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Abstract

Insurers and pension funds face the challenges of historically low interest rates and volatility in equity markets, that have been accentuated due to the COVID-19 pandemic. Recent advances in equity portfolio management with a target volatility have been shown to deliver improved on average risk adjusted return, after transaction costs. This paper studies these targeted volatility portfolios in applications to equity, balanced and target-date funds with varying constraints on leverage. Conservative leverage constraints are particularly relevant to pension funds and insurance companies, with more aggressive leverage levels appropriate for alternative investments. We show substantial improvements in fund performance for differing leverage levels and that the return per unit of risk is not significantly impacted by the leverage constraint. Of most interest to insurers and pensions funds, we show that the highest return per unit of risk is in targeted volatility balanced portfolios with equity and bond allocations. Furthermore, we demonstrate the

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outperformance of targeted volatility portfolios during major stock market crashes, including the crash from the COVID-19 pandemic.

Key Words: COVID-19 pandemic, Equity investment, Portfolio management, Target-date funds, Volatility management
JEL classification: C53, G17.

1 Introduction

The current low interest rate environment and equity market volatility, that have been accentuated due to the COVID-19 pandemic, are a challenge for insurers and pension funds. Although equity investments offer the potential for higher returns, the increased volatility of these investments must be taken into account. Targeting a constant volatility for a broad market equity portfolio has become increasingly important for investment management in this market environment. Insurers can benefit significantly from targeted constant volatility portfolios for equity funds as can pension funds with balanced and target-date funds with declining glide paths of reduced equity exposure as the target-date approaches. Target volatility strategies should incorporate portfolio constraints on the degree of leverage for them to be relevant to insurers and pension funds. We consider these strategies with a range of leverage constraints from conservative levels, as seen commonly in pension funds and insurance companies, to more aggressive levels often associated with alternative investments.

The outperformance of targeted constant market volatility portfolios is driven from the well known negative relationship between equity market returns and conditional volatility, see Hocquard et al. (2013), Moreira and Muir (2017) and Doan et al. (2018). This relationship is primarily explained by the volatility feedback effect (see Poterba and Summers (1986)) where higher (lower) volatility results in a stock market price fall (rise) as the required rate of return on the stock market increases (decreases).[1]

[1]Further studies on models of volatility feedback include, Campbell and Hentschel (1992), Bekaert and Wu (2000), Wu (2001) and Bollerslev et al. (2006).

The approach to targeting a constant market volatility that was developed in Doan et al. (2018), showed how a simple univariate method could generate higher risk adjusted returns, relative to the existing literature.[2] Doan et al. (2018) apply their methodology without leverage constraints, and an important contribution in this paper is that we extend these results to include constraints on leverage. We focus on the U.S. market. The results in Doan et al. (2018) cover a broad range of equity markets so that the results extend to those other markets. Results for equity portfolios firstly demonstrate the outperformance of constant volatility portfolios over the different leverage constraints and secondly that the return per unit of risk is relatively constant with respect to the leverage constraints analyzed.

In addition, we also extend the target volatility analysis to traditional balanced portfolios, with a 65:35 split between equity and bonds. We find that the highest levels of return per unit of risk are in targeted volatility balanced portfolios and that the return per unit of risk is not significantly impacted by the leverage constraint. Applications to target-date funds over a range of investment life cycles (35, 25 and 15 year) are also studied. Three glide paths of declining equity exposure are examined; aggressive, moderate and conservative. Furthermore, three leverage constraints are examined; no leverage, conservative leverage and aggressive leverage. Outperformance is found in constant volatility target-date portfolios with both conservative and aggressive leverage constraints, where more aggressive leverage leads to higher average investment outcomes with higher variability.

Of significant current interest, we examine investment performance during major stock market crashes including COVID-19. We study the largest U.S. stock

[2] This literature was primarily based on multivariate methods, see for example, Fleming et al. (2001, 2003), Han (2006), Liu (2009), Kirby and Ostdiek (2012), and Clements and Silvennoinen (2013).

market crash in the last 100 years which is associated with the Great Depression in the 1930's. We also study the largest U.S. stock market crash in the last 50 years which is associated with the Global Credit Crisis of 2008. And finally we study the recent stock market crash from the COVID-19 crisis. Targeted volatility typically results in substantial limitation to portfolio drawdowns during these crashes. For example, in the COVID-19 crisis, the U.S. CRSP value-weighted index, including dividends fell 34.27 percent from 19 February 2020 to 23 March 2020, whereas the corresponding targeted constant average stock market volatility investment portfolio fell only 18.61 percent. Over this same time period, the balanced and targeted volatility balanced portfolios, fell 23.52 percent and 12.64 percent, respectfully. The minimization of portfolio drawdowns is an important criteria for many types of investments, including that in pension and insurance companies. In addition, drawdown minimization substantially contributes to the long run return outperformance of targeted volatility stock index portfolios and targeted volatility balanced portfolios. Targeted volatility results in volatility more consistent with historical average market volatility, avoiding the high volatility during and following market crashes, and also provides higher returns for periods including the crashes.

This paper is organized as follows. The next section describes the volatility forecasting, trading strategy and data. The third section is on average returns and risk for equity, balanced and target-date funds, over a range of leverage constraints. The fourth section is on investment performance during major stock market crashes and the final section concludes.

2 Volatility Forecasting and Trading Strategy

We begin by describing the asset class and portfolios that are analyzed. This is followed with a discussion of the volatility forecasting approach, the trading strategy and portfolio performance statistics, similar to Doan et al. (2018) and repeated here for completeness.

We implement the strategy for three different portfolios; equity, balanced and target-date. The balanced portfolio contains 65% of equity and 35% of bonds, and the target-date portfolio invests in equity with decreasing percentage over life cycles of 35, 25 and 15 years.

2.1 Volatility forecasting

One-day-ahead forecasts of portfolio return volatility are generated from an outlier-corrected GARCH(1,1) model, as the GARCH(1,1) model is the most widely utilized return volatility forecasting model in settings where daily returns are the highest frequency readily available. Bond volatility is not used in the volatility calculations as it is relatively small when compared to equity volatility. The GARCH model was originally proposed by Bollerslev (1986), though it has been shown to have biased parameter estimates and forecasts when outliers are present in the return series, see Gregory and Reeves (2010), Carnero et al. (2012) and Harvey (2013). To overcome this problem, we follow the approach in Doan et al. (2018), by first winsorizing returns at a specified level, r_{max} of 4%, such that the return series over the estimation period is in the range between $-r_{max}$ and r_{max}. The estimation then follows standard Gaussian quasi-maximum likelihood estimation

(QMLE) of the GARCH(1,1) model on the winsorized return series.

The weighted daily equity return (in percentage) at date t is given by;

$$r_t = \varepsilon_t, \tag{1}$$

where r_t is winsorized at $\pm 4\%$, ε_t is i.i.d $(0, \sigma_t^2)$, and the conditional variance σ_t^2 follows the GARCH(1,1) process;

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \tag{2}$$

with the following parameter constraints $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\alpha_2 \geq 0$ and $\alpha_1 + \alpha_2 < 1$. The starting values for $\hat{\varepsilon}_0^2$ and $\hat{\sigma}_0^2$ are the unconditional sample variance.

Given the estimated parameter set $\{\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2\}$ for each estimation window of 1,000 observations, we compute the one-day ahead volatility forecast using the following equation;

$$\hat{\sigma}_{t+1}^2 = \hat{\alpha}_0 + \hat{\alpha}_1 \hat{\varepsilon}_t^2 + \hat{\alpha}_2 \hat{\sigma}_t^2 \tag{3}$$

2.2 Trading strategy

To implement our targeted volatility approach, we first determine the desired absolute daily target volatility level of equity returns, which is constant over the period of analysis. We also introduce a daily participation ratio which is the weight w_t invested in the market equity portfolio:

$$w_t = \frac{target\ volatility}{\hat{\sigma}_t}, \tag{4}$$

where $\hat{\sigma}_t$ is the volatility forecast for trading day t.

