cases} \frac{\beta}{1-\gamma}\left(\phi + \frac{W_t}{\beta}\right)^{1-\gamma}, & W_t \geq 0 \\ \frac{\beta}{1-\gamma}\phi^{1-\gamma}, & W_t < 0 \end{cases} \tag{2}$$

where β measures the strength of the bequest motive, ϕ captures the degree to which bequests are regarded as a luxury good,[3] and W_t is the individual's bequest wealth at time t.

An individuals selects a certain amount of reverse mortgage loans and private LTC insurance to fund consumption and health care costs, with liquid wealth invested in the risk free asset. To simplify the problem, we do not include life annuities since reverse mortgage loans hedge longevity risk in a similar manner - individuals do not have to repay the loan as long as they are alive and continue living in their house.

2.1 Health status

We follow Ameriks *et al.* (2011) to categorize a retiree's health state and use a Markov transition model to capture the health dynamics of the retiree. Ameriks *et al.* (2011) only consider four states, i.e., good health, having minor medical problems but no need for LTC, LTC needed, and death. Since we consider both reverse mortgage and private LTC insurance, the health state categorization needs to reflect the exact timing of the payment of the reverse mortgage loan and private LTC insurance benefits. Private LTC insurance in the U.S. typically pays benefits when the insured has difficulties in two or more ADLs (Brown and Warshawsky, 2013; Shao *et al.*, 2017). Reverse mortgage borrowers (or their beneficiaries) need to sell their house and use the sale proceeds to pay off the loans when they move into LTC facilities or die. We therefore consider a five-state model by introducing two LTC states, LTC at home and LTC at nursing home. Retirees who are severely disabled but staying at home receive a base level of LTC benefits to cover care costs. Moving into a LTC facility, or nursing home, triggers the repayment of reverse mortgage loans and results in an escalated level of disability benefits from LTC insurance.

More specifically, health states are defined based on the individual's number of difficulties in ADLs and whether she is in a LTC facility. Health states at time t are denoted by Λ_t. State

[3] De Nardi *et al.* (2010) find that bequests are luxury goods and that bequest motives are potentially very important for the richest retirees. Ameriks *et al.* (2011) interprets ϕ as the consumption threshold above which the bequest motive becomes operative.

"1" is the healthy state, defined as no ADL difficulties. State "2" is the mildly disabled (defined as disabilities of 1 ADL) and staying-at-home state. State "3" is the severely disabled (defined as disabilities of 2 - 6 ADLs) and staying-at-home state. State "4" is the nursing home state where admittance is based on health status. State "5" is the death state. The transition diagram is shown in Figure 1. We assume that moving into a LTC facility or nursing home is non-reversible. We also assume that health status transitions occur at the end of each year and that at the beginning of the next year each individual has full knowledge of the LTC costs for the following year based on her current health status as well as the transition probabilities among different health states.

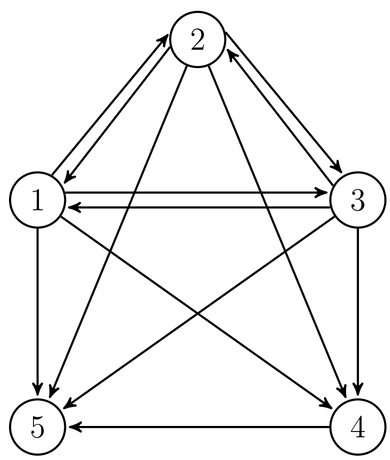

Figure 1. Five-state Markov transition diagram.

We calibrate health transitions to data from the University of Michigan Health and Retirement Study (HRS), which is a U.S. nationally representative ongoing survey of people aged 50 and above. Starting from 1992, the survey has been conducted biennially to collect information on physical and mental health function, health insurance, health expenditures, retirement plans, and assets. The data has detailed information on self-reported difficulties of ADLs and an assessment of mental function. There is also information on whether the respondent moves into a nursing home. Since an inconsistent structure of questions were asked before wave 1998, we use data from wave 1998 onward to the recent wave in 2010.

2.2 Health Transition Rates and Probabilities

Let $i \in \{1, 2, 3, 4\}$ denote the current health state of an individual and $j \in \{1, 2, 3, 4, 5\}$ denote the possible health state that the individual can transit into. μ_x^{ij} denotes the transition rate from State i to State j for an x-year-old individual. $p_{x:x+t}^{ij}$ denotes the probability of transitions from State i at age x to State j at age $x+t$. For $t = 0$, $p_{x:x}^{ij}$ is equal to 0 if $i \neq j$ and equal to 1 if $i = j$.

Crude transition rates are first estimated as the number of transitions divided by the central exposed to risk for each of the states. Following Fong *et al.* (2015), we use generalized linear models (GLM) to smooth, or graduate in actuarial terminology, the transition rates. These graduated transition rates are grouped into three categories: health decrements, health improvements, and mortality. Figure 2 shows the graduated transition rates for these three groups. The annual transition probability matrix is calculated as the matrix exponential of graduated transition rates. Annual transition probabilities are used in the model since we use an annual time period.

Using the graduated transition rates, we show simulated bio-metric dynamics of a given cohort in Figure 3. The impact of disability and mortality can be seen in the top figure. As the cohort ages, the number of individuals who are in each of the three disabled states (State 2, 3 and 4) initially increases, and then declines as some of them transit into the next disabled state or die. The number of people who are severely disabled and staying in LTC facilities reaches the peak right after age 85. The proportion of the cohort in each state is shown in the bottom figure. Those who survive into the older ages will mostly be in the LTC state in a nursing home (State 4) requiring substantial aged care costs to meet accommodation and living expenses.

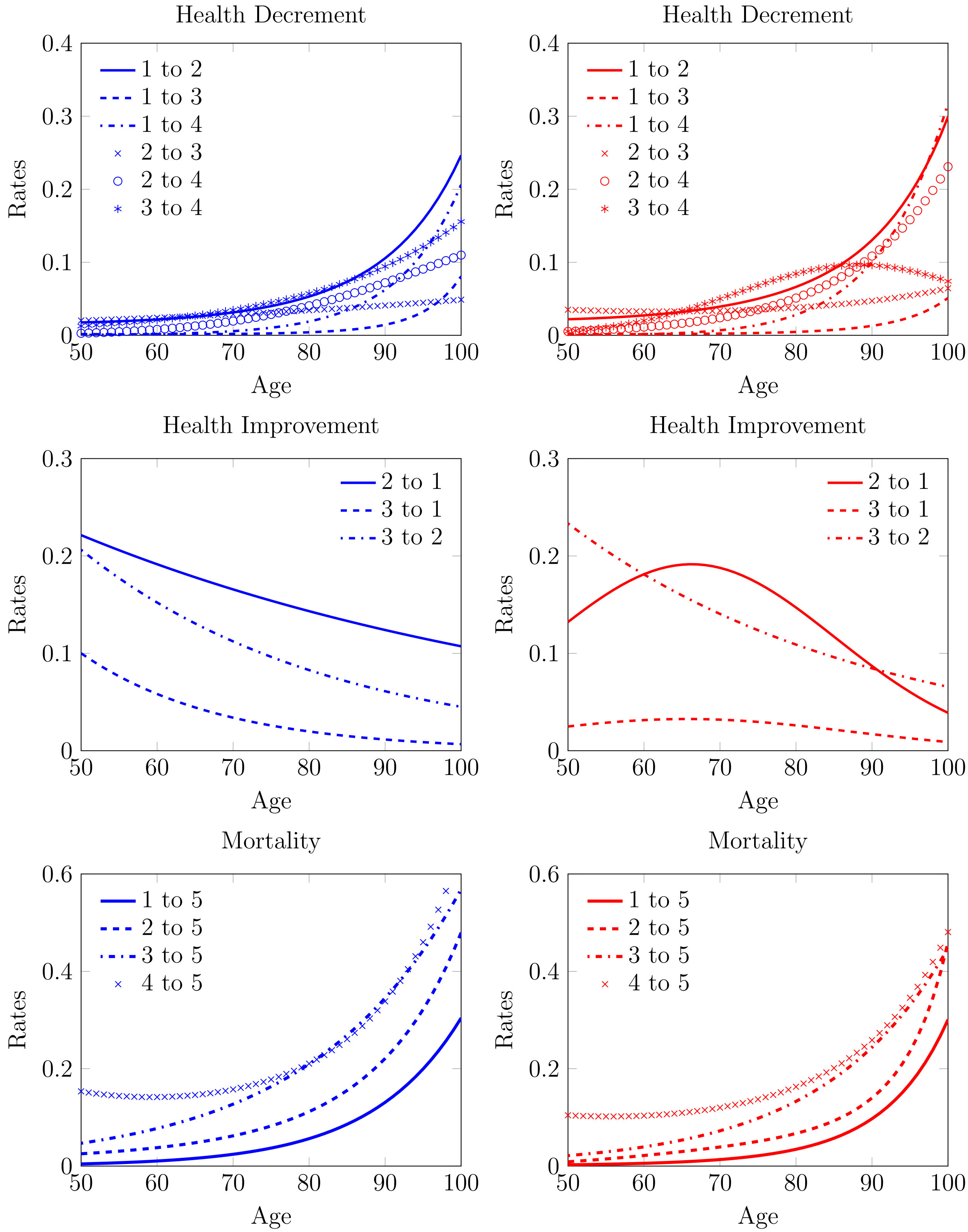

Figure 2. Graduated transition intensities for males (left) and females (right).

