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Taxation and policyholder behavior: the case of guaranteed minimum accumulation benefits

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Abstract

This paper considers variable annuity contracts embedded with guaranteed minimum accumulation benefit (GMAB) riders when policyholder's proceeds are taxed. These contracts promise the return of the premium paid by the policyholder, or a higher stepped up value, at the end of the investment period. A partial differential valuation framework, which exploits the numerical method of lines, is used to determine fair fees that render the policyholder and insurer profits neutral. Two taxation regimes are considered; one where capital gains are allowed to offset losses and a second where gains do not offset losses, reflecting stylized institutional arrangements in Australia and the US respectively. Most insurance providers highlight the tax-deferred feature of a variable annuity. We show that the regime under which the insurance provider is taxed significantly impacts supply and demand prices. If losses are allowed to offset gains then this enhances the market, narrowing the gap between fees, and even producing higher demand than supply fees. On the other hand, when losses are not allowed to offset gains, then the demand-supply gap increases. When charging the demand price, we show that insurance companies would be profitable on average. Low (high) Sharpe ratios are not as profitable as policyholders are more likely to stay long (surrender).

Keywords: taxation; retirement income; polycholder behavior; pricing; method of lines; surrender; variable annuity
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1 Introduction and Motivation

Variable Annuities are highly popular in the US where the net asset value is approximately \$1.93 trillion as of June 2019 (Insured Retirement Institute, 2019). Conversely, there is a very thin market for VAs in Australia and Europe. The VA market is relatively immature in Australia[1]. In Europe, the VAs' market was worth 188 billion in 2010 (EIOPA, 2011). However, after the Global Financial Crisis, their popularity decreased and various life insurers stopped offering such contracts[2]. VAs are among the few assets which grow tax-deferred within the US and Australia[3]. Indeed, investors willing to save more than the guaranteed pension employer contributions can invest in a VA, gaining exposure to the equity markets, profiting from a tax-deferred investment to then annuitize the account value upon retirement (Stanley, 2017).

VAs offer an opportunity to participate in the equity market while providing minimum guarantees in case of poor market performance. We focus on GMABs which promise the return of the premium payment, or a higher stepped up value at the end of the accumulation period of the contract[4]. The policyholder can surrender their contract anytime prior to maturity, incorporating often underestimated lapse risk. This is the risk that policyholders exercise their surrender options at a different rate than assumed at inception of the contract. Indeed, Moody's Investor Service (2013) highlights that underpricing lapse risk leads to significant write-downs and earnings charges for insurers.

Taxation levels are known to affect household financial behavior, yet few studies focus on the effect of institutional settings on the demand of insurance products including variable annuities (VAs). All proceeds for the policyholder, be it at maturity or surrender, are assumed to be taxed creating a valuation wedge between the insurer and policyholder. We study the effect of three taxation arrangements: no tax, losses offset (or not) other capital gains on variable annuities (VAs). We aim to identify the extent to which taxation structures affect supply and demand prices for VAs to ascertain whether this might explain the lower popularity of such contracts outside the US. We find that allowing for losses to offset gains narrows the gap between supply and demand fees, often yielding demand fees that exceed supply, whereas the no offset case increases the gap. Given the fee mismatch, we find a distorted demand of this product under our taxation and market assumptions which mirrors the Australian context. Yet, we find that insurance companies charging demand fees that lie lower than their supply break-even fees are profitable on average, benefit from any tax setting, and are particularly affected by the Sharpe ratio of the underlying fund.

The greater part of the existing literature has focused on risk-neutral valuation of VA

[1] There are only a few notable players which includes AMP Financial Services, BT Financial Group and MLC (Vassallo et al., 2016).

[2] For instance, Prudential's GMAB offering Pru Flexible retirement has been closed for business since 2018 (Prudential UK, 2018).

[3] Both countries have a high share of private occupational or private pension savings to finance retirement. The Australian superannuation system, similar to 401(k) plans, is valued at \$3 trillion as of December 2019 and is projected to increase to \$3.5 trillion by 2020 (The Association of Superannuation Funds of Australia Limited, 2020). Maximizing their retirement savings is a high stakes problem in both countries.

[4] There are various types of guarantees embedded in variable annuity contracts and these can be classified into two broad categories namely; guaranteed minimum living benefits (GMLBs) and guaranteed minimum death benefits (GMDBs). GMLBs can be further divided into four subcategories as follows: guaranteed minimum accumulation benefits (GMABs), guaranteed minimum income benefits (GMIBs) and guaranteed minimum withdrawal benefits (GMWBs). A GMIB guarantees an income stream upon maturity of a GMAB for a given term if the policyholder chooses to annuitize. A GMWB guarantees a certain level of withdrawals during the life of the contract.

contracts using a variety of techniques without considering income and wealth tax. Bauer et al. (2008); Bacinello et al. (2011) and Kélani and Quittard-Pinon (2013) provide universal pricing frameworks for various riders embedded in VA contracts when the underlying fund dynamics evolve under the influence of geometric brownian motion (GBM) and Lévy markets, respectively. Incorporating a surrender option is a recent development that addresses the underpricing of lapse risk[5]. Bernard et al. (2014) note that it can always be optimal for the policyholder to surrender the contract anytime prior to maturity if the underlying fund value exceeds a certain threshold. As a means of disincentivizing early surrender, the authors consider an exponentially decaying surrender charge and use numerical integration techniques to determine optimal surrender boundaries. Various authors have since extended the framework in Bernard et al. (2014) to incorporate realistic market dynamics and computationally efficient methods[6]. These valuation frameworks determine fees which lie much higher than those observed in the market partly because taxes are not considered. Our general setting, considering two tax regimes, can be simplified to assess the classical case in the literature where taxes are not considered.

However, it is well known that taxes affect household financial behavior. Souleles (1999); Johnson et al. (2006) and Parker (1999) show that US households' consumption is significantly affected by income tax refunds as well as changes in social security taxes, covering old age survivor and disability insurance (OASDI) and health insurance (DI), respectively. These findings contradict classical life-cycle theory as these tax-related cash-flows are expected and considered in their optimal decision making. Taxes also influence how to finance savings. Multiple studies show that taxes should affect portfolio allocation and asset holding in tax-deferred accounts[7]. However, as highlighted by Poterba (2002), little attention has been paid to the effect of institutional setting taxation on the demand of insurance products. Exceptions include Gruber and Poterba (1994); Gentry and Milano (1998) and Gentry and Rothschild (2010) who find that tax incentives enhance the demand of health insurance for self-employed, variable annuities and life annuities, respectively. Similarly, Horneff et al. (2015) show that purchasing VAs embedded with GMWB riders would increase when taxes are deferred, enhancing the welfare of retirees.

Taxation effects have been highlighted as possible explanation to the mismatch between theoretical and empirical values of variable annuities (Milevsky and Panyagometh, 2001; Brown and Poterba, 2006). Indeed, Moenig and Bauer (2015) resolve this partially by noting that incorporating taxation in the risk-neutral valuation of GMWB riders yields fees that closely match empirically observed values. In a subsequent paper, Moenig and Bauer (2017) find that providers can attach free death benefit riders to guaranteed minimum benefits as a strategy to disincentivize early surrender when income and capital gains taxation are considered. Ulm (2018) also highlight that, for the same taxation regime, the timing of tax affects VA policyholder's value, with taxation at maturity being more advantageous than taxation whenever proceeds are earned. In the same vein, this paper examines the impact of taxation

[5]Bauer et al. (2017) review the state of affairs with regards to the theoretical and empirical insights of policyholder behavior in variable annuities, including lapse risk.

[6]Examples are Ignatieva et al. (2016) who provide a fast and efficient framework for valuing guaranteed minimum benefits using the Fourier space time-stepping algorithm and Kang and Ziveyi (2018) who incorporate stochastic volatility and stochastic interest rates and solve the pricing with surrender resulting free-boundary problem using the method of lines.

[7]See Black (1980) and Tepper (1981) for their seminal work or Fischer and Gallmeyer (2016) for a recent review of the extensions to the Tepper-Black model. Chen et al. (2019) show that life insurance contracts with guarantees contracts lead to a higher expected utility level than traditional long positions in stocks when tax incentives are considered.

on the optimal surrender boundaries for a GMAB when the policyholder behaves rationally with respect to the post-tax value of the contract and we find that the presence of taxation drives a substantial wedge between policyholder and insurer valuations.

These recent findings indicate that individuals might behave rationally with respect to their aftertax benefits. However, a fruitful strand of literature indicates that households do not behave rationally with respect to their financial planning and accumulation of retirement savings or retirement income product purchase, and that this may be due to lack of financial literacy (Lusardi and Mitchell, 2011, 2014; Bateman et al., 2018) or limited opportunities for the current generation to engage in social learning Bernheim (2002). However, the same literature on financial literacy indicates that high-income individuals and households score higher in financial literacy and numeracy measures, and this holds across most developed countries (Lusardi and Mitchell, 2011). This also translates to complex product ownership[8] and better financial decision making (Agnew, 2006). Since VA ownership is more prevalent in high-income households Brown and Poterba (2006), we focus on high-income individuals marginal rate of taxation.

The remainder of the paper is structured as follows: Section 2 presents the partial differential valuation problem to be solved with the aid of the method of lines algorithm. Section 3 analyzes the case when capital losses incurred on the GMAB can be used to offset taxable income from other investments. Section 4 presents the extension in which capital loss made on the contract by the policyholder is not recognised by the government for taxation purposes. Finally, Section 5 analyzes the profit and loss statements of these products under the various tax regimes considered to assess the impact of the moneyness and tax. Concluding remarks are presented in Section 6.

2 Model and Valuation Approach

In this section we provide an overview of the tax treatment of pension funds and VA contracts across the markets that we will study, namely the US, Australia and Europe. The tax treatment of losses is raised as a major difference that may drive the differing demand of GMAB riders embedded in VAs. The valuation framework for a VA contract embedded with a GMAB is subsequently presented. It uses a partial differential equation approach that is solved by the fast and accurate method of lines algorithm. Details about the implementation are provided.

2.1 Tax treatment of pension savings and VAs

Pension funds, mutual funds and retirement income arrangements have differing tax treatment across countries and products. Whether VAs are viewed as a part of an occupational and second pillar retirement account or a third pillar account will affect their tax-treatment.

The tax-treatment of pension fund contributions and benefit payments can be broadly classified into two approaches: Taxed, Tax-Exempt (TTE) and Exempt, Exempt, Taxed (EET) (Bateman, 2017). The former indicates a taxation on contributions and earnings and pay tax-exempted benefits, whereas the latter indicates that contributions and earnings are exempted

[8]Indeed Poterba and Samwick (2003) indicate that households share of tax-advantaged assets increase with marginal income tax rate. Similarly, Inkmann et al. (2010) find that annuity ownership in the UK increases with financial wealth. These households are also more likely to seek financial advice (Finke et al., 2011; Hackethal et al., 2012; Calcagno and Monticone, 2015) and hence benefit from tax management (see e.g. Hackethal et al. (2012) for Germany and Cici et al. (2017) for the US.

and benefit payments are not. Australia's approach to the taxation of superannuation, similar to 401(k) plans in the US, has significantly evolved over the years, moving through many regimes. Currently, Australia operates under a TTE approach, which has only been adopted by a few nations[9]. The Australian approach of flat-rate taxation for contributions and earnings introduces inequity between low- and high-income earners. Most countries, including the US and most European countries[10], operate under a backloaded post-paid expenditure EET tax approach for pension and mutual funds, which encourages participation, creates higher voluntary contributions, delays retirement, and is simpler to tax (Bateman, 2017).