The intuition underlying the dynamic trading strategy follows Doan et al. (2018). When the forecast volatility for a given trading day is greater than the target volatility, we shift away from equity markets by selling futures contracts on the equity market, leading to a decrease in the portfolio volatility. When the forecast volatility is less than the target volatility, we purchase futures contracts on the equity market, in order to increase our equity exposure, leading to an increase in the portfolio volatility.

In implementation, we also set a threshold weight change (δ), to minimize excessive turnover, in that we only change our market exposure when the new participation ratio differs from the prior by an absolute amount greater than δ. To further control risk in leveraging the equity portfolio, we set different levels of maximum participation ratio, namely 1.5, 2, and unrestricted value.

In the case where we take positions in the futures market, the daily return at date t of the trading strategy on the equity portfolio is computed as;

$$r_{equity,t} = (w_t - 1)r_{futures,t} + r_{market,t} \tag{5}$$

where $r_{futures,t}$ is the index futures return at date t.

During the period around the stock market crash associated with the Great Depression, futures contracts on the stock index were not available so we instead compute the daily return at date t of the trading strategy on the targeted volatility equity portfolio as;

$$r_{equity,t} = w_t(1 + r_{market,t}) - 1_{w_t \geq 1}(w_t - 1)(1 + r_{f,t}) + 1_{w_t \leq 1}(1 - w_t)(1 + r_{f,t}) - 1 \tag{6}$$

where $r_{f,t}$ is the borrowing and lending rate at date t, $1_{w_t \geq 1} = 1$ if $w_t \geq 1$ and 0 otherwise, and $1_{w_t \leq 1} = 1$ if $w_t \leq 1$ and 0 otherwis

For the balanced portfolio, the daily returns at day t of the trading strategy are computed as;

$$r_{balanced,t} = 0.65 r_{equity,t} + 0.35 r_{bond,t} \tag{7}$$

where $r_{bond,t}$ is the daily bond return at date t.

With respect to the target-date portfolio, we consider a worker who contributes 9% of his/her salary to a target-date fund at the end of each year over his/her career. His/her initial annual wage is 20,000 USD, which grows by 4% in nominal terms every year.[3] The target-date fund equity contributions are reset at the end of each year, without considering tax implications or transaction costs. To determine the glide path of the target-date fund, we collect the asset allocation for aggressive, moderate, and conservative strategies from the Morningstar Lifetime Allocation Indexes as of June 2017. Given our focus on equity and bond investment, we aggregate the contribution of non-equity securities into the bond asset class. We also linearly interpolate the quinquennial values provided from Morningstar to obtain yearly data. The percentage of equity invested in the portfolio is presented in Figure 1. The daily returns of equity and bond components on day t are given by $r_{equity,t}$ and $r_{bond,t}$, respectively.

[3] The accumulation amount for any starting salary can be calculated by dividing the reported accumulation amount by 20,000 and multiplying by the starting salary.

2.3 Summary statistics

We define the annualized returns, μ, and annualized standard deviation, σ, of a given strategy as;

$$\mu = 100[(1+\hat{r})^{252} - 1] \text{ with } \hat{r} = Y^{\frac{1}{n}} - 1 \tag{8}$$

$$\sigma = 100\sqrt{252\frac{\sum_{t=1}^{n}(r_t - \bar{r})^2}{n-1}}, \tag{9}$$

where n is the total number of trading days in the investment sample period, r_t is the daily return of the equity portfolio or balanced portfolio, $\bar{r}$ and Y are the average daily return and cumulative amount from 1 initial value in the investment period, respectively. The return per unit of risk (ϕ) is expressed as;

$$\phi = \frac{\mu}{\sigma} \tag{10}$$

and the maximum daily drawdown (Min ret) is defined as;

$$\text{Min ret} = min\{r_t, t = 1, ..., n\}. \tag{11}$$

For the results on target-date portfolios, we calculate the ending market value of a target-date fund that starts to invest at every trading day in the sample and present their summary statistics of average, standard deviation, minimum and maximum values. We also report the internal rate of return (IRR) of the average target-date ending value by solving the following equation for IRR;

$$\text{Average ending value} = \sum_{t=1}^{T} C_t(1 + IRR)^{T-t} \tag{12}$$

where C_t is the end of year t contribution and T is the number of years in the target-date fund life.

2.4 Data

The data for this study is from the following sources. The U.S. market index returns are the CRSP value-weighted market returns. The series is adjusted to account for dividend re-investment and runs from 9 May 1978 to 30 June 2020. The same series over the period, 1 July 1926 to 30 July 1937, is also used to study the performance of the target volatility strategy during the stock market crash in the Great Depression. We obtain daily settlement price series of futures contracts on the S&P500 from Datastream. The daily returns of futures contracts start on 23 April 1982. For the bond data, we collect the U.S. bond return index that invests in a wide set of government and corporate bonds, provided by Barclays (mnemonic: LHAGGBD). The bond returns start at the same time as equity returns and the risk free rate is sourced from the Kenneth French Data Library.

3 Average Returns and Risk with Leverage Constraints

3.1 Equity portfolios

Summary statistics on U.S. equity market portfolio performance with a threshold weight change (δ) of 0.05 are displayed in Table 1. Four levels of maximum participation ratios are considered; 1, 1.5, 2 and unrestricted. The 1 (no leverage) and 1.5 levels correspond to settings common in pension funds and insurance companies. Whereas, the 2 level and unrestricted level are more applicable to alternative investments.

Over our sample period starting 26 April 1982 and finishing 30 June 2020, the annualized average return of the market portfolio is 11.59 percent, with an annualized standard deviation of 17.70 percent. For all four maximum participation ratio levels, all targeted volatility portfolios outperform the market portfolio. These targeted volatility portfolios all have return per unit of risk exceeding that of the market portfolio. Furthermore, lowering target volatility levels results in rises in the return per unit of risk.

When targeting the average daily market volatility[4] (a constant daily return volatility of 1 percent) with GARCH(1,1), the annualized average return rises to 12.58 percent, with a maximum participation ratio of 1.5. The return per unit of risk is 0.76, compared with 0.65 for the market portfolio. When the maximum

[4] Doan et al. (2018) found over their sample period from January 1926 to December 2013 that the daily return volatility of the CRSP value-weighted index, including dividends, was approximately 1 percent when outliers had been removed. We find this also the case over our sample period, corresponding to volatility being stationary over long time spans.

participation ratio rises to 2, the performance in terms of return per unit of risk and maximum daily drawdown, remains mostly unchanged. This is also the case when the participation ratio is unrestricted.

Table 2 displays the summary statistics on portfolio performance when the δ is 0.1, with results being very similar to Table 1. This favors the 0.1 threshold which is associated with lower transaction costs than the 0.05 threshold. When the δ is set at 0.2 (Table 3) there is a small deterioration in portfolio performance, with small declines in the return per unit of risk, compared with δ set at 0.1. For example, when targeting the average daily market volatility with GARCH(1,1) and a maximum participation ratio of 1.5, the return per unit of risk is 0.76, compared to 0.75 when δ is 0.1. The remainder of the analysis in this paper is conducted with δ set at 0.1.

3.2 Balanced portfolios

Summary statistics of U.S. balanced portfolio performance are displayed in Table 4. The annualized average return of the balanced portfolio is 11.94 percent and exceeds that of the equity market portfolio, before transaction costs with daily rebalancing, over the same sample period. It has an annualized standard deviation of 11.74 percent, compared to 17.70 percent for the equity market portfolio. This results in the balanced portfolio's return per unit of risk ($\phi = 1.02$) being substantially higher than the equity market portfolio's return per unit of risk ($\phi = 0.65$). In addition, the maximum daily drawdown of the balanced portfolio is -11.32 percent, compared with -17.41 percent for the equity market portfolio.

Overall performance of the balanced portfolio, dominates that of the equity market portfolio. This result highlights how the conventional approach to mitigating the volatility of equity market returns through balancing with bonds, is very effective in generating strong long run portfolio performance. In particular, the bond component of balanced portfolios provides a support to balanced portfolio values during periods of large drawdowns from falls in the equity market, that leads to strong growth in cumulative portfolio value over time.