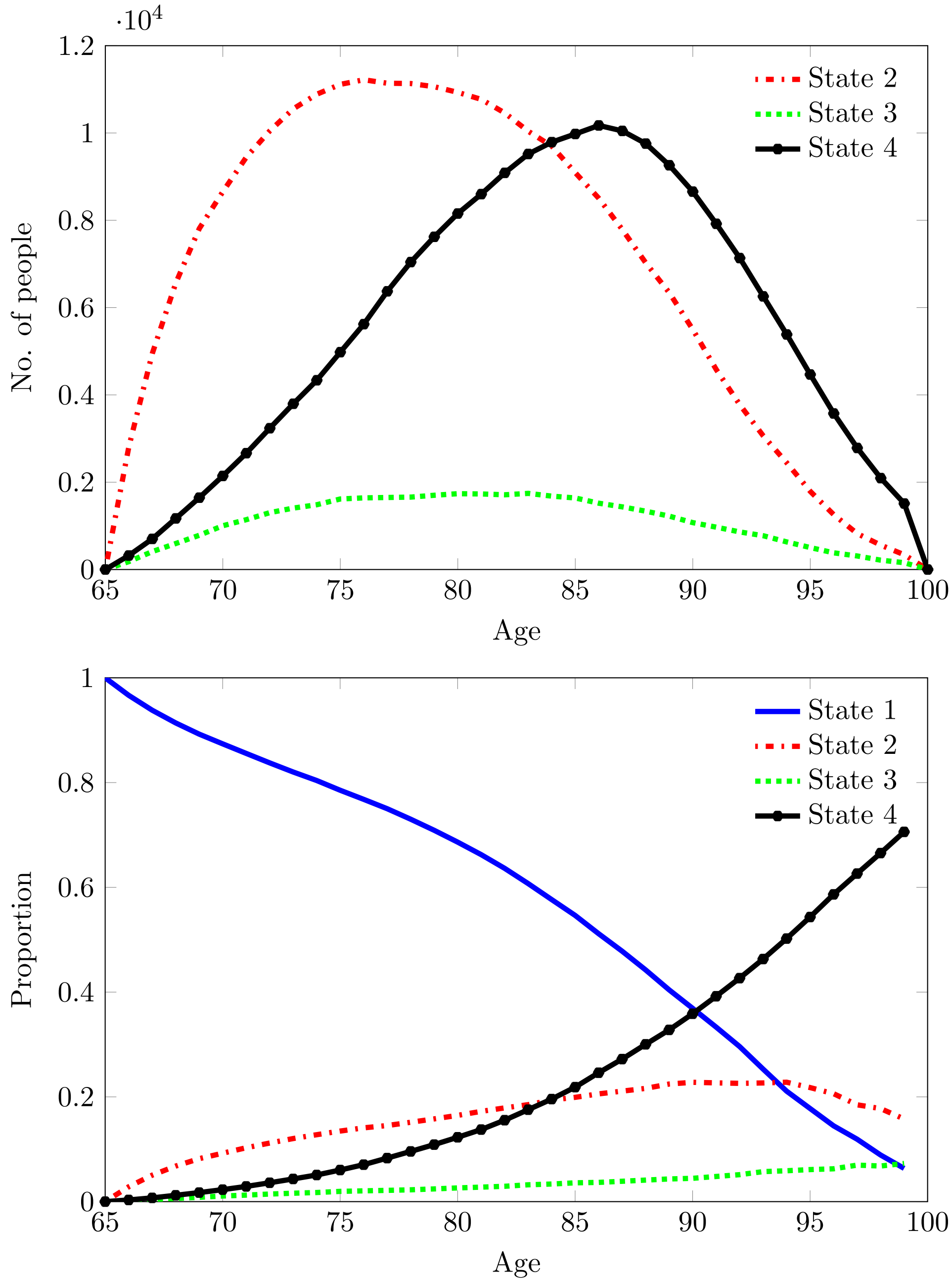

Figure 3. The number of people in disabled states across different ages and as a proportion of alive individuals, based on a simulated cohort of 100,000 65-year-old healthy females.

2.3 Long-Term Care Insurance

LTC costs depend on the health status and also increase with inflation. The annual LTC cost for an individual who is in State $i \in \{2, 3, 4\}$ at time t is denoted by LTC_t^i. We assume that

$$LTC_t^i = LTC^i \cdot e^{ft}, \tag{3}$$

where LTC^i is the LTC expense in State i in the base year and f is the continuously compounded inflation rate. For simplicity, we assume a constant inflation rate of 1% p.a. Based on the median annual LTC costs in different levels of facilities as of July 2015, the base LTC costs are assumed to be \$20,000 in the mildly disabled state, \$40,000 in the severely disabled but staying-at-home state, and \$80,000 in the nursing home state.[4]

We assume that only healthy individuals (in State 1) are eligible for purchasing private LTC insurance to cover LTC expenses in the severely disabled but staying-at-home state (State 3) and in the nursing home state (State 4). We denote by PI the percentage coverage of the private LTC insurance with a lump sum premium paid at the retirement age. Annual premiums will make little difference to the results since they must still be financed from consumption while in the healthy state at the early ages. No additional private LTC insurance is allowed to be purchased after retirement. In practice this is often the case. The lump-sum premium paid by an x-year-old to purchase a stand-alone LTC insurance policy is given by

$$\Pi_{PI,x} = PI\ \pi_x, \tag{4}$$

where π_x is the actuarially fair premium for a full coverage stand-alone LTC insurance policy sold to an x-year-old healthy individual (see Shao *et al.* (2017) for a detailed discussion of methods for pricing LTC insurance).

The formula for calculating the premium π_x is given by:

$$\pi_x = \sum_{s=1}^{\infty} \sum_{j=3}^{4} p_{x:x+s}^{1j}\ LTC^j\ e^{(f-r_f)s}, \tag{5}$$

[4] Data source: Genworth at `https://www.genworth.com/corporate/about-genworth/industry-expertise/cost-of-care.html`

where r_f denotes a constant risk-free interest rate. In the premium calculation, we do not explicitly include expense and profit loading since we wish to concentrate on the impact of risk rather than premium loading. However, there is an implicit loading included because we use a risk-free (not risk-adjusted) discount rate to determine the premium. This has only a minor impact on results. We will see later that full LTC insurance coverage is not purchased as would normally occur if the premium were actuarially fair. We assume that the annual effective risk-free interest rate is constant at 2.02% p.a., which is the 10-year Treasury yield curve rate on February 13, 2015.[5] The equivalent continuously compounded risk-free interest rate is 2% p.a.

The calculated lump sum LTC insurance premiums for different inception ages are shown in Figure 4. Reflecting the higher prevalence of disability, premiums for females are substantially higher than those for males. The premiums drop substantially at older inception ages when the elderly have a much shorter life expectancy, even though they are more likely to transit into the disabled states and thus claim more LTC costs.

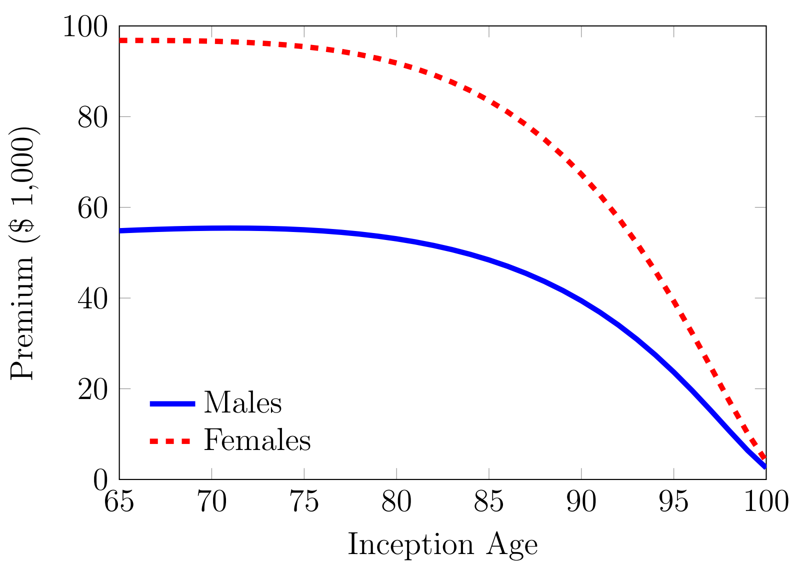

Figure 4. Lump sum premiums for purchasing LTC insurance at different inception ages.

[5]Data source: U.S. Department of the Treasury at `http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yieldYear&year=2015`

2.4 Housing

2.4.1 Housing Consumption

We include housing consumption in the utility function. Individuals typically adjust their housing consumption downward in the face of large financial and health shocks (Davidoff, 2010). To allow for this we assume that individuals liquidate their net home equity upon moving to LTC facilities, which is State 4 in our health transition model. The proceeds from the sale of the house net of any outstanding reverse mortgage loan, if any, are then available to fund the costs associated with staying in LTC facilities.

To simplify the problem, we assume that housing consumption at time t, H_t, is proportional to an individual's initial housing value at her retirement age, HV_0, with a larger multiplier, h_1, when she stays at home and a smaller multiplier, h_2, when she moves to a LTC facility, indicating she downsizes her housing consumption from State 1, 2, or 3 to State 4.

$$H_t = \begin{cases} h_1 HV_0, & \Lambda_t \in \{1,2,3\} \\ h_2 HV_0, & \Lambda_t = 4 \\ 0, & \Lambda_t = 5 \end{cases} \tag{6}$$

where $h_1 > h_2$. In the base model, we assume $h_1 = 5\%$ and $h_2 = 2.5\%$.

2.4.2 House Price Capital Growth

Housing is not only a significant component of individual consumption but also a form of precautionary savings against health shocks. Therefore, a house price growth model is needed in order to better predict house values in the future.

A consensus in the house price dynamics literature is that the house value is not stationary. Cunningham and Hendershott (1984) and Kau *et al.* (1993) model house values as a geometric Brownian motion. Under this model setup, nonstationarity arises because the cumulative house price growth rate is normally distributed, with the mean and standard deviation growing over time; the house price return is a random walk and thus has no memory, suggesting that previous house values do not help to predict future values. However, tests of efficient market hypothesis (EMH) in the real estate markets provide contradictory results. Positive autocorrelations in the house price indices have been found in the U.S. (Case and Shiller, 1989), Canada (Hosios and Pesando, 1991), and the U.K. (Institute and Faculty of Actuaries, 2005). Autocorrelation means that there exists some memory in the house price series (Szymanoski, 1994), so it is natural to apply time series analysis to the real estate market. Particularly, the development of ARCH and GARCH types of models relaxes the assumption of a constant error variance, which can nicely explain the increasing volatility of house price dynamics.