However, whether the product falls under an EET or TEE approach depends on the type of plan elected. In the US, a retiree may have three different account types, namely; (i) tax-deferred accounts, (ii) taxable accounts, and (iii) tax-free accounts. Tax-deferred accounts are those where withdrawals are taxed at the ordinary income rate and investment earnings are tax-deferred until the investment is withdrawn, taxable accounts are those where earnings such as capital gains are taxed at favourable rates while other earnings such as interest are taxed at the normal income rate. Tax-free accounts are those where earnings and withdrawals are not taxed. VA earnings typically fall under the first account type and an EET approach, effectively making private pension contributions to a VA account a tax-deferred investment (OECD, 2016; Moenig and Bauer, 2015). In Europe, earnings on VA contracts embedded with GMAB riders are typically tax-deferred, or in some cases tax-free after a certain waiting period[11]. For instance, in Switzerland earnings are taxable on the guarantee but the gains on the index-based return might be tax-free after a waiting term ranging between 7 to 10 years. France has a similar approach but decreases liable taxes over time from 35% if surrendered before the end of the fourth year to either tax free if earnings are below a given threshold or 7.5% if they lie above. In the UK, age, rather than the term is a factor for benefiting from tax-advantages. If you are aged 55 years or older, then you might take the VA as a tax-free lump sum[12].

Treatment of capital losses is another factor to consider when analysing retirement benefits. In the US, for retirement accounts that are receiving favourable tax treatment, it is generally not possible to offset capital gains with losses. However, losses under non-qualified plans such as commercial variable annuity contracts are deductible in the same manner as lump sum distributions (IRS, 2016). This is the case in Australia as well for certain VA products[13] as losses made on the product can be used to offset gains from other income sources. This is the case as commercial VAs in Australia have the same tax-treatment as unsegregated assets in a self-managed super fund (Australian Taxation Office, 2017a). These are defined as assets that are not specifically set aside for paying a superannuation income stream. In contrast the ordinary income that is earned from segregated pension assets, such as the compulsory superannuation, is tax-exempt. In Germany capital losses on the sale of stock can be transferred to future years (PKF-International, 2016).

The level of the capital gains tax varies across countries too. In Australia there is no sep-

[9]Denmark and New Zealand are among the few nations which have also adopted this approach to the taxation of private pensions.

[10]Excluding Denmark as mentioned above.

[11]These products are typically paid on a single premium basis and any subsequent additional purchase would be seen as a new product falling under new conditions. In Australia, VA products with GMAB riders tend to offer flexibility to partially withdraw or pay additional premiums whilst promising that fees will fall within a pre-specified range.

[12]Kalberer and Ravindran (2009); Junker and Ramezani (2010).

[13]For instance, the MyNorth product offered by AMP Financial Services `https://www.amp.com.au/personal/super-and-retirement/products/superannuation/mynorth-super`

arate tax rate for capital gains (Australian Taxation Office, 2017b). Instead the capital gains are added to taxable income and taxed at the regular marginal income tax rate. Furthermore, the withdrawal from retirement accounts (superannuation) can consist of tax-free and taxable components depending on the tax liability upon contribution into the fund. Tax-free components of superannuation include post-tax contributions, also known as non-concessional contributions. Concessional contributions, on the other hand, are liable to tax upon withdrawal as they are before tax contributions. These include employer contributions and salary sacrifice contributions. Furthermore, taxable decumulation withdrawals have taxed and untaxed elements. The actual tax rate depends on the age of the withdrawal and the proportion of the fund which is considered tax-free[14].

Similar to the approach in Moenig and Bauer (2015)[15], we abstract from these complications and define a unique marginal effective tax rate which the policyholder is liable to. Furthermore, we assume that the policyholder is a high net worth individual and therefore all investment earnings will fall under the highest tax bracket[16].

2.2 VA Embedded with a GMAB

A Guaranteed Minimum Accumulation Benefit (GMAB) rider discussed in this paper involves a policyholder entering into a VA contract by investing an initial amount x_0 into a mutual fund. Upon maturity of the contract, the policyholder is promised the greater of the minimum guarantee that is determined by a fixed roll-up rate δ, $G(\delta) = x_0 e^{\delta t}$, and the fund value. In order to finance this guarantee, the insurer charges a continuously compounded fee at rate q which is deducted as a percentage of the fund. Assume that the underlying (S_ν)[17] follows a standard geometric Brownian motion (GBM)[18] under the risk-neutral measure such that $dS_\nu = rS_\nu d\nu + \sigma S_\nu dW_\nu$. The VA fund value (x_ν) can be expressed as $x_\nu = e^{-q\nu} S_\nu$ where q is the fee rate. Applying Ito's Lemma to the process (x_ν) yields the following dynamics:

$$dx_\nu = (r - q)x_\nu d\nu + \sigma x_\nu dW_\nu. \tag{1}$$

Furthermore, although the income of the policyholder is taxable, they are not taxed until early surrender or maturity. This is consistent with the tax-deferred EET treatment of VAs. The taxable income of the policyholder at maturity can be re-written as:

$$[\max(G(\delta), x_T) - x_0 - C_0 - y(T)]_+, \tag{2}$$

where C_0 is any additional up front cost incurred on top of the premium x_0 which we will assume to be 0, $[z]_+ = \max(z, 0)$ and $y(T)$ is the numerical value of the total fees paid from

[14] The taxed element is the amount that the fund has already paid tax for at a rate of 15%. The taxed element of the taxable component is liable to a lower tax rate compared to untaxed element. In order to compute the total tax paid, a proportional basis is used, that is, the proportion of a withdrawal that is considered tax-free is equal to the proportion of the entire fund that is considered tax-free.

[15] As a contrast Moenig and Bauer (2015) considers the taxes on VA payouts as well as the taxes the policyholder would face in a potential replicating portfolio.

[16] As of 2018 in Australia, individuals in the highest tax bracket with income of $180,001 and over pay $54,097 tax plus 45c for each $1 over $180,000. Income is not taxed if it is under $18,200. Above this threshold, individuals pay 19c for each $1 over $18,200. If their taxable income lies within the range $37,001 and $90,000, they pay $3,572 plus 32.5c for each $1 over $37,000. For the income bracket $37,0001 to $90,000, individuals pay $20,797 plus 37c for each $1 over $90,000 (Australian Taxation Office, 2019).

[17] We reserve the use of t for the time to maturity and τ for the tax rate. Therefore, we use ν to denote the time elapsed since the inception of contract.

[18] We use GBM despite its pitfalls, such as the underestimation of the tails of the asset return distribution. However, an equivalent analysis would naturally follow with the use of sophisticated modelling frameworks.

the fund throughout the life of the contract. In general, $y(\nu) = \int_0^{\nu} q \cdot x(s)ds$ is the cumulative fees paid at time ν[19]. This specification reflects the institutional setting in Australia[20].

We assume two tax treatments for losses. First, we assume that the taxable income cannot be negative in this case because capital losses incurred on the variable annuity account cannot be offset against other income to reduce total taxes paid. This is in line with the approach in Moenig and Bauer (2015) in which there are no offsetting investments and capital losses are not incurred in the GMWB product. The case when losses offset gains is presented as an extension that reflects the tax treatment in Australia.

In addition to this, the GMAB contract permits the policyholder to surrender early. Policyholders are not eligible for the guarantee if they surrender early (Kang and Ziveyi, 2018). If the policyholder surrenders the contract at time ν from the inception of the contract, the insurer will pay $\gamma_\nu x_\nu$, where $(1-\gamma_\nu)$ is the surrender penalty that is charged as a percentage of the current fund value. In the event of early surrender at time ν, the taxable income will thus be

$$[\gamma_\nu x_\nu - x_0 - C_0 - y(\nu)]_+. \tag{3}$$

Let $u^p(x, y, \nu)$ be the value of the GMAB contract for the policyholder where x represents the fund value, the accumulative fees paid is $y(\nu) = \int_0^{\nu} q \cdot x(s)ds$ and the time elapsed since the inception of the contract is ν. Here, the definition of $y(\nu)$ is notationally equivalent to $dy = q \cdot x(\nu)d\nu$. Using risk-neutral hedging arguments akin to path-dependent options, such as Asian options[21] (Shreve, 2004), we obtain the following solution of the contract for the policyholder $u^p(x, y, \nu)$. Appendix A provides details about the steps followed to obtain the following pricing partial differential equation:

$$\frac{1}{2}\sigma^2 x^2 u^p_{xx} + xq \cdot u^p_y + (r-q) \cdot x u^p_x - ru^p - u^p_t = 0. \tag{4}$$

Note that we applied the transformation $t = T - \nu$ and t represents the time to maturity on the contract. We consider taxes on the boundary condition of the policyholder's valuation function, where the policyholder elects to surrender or receives the final payout from the GMAB contract. In order to obtain the contract value from the policyholder's perspective, we solve Equation (4) subject to the following boundary conditions:

$$u^p(x, y, 0) = \max(x, G(\delta)) - \tau\big[\max(x, G(\delta)) - y - x_0 - C_0\big]_+, \tag{5}$$

$$u^p(s(t, y), y, t) = s(t, y)\gamma_t - \tau\big[s(t, y)\gamma_t - y - x_0 - C_0]_+, \tag{6}$$

$$u^p(0, y, t) = (G(\delta) - \tau[G(\delta) - y - x_0 - C_0]_+)e^{-rt}, \tag{7}$$

$$u^p_x(s(t, y), y, t) = \gamma_t - \tau\gamma_t \mathbb{I}\big\{s(t, y)\gamma_t - y - x_0 - C_0 > 0\big\}, \tag{8}$$

[19] The MyNorth product disclosure statement offered by AMP Financial Services (AMP, 2017) states on p26 'guarantee fees you have paid represent the cost base of your asset. A capital gain may arise if the guarantee payments received exceed the guarantee fees you have paid.' Thus the fees paid are tax deductible

[20] Note that if $y(\nu)$ is removed from the cost base, as it is the case in the U.S., the product would not produce capital losses if the policyholder stays until maturity.

[21] Augmenting the state space in this manner is similar to the approach used in deriving pricing equations for path-dependent options, such as Asian options (Shreve, 2004). In the case of the Asian Option, one would typically introduce a new state variable $y(t) = \frac{1}{T} \cdot \int_0^t x(\nu)d\nu$ and the payoff at maturity would be given by $\max\{y(T) - K, 0\}$. Given this boundary condition and a linear partial differential equation for the function $u^p(x, y, t)$ obtained using risk neutral hedging arguments, one can obtain a solution for u^p. Here we follow a similar approach in introducing the new variable y.

where γ is the proportion that the policyholder is allowed to keep subsequent to surrender, x_0 is the initial fund value (that is, the 'premium'), C_0 is the initial upfront cost, τ is the tax rate, $G(\delta)$ is the guarantee amount and $s(t, y)$ is the the minimum fund value to trigger surrender, given that there are still t years to maturity and \$$y$ is accumulated values of fees paid thus far. The free boundary, $s(t, y)$, must be computed along with the valuation solution u. The first two boundary conditions, Equations (5) and (6), represent the post-tax payoff at maturity or upon surrender, which occurs for rational agents when the fund value x exceeds $s(t, y)$. Equation (7) is the present value of the taxable income at maturity when the fund value is zero as given by Equation (2) [22]. Hence, in this case the guarantee is paid with certainty and the payoff is deterministic. Also note that it is usually the case that $G(\delta) < y+x_0+C_0$, so there is no tax and the right hand side simplifies to $G(\delta)e^{-rt}$. The final boundary condition, Equation (8), enforces the continuity of u_x at the boundary $x = s(t, y)$.

The boundary conditions Equations (5) - (8) are in fact not sufficient for solving PDE (4) over the computational domain $\{x, y, t \in [0, X] \times [0, Y] \times [0, T]\}$, with X and Y being sufficiently large enough to be close to 'infinity'. Using Fichera Theory[23], we note that we must add an additional boundary condition to Equations (5) - (8) not only for theoretical reasons but also for practical reasons. Indeed, for sufficiently large fees paid, the taxable amount is zero. Hence, the option reduces to the case where taxation is not considered. In this case, the solution u^p must be independent of the fees paid. Thus we can approximate $u^p_y(x, Y, t) = 0$. We refer to Appendix B for more details on the additional boundary.

Allowance for capital losses to offset gains

Now we explore the case in which capital losses on the GMAB rider can immediately be used to offset other income sources, as is the case for nonqualified plans in the US and Australian variable annuities. In most European countries, these life insurance products fall under advantageous life insurance-specific tax treatments and hence no capital losses can be used to offset gains. Mathematically, this entails the following replacement

$$\tau(\gamma_t x - y - x_0)_+ \to \tau(\gamma_t x - y - x_0),$$

in the boundary conditions (5), (6), (7) and (8). In this case, however, the additional condition at $y = Y$ will be different as high levels of fees paid do not lead to the no-taxation case as high fees will incur losses that will offset any gains on the product. This is discussed in more detail in Appendix B.