Outperformance of balanced portfolios is extended further when the equity component of the portfolio has targeted volatility. Return per unit of risk rises through targeting volatility. There is a monotonically increasing rise in ϕ as the target level of equity return volatility is lowered. This occurs for all levels of the maximum participation ratio. When targeting the average daily market return volatility with GARCH(1,1), the annualized average return rises to 12.53 percent and the maximum daily drawdown is -9.16 percent, with the maximum participation ratio set at 1.5. Over the different target levels of volatility for the balanced portfolios, the ϕ and maximum daily drawdown, are relatively constant with respect to the leverage levels.

3.3 Target-date portfolios

We next move to analysis of target-date funds over life cycles of 35, 25 and 15 years. Table 5 displays results for the 35 year life cycle, with cumulative amounts calculated over the 35 year investments for the period of analysis. Panel A contains results when there is no targeting of a constant volatility for the equity component. The investment strategies considered are aggressive, moderate and conservative. The mean cumulative amount increases as the investment strategy has a greater equity exposure. These means are \$581,898, \$641,232 and \$692,538 for the conservative, moderate and aggressive strategies, respectively. The variability in the cumulative amounts are not too different, between the three investment strategies, when considering standard deviation and differences between the maximum and minimum cumulative amounts.

Results from targeting the sample volatility of daily equity returns (1 percent) with GARCH forecasts are displayed in Panel B. Cumulative amounts rise in all summary statistics on investment performance, relative to results with no volatility targeting on the equity component when the maximum participation ratio is 1.5 and 2. The mean cumulative amounts, with a maximum participation ratio of 1.5, are \$652,772, \$736,941 and \$809,166 for the conservative, moderate and aggressive strategies, respectively. These mean cumulative amounts rise further to \$690,271, \$784,743 and \$862,591 with a maximum participation ratio of 2. Furthermore, the variability of cumulative amounts with the GARCH volatility targeting rises when the maximum participation ratio is 1.5 and 2, relative to no volatility targeting. Volatility targeting with no leverage (maximum participation ratio of 1) results in cumulative amount means and variabilities reducing, relative to no volatility targeting. Internal rate of return (IRR) of mean cumulative amounts rises from volatility targeting with leverage. For example, an aggressive strategy with no volatility targeting has an IRR of 9.78 percent, whereas with volatility targeting and a maximum participation ratio of 2, this rises to 10.85 percent.

Tables 6 and 7 display results for the 25 and 15 year life cycles, respectively. Similar patterns to Table 5 are also observed in these tables, though cumulative amounts reduce as the life cycle years reduce. As an approximate summary, for every 5 year reduction in the life cycle, the cumulative amount reduces by half.

The lowest average investment outcomes are from the 15 year target date with a conservative glide path. In this case, with no volatility targeting, the mean cumulative amount is only \$71,633. Volatility targeting with a maximum participation ratio of 2 results in this mean cumulative amount rising to \$76,076. This also highlights the importance of a sufficiently long investment period.

4 Investment Performance During Major Stock Market Crashes

In this section we report results of the performance of targeted volatility portfolios during major U.S. stock market crashes. These include the crash from 3 September 1929 to 8 July 1932 which is associated with the Great Depression in the 1930's and is the largest U.S. stock market crash in the last 100 years. We also study the crash from 9 October 2007 to 9 March 2009 which is associated with the Global Credit Crisis and is the largest U.S. stock market crash in the last 50 years. Finally, we also report results on the targeted volatility portfolios during the recent COVID-19 pandemic stock market crash.

Tables 8 and 9 report results for the crashes associated with the Great Depression and the Global Credit Crisis. Table 8 contains holding period returns for the one year prior to the crash, the crash period, the five years post crash, and the period starting one year prior to the crash and ending five years after the crash. The market index is the value-weighted CRSP index, including dividends. The target volatility strategy targets a daily 1 percent or 0.8 percent standard deviation of market returns, with a daily threshold weight change of 0.1 and with maximum leverage of 50 percent.

The crash from 3 September 1929 to 8 July 1932 resulted in the CRSP value-weighted index, including dividends, falling 83.84 percent. During this crash the targeted volatility strategy resulted in a substantial mitigation of the drawdown. The corresponding CRSP portfolio that targeted average daily volatility at 1 percent had a reduced drawdown at 70.01 percent, and when targeting volatility at 0.8 percent the drawdown was 60.36 percent. Over the period starting one year prior to the crash and ending five years after the crash, the CRSP index had a negative return of 1.62 percent, whereas, the targeted 1 percent and 0.8 percent volatility portfolios had positive returns of 14.87 percent and 20.84 percent, respectively. Figure 2 displays the cumulative outperformance of the targeted 1 percent volatility portfolio, against the market portfolio over this period.

The crash associated with the Global Credit Crisis resulted in the CRSP value-weighted index, including dividends, falling 54.32 percent from 9 October 2007 to 9 March 2009. The corresponding CRSP portfolios that targeted 1 and 0.8 percent daily volatility had much less of a drawdown at 38.40 and 30.28 percent, respectively, over the same period. Over the period starting one year prior to the crash and ending five years after the crash, the CRSP index had a return of 71.06 percent, whereas, the targeted 1 percent and 0.8 percent volatility portfolios had returns of 88.23 percent and 73.84 percent, respectively. Figure 3 displays the cumulative outperformance of the targeted 1 percent volatility portfolio, against the market portfolio over this period.

Table 9 presents the annualized volatility of returns for the same portfolios and over the same periods as in Table 8. During the Great Depression and Global Credit Crisis crashes the annualized volatility of returns in the value-weighted

CRSP index was 33.76 and 37.11 percent, respectively. The targeted volatility portfolios had substantially less volatility, as expected. The 1 percent volatility portfolios over the Great Depression and Global Credit Crisis crashes had an annualized volatility of 20.02 and 20.91 percent, respectively. Over the periods starting one year prior to the crash and ending five years after the crash, the target volatility portfolios displayed volatility close to their volatility target.

Tables 10 and 11 report returns and volatilities for the equity and balanced portfolios and corresponding target 1 and 0.8 percent volatility portfolios that apply the targeted volatility strategy to the equity component with maximum leverage of 50 percent, over the first half of 2020 which includes the COVID-19 crash. In this episode, the pre crash period is 1 January 2020 to 19 February 2020, the crash period is 19 February 2020 to 23 March 2020, the post crash period is 23 March 2020 to 30 June 2020 and the full period is 1 January 2020 to 30 June 2020. The market index is again the value-weighted CRSP index, including dividends and the balanced portfolio invests 65% in equity markets and 35% in bond markets.

During this crash period the CRSP index, fell 34.27 percent from 19 February 2020 to 23 March 2020. Over the same period, the targeted volatility strategy again resulted in a substantial mitigation of the drawdown. The corresponding CRSP portfolio that targeted daily volatility at 1 percent had much less of a drawdown at 18.61 percent over this period. Figure 4 displays these two portfolios from the beginning of January 2020 to the end of June 2020. Over the months following the crash the stock index recovered substantially with the targeted volatility portfolio continuing to show outperformance. The CRSP index was down 1.89 percent over the period from 1 January 2020 to 30 June 2020, whereas the targeted 1 percent volatility portfolio was up 1.77 percent.

Outperformance in the balanced portfolio with targeted volatility on the equity component also occurred during the pandemic. The 65:35 equity bond split in the balanced portfolio resulted in a portfolio decline of 23.52 percent from 19 February 2020 to 23 March 2020. While targeting a daily 1 percent volatility on the equity component in the balanced portfolio, resulted in a decline of only 12.64 percent. Outperformance from targeting volatility continued over the extended period to the 30 June 2020. From 1 January 2020 to 30 June 2020 the balanced portfolio gained 2.26 percent, while the balanced portfolio with targeted volatility at 1 percent on the equity component, gained 3.80 percent. The performance of these portfolios are displayed in Figure 5 from the beginning of January 2020 to the end of June 2020.

Table 11 presents the annualized volatility of returns for these portfolios. The COVID-19 crash was extremely volatile with annualized volatility of the CRSP index during the crash being 79.28 percent. The corresponding CRSP target 1 and 0.8 percent volatility portfolios had substantially less volatility at 38.93 percent and 31.92 percent, respectively. The equity volatility also created substantial volatility in the balanced portfolio, which was 50.47 percent during the crash. Over the full period from 1 January 2020 to 30 June 2020, the volatility in the balanced portfolio with the equity component having a 1 percent target volatility, was approximately half that of the balanced portfolio without the targeted volatility strategy.