We follow the previous literature (see, for example, Chen *et al.*, 2010; Chinloy *et al.*, 1997; Li *et al.*, 2010) and use an ARMA-GARCH process to model house price growth rates, which can be expressed in the following equations:

$$\begin{aligned} y_t &= \psi_y + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j z_{t-j} + z_t, \\ \sigma_t^2 &= \psi_{\sigma^2} + \sum_{i=1}^{m} \mu_i \sigma_{t-i}^2 + \sum_{j=1}^{n} \nu_j z_{t-j}^2, \end{aligned} \tag{7}$$

where y_t is the continuously compounded house price growth rate at time t, ψ_y is the constant term for house price growth rate series, p is the lag length of the autocorrelation term, ϕ_i is the coefficient for the ith autocorrelation term, q is the lag length of the moving average term, θ_j is the coefficient for the jth moving average term, z_t is a series of independently distributed normal variables, σ_t^2 is the conditional variance of z_t given information up to time $t-1$, ψ_{σ^2} is the constant term for the conditional variance process, m is the lag length of the GARCH term, μ_i is the coefficient for the ith GARCH term, n is the lag length of the ARCH term, and ν_j is the coefficient for the jth ARCH term.

We estimate our house price model using the U.S. nationwide quarterly house price index from the first quarter of 1975 to the last quarter of 2014. The sample auto-correlation function of log returns of the house price index shows a lag of 9, so we perform the Augmented Dickey-Fuller (ADF) test with a lag of 9 on house price growth rates. The ADF test rejects the null hypothesis of a unit root at the 5% significance level, indicating that the series of the house price growth rates is stationary (see Li *et al.*, 2010).

Based on the approach in Box and Jenkins (1976), we select the optimal lags for the conditional mean and conditional variance models using the Bayesian Information Criteria (BIC). The optimal model specification is ARMA(2,4)-GARCH(1,1). Parameter estimates are shown in Table 1.

Table 1. Parameter estimates for the ARMA(2,4)-GARCH(1,1) model on house value growth rates.

Parameter	**Value**	**Standard Error**	***t*-Statistic**
ψ_y	0.0024	0.0011	2.1592
ϕ_1	0.2799	0.1576	1.7763
ϕ_2	0.5485	0.1610	3.4077
θ_1	0.2375	0.1677	1.4159
θ_2	-0.6136	0.1145	-5.3597
θ_3	-0.0012	0.1082	-0.0112
θ_4	0.4184	0.0880	4.7518
ψ_{σ^2}	4.16E-06	2.79E-06	1.4915
μ_1	0.7202	0.1201	5.9974
ν_1	0.2084	0.0858	2.4295

Future house values are simulated using this estimated ARMA-GARCH process, assuming an initial house value of $300,000 at age 65. Projected house values with 95% confidence intervals for the next 35 years are shown in Figure 5. The average house price increases significantly at older ages so that this illiquid asset becomes a substantial component of future wealth for individuals with home equity at retirement. We also observe a large variation in possible house prices at later ages, indicating that home equity is a significantly risky asset in our model.

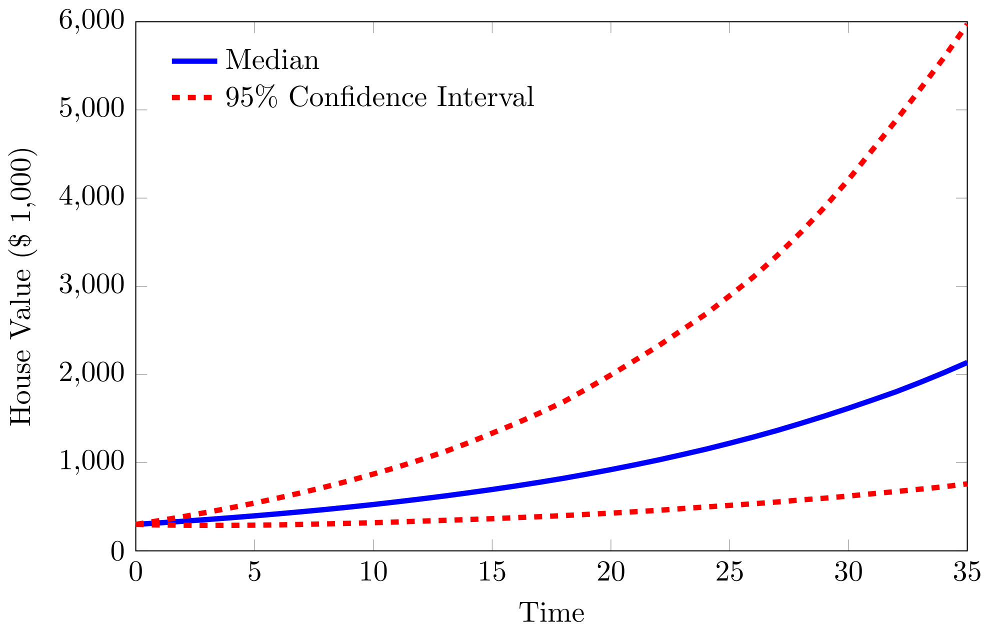

Figure 5. House value projections based on the ARMA(2,4)-GARCH(1,1) model of house value growth rates. The current house value is assumed to be $300,000.

2.5 Reverse Mortgage

We assume a retiree at age 65 takes out a lump sum reverse mortgage loan against home equity. The amount of this loan is one of the decision variables. The outstanding balance of the reverse mortgage loan accumulates at a rate which is the sum of the risk-free interest rate, a spread margin and a mortgage insurance premium (MIP) rate. The MIPs are paid by reverse mortgage borrowers to the lender for providing a no negative equity guarantee (see Chen *et al.*, 2010; Shao *et al.*, 2015). When the individual moves out of the house or dies, the reverse mortgage loan needs to be repaid using the sale proceeds of the house. In the event that the loan outstanding balance exceeds the house value, the no negative equity guarantee ensures that the lender cannot pursue the borrower or the beneficiary for the shortfall. Instead, the mortgage insurance will meet the shortfall, if any. The spread margin is assumed to be zero since we aim to minimize the effect of any product loading on our results. We assume an annual MIP rate of 1.5% that is an approximation based on current HECM charges.[6]

Let RM denote the percentage of the house value that an individual takes out at retirement as the reverse mortgage loan. Then the lump sum loan amount at retirement is $RM \cdot HV_0$. The dynamics for the outstanding loan balance at time t, $RMLB_t$, is given by the following equation:

$$RMLB_t = \begin{cases} RM \cdot HV_0 \cdot e^{\left(r_f+\pi\right)t}, & \Lambda_t \in \{1,2,3\} \\ 0, & \Lambda_t \in \{4,5\} \end{cases} \tag{8}$$

where r_f is the continuously compounded risk-free interest rate, and π is the annual MIP rate. The loan outstanding, $RMLB_t$, accrues with interests and MIPs as long as the individual still lives in the house (in State 1, 2, and 3). Once the individual moves to a nursing home or is dead (in State 4 or 5), the reverse mortgage loan is repaid and the value of $RMLB_t$ becomes zero.

We assume that individuals can only borrow against home equity at age 65. The loan amount forms part of their liquid assets which are optimally spread over time by investing at the risk-free interest rate to fund consumption.

[6] In practice, MIPs under the HECM program are composed of two parts: an initial charge that equals 0.5% or 2.5% of the appraised house value depending on the initial disbursement, and annual charges that equal 1.25% of the outstanding loan balance. The initial MIP is deducted from the loan proceeds and the annual insurance charges accrue with the loan balance over time. The initial MIP is 0.5% of the appraised house value when one does not take out 60% of the available fund (i.e., the loan-to-value ratio is less than 0.6) in the first year. If the loan-to-value ratio exceeds 0.6, the upfront MIP is 2.5% of the appraised house value. See `http://www.reversemortgage.org/gethelp/mostfrequentlyaskedquestions.aspx#mip` for detailed information. We limit loan-to-value ratios for all reverse mortgage loans to be lower than 0.6.

2.6 Optimization Problem

Dynamic programming is used to derive the optimal consumption after retirement. The probability space is denoted by $\left(\Omega, \mathbb{F} = \{\mathcal{F}_t\}_{0\leq t\leq T}, P\right)$, where Ω is the sample space, $\mathcal{F}_t$ represents the information up to time t, T is the time horizon, and P is the probability measure defined on $(\Omega, \mathbb{F})$. Portfolio decisions are $\mathcal{F}_t$-measurable. Based on the modeled health state transitions and the portfolio choices for reverse mortgages and LTC insurance, the utility maximization problem of an x-year-old individual currently in health state $\Lambda_0 = i \in \{1, 2, 3, 4\}$ is given in the following equation:

$$\max_{\{O_t\}_{t=0}^{T-1}} \left\{ \sum_{t=0}^{T-1} \alpha^t \mathbb{E}\Big[\sum_{j\neq 5} p_{x:x+t}^{ij} U(C_t, H_t) + \alpha \left(p_{x:x+t+1}^{i5} - p_{x:x+t}^{i5} \right) B(W_{t+1}) \Big] \right\}, \tag{9}$$

where $O_t = (C_t,\ RM,\ PI)$ denotes the choice decision vector, including consumption at time t, the percentage of home value for a reverse mortgage loan taken at retirement and private LTC insurance coverage at retirement, and α is the utility discount parameter.

At retirement, an individual makes a decision with respect to the first year's consumption, how much reverse mortgage to take out, and how much LTC insurance to purchase. At the end of each period, the individual's health status changes and she stays in the updated health state for the following period. If the individual dies, the reverse mortgage loan is repaid and the total wealth (including liquid wealth and the net housing asset) becomes a bequest to the heir. If the individual moves into a LTC facility, the reverse mortgage loan is also repaid so that net home equity is liquidated. As long as the individual survives, at the beginning of each period she first sets aside the LTC expenses for the whole year, if any, based on her health state at that time and then she determines how much to consume for the following year.