Insurer's perspective

As highlighted above, tax is a friction that distorts the valuation of the contract. This yields different results for the policyholder and insurers. The government receives a proportion of the payout, either at surrender or maturity, creating a gap between the value for the policyholder and the insurer's liabilities. To obtain the value of the contract from the insurer's perspective, henceforth to be referred to as the insurer's liabilities, the partial differential equation (4) must be solved subject to boundary conditions which reflect the total before tax payments the insurer must make to the policyholder. The boundary conditions are equal to the boundary conditions in Equations (5) - (8) when $\tau = 0$. In this case, the initial net profit of the insurer is $x_0 - u^i$, where x_0 is the initial premium paid by the policyholder and u^i is the value of the insurer's liabilitites.

[22] This scenario is unlikely in practice, however the boundary condition is necessary for a well-posed problem.

[23] Fichera Theory, as discussed in Oleinik (2012), provides a mathematically sound means of determining which boundaries require a Dirichlet condition in order to have a well-posed problem. The interested reader is referred to Appendix B.

Fair fee

In presence of taxation, the fee that renders the contract fair for the policyholder will differ from the insurer's fee. We obtain these fees by solving the PDE (4) subject to the policyholder boundary conditions, and subject to the insurer's boundary conditions assuming $\tau = 0$. We denote the policyholder fair fee q^p as:

$$q^p = \min \{q : x_0 = u^p(x_0, 0, T)\}.$$

This is the minimum fee rate such that the value of the contract at its inception, when the time to maturity t is T and $y = 0$, is equal to the initial premium paid by the policyholder. In other words, the net profit to the policyholder is zero. Similarly, the insurer perspective fair fee rate q^i can be determined implicitly as

$$q^i = \min \{q : x_0 = u^i(x_0, 0, T)\}.$$

It is the smallest fee rate such that at the inception of the contract, when $t = T$ and $y = 0$, the liabilities of the insurer are equal to the initial amount they receive from the policyholder. This sets the net profit of the insurer to be zero.

Roll-up guarantee

GMAB have typically long maturities. Keeping the guarantee fixed over time may dilute the value of the insurance feature as years pass by. Hence, GMAB products commonly include roll-up features that increase the value of the guarantee at a pre-specified rate, increasing the attractiveness of the product over time. We also analyse the case when the product offers a roll-up guarantee. In this case, upon maturity of the contract the policyholder receives an amount $G(\delta) = x_0 e^{\delta T}$ where δ is a continuously compounded guarantee growth rate amount. This guaranteed growth rate is commonly known as a 'roll up', which locks in a growth rate for the investment. Due to no-arbitrage, we require that $\delta \leq r$. The existence of fair fees may impose an even stronger constraint on δ. In our analysis, we choose values of δ for which a reasonable fair fee can be found using the algorithm.

2.3 Implementation and calibration

In order to solve the Equation (4) subject to the initial and boundary conditions (5) - (8), we utilize the numerical method of lines algorithm. This is accomplished by truncating to the computational domain $\{(x, y, t) \in [0, X] \times [0, Y] \times [0, T]\}$.

It is well known that the method of lines is a fast, accurate and efficient algorithm for solving such free-boundary problems (Kang and Ziveyi, 2018; Meyer and Van der Hoek, 1997; Chiarella et al., 2009). To obtain the contract values, Equation (4) is discretised in the t and y directions and continuity is maintained in x. Time is discretized uniformly starting at inception t_0 up to maturity T, and the spatial variable y representing the total fees paid is discretized between $y_0 = 0$ and Y which is a sufficiently large number. Appendix C describes the step-by-step implementation of the method of lines algorithm used for the valuation of the contract. Since this algorithm provides contract values, we can find fair fees using the bisection method.

In order to assess the attractiveness of GMAB riders in presence of taxation, we perform a numerical analysis based on the calibrated parameters presented in Table 1. Unless otherwise stated, these parameters will be used throughout the remainder of the paper. Table 1 presents the calibrated risk-free rate r, volatility σ, tax rate τ, initial premium x_0, roll-up rate δ, maturity T and penalty κ for Australia, US and Europe. In the remainder of the paper we will denote these markets as *Australia*, *United States* and *Europe* despite the valuation framework not reflecting the array of particularities present in the different economies. We are rather interested in assessing to which extend the financial market and institutional characteristics, reflected by the risk-free rate r and volatility σ as well as marginal tax rate levels, affect our valuation exercise which includes the guarantee fees paid as a part of the cost base. The values of x_0 are chosen to be 100 as a convenient numerical value, since it is only the ratio $\frac{G(\delta)}{x_0} = e^{\delta T}$ which affects pricing. The maturity in all markets is assumed to be 15 and the surrender penalty is arbitrarily chosen to be $\kappa = 0.005$.

Table 1: Financial base case parameters for Australia, US and Europe and sensitivity analysis

	r	σ	τ	x_0	δ	T	κ
Australia	3%	20%	22.5%	100	1.5%	15	0.005
US	3%	19%	15%	100	1.5%	15	0.005
Europe	3.4%	31%	20%	100	1.5%	15	0.005
Sensitivity	2.5%, 3.5%	15%, 25%	17.5%, 27.5%	-	1%,2%	-	0.010

For Australia, we select r based on the historical average of the cash rate in Australia, from 2009 - 2018, and σ based on ASX200 VIX, from 2009-2018. The marginal tax rate τ is calculated based on the 0.45 marginal income tax rate, multiplied by the discount of 0.50 for capital gains. This is the case for such investments in the Australian tax system (Australian Taxation Office, 2018). For the US, r is chosen to be the average 3 month T-Bill rates from 1988 - 2018, and volatility is selected as the historical average of the VIX index from 1991 - 2018. The marginal tax rate is chosen to be the capital gains tax rate in the US (Policy, 2010). Under Europe, we provide calibrated parameters based on the market data from Germany, being the largest economy in the EU. The risk-free rate is the average of interest rates from 1988 - 2017. Volatility is estimated as the average of the VDAX-NEW index from 2009 onwards. As before, τ is taken to be the capital gains tax rate in Germany (PKF-International, 2016).[24] Appendix E shows the sensitivity analysis performed for the alternative parameterization. Overall the results are robust to the alternative specifications and are hence omitted of the main manuscript.

In addition, the following numerical parameters are used for the method of lines algorithm with the number of points in the x grid given by $J1 = 300$ and the number of points used in the y grid being $J2 = 100$. The upper limits of the x and y grids are set to be four times the strike, that is, $X = Y = 4G$. The time grid contains $N = 52 \times 15 = 780$ points such that Δt is one week. We also provide some justification for the choice of $J1$ and $J2$ in Table 2. As evident in Table 2 it is reasonable to assume that the solution converges to the nearest dollar for $J1 = 300$ and $J2 = 100$.

3 VA when capital losses can offset other gains

We first consider the case where capital losses can be used to offset capital gains from other investments. We are interested in the value of the contract from the policyholder and insurer

[24] We use different time periods for the analysis of the risk-free rate because of lack of available data for Australia. However, we do not view an analysis based on pre and post-GFC for US and Europe, compared to a post-GFC for Australia as a shortcoming as Australia was virtually unaffected by the GFC (Reserve Bank of Australia, 2017).

Table 2: Contract values for parameters in Table 1 and policyholder fair fee $q = 0.0198$ which illustrates numerical convergence

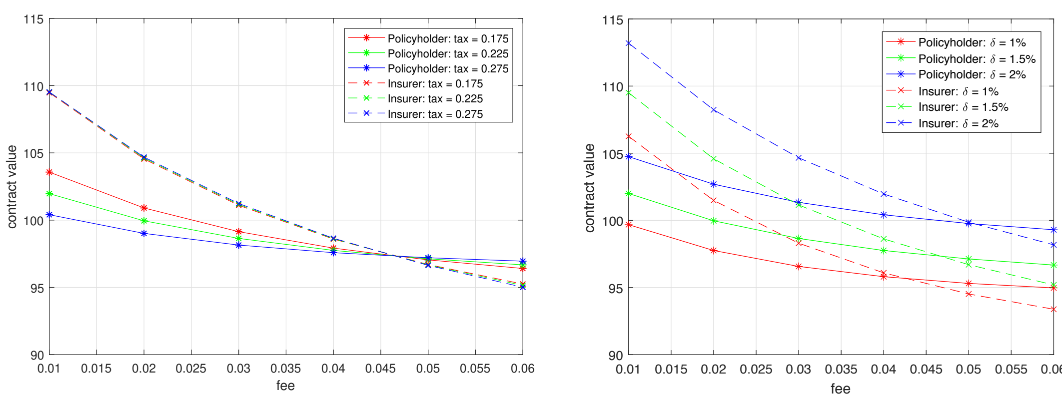
$J1$ \ $J2$	100	200	300	400
100	100.110	100.108	100.105	100.102
200	100.035	100.033	100.032	100.030
300	100.001	99.999	99.999	99.998
400	99.983	99.982	99.981	99.980

perspectives where we will infer surrender boundaries in assessing the attractiveness of the GMAB rider in various financial market settings.

3.1 Insurer liabilities and policyholder contract values

In this subsection we discuss the impact of increasing the level of taxation to the insurer liabilities and policyholder contract values. From an insurer's perspective, the value is the one that applies for accounting and regulatory capital considerations. Indeed, insurers have to hold certain funds notwithstanding the marginal tax rate that policyholders have to pay to the government.

Figure 1: Contract values from the policyholder and insurer perspective as a function of fees charged for the Australian base case parameters as presented in Table 1.



(a) Varying marginal rates of taxation τ

(b) Varying roll-up guarantee rates δ

In Figure 1(a) we present contract values from the policyholder and insurer perspective as a function of fees charged for the Australian[25] base case parameter (first row in Table 1). We observe that the insurer's liability curve represented by the broken lines appears to be quite robust to changes in the tax rate. This suggests that it is the policyholder who bears most of the burden of taxation. Policyholders are more likely to alter their surrender behavior so as to obtain a certain post tax value which results in a higher fee income for the variable annuity provider. We also remark that, for any given tax rate, the insurer's liability and policyholder derived value decreases as the fee rate increases. This is quite intuitive as high fee rates

[25] Rationale for US and Europe is similar and is omitted for a concise discussion.

allow the insurer to collect a large amount of fees during the lifetime of the contract, typically exceeding what the insurer has to pay upon maturity or surrender. A less obvious fact that we find is that the policyholder's option to surrender does not alter the aforementioned intuitive conclusion.

We also observe that the policyholder is not always going to prefer a lower tax rate. For higher fee rates, the policyholder value actually shifts up as the tax rate increases, suggesting that they are better off. This is because at fee rates much larger than q^p, for which the numerical values are shown in Table 3, the policyholder can expect to pay a large amount of fees, implying a larger value for y. Therefore if the tax rate decreases, the policyholder obtains less value from the tax deduction associated with having paid these fees.

At fee rates considerably lower than q^p, the cumulative fees paid will be low so there is no significant tax advantage. Under these circumstances, a higher tax rate decreases the policyholder value of the contract. Figure 1(b) demonstrates that these conclusions also hold for various roll-up rates of the guarantees. As the roll-up rate increases, the insurer's cost of provisions and policyholder value increases, conforming to expectations as the guarantee increases with δ. We observe as well that higher roll-up fee rates reduce the gap between the policyholder's and insurer's fee[26]. In other words, the roll-up guarantee decreases the gap between the fee that the policyholder is willing to pay and the insurer is willing to accept.