5 Conclusion

This paper has analyzed targeted volatility investment portfolios over a range of leverage constraints; from no leverage, to conservative and aggressive leverage, to unconstrained leverage. Results are presented for equity, balanced and target-date portfolios. We show how different constraints, result in relatively small changes in the return per unit of risk for the targeted volatility portfolios. We show the extent to which for target-date funds with targeted volatility, less conservative leverage constraints result in higher average investment outcomes. Significantly, substantial risk adjusted return outperformance is shown for the targeted volatility portfolios with leverage constraints. The highest return per unit of risk is found in targeted volatility balanced portfolios with equity and bond allocations.

Of significant current interest, the paper has also demonstrated outperformance of targeted volatility investment portfolios during and after major stock market crashes. A primary generator of outperformance comes from the mitigation of the drawdown during the crash, particularly in highly volatile crashes. For example, during the extreme volatility in the recent COVID-19 crash the targeted volatility strategy approximately halved the drawdown, relative to the benchmark stock index.

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Table 1: **Equity portfolio performance statistics with $\delta = 0.05$**

	μ	σ	ϕ	Min ret
market portfolio	11.59	17.70	0.65	-17.41
maximum participation ratio of 1				
daily market volatility 0.6%	8.78	10.25	0.86	-6.10
daily market volatility 0.8%	10.06	12.38	0.81	-9.03
daily market volatility 1.0%	10.50	13.84	0.76	-14.09
daily market volatility 1.1%	10.65	14.42	0.74	-17.41
daily market volatility 1.2%	10.84	14.86	0.73	-17.41
maximum participation ratio of 1.5				
daily market volatility 0.6%	9.15	10.53	0.87	-6.10
daily market volatility 0.8%	11.09	13.71	0.81	-9.03
daily market volatility 1.0%	12.58	16.56	0.76	-14.09
daily market volatility 1.1%	13.19	17.71	0.74	-16.62
daily market volatility 1.2%	13.61	18.72	0.73	-19.15
maximum participation ratio of 2				
daily market volatility 0.6%	9.16	10.54	0.87	-6.10
daily market volatility 0.8%	11.34	13.88	0.82	-9.03
daily market volatility 1.0%	13.21	17.31	0.76	-14.09
daily market volatility 1.1%	14.02	18.92	0.74	-16.62
daily market volatility 1.2%	14.75	20.47	0.72	-19.15
unrestricted participation ratio				
daily market volatility 0.6%	9.16	10.54	0.87	-6.10
daily market volatility 0.8%	11.36	13.89	0.82	-9.03
daily market volatility 1.0%	13.41	17.44	0.77	-14.09
daily market volatility 1.1%	14.46	19.24	0.75	-16.62
daily market volatility 1.2%	15.44	21.08	0.73	-19.15

This table presents the summary statistics of equity portfolios over the sample period of 26 April 1982 to 30 June 2020. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model. The statistics include the annualized average return in percentage (μ), annualized standard deviation in percentage (σ), return per unit of risk (ϕ), and maximum daily drawdown (Min ret). The daily target constant volatility portfolios have a threshold weight change of 0.05, and the panels present the results of different levels of maximum participation ratios of 1, 1.5, 2 and unrestricted. In each panel, the rows present the results from the trading strategy that targets a constant level of daily market volatility.

Table 2: **Equity portfolio performance statistics with $\delta = 0.1$**

	μ	σ	ϕ	Min ret
market portfolio	11.59	17.70	0.65	-17.41
maximum participation ratio of 1				
daily market volatility 0.6%	8.94	10.33	0.87	-6.10
daily market volatility 0.8%	10.12	12.49	0.81	-9.03
daily market volatility 1.0%	10.60	13.93	0.76	-14.09
daily market volatility 1.1%	10.64	14.52	0.73	-17.41
daily market volatility 1.2%	10.82	14.90	0.73	-17.41
maximum participation ratio of 1.5				
daily market volatility 0.6%	9.25	10.56	0.88	-6.10
daily market volatility 0.8%	11.25	13.81	0.81	-9.03
daily market volatility 1.0%	12.60	16.64	0.76	-14.09
daily market volatility 1.1%	13.14	17.82	0.74	-16.62
daily market volatility 1.2%	13.53	18.84	0.72	-19.15
maximum participation ratio of 2				
daily market volatility 0.6%	9.26	10.56	0.88	-6.10
daily market volatility 0.8%	11.45	13.98	0.82	-9.03
daily market volatility 1.0%	13.29	17.39	0.76	-14.09
daily market volatility 1.1%	13.97	19.03	0.73	-16.62
daily market volatility 1.2%	14.61	20.60	0.71	-19.15
unrestricted participation ratio				
daily market volatility 0.6%	9.26	10.56	0.88	-6.10
daily market volatility 0.8%	11.47	13.99	0.82	-9.03
daily market volatility 1.0%	13.53	17.51	0.77	-14.09
daily market volatility 1.1%	14.44	19.33	0.75	-16.62
daily market volatility 1.2%	15.33	21.19	0.72	-19.15

This table presents the summary statistics of equity portfolios over the sample period of 26 April 1982 to 30 June 2020. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model. The statistics include the annualized average return in percentage (μ), annualized standard deviation in percentage (σ), return per unit of risk (ϕ), and maximum daily drawdown (Min ret). The daily target constant volatility portfolios have a threshold weight change of 0.1, and the panels present the results of different levels of maximum participation ratios of 1, 1.5, 2 and unrestricted. In each panel, the rows present the results from the trading strategy that targets a constant level of daily market volatility.

Table 3: **Equity portfolio performance statistics with $\delta = 0.2$**

	μ	σ	ϕ	Min ret
market portfolio	11.59	17.70	0.65	-17.41
maximum participation ratio of 1				
daily market volatility 0.6%	9.09	10.64	0.85	-6.72
daily market volatility 0.8%	9.89	12.77	0.77	-9.03
daily market volatility 1.0%	10.57	14.28	0.74	-17.41
daily market volatility 1.1%	10.55	14.72	0.72	-17.41
daily market volatility 1.2%	10.91	15.13	0.72	-17.41
maximum participation ratio of 1.5				
daily market volatility 0.6%	9.46	10.90	0.87	-6.10
daily market volatility 0.8%	10.93	14.06	0.78	-9.03
daily market volatility 1.0%	12.72	17.08	0.75	-18.66
daily market volatility 1.1%	13.13	18.12	0.72	-16.62
daily market volatility 1.2%	13.44	19.16	0.70	-24.63
maximum participation ratio of 2				
daily market volatility 0.6%	9.45	10.90	0.87	-6.10
daily market volatility 0.8%	11.27	14.24	0.79	-9.03
daily market volatility 1.0%	13.50	17.64	0.77	-14.09
daily market volatility 1.1%	14.05	19.30	0.73	-16.62
daily market volatility 1.2%	14.92	20.76	0.72	-19.15
unrestricted participation ratio				
daily market volatility 0.6%	9.45	10.90	0.87	-6.10
daily market volatility 0.8%	11.30	14.24	0.79	-9.03
daily market volatility 1.0%	13.74	17.76	0.77	-14.09
daily market volatility 1.1%	14.60	19.60	0.74	-16.62
daily market volatility 1.2%	15.53	21.27	0.73	-19.15

This table presents the summary statistics of equity portfolios over the sample period of 26 April 1982 to 30 June 2020. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model. The statistics include the annualized average return in percentage (μ), annualized standard deviation in percentage (σ), return per unit of risk (ϕ), and maximum daily drawdown (Min ret). The daily target constant volatility portfolios have a threshold weight change of 0.2, and the panels present the results of different levels of maximum participation ratios of 1, 1.5, 2 and unrestricted. In each panel, the rows present the results from the trading strategy that targets a constant level of daily market volatility.