The utility maximization problem of an x-year-old, as described in Equation (9), is solved using the following Bellman equation:

$$\begin{aligned} V(t,i,G_t) &= \max_{C_t} \mathbb{E}\left[U(C_t,H_t) + \alpha \left(\sum_{j\neq 5} p_{x+t}^{ij} V(t+1,j,G_{t+1}) + p_{x+t}^{i5} B(W_{t+1}) \right) \Big| \mathcal{F}_t \right], \\ 0 &\leq t \leq T-1, \end{aligned} \quad (10)$$

where the value function V at time t for health state i depends on a vector of non-health state variables, $G_t = (B_t,\ RM,\ PI,\ HV_{1:t})$, including liquid wealth B_t , the percentage of the reverse mortgage loan taken out of house equity, the coverage of private LTC insurance, and the house value up to time t. p_{x+t}^{ij} is the annual transition probability from State i to State j for an individual that is $x+t$ years old, i.e., $p_{x+t}^{ij} = p_{x+t:x+t+1}^{ij}$. Other notations are the same as those in Equation (9).

Let $\Lambda_t = i \in \{1,2,3,4,5\}$ denote the health status at time t. The utility maximization problem is subject to budget constraints in Equations (11) - (17).

$$B_1 = \begin{cases} e^{r_f}\Big[B_0 - C_0 + RM - \pi_x PI\Big], & i=1 \\ e^{r_f}\Big[B_0 - C_0 + RM - (1-GI)\cdot LTC_0^i\Big], & i=2 \\ e^{r_f}\Big[B_0 - C_0 + RM - (1-GI)\cdot LTC_0^i\Big], & i=3 \\ e^{r_f}\Big[B_0 - C_0 - (1-GI)\cdot LTC_0^i\Big], & i=4 \end{cases} \quad (11)$$

For $1 \leq t \leq T-1$

$$B_{t+1} = \begin{cases} e^{r_f}\Big[B_t - C_t\Big], & i=1 \\ e^{r_f}\Big[B_t - C_t - (1-GI)\cdot LTC_t^i\Big], & i=2 \\ e^{r_f}\Big[B_t - C_t - (1-GI-PI)\cdot LTC_t^i\Big], & i=3 \\ e^{r_f}\Big[B_t - C_t + \mathbb{1}_{\{\Lambda_{t-1}\in\{1,2,3\}\}}\cdot \max\left\{HV_t - RMLB_t,\ 0\right\} & \\ \quad -(1-GI-PI)\cdot LTC_t^i\Big], & i=4 \end{cases} \tag{12}$$

$$W_{t+1} = \begin{cases} B_{t+1} + \max\left\{HV_{t+1} - RMLB_{t+1},\ 0\right\}, & i \in \{1,2,3\} \\ B_{t+1}, & i=4 \end{cases} \tag{13}$$

$$B_t \geq 0, \tag{14}$$

$$C_t \geq \underline{C}, \tag{15}$$

$$0 \leq RM \leq MLTV_x, \tag{16}$$

$$0 \leq PI \leq 1-GI, \tag{17}$$

where $\underline{C}$ is the consumption floor that guarantees a minimum standard of living, $MLTV_x$ is the age-dependent maximum loan-to-value ratio of reverse mortgage loans, and GI is the percentage cover provided by the government compulsory LTC program. In the U.S., about 71% of LTC expenses are covered by the public funded program (i.e., Medicaid and Medicare) and depend on an individual's wealth (Lewin Group, 2010). However, with funding deficiencies at both the state and federal government levels, there is an increasing need to fund LTC costs by individuals through either out-of-pocket savings or private insurance plans. Therefore, we assume that the government only funds a relatively small percentage of LTC costs in our model (i.e., $GI = 0.1$) in order to explore the role of private LTC insurance in retirement planning and its welfare implication.

Equation (12) describes the dynamics of liquid wealth for each of the four alive health states at time t. Equation (13) links bequest wealth to liquid wealth. Equation (14) avoids negative liquid assets. Equation (15) imposes a consumption floor. Equation (16) ensures that the retiree cannot borrow from home equity more than the maximum loan-to-value ratio. Equation (17) eliminates short-sales of private LTC insurance and ensures that individuals do not purchase private LTC insurance that is already covered by the government program.

2.7 Optimization Conditions

In this subsection, we derive the first-order condition and the envelope condition. These conditions provide the basis for the computational approach that we apply in order to solve the optimization problem numerically.

2.7.1 First-Order Condition

Denote $F(t,i) = \mathbb{E}\left[U(C_t, H_t) + \alpha\left(\sum_{j\neq 5} p_{x+t}^{ij} V(t+1, j, G_{t+1}) + p_{x+t}^{i5} B(W_{t+1})\right) \,\Big|\, \mathcal{F}_t\right]$, which would become the value function expressed in Equation (10) if the optimal values of C_t were used. The first-order condition is obtained by equating to zero the first-order partial derivative of $F(t,i)$ with respect to C_t.

For $i \in \{1, 2, 3, 4\}$, the first-order partial derivative of $F(t,i)$ with respect to C_t is as follows:

$$\begin{aligned}
\frac{\partial F(t,i)}{\partial C_t} &= \frac{\partial U(C_t, H_t)}{\partial C_t} + \alpha \mathbb{E}\left[\sum_{j\neq 5} p_{x+t}^{ij} \frac{\partial V(t+1, j, G_{t+1})}{\partial C_t} + p_{x+t}^{i5} \frac{\partial B(W_{t+1})}{\partial C_t} \,\Big|\, \mathcal{F}_t\right] \\
&= \eta C_t^{\nu-\gamma} H_t^{-\nu} + \alpha \mathbb{E}\left[\sum_{j\neq 5} p_{x+t}^{ij} V_B^{'}(t+1, j) \frac{\partial B_{t+1}}{\partial C_t} + p_{x+t}^{i5} \left(\phi + \frac{W_{t+1}}{\beta}\right)^{-\gamma} \frac{\partial W_{t+1}}{\partial C_t} \,\Big|\, \mathcal{F}_t\right],
\end{aligned}$$

where $\nu = (\gamma - 1)(1 - \eta)$, and $V_B'(t+1, j)$ denotes the partial derivative of $V(t+1, j, G_{t+1})$ with respect to B_{t+1}.

From Equation (12) and Equation (13), the partial derivatives of B_{t+1} and W_{t+1} with respect to C_t are

$$\frac{\partial W_{t+1}}{\partial C_t} = \frac{\partial B_{t+1}}{\partial C_t} = -e^{r_f}.$$

Therefore, the partial derivative of $F(t,i)$ with respect to C_t becomes

$$\frac{\partial F(t,i)}{\partial C_t} = \eta C_t^{\nu-\gamma} H_t^{-\nu} - \alpha \mathbb{E}\left[e^{r_f}\left(\sum_{j\neq 5} p_{x+t}^{ij} V_B^{'}(t+1,j) + p_{x+t}^{i5}\left(\phi + \frac{W_{t+1}}{\beta}\right)^{-\gamma}\right) \Big| \mathcal{F}_t\right]. \tag{18}$$

By equating $\partial F(t,i)/\partial C_t$ to zero, we get the first-order condition with respect to C_t. For $0 \leq t \leq T-1$,

$$\eta C_t^{\nu-\gamma} H_t^{-\nu} = \alpha \mathbb{E}\left[e^{r_f}\left(\sum_{j\neq 5} p_{x+t}^{ij} V_B^{'}(t+1,j) + p_{x+t}^{i5}\left(\phi + \frac{W_{t+1}}{\beta}\right)^{-\gamma}\right) \Big| \mathcal{F}_t\right], \tag{19}$$

or,

$$C_t = \left\{\frac{\alpha H_t^{\nu}}{\eta} E_t\left[e^{r_f}\left(\sum_{j\neq 5} p_{x+t}^{ij} V_B^{\prime}(t+1,j) + p_{x+t}^{i5}\left(\phi + \frac{W_{t+1}}{\beta}\right)^{-\gamma}\right)\right]\right\}^{\frac{1}{\nu-\gamma}}. \tag{20}$$

For the initial period (i.e., $t = 0$), we need to select the optimal grids for the reverse mortgage loan, RM, and private LTC insurance coverage, PI, that maximize the individual's utility.

2.7.2 Envelope Conditions

Using Equation (10), for $i \in \{1,2,3,4\}$ the first-order partial derivative of $V(t,i,G_t)$ with respect to B_t is

$$\begin{aligned} V_B^{'}(t,i) &= \frac{\partial U(C_t,H_t)}{\partial C_t}\frac{\partial C_t}{\partial B_t} + \alpha \mathbb{E}\left[\sum_{j\neq 5} p_{x+t}^{ij} V_B^{'}(t+1,j)\frac{\partial B_{t+1}}{\partial B_t} + p_{x+t}^{i5}\frac{\partial B(W_{t+1})}{\partial B_t} \,\Big|\, \mathcal{F}_t\right] \\ &= \eta C_t^{\nu-\gamma} H_t^{-\nu}\frac{\partial C_t}{\partial B_t} + \alpha \mathbb{E}\left[\sum_{j\neq 5} p_{x+t}^{ij} V_B^{'}(t+1,j)\frac{\partial B_{t+1}}{\partial B_t} + p_{x+t}^{i5}\left(\phi + \frac{W_{t+1}}{\beta}\right)^{-\gamma}\frac{\partial W_{t+1}}{\partial B_t} \,\Big|\, \mathcal{F}_t\right]. \end{aligned}$$

From Equation (12) and Equation (13), the partial derivatives of B_{t+1} and W_{t+1} with respect to B_t are

$$\frac{\partial W_{t+1}}{\partial B_t} = \frac{\partial B_{t+1}}{\partial B_t} = e^{r_f}\left[1 - \frac{\partial C_t}{\partial B_t}\right].$$

Therefore, the envelope condition is:

$$\begin{aligned} V_B^{'}(t,i) &= \eta C_t^{\nu-\gamma} H_t^{-\nu}\frac{\partial C_t}{\partial B_t} + \alpha \mathbb{E}\left[\sum_{j\neq 5} p_{x+t}^{ij} V_B^{'}(t+1,j) e^{r_f} + p_{x+t}^{i5}\left(\phi + \frac{W_{t+1}}{\beta}\right)^{-\gamma} e^{r_f} \,\Big|\, \mathcal{F}_t\right]\left[1 - \frac{\partial C_t}{\partial B_t}\right] \\ &= \eta C_t^{\nu-\gamma} H_t^{-\nu}\frac{\partial C_t}{\partial B_t} + \eta C_t^{\nu-\gamma} H_t^{-\nu}\left[1 - \frac{\partial C_t}{\partial B_t}\right] \\ &= \eta C_t^{\nu-\gamma} H_t^{-\nu}, \end{aligned} \tag{21}$$

where the second last equation is based on the first-order condition with respect to consumption in Equation (19).