Furthermore we note in Figure 1 that for larger (than fair) fee rates, the policyholder value curve is above the insurer liabilities curve. Using the simple identity

$$\texttt{GovtValue} = \texttt{InsurerLiabilities} - \texttt{PolicyholderValue},$$

we can note that for these large fee rates, the value to the government (`GovtValue`) is negative. A large fee rate is beneficial to both the insurer and the policyholder in that the policyholder is able to use the higher fee paid to offset their taxable income, while the insurer can obviously collect more fees. This is akin to Moenig and Bauer (2017) who note that adding a free death benefit nudges a favorable policyholder behavior, increasing the value to the insurer. In both the aforementioned cases, the policyholder and insurer both gain at the expense of the government. In particular, the total tax revenue from the policyholder will be reduced but the effect of the insurer should be analysed further as done in Section 5. Indeed, the delayed surrender will increase the fee revenue of the insurer, increasing its profits. Depending on the corporate taxation regime the insurer falls under, the additional revenue will yield to additional tax liabilities towards the government. In Figure 1 the intersection between the insurer liability curve and the policyholder contract value curve represents the fee at which the value of the contract to the government is zero.

Table 3 summarizes the policyholder and insurer fair fees (q^p and q^i respectively), that is, the fees that render the contract fair for either the policyholder and insurer in three financial markets and policy settings. Before inspecting the effect of the tax wedge in the valuation of the product, we highlight that the fair fees under the no taxation case, classical in the VA pricing literature, yields $q^p = q^i$. As expected, in the no tax case the valuation curves coincide and the value that both policyholder and insurer are willing to pay or accept in order to enter the contract coincide. Furthermore, we note that both the policyholder and insurer fair fee increases for higher roll-up guarantee δ as suggested in Figure 1(b). Indeed, higher δ increase the minimum accumulation benefit, making the product more attractive for the policyholder and more expensive to provide for the insurer, especially as the spread between the risk-free rate and δ decreases.

[26] This is the policyholder q^p and insurer fee q^i that yield a value of $x_0 = 100$.

Table 3: Fair fees (% p.a.) from the policyholder (q^p) and insurer (q^i) perspectives when capital losses can be used to offset gains from other investments, at various tax rates (τ) and roll-up guarantees (δ).

a Australia

	δ=0.0		δ=0.010		δ=0.015		δ=0.020	
τ	q^p	q^i	q^p	q^i	q^p	q^i	q^p	q^i
0	1.17	1.17	2.33	2.33	3.35	3.35	5.04	5.04
0.175	n.a.	n.a.	1.35	2.41	2.47	3.40	4.80	4.94
0.225	n.a.	n.a.	0.80	2.44	1.97	3.42	4.61	4.91
0.275	n.a.	n.a.	0.24	2.45	1.25	3.44	4.14	4.87

b United States

	δ=0.0		δ=0.010		δ=0.015		δ=0.020	
τ	q^p	q^i	q^p	q^i	q^p	q^i	q^p	q^i
0	1.00	1.00	2.06	2.06	2.99	2.99	4.53	4.53
0.10	0.57	1.03	1.59	2.10	2.56	3.02	4.34	4.51
0.15	0.30	1.04	1.26	2.13	2.25	3.04	4.16	4.50
0.20	n.a.	1.02	0.84	2.15	1.81	3.06	3.87	4.48

c Europe

	δ=0.0		δ=0.010		δ=0.015		δ=0.020	
τ	q^p	q^i	q^p	q^i	q^p	q^i	q^p	q^i
0	3.01	3.01	4.98	4.98	6.56	6.56	8.93	8.93
0.15	2.05	3.17	4.17	5.07	6.02	6.51	9.08	8.59
0.20	1.57	3.24	3.73	5.10	5.70	6.49	9.22	8.47
0.25	0.97	3.33	3.13	5.14	5.23	6.47	9.45	8.32

Notes: 'n.a.' implies that a fair fee does not exist. In other words, for all fee rates q, the value of the contract is less than $x_0 = 100$ due to the absence of a guarantee roll up.

We note that the insurer fair fee, q^i, is more robust to the taxation rate as compared to the policyholder fair fee, q^p. This aligns with the illustrations in Figure 1, where the valuation curve of the insurer remained stable for a given δ. We observe that the policyholder fee, q^p, typically decreases with tax rates, whereas insurer fee, q^i, increases for higher tax rates. As earlier discussed, policyholders act so as to maximize post-tax value, and increasing tax rates reduce the potential gains for the market. This affects their surrender behavior, which increases the uncertainty on the insurer side, increasing q^i. However, the roll-up fee affects this conclusions. For high roll-up fees, such as $\delta = 2\%$ for the three markets considered, the insurer fee decreases with tax rates. We hypothesize that this is due to the advantageous surrender behavior. Higher δ will incentivize policyholders to stay in the contract, reducing the uncertainty on the insurer's side, decreasing q^i.

Considering taxation and the potential for losses to offset gains creates a gap between q^p and q^i. We observe that the absolute gap increases as tax increases. If we perform our analysis solely on the grounds of this gap, we could argue that *a market could not exist* as the inequality $q^p > q^i$ rarely holds, except whenever we find ourselves in the European market, Table 3c with a roll-up guarantee fee of $\delta = 2\%$. However, in Section 5 it becomes clear that a profitable market can still exist on average. Note, however, that whenever $q^p > q^i$ holds the policyholder and insurer will be satisfied with any fee rate $q \in [q^i, q^p]$. This corresponds to a net of δ reward over variability strictly under 5%. As the roll-up rate δ increases, and gets closer to the risk-free rate, r, the guarantee provided becomes more attractive to the policyholder as virtually guarantees the risk-free rate, with no risk and unlimited (but bounded) upward potential from the financial markets as indicated by the boundary conditions. This suggests that, for higher δ, the policyholder is more willing to enter this market for higher tax rates τ, increasing q^i and potential losses under the EET tax regime on scope, receiving both a higher minimum maturity guarantee and higher tax reimbursements from the government.

In summary, the results in Table 3 suggest that the attractivennes of the rider is very closely related to the relationship between the roll-up guarantee fee δ, risk-free rate r and volatility σ, more so than the the taxation level, τ. Allowing for losses to offset gains enhances the market for high roll-up rates, as the policyholder fee q^p increases with tax rates, indicating that a policyholder is more willing to enter a contract under such a taxation regime compared to the no tax case. Indeed, the insurer can gain up to 52bps in this context, at the expense of the government reimbursing part of the losses to the policyholder. In contrast, as suggested by the analysis in Section 4, not allowing for losses to offset gains widens the valuation gap and lowers the product's attractiveness.

3.2 Optimal surrender behaviour

An insightful way of visualizing policyholder surrender behaviour is to plot the surrender boundaries. Recall that the surrender boundary is represented by the function $s(t, y)$ as discussed in Section 2.2. It is the minimum fund value required to trigger rational surrender, as a function of the time to maturity of the contract, t, and y which is the cumulative fees paid by the policyholder up to time t.

For the purpose of presenting surrender profiles, we assume that the fee rate that is actually charged on the contract is q^p, which delivers zero profit to the policyholder. We assume $\delta = 0.015$[27] and the base case parameters for each financial market as described in

[27] Results are highly sensitive to δ and these values were chosen arbitrarily for the countries since roll-up rate for purely GMAB products are not available.

Table 1, that is q^p equals 1.97, 2.25 and 5.7 for Australia, US and Europe as depicted in Table 3, respectively.

Figure 2: Optimal surrender boundary for parameters calibrated to various markets (offset allowed)

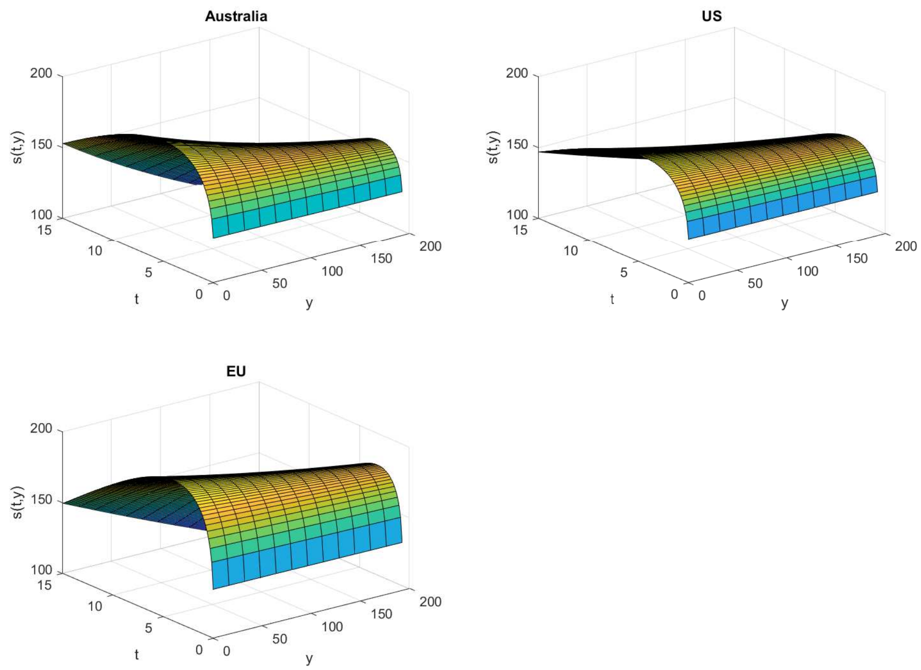
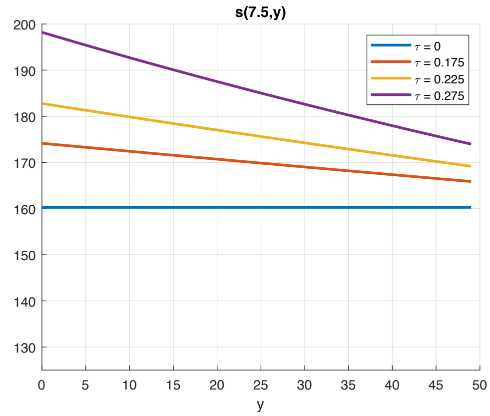
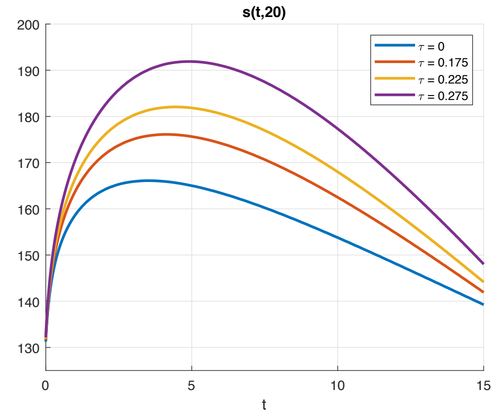


Figure 2 indicates that the surrender surface $s(t, y)$ is monotonically decreasing in y. This is because, all else equal, having already paid a greater sum of fees will reduce taxable income. Hence the policyholder is willing to tolerate a smaller fund value upon surrender. This effect is amplified for greater marginal rates of taxation. We also note that the surrender behavior is always beneficial for the insurer, since the policyholder is only likely to surrender after having paid a lot of fees. Therefore the insurer also benefits, owing to greater fee collections as long as the policyholder is invested in the contract. Hence, if early surrender is contingent on the policyholder having paid a large amount of fees, then the insurer is not disadvantaged as much. Taxation causes the rational policyholder to act in a manner that turns out to be of neither benefit or detriment to the insurer, as demonstrated in Figure 1(a).

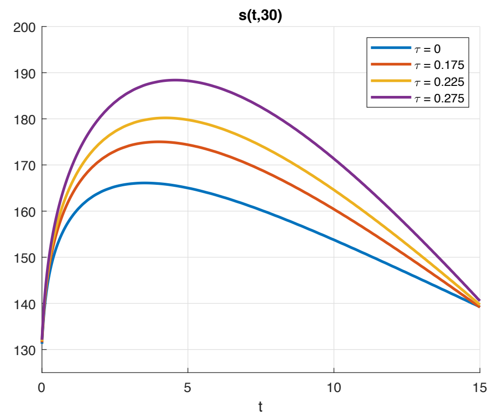
Figure 3: Surrender boundaries for fixed time t and total fee paid y (offset allowed)



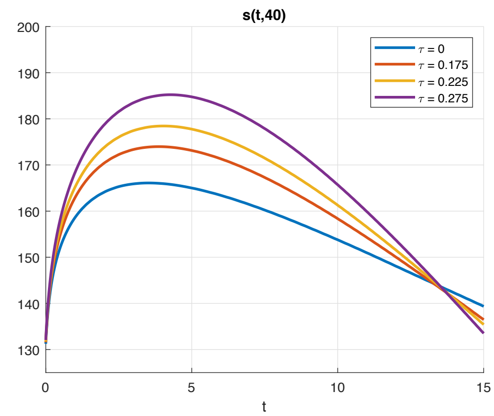
(a) $t = 7.5$ for varying cumulative fees y paid.