Table 4: **Balanced portfolio performance statistics with $\delta = 0.1$**

	μ	σ	ϕ	Min ret
balanced portfolio	11.94	11.74	1.02	-11.32
maximum participation ratio of 1				
daily market volatility 0.6%	9.93	7.24	1.37	-3.97
daily market volatility 0.8%	10.76	8.54	1.26	-5.87
daily market volatility 1.0%	11.13	9.41	1.18	-9.16
daily market volatility 1.1%	11.18	9.77	1.14	-11.32
daily market volatility 1.2%	11.31	10.00	1.13	-11.32
maximum participation ratio of 1.5				
daily market volatility 0.6%	10.14	7.39	1.37	-3.97
daily market volatility 0.8%	11.55	9.38	1.23	-5.87
daily market volatility 1.0%	12.53	11.14	1.13	-9.16
daily market volatility 1.1%	12.94	11.88	1.09	-10.80
daily market volatility 1.2%	13.24	12.52	1.06	-12.45
maximum participation ratio of 2				
daily market volatility 0.6%	10.14	7.39	1.37	-3.97
daily market volatility 0.8%	11.68	9.49	1.23	-5.87
daily market volatility 1.0%	13.02	11.62	1.12	-9.16
daily market volatility 1.1%	13.54	12.66	1.07	-10.80
daily market volatility 1.2%	14.03	13.65	1.03	-12.45
unrestricted participation ratio				
daily market volatility 0.6%	10.14	7.39	1.37	-3.97
daily market volatility 0.8%	11.69	9.49	1.23	-5.87
daily market volatility 1.0%	13.18	11.70	1.13	-9.16
daily market volatility 1.1%	13.86	12.86	1.08	-10.80
daily market volatility 1.2%	14.53	14.04	1.03	-12.45

This table presents the summary statistics of balanced portfolios over the sample period of 26 April 1982 to 30 June 2020. The balanced portfolio invests 65% in equity markets and 35% in bond markets. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model. The statistics include the annualized average return in percentage (μ), annualized standard deviation in percentage (σ), return per unit of risk (ϕ), and maximum daily drawdown (Min ret). The daily target constant volatility portfolios have a threshold weight change of 0.1, and the panels present the results of different levels of maximum participation ratios of 1, 1.5, 2 and unrestricted. In each panel, the rows present the results from the trading strategy that targets a constant level of daily market volatility for the equity component.

Table 5: **35 year target-date fund performance statistics**

	aggressive	moderate	conservative
Panel A: No volatility targeting			
mean	692,538	641,232	581,898
stdev	37,026	36,802	37,381
min	525,889	509,902	480,159
max	769,065	741,362	690,985
IRR	9.78%	9.39%	8.91%
Panel B: Volatility targeting			
maximum participation ratio of 1			
mean	612,227	569,872	526,205
stdev	26,821	27,953	30,606
min	529,929	496,744	461,361
max	669,031	627,678	595,134
IRR	9.16%	8.80%	8.39%
maximum participation ratio of 1.5			
mean	809,166	736,941	652,772
stdev	46,136	46,145	46,171
min	672,816	616,689	551,781
max	921,491	828,204	750,074
IRR	10.54%	10.08%	9.48%
maximum participation ratio of 2			
mean	862,591	784,743	690,271
stdev	51,736	50,963	49,731
min	714,500	653,406	580,773
max	996,136	892,569	793,326
IRR	10.85%	10.39%	9.75%

This table presents summary statistics on cumulative amounts and the internal rate of return (IRR) at the end of target-date funds over 35 year life cycles in the U.S.. The period of analysis is from 26 April 1982 to 30 June 2020. The columns present the results under aggressive, moderate, and conservative strategies. Panel A presents results without volatility targeting and Panel B presents results with targeting daily market volatility at 1 percent, with maximum participation ratios of 1, 1.5 and 2. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model and the threshold weight change is 0.1. The 35 year cumulative amount that starts at each trading day in the period of analysis is calculated, where the equity contribution is sourced from Morningstar Lifetime Allocation Indexes. It is assumed that the worker (investor) starts with a salary of 20,000 USD, which grows by 4% in nominal terms each year during a 35 year career. The worker contributes 9% of his/her salary to target-date fund at the end of each year for their 35 years of work. The target-date fund rebalances daily without considering tax implications or transaction costs.

Table 6: **25 year target-date fund performance statistics**

	aggressive	moderate	conservative
Panel A: No volatility targeting			
mean	221,360	216,713	207,760
stdev	32,905	32,602	33,025
min	165,877	160,985	153,653
max	337,987	326,808	311,001
IRR	9.24%	9.08%	8.76%
Panel B: Volatility targeting			
maximum participation ratio of 1			
mean	206,571	202,940	197,165
stdev	24,041	26,236	29,242
min	174,096	164,185	154,229
max	294,157	289,274	284,497
IRR	8.72%	8.58%	8.36%
maximum participation ratio of 1.5			
mean	245,103	236,035	221,209
stdev	46,906	46,457	43,805
min	189,991	177,031	162,734
max	410,294	383,739	347,005
IRR	10%	9.72%	9.23%
maximum participation ratio of 2			
mean	256,452	246,136	228,541
stdev	55,521	53,762	48,709
min	190,335	176,952	162,743
max	444,875	411,799	367,215
IRR	10.34%	10.03%	9.48%

This table presents summary statistics on cumulative amounts and the internal rate of return (IRR) at the end of target-date funds over 25 year life cycles in the U.S.. The period of analysis is from 26 April 1982 to 30 June 2020. The columns present the results under aggressive, moderate, and conservative strategies. Panel A presents results without volatility targeting and Panel B presents results with targeting daily market volatility at 1 percent, with maximum participation ratios of 1, 1.5 and 2. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model and the threshold weight change is 0.1. The 25 year cumulative amount that starts at each trading day in the period of analysis is calculated, where the equity contribution is sourced from Morningstar Lifetime Allocation Indexes. It is assumed that the worker (investor) starts with a salary of 20,000 USD, which grows by 4% in nominal terms each year during a 25 year career. The worker contributes 9% of his/her salary to target-date fund at the end of each year for their 25 years of work. The target-date fund rebalances daily without considering tax implications or transaction costs.

Table 7: **15 year target-date fund performance statistics**

	aggressive	moderate	conservative
Panel A: No volatility targeting			
mean	77,728	74,829	71,633
stdev	23,413	20,129	17,649
min	41,696	47,111	51,528
max	132,808	124,316	118,870
IRR	11.18%	10.66%	10.07%
Panel B: Volatility targeting			
maximum participation ratio of 1			
mean	73,743	71,722	69,526
stdev	20,246	17,973	16,400
min	46,718	50,449	53,265
max	126,542	121,045	116,756
IRR	10.46%	10.08%	9.66%
maximum participation ratio of 1.5			
mean	83,736	79,227	74,399
stdev	31,836	26,444	21,797
min	49,718	53,092	55,279
max	173,788	156,519	139,846
IRR	12.18%	11.43%	10.58%
maximum participation ratio of 2			
mean	87,462	81,922	76,076
stdev	37,074	30,131	24,069
min	50,258	53,626	54,845
max	190,495	168,149	146,964
IRR	12.76%	11.88%	10.89%

This table presents summary statistics on cumulative amounts and the internal rate of return (IRR) at the end of target-date funds over 15 year life cycles in the U.S.. The period of analysis is from 26 April 1982 to 30 June 2020. The columns present the results under aggressive, moderate, and conservative strategies. Panel A presents results without volatility targeting and Panel B presents results with targeting daily market volatility at 1 percent, with maximum participation ratios of 1, 1.5 and 2. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model and the threshold weight change is 0.1. The 15 year cumulative amount that starts at each trading day in the period of analysis is calculated, where the equity contribution is sourced from Morningstar Lifetime Allocation Indexes. It is assumed that the worker (investor) starts with a salary of 20,000 USD, which grows by 4% in nominal terms each year during a 15 year career. The worker contributes 9% of his/her salary to target-date fund at the end of each year for their 15 years of work. The target-date fund rebalances daily without considering tax implications or transaction costs.