Incorporating Equation (21) into Equation (20), we get the following Euler equation:

$$C_t = \left\{\frac{\alpha H_t^{\nu}}{\eta} E_t\left[e^{r_f}\left(\sum_{j\neq 5} p_{x+t}^{ij}\eta C_{t+1}^{\nu-\gamma} H_{t+1}^{-\nu} + p_{x+t}^{i5}\left(\phi + \frac{W_{t+1}}{\beta}\right)^{-\gamma}\right)\right]\right\}^{\frac{1}{\nu-\gamma}}. \tag{22}$$

3 Algorithm: Endogenous Grid Method with Simulation

The algorithm used in our utility maximization is a novel combination of methods developed in the economic literature to improve computational efficiency and avoid the curse of dimensionality. In this section, we set out the algorithm steps since this is new to research on health state transition and optimal portfolio choice models involving house price dynamics.

3.1 Endogenous Grid Method

The conventional approach to solve optimal portfolio and consumption problems is to use grids for the state variables and to solve recursively on the grid based on the first-order conditions derived from the Bellman equation. Although this approach is intuitive, it becomes excessively time-consuming when the number of state variables increases because multi-dimensional root-finding is required to solve the first-order conditions. Our approach is to incorporate the endogenous grid method (EGM) that was first introduced by Carroll (2005) to improve computing efficiency. Compared to conventional dynamic programming, EGM avoids the time-consuming process of finding roots of non-linear equations by predefining after-consumption wealth.

Note that consumption at time t appears on both sides of Equation (22) because W_{t+1} is a function of C_t. Solving for the first-order condition would require a time-consuming root-finding process. Instead, we define the grid as after-consumption wealth, the current wealth subtracting all payouts including expenses and consumption. Let $\tilde{B}_t$ denote the after-consumption wealth at time t. Then the next period's wealth B_{t+1} can be calculated as $\tilde{B}_t$ accumulated by the risk-free interest rate,

$$B_{t+1} = \tilde{B}_t \ e^{r_f}. \tag{23}$$

Now W_{t+1} becomes a function of $\tilde{B}_t$, so C_t does not appear on the right hand side of Equation (22), which can then be easily solved for. Refer to Carroll (2005) for more detail.

We pre-define grids for state variables that include three grids for our continuous state variables and one grid for our discrete state variable. The three-dimensional grids for continuous state variables are defined as

$$\mathcal{G} := \mathcal{G}^{\tilde{B}} \otimes \mathcal{G}^{RM} \otimes \mathcal{G}^{PI}, \tag{24}$$

$$\mathcal{G}^{\tilde{B}} := \{\tilde{B}^1, \tilde{B}^2, \cdots, \tilde{B}^J\}, \tag{25}$$

$$\mathcal{G}^{RM} := \{RM^1, RM^2, \cdots, RM^K\}, \tag{26}$$

$$\mathcal{G}^{PI} := \{PI^1, PI^2, \cdots, PI^L\}, \tag{27}$$

where $\mathcal{G}^{\tilde{B}}$ denotes grids for after-consumption wealth, $\mathcal{G}^{RM}$ denotes grids for the reverse mortgage loan-to-value ratio, $\mathcal{G}^{PI}$ denotes grids for the private LTC insurance coverage, and $\otimes$ denotes the outer product operation. The grid for the discrete state variable (i.e., health status) is defined as

$$\mathcal{G}^{\Lambda} = \{1, 2, 3, 4\}. \tag{28}$$

The four-dimensional grids are then constructed as

$$\mathcal{G} \otimes \mathcal{G}^{\Lambda} := \mathcal{G}^{\tilde{B}} \otimes \mathcal{G}^{RM} \otimes \mathcal{G}^{PI} \otimes \mathcal{G}^{\Lambda}. \tag{29}$$

3.2 A Regression Approach

The path-dependent house price model would normally require increased dimensions in the grids for house value paths, requiring additional state variables for lagged house values. To avoid this and improve computational efficiency we use a linear regression method to estimate a parametric function for the optimal choice variables with respect to past and current house values, based on simulated house value paths. This method was first proposed by Brandt *et al.* (2005). This regression method also avoids computationally intensive multi-dimensional simulations in calculating conditional expectations.

We first simulate N house value sample paths with $HV_{1:t}^{(l)}$ denoting the lth sample path of simulated house values from time 1 to t, where $l = 1, 2, \cdots, N$. At each grid point $g \in \mathcal{G}$ and health state grid point $i \in \mathcal{G}^{\Lambda}$, and for each simulated house value path, the optimal consumption can be expressed as a function of an expectation function that is conditional on realized house values. This is given by the following equations:

$$\begin{aligned} \tilde{C}_t(g, i, HV_{1:t}^{(l)}) &= \zeta\Big(\varepsilon_t\big(g, i, HV_{1:t}^{(l)}\big)\Big), \qquad (30) \\ \varepsilon_t\big(g, i, HV_{1:t}^{(l)}\big) &= E_t\Big[\eta\left(g, i, HV_{1:t}^{(l)}, HV_{t+1}\right) \Big| \mathcal{F}_t\Big], \qquad (31) \end{aligned}$$

where $\tilde{C}_t(g, i, HV_{1:t}^{(l)})$ is the optimal consumption at time t, $\varepsilon_t\big(g, i, HV_{1:t}^{(l)}\big)$ denotes the expectation function, $\zeta(\cdot)$ denotes the function as above, and $\eta(\cdot)$ denotes the function within the conditional expectation operator.

Based on the method described in Brandt *et al.* (2005), we approximate the function within the expectation operator using the following parametric formula:

$$\eta\left(g, i, HV_{1:t}^{(l)}, HV_{t+1}\right) = a(g, i) + \sum_{k=0}^{K}\sum_{d=1}^{D} b_{kd}(g, i)\left(HV_{t-k}^{(l)}\right)^d + e_l, \qquad (32)$$

where K is the lag length of past house values that have impacts on the optimal portfolio choice, D is the polynomial order of house values, $a(g, i)$ and $b_{kd}(g, i)$ are parameters to be estimated, and e_l is a random error that corresponds to uncertainties in HV_{t+1}.

Therefore, instead of evaluating the conditional expected values in Equation (31), we perform a linear regression of $\eta\left(g, i, HV_{1:t}^{(l)}, HV_{t+1}\right)$ against simulated house value paths. The fitted dependent variable is then used as the conditional expectation. The optimal choice variable is calculated as follows:

$$\hat{C}_t(g, i, HV_{1:t}) = \zeta\Big(\hat{\varepsilon}_t\big(g, i, HV_{t-K:t}\big)\Big), \tag{33}$$

$$\hat{\varepsilon}_t(g, i, HV_{t-K:t}) = \hat{a}(g, i) + \sum_{k=0}^{K}\sum_{d=1}^{D} \hat{b}_{kd}(g, i)\,(HV_{t-k})^d\,, \tag{34}$$

where $\big(\hat{a}(g, i),\ \hat{b}_{kd}(g, i)\big)$ are fitted values of the parameters in the linear regression expressed in Equation (32).

For continuous state variable values that are not equal to a grid point, we use linear interpolations of the estimated parameters. If $g' \notin \mathcal{G}$, we can always find $g_1 \in \mathcal{G}$ and $g_2 \in \mathcal{G}$ that are two neighbour points of g'. The value of the conditional expectation function at g' is then estimated using the following equation:

$$\hat{\varepsilon}_t(g', i, HV_{t-K:t}) = \hat{a}(g', i) + \sum_{k=0}^{K}\sum_{d=1}^{D} \hat{b}_{kd}(g', i)\,(HV_{t-k})^d\,, \tag{35}$$

where $\big(\hat{a}(g', i),\ \hat{b}_{kd}(g', i)\big)$ are linear interpolations of $\big(\hat{a}(g_1, i),\ \hat{b}_{kd}(g_1, i)\big)$ and $\big(\hat{a}(g_2, i),\ \hat{b}_{kd}(g_2, i)\big)$. The optimal choice variable at grid point g' can then be determined using Equation (33).

3.3 Algorithm for Terminal Period Optimization

Based on the grids specified in Section 3.1, and given any $g \in \mathcal{G}$ and the health state $i \in \mathcal{G}^{\Lambda}$, the optimal consumption at time $T-1$ can be calculated using the first-order condition.

3.3.1 For $i \in \{1,\ 2,\ 3\}$

$$\begin{aligned} & C_{T-1}(g, i, HV_{1:T-1}) \\ = \ & \left\{ \frac{\alpha H_{T-1}^{\nu}}{\eta} E_{T-1}\Big[e^{r_f} \Big(\phi + \frac{e^{r_f} \tilde{B}_{T-1}^{g} + \max\left\{HV_T - RMLB_T,\ 0\right\}}{\beta} \Big)^{-\gamma} \Big] \right\}^{\frac{1}{\nu-\gamma}}, \end{aligned} \tag{36}$$

where $\tilde{B}_t^g$ denotes the after-consumption wealth on grid point g at time t.

We then use the regression method in Section 3.2 to evaluate the optimal consumption.

3.3.2 For $i = 4$

$$\begin{aligned} C_{T-1}(g,i,HV_{1:T-1}) &= \left\{\frac{\alpha H_{T-1}^{\nu}}{\eta} E_{T-1}\left[e^{r_f}\left(\phi + \frac{e^{r_f}\tilde{B}_{T-1}^g}{\beta}\right)^{-\gamma}\right]\right\}^{\frac{1}{\nu-\gamma}} \\ &= \left\{\frac{\alpha H_{T-1}^{\nu} e^{r_f}}{\eta\left(\phi + \frac{e^{r_f}\tilde{B}_{T-1}^g}{\beta}\right)^{\gamma}}\right\}^{\frac{1}{\nu-\gamma}}. \end{aligned} \tag{37}$$

For health state 4, the optimal consumption is independent of next period's house value and becomes deterministic. Therefore, we can directly evaluate the optimal consumption when the individual is in a nursing home.