(b) $y = 20$, for varying time to maturity t



(c) $y = 30$, for varying time to maturity t



(d) $y = 40$, for varying time to maturity t

Note: the Australian market parameters as of table 1 are considered.

Figure 3 shows the impact of taxation on surrender for varying times-to-maturities and cumulative fees paid. Figure 3(a) shows the evolution of the surrender value when the contract is halfway from maturity. We note that as the taxation rate increases, the policyholder is less willing to surrender for smaller values of y due to the lower fee rate being charged. However, after a given value of y that remains virtually the same under the various tax levels, higher fees make policyholders more willing to surrender for higher fee rates as the benefits of tax reimbursement from the government are greater.

Figures 3(b), 3(c) and 3(d) show the evolution of the surrender boundaries for three levels of accumulated fees, $y = 20, 30, 40$ for varying time to maturity t. We note that $t = 15$ and $t = 0$ indicate that the contract has just started or is about to mature, respectively. First, we observe that the policyholder is more incentivized to surrender for larges values of y as indicated by the lower boundaries that trigger rational surrender. This behavior follows from the greater tax benefit.

However, later on in the contract, when t approaches zero, the presence of tax reduces the volatility in the final payoff, since the government absorbs a portion of both losses and gains. Indeed, as $t \to 0$ the surrender penalty approaches zero and hence the boundaries approach the guaranteed amount $G(\delta)$. Thus the policyholder is more willing to remain invested at higher τ

for smaller t, which is indicated by the surrender boundary being shifted up. This is consistent with Bernard et al. (2014). If the policyholder has any amount in the fund exceeding 100, then they would prefer to surrender δt (and keep the fraction $e^{-\kappa\delta t}$) before maturity rather than pay fees in the time interval δt for a guarantee which has a low probability of ending up in the money. When the tax rate, τ, is set equal to zero, we reproduce results from the setting which has been extensively studied in the literature (Bernard et al., 2014; Shen et al., 2016).

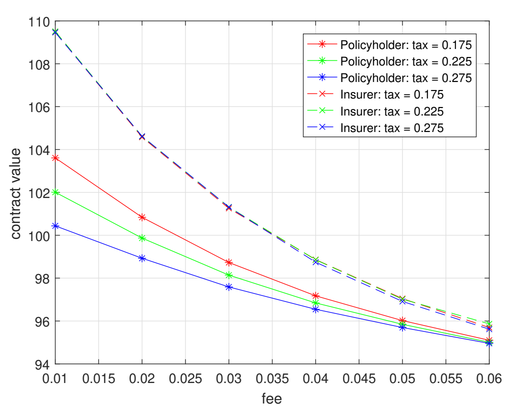
4 VA when capital losses do not offset other gains

In the following section we consider the case where capital losses cannot offset gains from other investments, which is the case in previous literature (Moenig and Bauer, 2015). More specifically, if the tax base exceeds the pay-off of the asset then the difference may not be claimed as a capital loss to take advantage of tax benefits. This implies that the value of the contract is always positive to the government.

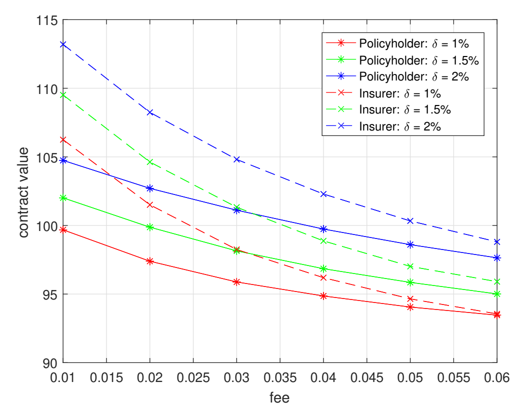
4.1 Insurer liabilities and policyholder contract values

In this subsection we discuss the effect of taxation to the insurer liabilities and policyholder contract values. Contrary to Section 3, losses cannot offset capital gains and therefore tax is only paid when gains are incurred. This will affect the behavior of the policyholder as they will try to avoid losses, that is, they will try to receive as much value of their contract while minimizing the fees paid. This will have a distortionary effect in the viability of such products in this taxation regime, especially for high marginal tax rates.

Figure 4: Contract values from the policyholder and insurer perspective as a function of fees charged, when no offsets allowed



(a) Contract values for various tax rates



(b) Contract values for various roll-up guarantee rates

Figure 4 presents the contract values from the policyholder and insurer perspective as a function of fees charged when no offset is allowed. In Figure 4(a) we observe three clear patterns. Firstly, we observe that the insurer and policyholder value functions decrease when the fee charged increases (for a given tax rate). Contrary to what is observed in Figure 1, charging a higher, than the fair, fee does not stabilize the value function as the increasing value

Table 4: Fair fees (% p.a.) from the policyholder and insurer perspectives when capital losses cannot be used to offset gains from other investments, at various tax rates

a AU

	δ=0.0		δ=0.010		δ=0.015		δ=0.020	
τ	q^p	q^i	q^p	q^i	q^p	q^i	q^p	q^i
0	1.17	1.17	2.33	2.33	3.35	3.35	5.04	5.04
0.175	0.25	1.21	1.32	2.47	2.35	3.53	4.15	5.21
0.225	n.a.	1.13	0.87	2.46	1.92	3.54	3.79	5.22
0.275	n.a.	0.88	0.24	2.37	1.28	3.51	3.30	5.22

b US

	δ=0.0		δ=0.010		δ=0.015		δ=0.020	
τ	q^p	q^i	q^p	q^i	q^p	q^i	q^p	q^i
0	1.00	1.00	2.06	2.06	2.99	2.99	4.53	4.53
0.10	0.14	1.02	1.55	2.12	2.49	3.08	4.08	4.64
0.15	n.a.	0.93	1.24	2.16	2.18	3.13	3.80	4.68
0.20	n.a.	0.68	0.85	2.16	1.79	3.14	3.46	4.70

c EU

	δ=0.0		δ=0.010		δ=0.015		δ=0.020	
τ	q^p	q^i	q^p	q^i	q^p	q^i	q^p	q^i
0	3.01	3.01	4.98	4.98	6.56	6.56	8.93	8.93
0.15	1.54	3.39	3.75	5.45	5.35	7.02	7.77	9.34
0.20	1.02	3.30	3.24	5.53	4.84	7.12	7.30	9.41
0.25	0.45	2.99	2.66	5.53	4.27	7.15	6.76	9.47

Notes: 'n.a.' implies that a fair fee does not exist. In other words, for all fee rates q, the value of the contract is less than $x_0 = 100$ due to the absence of a guarantee roll up.

of the cumulative fees paid, y, is of no advantage to the policyholder. Indeed, contrary to the offset case, higher fee payments yield lower gains with no reimbursement from the government. Hence, the value decreases with the fee. The same trend holds for the insurer, however, the reasoning differs. The insurer value function is calculated as the difference between the outgoing cash-flows paid to the policyholder and the in-flowing payments from the policyholder. The greater fee income, for a given guarantee, the lower this difference will be.

Secondly, we observe that the separation between the insurer and policyholder valuation curves (for a given tax rate) increases as tax increases. This is partially caused by the limited variation in the insurer valuation curve. Indeed, Figure 4(a) suggests that the insurer valuation curve is robust to the taxation rate. However, the value to the policyholder decreases with tax rate (for a given fee) as all gains are taxed at a higher rate. This wedge between the policyholder and insurer value reflects the increasing value of the contract to the government as the tax rate increases. Thirdly, we observe that the fee at which the policyholder value curve hits 100 is 30 to 50% lower than the fee the insurer is willing to charge in order to break even.

In Figure 4(b) we observe that the values to the policyholder and insurer increase as the roll-up guarantee rate, δ, increases. The higher guarantee, the more valuable the contract will be and the higher fee the policyholder will be willing to pay. As a counterparty, the product becomes more expensive for the insurance company to offer, increasing the out-flow payments, and liability, accordingly. Similar to Figure 1(b), the valuation gap decreases as the guarantee increases, narrowing the gap between the fair fees, and liabilities for the policyholder and insurer.

This behavior becomes clear in Table 4 where we observe three trends. Firstly, the fair policyholder fee, q^p, decreases as the tax rate increases, for the three financial and policy settings studied (Table 4a, 4b and 4c). As observed in the valuation curves, the after-tax capital gains decrease with higher tax rates, lowering the attractiveness of the product to the policyholder. This holds for all studied roll-up rates, δ[28]. Note, however, that the no roll-up case is very unattractive to the policyholder, to the extend that fair fees do not exist. In other words, the value of the product is always lower than the initial premium 100.

Secondly, we observe that the interaction between the fair fee of the insurer, q^i, and the tax rate, τ, highly depends on the roll-up rate δ. Whenever the product does not offer a roll-up, the fee decreases as tax-rate increases. This follows from the surrender behavior of the policyholder. The non-decreasing guarantee, $\delta = 0$, degrades the value of the product soon for the policyholder, which leads to undesirable early surrender. This effect is starker the greater the tax rate is. However, when the roll-up guarantee rate is positive, we observe the opposite behavior, that is, q^i increases with the tax rate. Note that this increase is not substantial and seems to plateau after a certain level is reached. This is a direct consequence of the taxation regime considered. For higher tax rates, the policyholder acts as to maximize post-tax value. This leads them to stay longer, or up to maturity, invested in the contract as to benefit from a positive market performance of the underlying fund, or receive the minimum guarantee amount. This renders the contract being more expensive for the insurer to provide, hence increase q^i. We observe, however, that this is not a monotonic increase in all cases. For the Australian base case parameters, Table 4a, we observe that the fees increase for low tax rates and subsequently decrease. However, the level never reaches the policyholder demand price. This is further analyzed in Section 4.2.

[28]We note that the risk-free rate for the European setting is much greater than δ as given in Table 1. However, performing this analysis for δ closer to r yield an analogous interpretation.

Thirdly, we observe that the maximum fee, q^p that the policyholder is willing to accept is always smaller than the minimum fee, q^i the insurer is willing to offer. This difference becomes greater as tax rates increase as q^p generally decreases and q^i generally increases.

4.2 Optimal surrender behavior

We observed in the previous subsection that there were a few non-monotonic relationships between the roll-up guarantee, δ, tax rate τ and the financial parameters. Here we present the early surrender surface $s(t, y)$ as discussed in Section 2.2. Similar to Subsection 3.2, we assume that the fee rate that is actually charged on the contract is q^p, which delivers zero profit to the policyholder and $\delta = 0.015$. Furthermore, we assume the base case parameters for each financial market as described in Table 1, that is q^p equals 1.97, 2.25 and 5.7 for Australia, US and Europe as depicted in Table 3, respectively.

Figure 5 depicts the surrender behaviour $s(t, y)$ for varying cumulative fees paid and time to maturity. The 'valley of surrender', a combination of values of cumulative fees paid y and time to maturity t is driven by the fact that capital losses cannot be claimed on the contract. From Figure 5, we note that the policyholder will be less eager to surrender as the cumulative fees paid stops increasing beyond a certain point. This is because the tax advantages associated with paying a large amount of fees are gradually fading away due to capital losses not impacting the policyholder's taxable income. A brief financial justification of the valley of surrender is provided in Appendix D. We find that the phenomenon is present in all countries considered by the depth and monotonicity depend on the market parameters considered. We observe as well that the valley of surrender is not a smooth line which might explain the results of the fees in Table 4.