Table 8: **Holding period returns pre, during and post Great Depression and Global Credit Crisis crashes**

crash period	economic event	before	the crash	after	full sample
		market index			
Sep 3, 1929-Jul 8, 1932	Great Depression	40.23	-83.84	334.01	-1.62
		[40.23]	[-41.89]	[34.12]	[-0.17]
Oct 9, 2007-Mar 9, 2009	Global Credit Crisis	17.53	-54.32	218.65	71.06
		[17.53]	[-42.57]	[26.08]	[7.51]
		target volatility of daily standard deviation of 1%			
Sep 3, 1929-Jul 8, 1932	Great Depression	43.90	-70.01	166.21	14.87
		[43.90]	[-30.15]	[21.63]	[1.49]
Oct 9, 2007-Mar 9, 2009	Global Credit Crisis	22.56	-38.40	149.31	88.23
		[22.56]	[-29.03]	[20.05]	[8.91]
		target volatility of daily standard deviation of 0.8%			
Sep 3, 1929-Jul 8, 1932	Great Depression	36.83	-60.36	122.79	20.84
		[36.83]	[-24.09]	[17.38]	[2.04]
Oct 9, 2007-Mar 9, 2009	Global Credit Crisis	17.63	-30.28	111.99	73.84
		[17.63]	[-22.54]	[16.21]	[7.75]

This table presents the holding period returns (in percent) for the one year prior to the crash, the crash period, the five years post crash, and the period starting one year prior to the crash and ending five years after the crash. The annualized returns are presented in square brackets. The market index is the value-weighted CRSP index, including dividends. The target volatility strategy targets a daily 1% or 0.8% standard deviation of market returns, with a daily threshold weight change of 0.1 and with maximum leverage of 50%.

Table 9: **Annualized volatility pre, during and post Great Depression and Global Credit Crisis crashes**

crash period	economic event	before	the crash	after	full sample
		market index			
Sep 3, 1929-Jul 8, 1932	Great Depression	15.65	33.76	28.88	29.77
Oct 9, 2007-Mar 9, 2009	Global Credit Crisis	13.05	37.11	18.65	22.86
		target volatility of daily standard deviation of 1%			
Sep 3, 1929-Jul 8, 1932	Great Depression	19.05	20.02	17.83	18.85
Oct 9, 2007-Mar 9, 2009	Global Credit Crisis	16.14	20.91	15.18	16.59
		target volatility of daily standard deviation of 0.8%			
Sep 3, 1929-Jul 8, 1932	Great Depression	15.80	16.07	14.28	15.17
Oct 9, 2007-Mar 9, 2009	Global Credit Crisis	13.64	16.56	12.03	13.26

This table presents the annualized volatility of returns (in percent) for the one year prior to the crash, the crash period, the five years post crash, and the period starting one year prior to the crash and ending five years after the crash. The market index is the value-weighted CRSP index, including dividends. The target volatility strategy targets a daily 1% or 0.8% standard deviation of market returns, with a daily threshold weight change of 0.1 and with maximum leverage of 50%.

Table 10: **Holding period returns pre, during and post COVID-19 crash**

portfolio	before	the crash	after	full sample
		market index		
equity	5.46	-34.27	41.55	-1.89
	[50.02]	[-98.99]	[255.73]	[-3.78]
balanced	4.29	-23.52	28.20	2.26
	[37.85]	[-94.71]	[147.79]	[4.60]
	target volatility of daily standard deviation of 1%			
equity	5.51	-18.61	18.51	1.77
	[50.62]	[-89.52]	[85.92]	[3.60]
balanced	4.35	-12.64	13.86	3.80
	[38.38]	[-77.24]	[60.67]	[7.80]
	target volatility of daily standard deviation of 0.8%			
equity	4.74	-16.19	13.44	-0.42
	[42.44]	[-85.56]	[58.52]	[-0.84]
balanced	3.84	-11.02	10.62	2.21
	[33.33]	[-72.17]	[44.57]	[4.51]

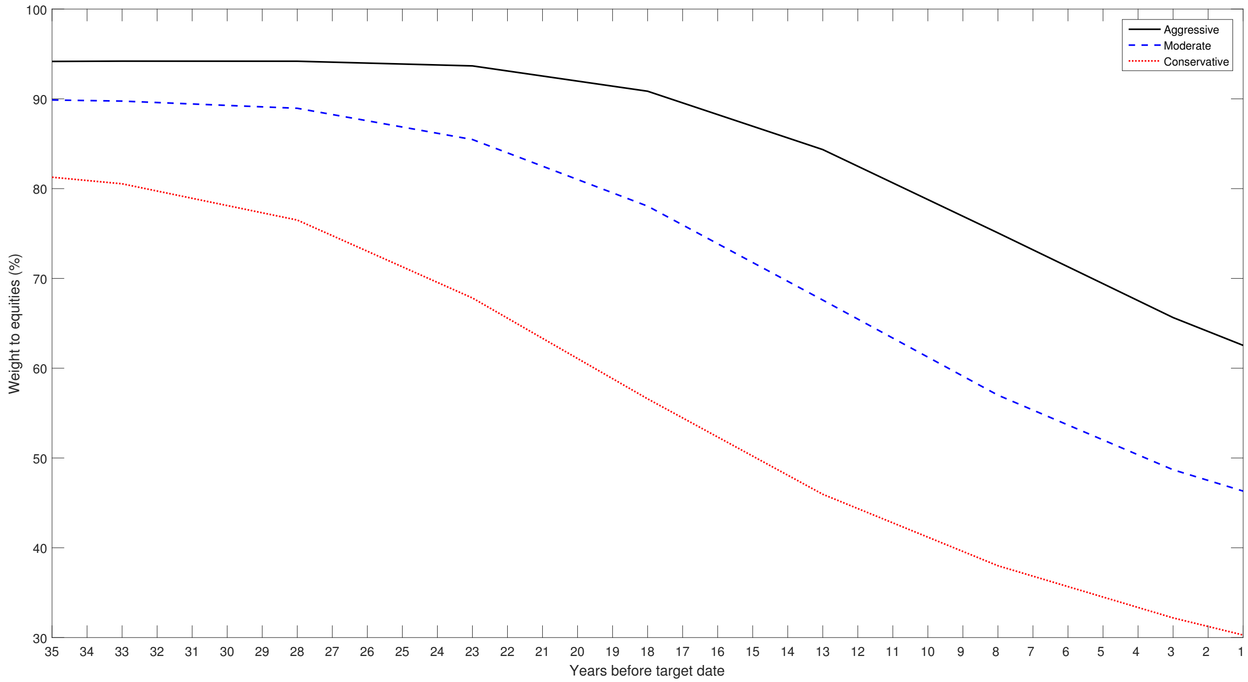
This table presents the holding period returns (in percent) for the pre crash period (1 January 2020 to 19 February 2020), the crash period (19 February 2020 to 23 March 2020), the post crash period (23 March 2020 to 30 June 2020) and the full period (1 January 2020 to 30 June 2020). The annualized returns are presented in square brackets. The market index is the value-weighted CRSP index, including dividends and the balanced portfolio invests 65% in equity markets and 35% in bond markets. The target volatility strategy targets a daily 1% or 0.8% standard deviation of market returns, with a daily threshold weight change of 0.1 and with maximum leverage of 50%.