3.4 Algorithm for Intermediate Period Optimization

We then solve for the optimal portfolio choice for intermediate periods using backward induction. At each intermediate time t, grid point $g \in \mathcal{G}$ and health state $i \in \mathcal{G}^{\Lambda}$, we evaluate the optimal consumption at time t using the regression method.

3.4.1 For $i \in \{1, 2, 3\}$

The optimal consumption at time t amounts to solving the following equation:

$$\begin{aligned} C_t(g,i,HV_{1:t}) = &\left\{\frac{\alpha H_t^{\nu}}{\eta} E_t\left[e^{r_f}\left(\sum_{j\neq 5} p_{x+t}^{ij}\eta C_{t+1}(g',j,HV_{1:t+1})^{\nu-\gamma} H_{t+1}^{-\nu}\right.\right.\right. \\ &\left.\left.\left. + p_{x+t}^{i5}\left(\phi + \frac{e^{r_f}\tilde{B}_t^g + \max\left\{HV_{t+1} - RMLB_{t+1}, 0\right\}}{\beta}\right)^{-\gamma}\right)\right]\right\}^{\frac{1}{\nu-\gamma}} \end{aligned} \tag{38}$$

where g' denotes the grid point at time $t+1$, and $C_{t+1}(g', j, HV_{1:t+1})$ is the optimal consumption at time $t+1$ at grid point g' and health state j.

3.4.2 For $i = 4$

For an individual staying in a nursing home, we calculate the optimal consumption as follows:

$$\begin{aligned} & C_t(g, i, HV_{1:t}) \\ = & \left\{ \frac{\alpha H_t^{\nu}}{\eta} E_t \left[e^{r_f} \left(p_{x+t}^{i4} \eta C_{t+1}(g', 4, HV_{1:t+1})^{\nu-\gamma} H_{t+1}^{-\nu} + p_{x+t}^{i5} \left(\phi + \frac{e^{r_f} \tilde{B}_t^g}{\beta} \right)^{-\gamma} \right) \right] \right\}^{\frac{1}{\nu-\gamma}}. \end{aligned} \tag{39}$$

Note that the optimal consumption at time $t+1$ does not depend on the house value if the housing asset was released into liquid wealth in the previous period,

$$C_{t+1}\big(g', \Lambda_{t+1} = 4, HV_{1:t+1}\big) = C_{t+1}\big(g', \Lambda_{t+1} = 4\big), \quad \text{for } \Lambda_t = 4. \tag{40}$$

Therefore, given a grid for after-consumption wealth at time t, the time-t optimal consumption for an individual staying in a nursing home is also independent of house value.

4 Optimal Borrowing, Insurance and Consumption

In this section, we first present the optimization results for a base case representing a reasonable set of assumptions for parameters. We then conduct sensitivity analysis by changing the level and composition of total wealth and other parameter values.

4.1 Preference Parameters

The parameter values used in the optimization are given in Table 2. We set these parameter values based on current market conditions and prior studies in this area (e.g, Ameriks *et al.*, 2011; Nakajima and Telyukova, 2017; Yogo, 2016).

Table 2. Parameter values for base-case analysis. Sources are cited in brackets.

Parameter	Explanation	Value	Source
r_f	Risk-free rate	2.00%	U.S.Treasury data
f_t	Inflation rate	1.00%	Assumption
γ	Risk aversion	5	All 3 studies
α	Utility discount factor	0.96	All 3 studies
η	Non-housing consumption aggregation	0.736	Nakajima and Telyukova (2017)
β	Bequest motive strength	32.30	Ameriks *et al.* (2011)
ϕ	Degree of bequest as luxury goods	7.55	Ameriks *et al.* (2011)
W_0	Initial wealth	\$200k	Assumption
H_0	Initial house value	\$300k	Assumption
$\underline{C}$	Consumption floor	\$4,630	Ameriks *et al.* (2011)
h_1	Housing consumption as a proportion of house value if living in the house	5%	Assumption
h_2	Housing consumption as a proportion of house value if living in nursing homes	2.5%	Assumption
LTC^2	Initial annual LTC cost in State 2	\$20k	Genworth data
LTC^3	Initial annual LTC cost in State 3	\$40k	Genworth data
LTC^4	Initial annual LTC cost in State 4	\$80k	Genworth data

4.2 Base Case Analysis

Our analysis is based on a 65-year-old female with an initial endowment of \$200k (which could be the value of the pension at retirement or personal savings) and a house that is worth \$300k. We focus our analysis on females because with longer lives, higher rates of disability and chronic health problems, and lower incomes than men on average, women face greater challenges of health shocks and longevity risk than men. Women who reach age 65 can expect to live an additional 20.5 years, outliving men by 2.5 years on average (CDC, 2015).[7] Women spend twice as many years in a disabled state as men at the end of their lives: 2.8 years if they live past 65, and 3.0 years if they live past 80. More than 70% of nursing home residents, 75.7% of assisted living community residents, and almost two-thirds of formal (paid) home care users and informal (unpaid) care recipients are women.[8]

[7] The life expectancy data is obtained from https://www.cdc.gov/nchs/data/hus/hus15.pdf#015

[8] Data comes from American Association of Long-Term Care Insurance. http://www.aaltci.org/long-term-care-insurance/learning-center/for-women.php

We first solve the optimization problem assuming that an individual does not have access to reverse mortgages or LTC insurance. The optimal consumption paths for different health states are shown in Figure 6. Optimal consumption paths in States 1, 2 and 3 are relatively stable. Those who move into LTC facilities have increased consumption from liquidating their home even after paying for a higher level of LTC costs. This highlights the role that a housing asset plays in meeting LTC costs.

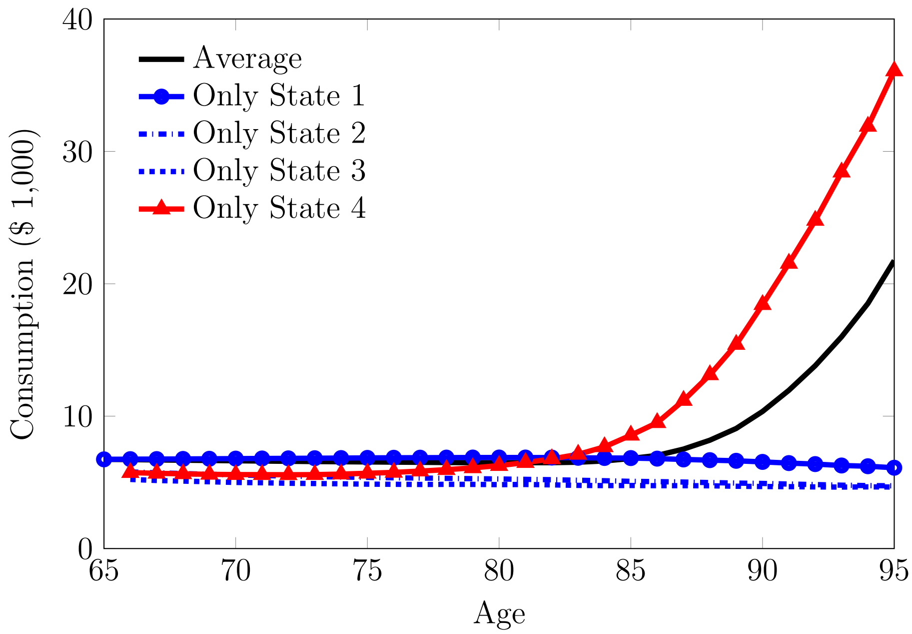

Figure 6. Optimal consumption paths for the 65-year-old female endowed with $200k liquid wealth and a house worth $300k.

We then introduce a reverse mortgage and private LTC insurance into our model framework and solve for optimal consumption at differing levels of loan-to-value ratios (LTVR) and private LTC insurance coverage (PI). The certainty equivalent consumption (CEC), defined as a constant amount of consumption from which an individual can derive the same utility level as the lifetime utility resulting from the optimal consumption path, is determined. We use CEC as the main measure for welfare analysis. Plots of the CEC for each LTVR-PI combination are given in Figure 7. We also report in Table 3 the levels of CEC corresponding to the optimal demand of reverse mortgage or private LTC insurance when these product are added separately, or jointly, to our model framework .

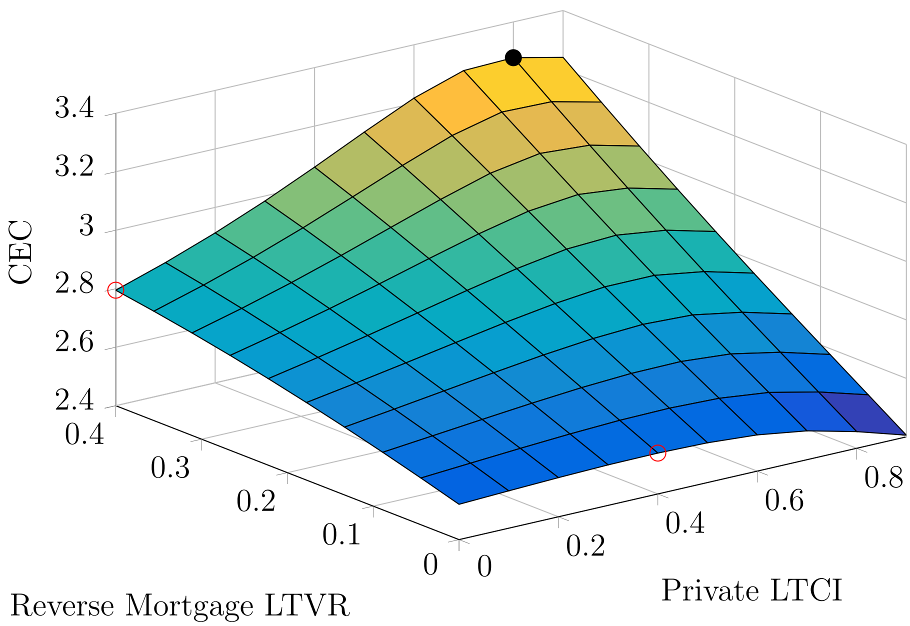

Figure 7. Certainty Equivalent Consumption (CEC) in \$1,000 for a 65-year-old female endowed with \$200k liquid wealth and a house that is worth \$300k. The black point indicates the optimal LTVR-PI combination that yields the highest level of CEC. The red circles indicate the optimal reverse mortgage LTVR or private LTC insurance coverage when individuals have access to only one of these two products.