Figure 6 shows the impact of taxation on surrender for varying times-to-maturities and cumulative fees paid. Figure 6(a) shows the effect of total fee paid y when the contract is halfway through maturity. We observe that for low fee paid, the policyholder's boundary always exceeds the no tax case. Furthermore, the surrender boundary increases for higher tax rates. This resonates with the explanation of Subsection 3.2 and Sections 4. Indeed, reducing the post-tax value through higher taxes delays surrender as individuals are maximizing their post-tax value. However, for too high rates, this effect dissipates. We observe that, for the parameters considered, the surrender boundary is lower than the $\tau = 0$ case for $y > 32$ for the lowest tax considered. Hence, Figures 6(b), 6(c), 6(d) highlight this distortion and its relationship with the time to maturity. Overall the surrender boundaries in the presence of tax exceed those with taxes except when paid fees are too high (Figure 6(d)). From these figures we observe two features of the valley of surrender; firstly, it is a phenomenon that appears for particular fee ranges. Secondly, it causes a break in the surrender boundary that is monotone but not smooth. For higher t, that is, when the contract is closer to inception that it is to maturity ($t = 0$), high fee rates disrupt the surrender boundary, increasing it beyond the $\tau = 0$ case. For high fees, the non-linearity introduced through the non-allowance of losses, distorts the relationship between fees paid, tax rates and maturity. This translates to non-monotone relationships in the fair fees for some parameterizations.

Figure 5: Optimal surrender boundary for various markets (capital gains only)

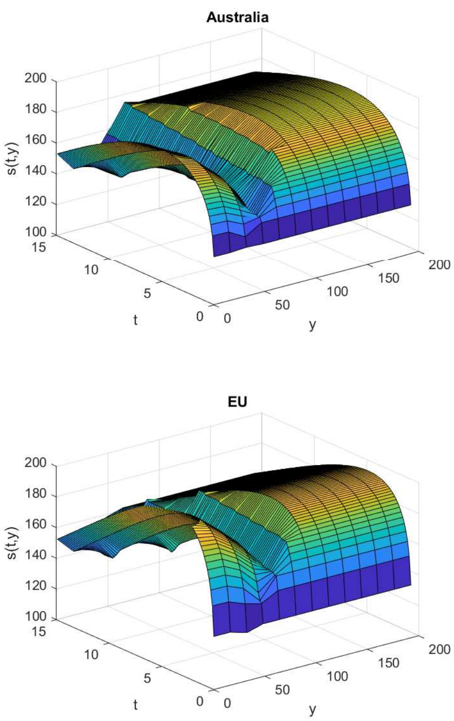
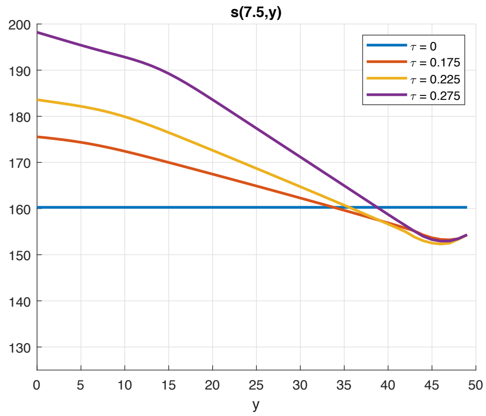
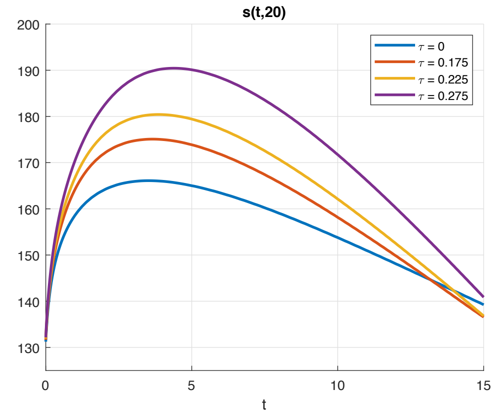


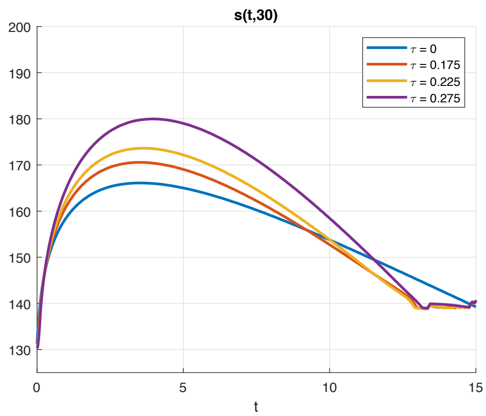
Figure 6: Surrender boundaries for fixed time t and total fee paid y (no offset allowed)



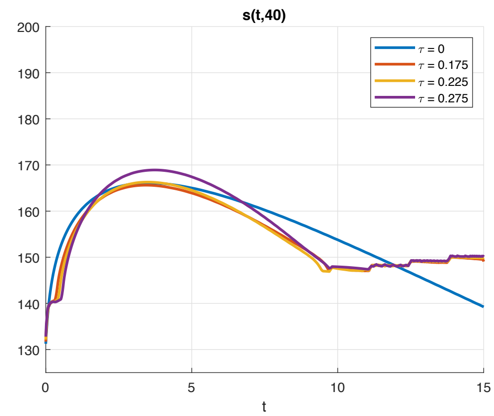
(a) $t = 7.5$ for varying cumulative fees y paid.



(b) $y = 20$, for varying time to maturity t



(c) $y = 30$, for varying time to maturity t



(d) $y = 40$, for varying time to maturity t

5 Profit and Loss analysis

To complement our analysis of the viability of the product, we are interested in the profit and loss statements for varying interest rates r, volatility σ and tax rates τ. These values provided in Table 5[29] result from the simulation of the fund under the real-world measure. The surrender boundary, already precomputed from the method of lines solution of the PDE, was used to determine whether the surrender condition is met. The average of 10 000 Monte Carlo simulations is shown, hence, the results in Table 5 are expectations under the real-world measure.

The profit and loss (P&L) tables provide an overview of the surrender fee that the insurer receives upon early surrender, the cost of providing the guarantee, as well as the management fees required to fund the insurance product. The implicit surrender behavior leads to the calculation of a surrender rate, defined as the proportion of insurance contracts that are terminated prior to maturity. Also, our simulations allow us to analyze the average time elapsed in the contract before surrender (if any). Finally, the net profit is calculated as the management fees, complemented by the surrender fee reduced by the guarantee cost. The last

[29]Full details are provided in the online appendix F in Tables F.2,F.3, F.4 and F.5.

row also shows the net profit quantiles in order to inform about their skewness and level. The results presented in the second, third set of results correspond to the case without taxation, with taxation and without the possibility to offset losses, and taxation with the option to offset, respectively. Furthermore, varying Sharpe ratios (SR) are considered.

First, we analyze the top left cell of Table 5. For a given risk-free rate, taxation rate and implied fair fee (q^p), we observe that surrender is more likely for higher SRs, that is, higher rewards-to-variability ratios. This is intuitive as a higher potential outside of the insurance contract, for given parameters, will incentivize the policyholder to leave the contract. More frequent lapses are associated with more surrender fees being collected and a lower average time in the contract. However, this increase in surrender fees is insufficient to overcome the loss of regular management fees, reducing the overall net profit of the product. This behavior is consistent across risk-free rates. We should note that the early surrender in the high SR market is undesirable for the insurance company. As the underlying fund performs well, the probability of having to pay the guarantee if the policyholder stays up to maturity is significantly reduced. However, as the fees are linked to the performance of the underlying, the fee income would increase with the market performance. In that case, the insurance contract would be very profitable. It turns out that the policyholder only stays until maturity, according to our rational boundaries, if the guarantee option is non-zero. In that case, it is detrimental for the insurer as the net profit is decreased by the guarantee cost. This is the case for low SRs. In Table F.2 we find that for low SR and lower (higher) r the net profit reduces (increases) as the guarantee cost becomes more (less) expensive. For high SR and lower (higher) r we find that surrenders happen less (more) often, increasing (decreasing) the net profit.

Taxation yields lower net profit in our base case[30] as reflected by a substantial increase in surrender, to the extent that it can amount 100% of the cases. Again, surrender fees do not suffice to compensate the loss of regular fee income, decreasing the net profits. For high SR, we find that the average time elapsed can be less than half the maturity. For the same risk-free rate, r, we find that allowing for losses to offset gains increases the net profit of the insurer, especially for high SR cases. Overall, it was clear from the previous sections that insurers could charge a higher fee under this tax regime, increasing their management fees accordingly. This, together with comparable surrender fees, increases the net profit of the product. Table F.4 shows that this effect is greater for higher taxes, that is, higher taxes lead to more surrenders, lower management fees, and overall lower net profit.

For a given σ, the SR affects the μ drift under the real-world, and the average return of the underlying. The second half of Table 5 shows the interaction of σ and SR. Similarly to what happened for the r analysis, when $\tau = 0$, we find that higher SRs increase the net profit as the fees are linked to the performance of the fund. However, after attaining a certain SR, rational surrender starts to kick-in, decimating the management fees, and subsequent net profit. Overall, an insurer is better off in a scenario with SR=0.25 for the base case parameters as the policyholders will stay until maturity, and the management fees will greatly exceed any guarantee cost paid. Low reward-to-variability ratios entail high guarantee cost which correspond almost one-to-one with the management fees received, providing a very low net profit. Whenever tax is considered, we observe that taxing gains, for both the no offset and offset case affect the gains in a similar manner as r.

[30] We note that for $r = 0.025$ the net profit slightly increases, rising the interesting question of what happens in a low interest rate environment. However, this analysis is out of scope and we analyze scenarios close to the calibrated parameters in Table 1.

Finally, considering higher roll-up rates, $\delta = 0.02$ leads to higher guarantee cost in all scenarios, which intuitively follows from the greater growth rate. However, as suggested in the previous sections, the policyholder and insurer fee reflect this already, demanding and supplying a higher fee. This yields higher management fees, which overall increase the net profit for high SR (SR=0.25 and 0.45), but decrease the net profit for low SRs as the guarantee becomes more expensive to supply. We note as well that increased roll-up rates have a positive effect in surrender. Indeed, people stay longer on average (compare first and third row). For high roll-up rates, we observed a limited effect of taxes on the net profit. Indeed, providing a guarantee which is close to the risk-free rate affects policyholder behavior as to make them stay longer invested in the contract, paying higher fees. From a P&L perspective, it seems that offering high roll-ups is the best way to mitigate the effect of taxation on the product. However, as shown in Table F.5, offering a lower roll-up has the opposite effect. Higher net profit for low SR and lower net profit for higher SR. Here the effect of taxation is greater as low roll-ups lead to 100% of the policyholder to surrender before maturity.

Table 5: Profit and Loss profiles for $r = 0.03$ (first half) and $\sigma = base$ (second half). Here SR denotes the Sharpe ratio, and 'mgmt fees' denotes the management fees in basis fees. q^p is the fair fee implied by the parameter set considered in each block.