Table 11: **Annualized volatility pre, during and post COVID-19 crash**

portfolio	before	the crash	after	full sample
market index				
equity	11.35	79.28	38.19	46.07
balanced	6.71	50.47	24.79	29.64
target volatility of daily standard deviation of 1%				
equity	15.13	38.93	20.52	24.58
balanced	9.19	24.44	13.14	15.62
target volatility of daily standard deviation of 0.8%				
equity	12.68	31.92	16.16	19.94
balanced	7.62	20.10	10.34	12.68

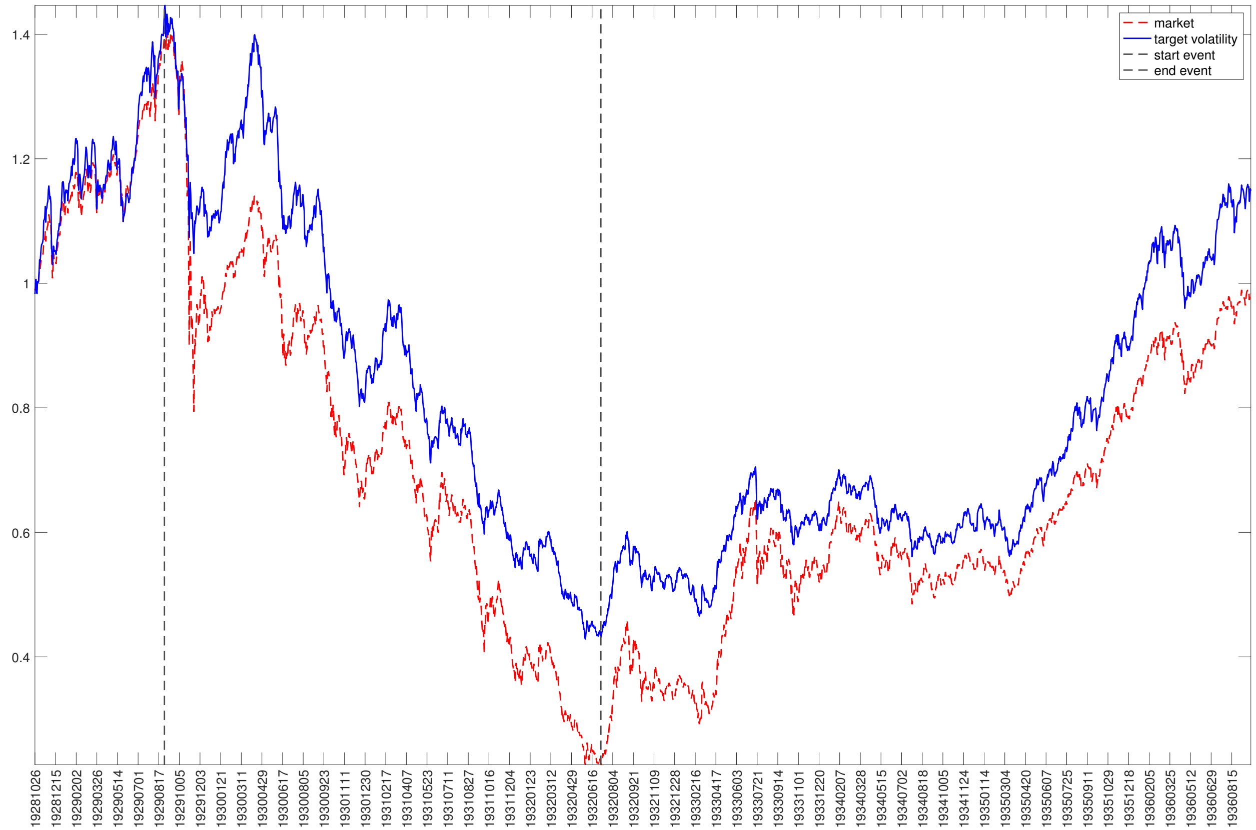
This table presents the annualized volatility of returns (in percent) for the pre crash period (1 January 2020 to 19 February 2020), the crash period (19 February 2020 to 23 March 2020), the post crash period (23 March 2020 to 30 June 2020) and the full period (1 January 2020 to 30 June 2020). The market index is the value-weighted CRSP index, including dividends and the balanced portfolio invests 65% in equity markets and 35% in bond markets. The target volatility strategy targets a daily 1% or 0.8% standard deviation of market returns, with a daily threshold weight change of 0.1 and with maximum leverage of 50%.

Figure 1: **Equity contribution to target-date portfolio**



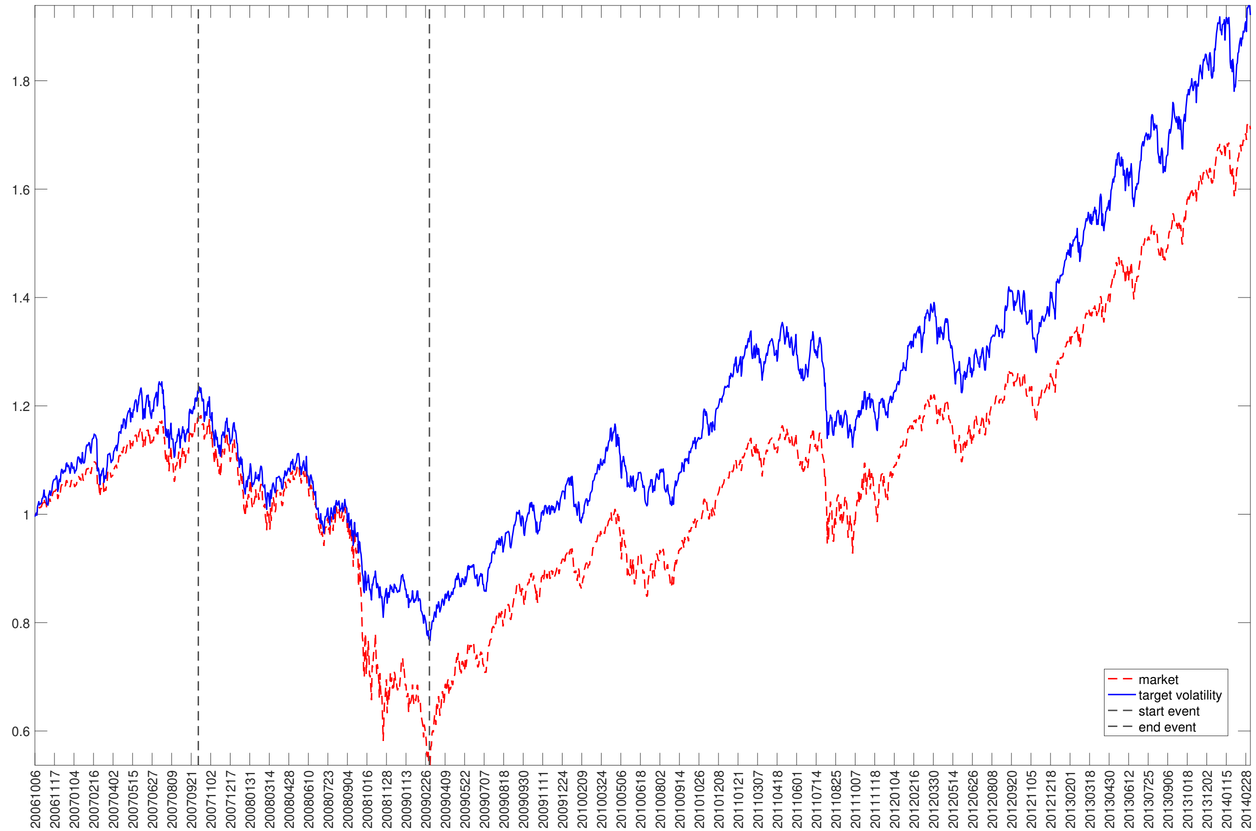
This figure displays the equity contribution to the target-date fund from Morningstar Lifetime Allocation Indexes.