Table 3. Certainty Equivalent Consumption (CEC) for retirees that have access to reverse mortgage and/or LTC insurance.

	No Private LTCI	**With Private LTCI**
No Reverse Mortgage	\$2,521	\$2,540
With Reverse Mortgage	\$2,795	\$3,278

When the retiree has no access to either reverse mortgage or private LTC insurace, her CEC is \$2,521. If she is offered private LTC insurance without a reverse mortgage, it is optimal for her to purchase 40% of LTC insurance coverage with a slight increase in CEC to \$2,540. When she has access to a reverse mortgage only, her optimal strategy is to take out as much as possible (i.e., 40%) of her house value to fund consumption and possible LTC costs, resulting in a CEC of \$2,795. When both products are available, she will still take out 40% of her current home value but will buy 80% of LTC insurance coverage, driving her CEC to reach the highest level, \$3,278. This demonstrates how having access to a reverse mortgage greatly reduces the retiree's liquidity constraint and therefore doubles her demand for private LTC insurance.

Figure 8 shows the optimal average consumption paths in four cases: (1) no access to either reverse mortgage or private LTC insurance; (2) only access to private LTC insurance; (3) only access to reverse mortgage; and (4) access to both reverse mortgage and private LTC insurance. The corresponding average liquid wealth and bequest wealth paths are shown in Figure 9.

When the retiree has no access to either product, her optimal consumption path is relatively stable for the first 20 years but increases steadily after age 85, reflecting a high probability of moving into a LTC facility and thus liquidating her housing asset at older ages as shown in Figure 3. Her liquid wealth decreases slightly when she ages, but increases significantly after age 85 due to the same reason. We also see a steady increase in her bequest wealth as the retiree becomes older. After she takes out a reverse mortgage loan, her liquid wealth increases for most of the years, compared to the case (1) where no product is available. So she is able to consume a bit more at earlier ages at a cost of a little less consumption at older years. Her bequest wealth decreases for all ages. Having access to private LTC insurance reduces initial liquid wealth and bequest wealth because of the payment of the insurance premium at retirement, but the consumption level in the earlier years is not largely affected. Nevertheless, the consumption level at older ages becomes much higher because LTC insurance transfers wealth, and thus consumption, from the healthy state to mildly or severely disabled states through the insurance mechanism. When the individual is provided with both products, her consumption level increases significantly at all ages, leading to a much higher lifetime utility.

The welfare gains of each product and the bundled product can also be evaluated as the amount of money that an individual would be willing to pay for having access to a reverse mortgage loan and/or private LTC insurance. To calculate the willingness to pay, we first determine a benchmark utility level as the maximum lifetime utility achieved when the individual has access to either product or both. We then calculate the incremental initial

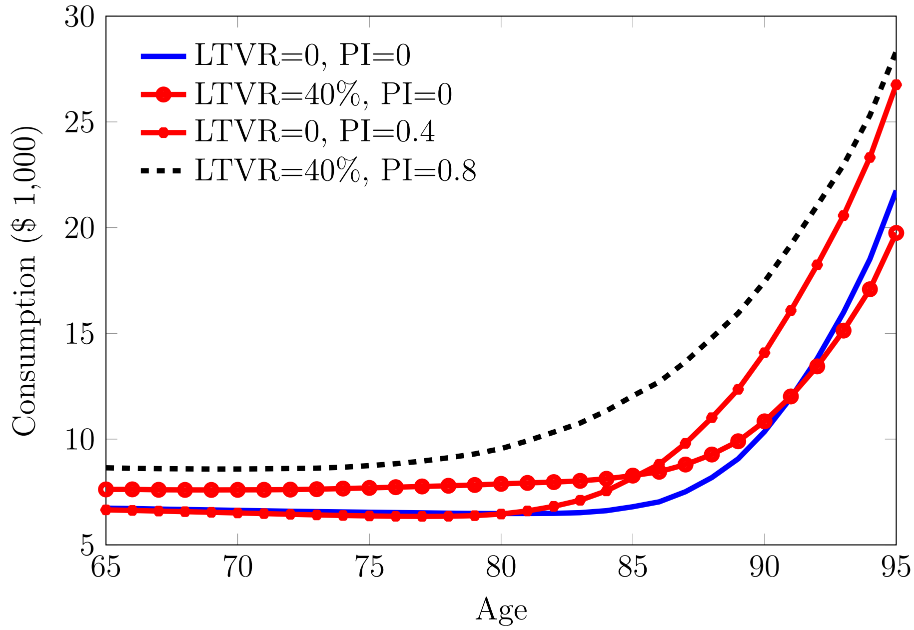

Figure 8. Optimal consumption paths for a 65-year-old female with $200k initial liquid wealth and a house worth $300k.

wealth needed in order to achieve this benchmark utility level when the individual has no access to either product. The increase in initial wealth is defined as the willingness to pay. The results are reported in Table 4.

Table 4. A retiree's willingness to pay (in $1,000) for having access to reverse mortgage and/or LTC insurance.

Reverse Mortgage	**Private LTCI**	**Reverse Mortgage and Private LTCI**
66.09	4.74	175.85

Table 4 shows that the retiree's willingness to pay for having access to private LTC insurance is relatively low, less than $5,000. However, she is willing to pay over $66,000 for having access to a reverse mortgage loan, about fourteen times the amount she is willing to pay for having access to private LTC insurance. Her willingness to pay for both products is as high as $175,850, much larger than the sum of the amount she is willing to pay for having access to the two products separately. This result shows the complementary nature of reverse mortgage and private LTC insurance. It also clearly demonstrates the benefits

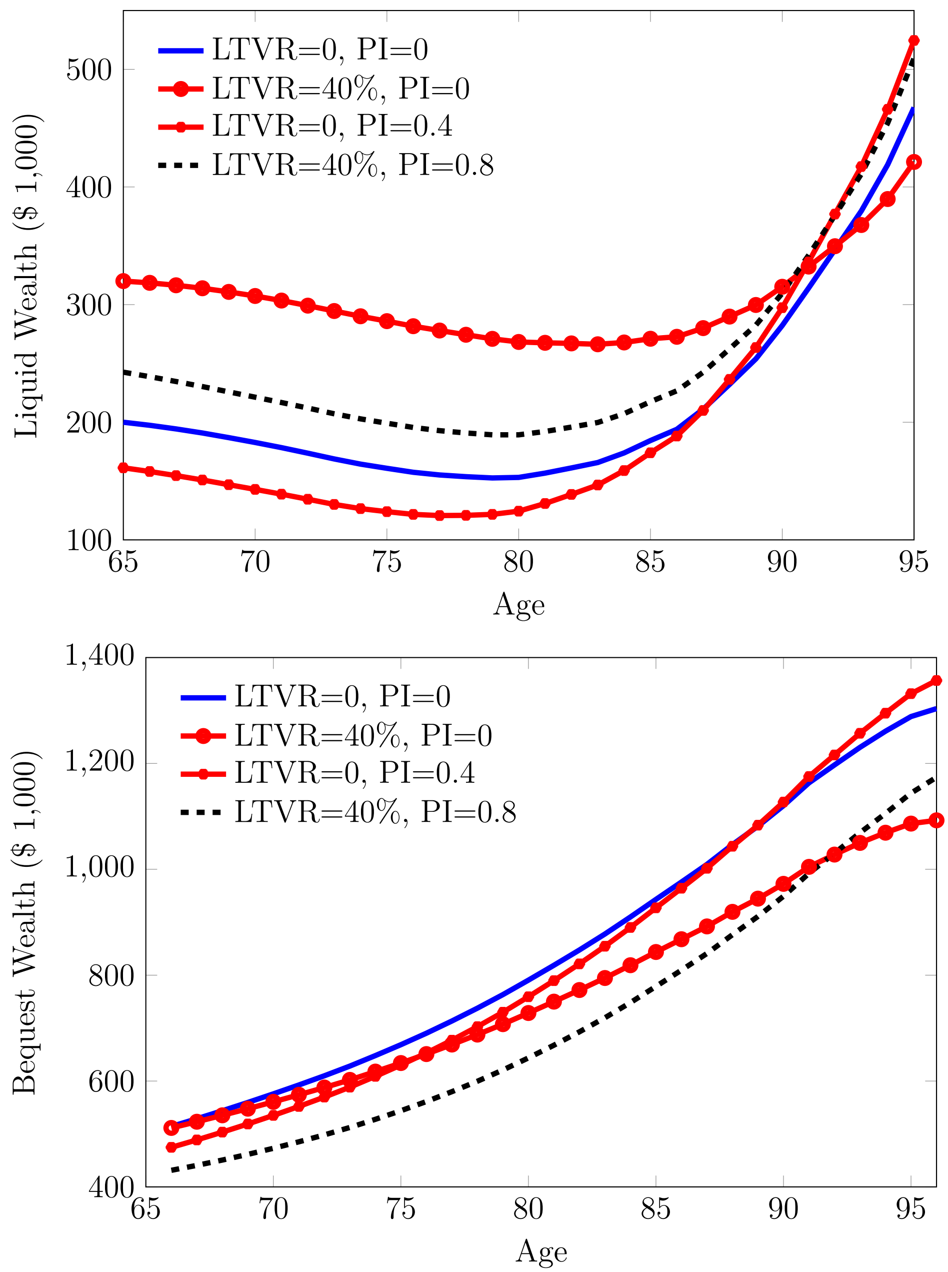

Figure 9. Average wealth paths for a 65-year-old female with $200k initial liquid wealth and a house worth $300k.

from a product that bundles these two products together. A similar conclusion was drawn in Davidoff (2010) with a one-period model.

4.3 Sensitivity Analysis: Wealth Endowment

In the baseline analysis we assume an initial wealth endowment of $500k with 40% in liquid wealth and 60% in housing asset. In this subsection, we change the level of total wealth and the wealth allocation between liquid asset and housing asset in order to allow for individual wealth heterogeneity. We report the optimal demand for reverse mortgage and/or private LTC insurance at different wealth levels and wealth allocations in Table 5.