(BASE Parameters)	r=0.03, $\tau = 0$ ($q^p = 3.32\%$)			r=0.03, no offset ($q^p = 1.92\%$)			r=0.03, offset ($q^p = 1.97\%$)		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	0.3831	6.8623	5.06E-06	2.3287	7.4236	0	1.8858	7.3462
Guarantee cost	29.998	0	0	7.2757	0	0	7.9411	0	0
Mgmt fees	48.917	59.245	24.8215	31.1907	30.5879	15.2211	31.73	32.9456	16.405
Surrender Rate	0%	100%	100%	0.02%	100%	100%	0%	100%	100%
Avg time elapsed	15	14.497	6.0104	15	12.2795	6.1216	15	12.8382	6.3975
Net profit	18.92	59.63	31.68	23.915	32.91	22.64	23.79	34.8314	23.7512
Net Profit Qtiles	17.2,18.9,20.7	58.9,59.3,59.6	30.9,31.7,32.5	21.7,23.9,26.0	32.6,33.0,33.3	22.3,22.7,23.0	21.6,23.8,26.0	34.5,34.9,35.2	23.4,23.8,24.1
	$\sigma = 0.25$, tax free ($q^p = 5.43\%$)			$\sigma = 0.25$, no offset ($q^p = 3.91\%$)			$\sigma = 0.25$, offset ($q^p = 5.00\%$)		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	0	6.211	0	0.0564	7.2973	0	3.90E-06	6.3151
Guarantee cost	61.46	13.47	0	45.6133	2.09E-05	0	56.7	5.3872	0
Mgmt fees	65.17	85.62	47.16	52.4865	69.3425	26.4297	61.4143	80.9474	45.5969
Surrender Rate	0%	0%	100%	0%	97.15%	100%	0%	0.03%	100%
Avg time elapsed	15	15	7.06	15	14.92	5.4782	15	15	7.296
Net profit	3.72	72.15	53.37	6.8733	69.3989	33.727	4.7143	75.5603	51.912
Net Profit Qtiles	2.2,3.7,5.1	69.3,71.8,74.5	51.7,54.0,55.9	5.0,6.8,8.6	69.7,69.8,70.0	33.3,33.9,34.7	3.1,4.7,6.3	73.0,75.7,78.5	50.3,52.1,53.7
	$\delta = 0.02$, $\tau = 0$ ($q^p = 5.04\%$)			$\delta = 0.02$, no offset ($q^p = 3.79\%$)			$\delta = 0.02$, offset ($q^p = 4.61\%$)		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	0	5.3834	0	0.0051	6.6672	0	0	5.54
Guarantee cost	60.8298	18.8222	0	45.71	0.1564	0	55.99	11.3	0
Mgmt fees	65.0776	81.2308	49.6514	53.62	67.3915	27.8996	61.46	76.9	46.515
Surrender Rate	0%	0.00%	100%	0%	21.74%	100%	0%	0%	100%
Avg time elapsed	15	15	8.0086	15	14.993	6.042	15	15	8.04
Net profit	4.248	62.4086	55.0348	7.91	67.24	34.56	5.47	65.6	52.06
Net Profit Qtiles	2.8,4.2,5.6	60.1,62.2,64.4	53.3,55.5,57.4	6.3,7.9,9.6	67.3,67.4,67.4	33.7,34.7,35.5	3.9,5.4,6.9	63.3,65.6,67.9	50.5,52.2,53.6

Notes: the other parameters are given as in the base case in Table 1.

6 Conclusions

Insurance providers benefit from the tax-deferred nature of variable annuities. However, the popularity of these products varies widely across countries. We show that the taxation regime, institutional setting and Sharpe ratio of the market are some of the key drivers of such demand. In this paper, we illustrate the impact of different tax systems on policyholder behaviour and the implications for insurers. In particular, we assess and compare the cases when losses are allowed to offset gains, and where losses are not allowed to offset gains. These two regimes reflect features of institutional arrangements in Australia, US and most European countries.

Most modeling frameworks abstract from the friction induced by taxation. Hence, they obtain that the fee that the policyholder is willing to pay (*demand fee*) coincides with the fee that the insurer is willing to charge (*supply fee*). However, upon introducing taxation, we show there are wide gaps between these two fees heavily influenced by underlying policyholder behavior reflecting our allowance for the policyholder to surrender.

We formulate the valuation of a GMAB contract from the policyholder and insurer's perspective as a free boundary problem which is solved using the method of lines. The corresponding policyholder fair fee and insurer fair fee are computed. The numerical results show how the risk-free rate r, volatility σ and surrender penalty κ impact the optimal surrender behaviour. This impact is determined by the fees that the policyholder has cumulatively paid and also on the particular taxation system.

We show that allowing for losses to offset gains enhances the market, yielding for some market parameters, a demand fee that exceeds the supply. However, the tax regime alone is not a sole driver of the attractiveness of the product. We show that it is possible for the demand to lie below the supply fee. This is the case where losses are not allowed to offset gains. In this case, the gap between supply and demand widens, and policyholders are only willing to enter the contract at very low fee levels. This also impacts their surrender behaviour since they are more likely to maintain the contract to its maturity in order to receive the guarantee.

The profit and loss analysis shows that, despite charging the (low) demand fee, insurance providers would break even and return a profit on average. Introducing taxes benefits the insurance company at the expense of the government. When losses offset gains, policyholders retain their policies for longer in order to receive the higher tax reductions reflecting their increased losses. If losses are not allowed to offset gains, then individuals retain their contracts longer in order to maximize the post-tax value. Profitability of the insurer varies with the Sharpe ratio. Low and high Sharpe ratio markets are not as profitable. For low Sharpe ratios, policyholders are more likely to retain the policy until maturity and receive the benefit of the guarantee, diluting the insurer's profit. For high Sharpe ratios, the higher returns outside of the product incentivizes the policyholder to surrender since the guarantee offered can quickly become out of the money.

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Appendices

A The Governing Partial Differential Equation

Applying Ito's Lemma in conjunction with the formula for dx in Equation (1) yields

$$\begin{aligned} du &= u_x dx + \frac{1}{2} u_{xx} (dx)^2 + u_\nu d\nu + u_y dy \\ &= u_x dx + \frac{1}{2} \sigma^2 x^2 u_{xx} d\nu + u_\nu d\nu + q x u_y d\nu. \end{aligned} \tag{A.1}$$

Now consider a portfolio consisting of a long position in the GMAB contract and u_x units short in the fund. The value of the portfolio can then be represented by $\Pi = u - u_x x$. Over a small time interval $d\nu$ the corresponding change in portfolio value, given that a continuously compounded fee at rate q is paid, is

$$d\Pi = du - u_x(dx + qxd\nu). \tag{A.2}$$

Substituting the known value of du from Equation (A.1) into Equation (A.2) implies that

$$d\Pi = \frac{1}{2}\sigma^2 x^2 \cdot u_{xx} d\nu + u_\nu d\nu + qx \cdot u_y d\nu - qx \cdot u_x d\nu.$$

Since this portfolio has no random component, that is, it does not have a dx term, it must accumulate at the pre-tax risk-free rate. Thus,

$$r(u - xu_x) = \frac{1}{2}\sigma^2 x^2 \cdot u_{xx} + u_\nu + qx \cdot u_y - qx \cdot u_x. \tag{A.3}$$

In deriving the PDE (A.3), we assume the existence of a complete, no-arbitrage market in which the participants (the policyholder and the insurer) can rebalance their portfolios without transaction costs. Instead, taxation is considered from an individual's perspective as it manifests at the boundary conditions of (A.3), when the policyholder elects to surrender or receives the final payout from the GMAB contract.

Re-arranging Equation (A.3) and applying the transformation $t = T - \nu$ where t represents the time to maturity on the contract, u will satisfy the PDE

$$\frac{1}{2}\sigma^2 x^2 u_{xx} + xq \cdot u_y + (r - q) \cdot xu_x - ru - u_t = 0. \tag{A.4}$$

B Fichera theory

The Fichera function corresponding to a general PDE for $u(x_1, x_2, ..., x_n)$ of the form

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i \frac{\partial u}{\partial x_i} + c = 0$$

on a boundary with inward pointing unit normal vector $\hat{n} = (n_1, n_2, .., n_n)$ is defined in Meyer (2001) as:

$$b(x) = \sum_{i=1}^{n}\left(b_i(x) - \sum_{j=1}^{n}\frac{\partial a_{ij}}{\partial x_j}\right)n_i$$

If we choose to label our variables as $x_1 = x$, $x_2 = y$ and $x_3 = t$, then for the PDE that we are solving (equation (4)), the Fichera function at the boundary $y = Y$ depends on the following quantities:

- $\hat{n} = (n_1, n_2, n_3) = (0, -1, 0)$ (the inward pointing unit normal vector of the plane $y = Y$)
- $b_2(x) = q \cdot x$ (the coefficient of u_y)
- $a_{21}(x) = a_{22}(x) = a_{23}(x) = 0$ (coefficient of u_{xy}, u_{yy} and u_{ty} respectively)

The other terms do not matter because $n_1 = n_3 = 0$. Thus the Fichera function is simply $b(x) = -q \cdot x$ which is always negative on the solution domain (x and q are always positive). According to the theory (Meyer (2001); Oleinik (2012)), if the Fichera function has negative sign then it is possible to supply a Dirichlet condition at that particular boundary and have a well posed problem. Thus, we permit ourselves to introduce a condition for $u(x, Y, t)$ as it is consistent with the mathematical theory.

B.1 Justification of the Additional Boundary condition where $y \to \infty$

We justify the approximation made to the case when large fees are paid and the taxable income is zero, that is, $u_y(x, Y, t) = 0$. This Neumann condition (a restriction imposed on the derivative) could be re-interpreted as a Dirichlet condition (a restriction imposed on the value of the solution, u, itself) with some additional financial reasoning as follows; putting $u_y = 0$ into equation (4), we recover the following 2-dimensional PDE

$$\frac{1}{2}\sigma^2 x^2 u_{xx} + (r - q) \cdot x u_x - ru - u_t = 0. \tag{B.1}$$

which must be solved subject to the following boundary conditions

$$u(x, Y, 0) = \max(x, G), \tag{B.2}$$
$$u(s(t, Y), Y, t) = s(t, Y)\gamma_t, \tag{B.3}$$
$$u(0, Y, t) = Ge^{-rt}, \tag{B.4}$$
$$u_x(s(t, Y), Y, t) = \gamma_t. \tag{B.5}$$

The boundary conditions (B.2) - (B.5) are obtained from the equations (5) - (8) in the limit as $y \to \infty$.

Solving (B.1) subject to boundary conditions (B.2) - (B.5) is equivalent to a GMAB rider subject to the same fee rate and surrender penalties however without taxes. This is a simplified version of the problem presented in Kang and Ziveyi (2018) where the authors also incorporate stochastic volatility and interest. This GMAB can be priced using the method of lines by discretizing equation (B.1) in t and maintaining continuity in x. The values of the solution to (B.1) can be used as an approximate Dirichlet boundary condition for $u(x, Y, t)$.

Note that this no longer holds whenever losses can offset capital gains as the value function will be dependent on the tax rate even for high fees paid. However, a boundary condition at $y = Y$ is still required. Here, we use extrapolation to estimate the far field boundary at $y = Y$, rather than supplying one as a Dirichlet condition, since it is difficult to specify one using financial reasoning as presented earlier in Section 2.2. The extrapolation proceeds as follows: before starting the sweep from $y = Y = y_K$ to $y = 0$, we approximate the value $u(x, Y, t_n)$ as the discounted value of $u(x, Y, t_{n-1})$. That is,

$$u(x, Y, t_n) = u(x, Y, t_{n-1})e^{-r\Delta t}.$$

After the algorithm obtains a solution along the plane $t = t_n$ using this crude approximation, we then modify the boundary value at $u(x, Y, t)$ using linear extrapolation

$$u(x, Y, t_n) = 2u(x, y_{K-1}, t_n) - u(x, y_{K-2}, t_n).$$

Numerical experiments suggest that further Gauss-Seidel iterations as described in Meyer (2015) is not necessary as it does not greatly alter the results obtained.

C Method of lines implementation

In order to solve Equation (4), it is discretised in t and y directions and continuity is maintained in x. Let $0 = t_0 < t_1 < ... < t_n < ..., t_N = T$ be a uniformly space time grid and $0 = y_0 < y_1.... < y_K = Y$ be the (uniform) grid for the spatial variable y. Denote $u(x, y_{k'}, t_{n'}) = u_{k',n'}(x) = u_{k',n'}$. The following finite difference approximations are used along the line $t = t_n, y = y_k$ (where we let $u_{k,n} = u$ to emphasise that u is presently being solved for as a function of x):

$$u_t = \begin{cases} \frac{u-u_{k,n-1}}{\Delta t} & \text{if } n = 1, 2 \\ \frac{3}{2}\frac{u-u_{k,n-1}}{\Delta t} - \frac{1}{2}\frac{u_{k,n-1}-u_{k,n-2}}{\Delta t} & \text{if } n \geq 3, \end{cases} \tag{C.1}$$

and

$$u_y = \begin{cases} \frac{u_{k+1,n}-u}{\Delta y} & \text{if } k = K-1, K-2 \\ \frac{3}{2}\frac{u_{k+1,n}-u}{\Delta y} - \frac{1}{2}\frac{u_{k+2,n}-u_{k+1,n}}{\Delta y} & \text{if } k \leq K-3, \end{cases} \tag{C.2}$$

The method of lines as presented in Meyer (2015) can be used to solve the system of equations generated when Equations (C.1) and (C.2) are used to approximate a solution for the partial differential equation (4).