Figure 2: **Crash from Great Depression**



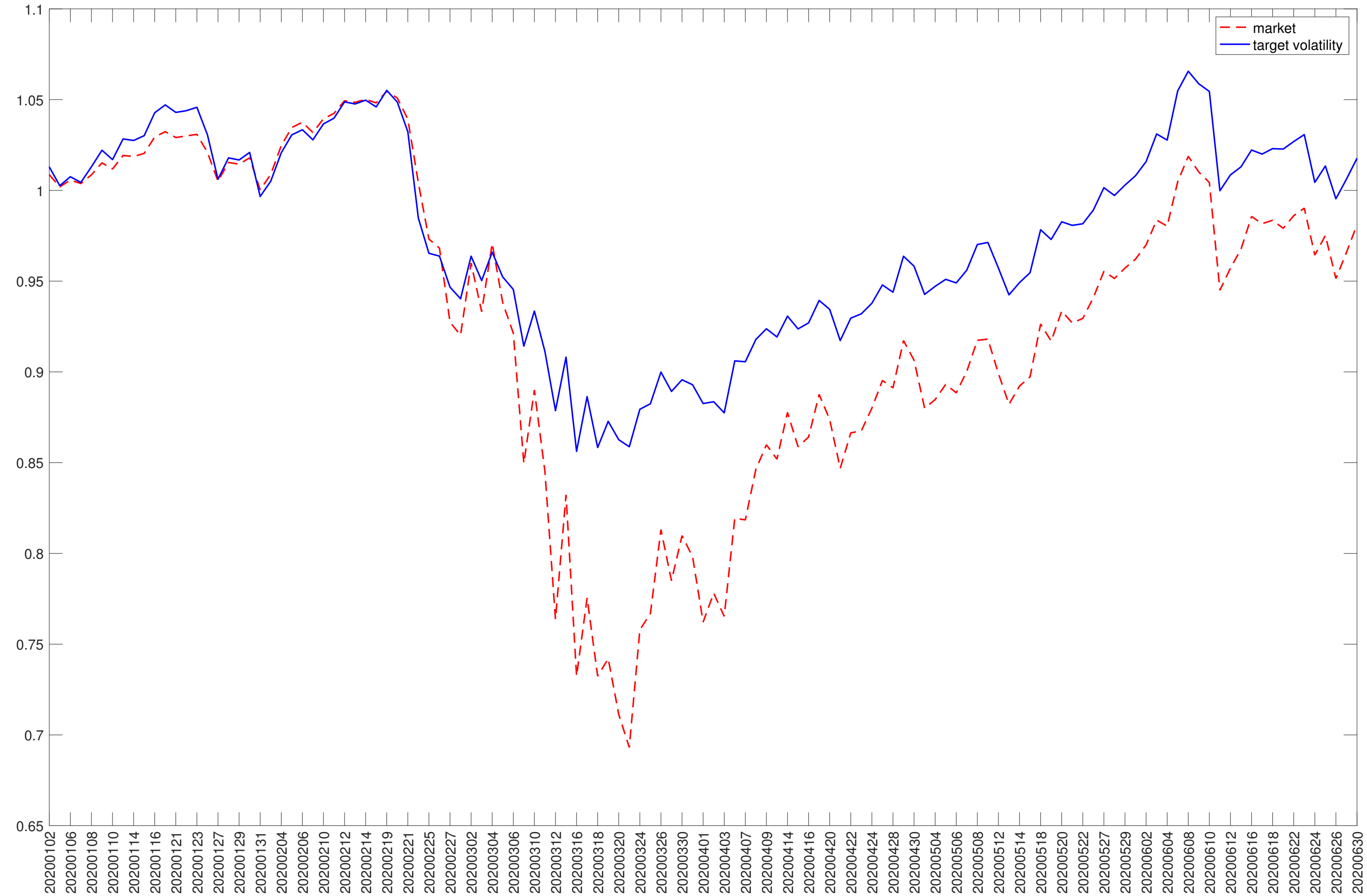
This figure displays the cumulative value from a one dollar investment starting one year prior to the U.S. crash 3 September 1929 to 8 July 1932. The market index is the value-weighted CRSP index, including dividends. The target volatility strategy targets a daily 1% standard deviation of market returns, with a daily threshold weight change of 0.1 and with maximum leverage of 50%. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model.

Figure 3: **Crash from Global Credit Crisis**



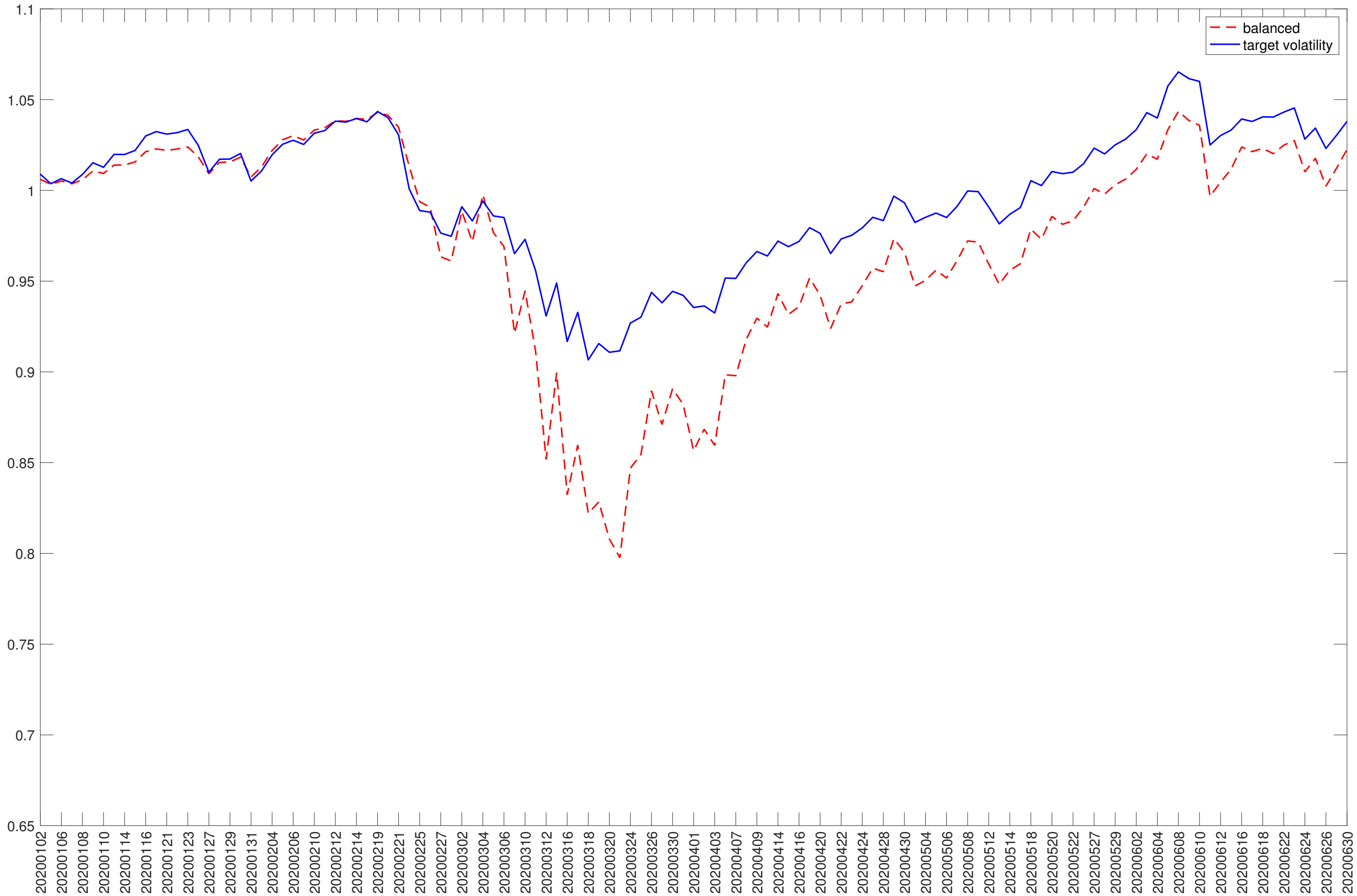
This figure displays the cumulative value from a one dollar investment starting one year prior to the U.S. crash 9 October 2007 to 9 March 2009. The market index is the value-weighted CRSP index, including dividends. The target volatility strategy targets a daily 1% standard deviation of market returns, with a daily threshold weight change of 0.1 and with maximum leverage of 50%. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model.

Figure 4: **Equity portfolios over the COVID-19 pandemic**



This figure displays the cumulative value in equity portfolios from a one dollar investment starting 1 January 2020 and ending 30 June 2020. The market index is the value-weighted CRSP index, including dividends. The target volatility strategy targets a daily 1% standard deviation of market returns, with a daily threshold weight change of 0.1 and with maximum leverage of 50%. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model.

Figure 5: **Balanced portfolios over the COVID-19 pandemic**



This figure displays the cumulative value in balanced portfolios from a one dollar investment starting 1 January 2020 and ending 30 June 2020. The portfolios invest 65% in equity markets and 35% in bond markets. The market index is the value-weighted CRSP index, including dividends. The target volatility strategy in the equity component targets a daily 1% standard deviation of market returns, with a daily threshold weight change of 0.1 and with maximum leverage of 50%. The volatility forecasts are computed from the outlier-corrected GARCH(1,1) model.