Table 5. Optimal reverse mortgage LTVR and private LTC insurance coverage PI for different wealth levels (in $1,000) and wealth allocations between liquid asset and housing asset.

Scenario	**Wealth**			**Only RM**	**Only LTCI**	**Both**	
	Total	Liquid	Housing	LTVR	PI	LTVR	PI
Scen 1.1	500	20%	80%	0.4	0	0.4	0.7
Base	500	40%	60%	0.4	0.4	0.4	0.8
Scen 1.2	500	80%	20%	0.1	0.8	0.4	0.9
Scen 2.1	200	20%	80%	0.4	0	0.4	0
Scen 2.2	200	40%	60%	0.4	0	0.4	0
Scen 2.3	200	80%	20%	0.4	0.9	0.4	0.9
Scen 3.1	1,000	20%	80%	0.4	0	0.4	0.6
Scen 3.2	1,000	40%	60%	0.4	0.4	0.4	0.7
Scen 3.3	1,000	80%	20%	0	0.8	0.4	0.9

Table 5 shows that the coverage from LTC insurance increases as the housing asset decreases as a percentage of total wealth, regardless of whether the individual has access to a reverse mortgage or not. When only private LTC insurance is available, and the retiree is extremely asset rich and cash poor (80% of housing asset and 20% of liquid asset), LTC insurance is not purchased since liquid wealth is insufficient to fund consumption as well as the relatively high insurance premiums. Individuals endowed with relatively more liquid assets in their portfolio can purchase more coverage of LTC insurance to hedge potential health shocks. This pattern remains unchanged when individuals have access to both reverse mortgage and private LTC insurance. For example, at a total wealth level of $500k an individual's optimal LTC insurance coverage increases from 0.7 to 0.9 as the percentage of housing asset decreases from 80% to 20%, while this number increases from 0.6 to 0.9 when the total wealth is $1,000k.

The effect of total wealth on the demand of private LTC insurance is nonlinear. To see this, fix the proportion of housing asset (for example, 80%) and only consider the change in total wealth. When an individual is offered both products and has an extremely low level of total wealth, $200k, she chooses not to buy LTC insurance due to the liquidity constraint. When her wealth increases to $500k, the liquidity constraint relaxes and she buys 70% of LTC insurance coverage. When her total wealth amounts to $1,000k, her optimal LTC insurance coverage decreases to 60%. This demonstrates the crowding out effect of housing on LTC insurance. Even when the percentage of housing asset does not change, the absolute level of housing asset increases as total wealth increases. This reduces the demand for LTC insurance since housing equity can be liquidated to cover the expensive LTC costs.

Table 5 also shows that allowing access to reverse mortgage generally increases the demand for LTC insurance, except when the total wealth is low (e.g., $200k) where the demand for LTC insurance remains unchanged. Because for a low level of total wealth the liquidity constraint dominates, having reverse mortgage does not significantly increase the available cash to buy private LTC insurance at all or afford a higher coverage of LTC insurance. At higher total wealth levels, individuals can cash out a significant portion of their housing equity via a reverse mortgage loan, which relaxes the liquidity constraint and thus increases their demand for LTC insurance.

Without a private LTC insurance market, the demand for reverse mortgage is affected by the level and composition of total wealth. For a low level of total wealth, $200k, an individual's optimal strategy is to borrow using a reverse mortgage as much as possible (in our case, 40% of house value) in order to provide additional financial resources for her retirement and potential LTC expenses, as shown in Scenarios 2.1 - 2.3. When the individual is endowed with a high level of total wealth and she is cash rich but asset poor (such as in Scenarios 1.2 and 3.3), her optimal demand for a reverse mortgage loan is reduced to 10% of the house value or even 0, because now she has more liquid wealth to fund her consumption and LTC costs.

However, having access to private LTC insurance can increase an individual's demand for reverse mortgage even when she is cash rich but asset poor and endowed with high levels of total wealth. As can be seen from Scenario 1.2 and 3.3, when both products are offered on the market the individual's optimal strategy is to take out a reverse mortgage loan as much as possible (40% of house value).

4.4 Sensitivity Analysis: Other Parameters

We also conduct sensitivity analysis by changing the values of other parameters, including risk aversion γ, utility discount factor α, bequest motive strength β, degree of bequest as luxury goods ϕ, and base-year LTC expenses. Table 6 shows the various scenarios considered along with the results.

We see that the demand for reverse mortgage is very robust when we use alternative parameter values, regardless of whether the individual has access to LTC insurance or not. The optimal strategy is to borrow as much as possible, 40% of the house value, to maximize consumption.

The demand for private LTC insurance is very sensitive to changes in parameter values when a reverse mortgage loan is not available to the retiree. For example, the retiree buys more coverage of LTC insurance when she becomes more risk averse. In the baseline case where the risk aversion parameter γ is equal to 5, the individual purchases private LTC insurance with 40% of coverage. However, for a less risk-averse individual (i.e., $\gamma = 2$), the optimal strategy is not to purchase private LTC insurance; for a more risk-averse individual (i.e.,

Table 6. Optimal reverse mortgage LTVR and private LTC insurance coverage for different parameter values. Initial wealth endowment is $200k liquid asset and $300k housing asset. Note: In the baseline scenario, we set $\gamma = 5$, $\alpha = 0.96$, $\beta = 32.30$, $\phi = 7.55$, and $LTC = \$[20k, 40k, 80k]$.

Scenario	**Only RM**	**Only LTCI**	**Both**	
	LTVR	PI	LTVR	PI
Base	0.4	0.4	0.4	0.8
Risk aversion parameter				
$\gamma = 2$	0.4	0	0.4	0.8
$\gamma = 10$	0.4	0.6	0.4	0.8
Utility discount factor				
$\alpha = 0.93$	0.4	0.2	0.4	0.7
$\alpha = 0.99$	0.4	0.5	0.4	0.8
Bequest motive strength				
$\beta = 20$	0.4	0.3	0.4	0.7
$\beta = 50$	0.4	0.5	0.4	0.8
Degree of bequest as luxury goods				
$\phi = 6$	0.4	0.6	0.4	0.8
$\phi = 9$	0.4	0.2	0.4	0.7
Initial annual LTC expenses: $[LTC^2,\ LTC^3,\ LTC^4]$				
$[10k 20k 40k]	0.4	0.2	0.4	0.7
$[50k 100k 200k]	0.4	0.1	0.4	0.8

$\gamma = 10$), it is optimal to purchase private LTC insurance that covers 60% of LTC expenses.

We can also see that the demand for private LTC insurance increases with the utility discount factor α and the bequest motive β, but decreases with the degree to which bequests are luxury goods ϕ. When it comes to the effect of LTC expenses, we see that becoming disabled would have less financial impact on the individual's well-being if LTC expenses in various health states were cut in half, so the optimal LTC insurance coverage is reduced to 20%. Interestingly, we find an even larger drop of the private LTC insurance coverage to 10% if LTC expenses were more than doubled. This is because high LTC expenses raise the lump sum insurance premium significantly, leading to a larger negative impact on liquid wealth and thus a smaller coverage for LTC insurance.

The demand for private LTC insurance, however, becomes very robust and stays at a much higher level (either 70% or 80% of coverage) if the individual can liquidate a significant portion of her housing asset via a reverse mortgage.

5 Conclusions

With the retirement of baby-boomers and the increase in the proportion of the elderly in the population, retirement programs in many countries face an aging-population tsunami and significant future imbalances. On the individual side, the increased life expectancy, the shift of pension plans from defined benefit (DB) to defined contribution (DC) and the declining contribution levels from employers impose significant challenges on financial budgets for the aged population after their retirement (Chen *et al.*, 2010). While the elderly may receive reduced monthly incomes, they may also experience deteriorating health conditions and rising health-care costs. It is increasingly difficult for them to maintain financial independence and their standard of living.

In this paper, we investigate a retirees' optimal consumption and portfolio selection problem with a discrete time life cycle model that takes into account longevity risk, health shocks, and house price risk simultaneously. We focus on the roles of LTC insurance and reverse mortgage loans in an individual's retirement planning. We use an endogenous grid method (EGM) along with a regression based approach to improve computational efficiency and avoid the curse of dimensionality. Our computational methodology allows us to extend the prior one-period or two-period model in this literature to a more realistic multi-period setting and provide a more informative assessment of the impact of these products on individual consumption and welfare.

Our baseline results show that, compared to the case that a retiree has no access to either reverse mortgage or private LTC insurance, her welfare gain is marginal (about $20 increase in CEC) if only private LTC insurance can be purchased. Her welfare gain is sizeable (about $270 increase in CEC) when only a reverse mortgage is offered. Therefore, borrowing against home equity dominates private LTC insurance, because it leads to higher consumption in the earlier stage in the life cycle model as well as providing some longevity insurance. LTC insurance, with lump sum premiums, transfers wealth from good health states to bad health states but reduces earlier consumption. These effects result in a strong demand for reverse mortgages for those endowed with significant illiquid home equity. Those with high levels of liquid wealth purchase more LTC insurance, since they do not face liquidity constraints.

The highest welfare benefits come from a combination of reverse mortgage and LTC insurance. When both products are available, the welfare gain is substantially higher, with a $757 increase in CEC. We also show strong complementary effects between LTC insurance and reverse mortgages, which provides motivation for new products that bundle these two products together. We show that the optimal bundling approach depends on the level of total wealth as well as the composition of wealth endowment. Our results not only confirm the traditional view that reverse mortgage is valuable for those who are asset rich but cash poor, but also show that bundling reverse mortgage with private LTC insurance would be of significant value to many individuals who are cash rich but asset poor.

Selling bundled products has many advantages from the provider's view as well. First, the demand for private LTC insurance is very sensitive to preference parameters, such as risk aversion and bequest motive. However, its demand becomes very robust when reverse mortgage is offered at the same time. It is an attractive feature because the provider can better predict the demand for a bundled product if it is provided on the market. Second, combining reverse mortgage and private LTC insurance is also beneficial because of reduced adverse selection (Davidoff, 2010). For example, people with higher potential disability risk may claim more LTC benefits which at the same time would trigger the repayment of reverse mortgage loans earlier.

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