Substituting (C.1) into (4) will give:

$$\frac{1}{2}\sigma^2 x^2 u_{xx} + xq \cdot u_y + (r-q) \cdot xu_x - \tilde{c}u = \hat{f} \tag{C.3}$$

where $\tilde{c} = r + \begin{cases} \frac{1}{\Delta t} & \text{if } n = 1, 2 \\ \frac{3}{2\Delta t} & \text{if } n \geq 3 \end{cases}$

and $\hat{f} = \begin{cases} -\frac{u_{k,n-1}}{\Delta t} & \text{if } n = 1, 2 \\ -\frac{4u_{k,n-1}-u_{k,n-2}}{2\Delta t} & \text{if } n \geq 3 \end{cases}$

Similarly (C.2) can be substituted into (C.3) to recover the equation:

$$\frac{1}{2}\sigma^2 x^2 u_{xx} + (r-q) \cdot xu_x - \hat{c}u = F \tag{C.4}$$

which is subject to equations (5) - (8) as boundary conditions on u, where

$\hat{c} = \tilde{c} + \begin{cases} \frac{xq}{\Delta y} & \text{if } k = K-1, K-2 \\ \frac{3xq}{2\Delta y} & \text{if } k \leq K-3 \end{cases}$

and $F = \hat{f} - \begin{cases} x \cdot q \cdot (\frac{u_{k+1,n}}{\Delta y}) & \text{if } k = K-1, K-2 \\ x \cdot q \cdot (\frac{2u_{k+1,n}}{\Delta y} - \frac{u_{k+2,n}}{2\Delta y}) & \text{if } k \leq K-3 \end{cases}$

These difference schemes were chosen in light of the fact that u is known along the plane $t = 0$ (the payoff boundary condition) and also along the plane $y = Y$.

Solving Equation (C.4) requires the one dimensional Method of Line solution, which is already discussed in great detail in Meyer (2015), which the following discussion is based on. We first rewrite (C.4) as the two point boundary value problem

$$u'(x) = v(x), \qquad u(0) = (G - \tau[G - y_k - x_0 - C_0]_+)e^{-rt_n} \tag{C.5}$$

$$v'(x) = C(x)u + D(x)v + g(x), \qquad v(S) = \gamma_t - \tau\gamma_t \mathbb{I}\{[S\gamma_t - y - x_0 - C_0 > 0\} \tag{C.6}$$

where $S = s(t_n, y_k)$ is the free boundary that needs to be computed along with the solution and

$$C(x) = \frac{2\hat{c}(x)}{\sigma^2 x^2}$$

$$D(x) = \frac{2(q-r)}{\sigma^2 x^2}$$

$$g(x) = \frac{2F}{\sigma^2 x^2}$$

The solution method of the system in (C.5) , (C.6) requires us to observe that the functions $u(x), v(x)$ are related through the Riccati transformation $u(x) = R(x)v(x) + w(x)$. $R(x)$ and $w(x)$ are solutions to the initial value problems

$$R' = 1 - D(x)R - C(x)R^2, \qquad R(0) = 0 \tag{C.7}$$
$$w' = -C(x)R(x)w - R(x)g(x), \qquad w(0) = (G - \tau[G - y_k - x_0 - C_0]_+)e^{-rt_n} \tag{C.8}$$

We first solve equation (C.7) using the implicit Trapezoidal rule as detailed in Meyer (2015), although in principle any standard technique for first order initial value problems can be employed. Equation (C.7) depends only on the order of the difference schemes being used. Hence, in this case, there are actually only 4 possible solutions for $R(x)$ (depending on if k is greater than or less than $K - 3$, and if n is greater than or less than 2). Thus we solve for $R(x)$ outside the main loop and store the 4 separate solutions off-line. Once the values of $R(x)$ along the grid points are obtained, these known values can be used to solve equation (C.8). This is also done using the trapezoidal rule for ODEs described in Chapter 3 of Meyer (2015).
Now we turn our attention to finding the exercise point $S = s(t_n, y_k)$. This is done by considering the function $\phi(x) = u(x) - R(x)w(x) - v(x)$ and noting that, by definition, it equals zero for $0 \leq x \leq S$. Thus $\phi(S) = u(S) - R(S)w(S) - v(S) = 0$. Moreover, the boundary conditions of equation (6) and (8) define what values $u(S)$ and $v(S)$ must take. In order to compute the appropriate S, we define the functions:

$$\begin{aligned} v_b(x) &= \gamma_{t_n} - \tau\gamma_{t_n}\mathbb{I}\{x - y_k - x_0 - C_0 > 0\} \\ u_b(x) &= \gamma_{t_n}x - \tau[\gamma_{t_n}x - y - x_0 - C_0]_+ \end{aligned}$$

and see that value of S is the root of the equation $\tilde{\phi}(x) = u_b(x) - v_b(x)R(x) - w(x)$.
These values are known on the points along x, so we find S by identifying where a sign change occurs in function $\tilde{\phi}$. More specifically, one uses the fact that $\tilde{\phi}(x_s) \cdot \tilde{\phi}(x_{s+1}) < 0$ then S occurs in the interval $[x_s, x_{s+1}]$. We use linear interpolation to estimate S. If there are multiple sign changes, we refer to the root computed at the previous iteration and choose the one that is closest to it, as $s(t, y)$ must be continuous for this particular problem. From general financial reasoning, a small change in t or y should not produce a discontinuous jump in the surrender behaviour for the GMAB.
Once S is found, the reverse sweep can proceed to solve for $v(x)$. Using the same linear implicit method used to find $w(x)$, the initial value problem in equation (C.6) can be solved. Since $x = S$ is not a point in the chosen grid, in order to perform the first backward step from $x = S$ to the nearest grid-point, we estimate the values of $C(S), R(S), g(S)$ and $D(S)$ using linear interpolation.
Since $v(x)$ is computed for $x < S$, we set the solution as:

$$u(x) = \begin{cases} R(x)v(x) + w(x) & \text{if } x < S \\ u_b(x) & \text{if } x \geq S \end{cases}$$

One sweeps backwards from $y = Y$ to $y = 0$ for each time level, solving along a plane of constant $t = t_n$. Once this plane is computed, then we then move onto the next time step $t = t_{n+1}$ and repeat this process sweeping backwards along lines of constant y. Since we are sweeping backwards in the grid for y and the initial condition at $y = Y = y_k$ is provided in Section 2.2, this means that in Equation (C.4), the source term F will always be known. One advantage of this scheme is that Gauss-Seidel line iteration is not required since a forward difference approximation is used for u_y and u is known along $y = Y = y_K$. Typically Gauss Seidel line iteration is required for PDEs with more than one spatial variable (Kang and Ziveyi, 2018).

D Financial arguments explaining the origin of the valley of surrender

Let $CV(x, y, t)$ be the continuation value of the GMAB product. The surrender boundary is represented by the set of points $\{(x, y, t) \in \mathbb{R}^3 | CV(x, y, t) = \gamma_t x - \tau(\gamma_t x - y - x - C_0)_+\}$ We define the cross section as $\{(x, y) \in \mathbb{R}^2 | CV^t(x, y) = \gamma_t x - \tau(\gamma_t x - y - x - C_0)_+\}$ Where $CV^t(x, y)$ is a continuation value function defined for a specific point in time t (here we consider cross sections of the surrender boundary are plotted along planes of constant t). A relevant example is Figure 3(a).

Consider a point (x_0, y_0) that lies on the surrender boundary. Let $(x_0 + \Delta x, y_0 + \Delta y)$ also lie on the surrender boundary. Consider the case where (x_0, y_0) is chosen such that the taxable income is equal to zero. Then for small Δx and Δy, applying the multivariable chain rule we have:

$$\begin{aligned} CV^t(x_0 + \Delta x, y_0 + \Delta y) - CV^t(x_0, y_0) &\approx \gamma_t \Delta x. - \mathbb{I}_\tau \gamma_t \tau \\ CV^t_x \Delta x + CV^t_y \Delta y &\approx \gamma_t \Delta x. - \mathbb{I}_\tau \gamma_t \tau. \end{aligned}$$

where the indicator function $\mathbb{I}_\tau$ is 1 if taxable income $(\gamma_t x - y - x_0 - C_0)$ is positive and is equal to zero otherwise.

Re-arranging and solving for the slope gives:

$$\frac{\Delta x}{\Delta y} = \frac{\tau \mathbb{I}_\tau - CV^t_y}{CV^t_x - (1 - \tau \mathbb{I}_\tau)\gamma_t}$$

There are several things to note here:

1. The denominator is always negative
2. A discontinuity in $\frac{\Delta x}{\Delta y}$ will be present due to the presence of the indicator function
3. The numerator is negative for $\mathbb{I}_\tau = 0$ and positive otherwise.

Further justification is provided for points 1 and 3.

Increasing the 'fund value' x will push the policyholder 'deeper' into the surrender region (since it is a minimum fund value that triggers surrender). In order for the the policyholder to prefer surrender, the change in the continuation value by adding 1 dollar to the fund, which is (CV^t_x), must be less than the value yielded by that extra dollar upon immediate surrender, which is $(1 - \tau \mathbb{I}_\tau)\gamma_t$. This proves point 1.

We also note that $CV^t_y > 0$, so the numerator is negative for $\mathbb{I}_\tau = 0$. All else equal, if more fees have been paid then this will allow for more tax deductions in the future. It is also true that $CV^t_y < \tau$. This is because the added value of an extra dollar of fee paid CV^t_y will increase future payoffs by at most τ (due to the tax deduction). However due to the time value of money, the increase in value is bound above by τ. Hence the numerator is positive for $\mathbb{I}_\tau = 1$ This proves point 3.

Using points 1,2,3 the surrender cross section must be decreasing initial (for small y where taxable income is greater than zero, $\mathbb{I}_\tau = 0$), then make a sharp turn (derivative doesn't exist), and then increase after that (for large y where taxable income exceeds 0, $\mathbb{I}_\tau = 1$). As y approaches infinity the slope should go to zero since taxation is irrelevant at this point (and we return to the y independent surrender boundary observed with a tax rate of zero).

Online supplementary material

E Sensitivity analysis: impact of interest rates, volatility and surrender penalties on surrender and fair fees

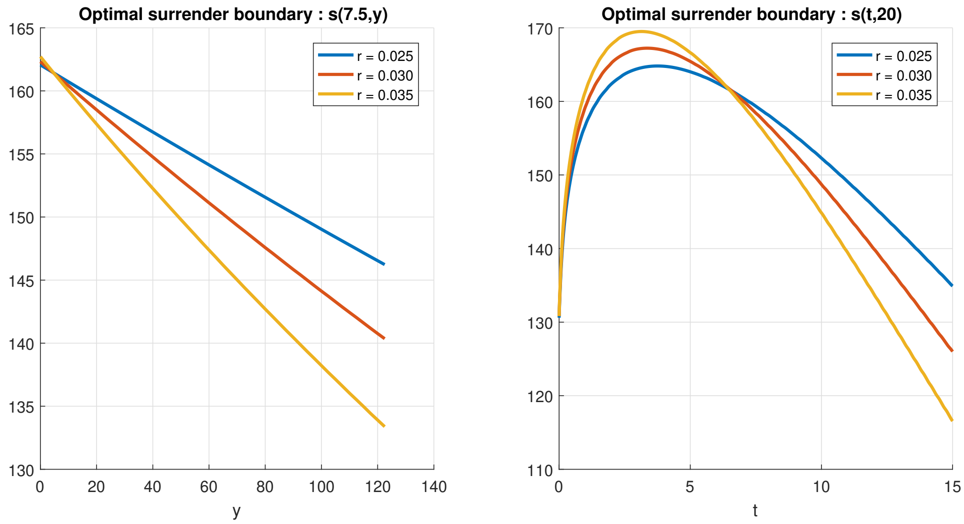
Table E.1: Sensitivity analysis: analysis of the impact of r, σ and κ on policyholder (q^p) and insurer (q^i) fair fees. The rest of the parameters are given by Table 1.

	Losses offset gains						Capital gains only				
	σ=0.20			r=0.03			σ=0.20			r=0.03	
	q^p	q^i		q^p	q^i		q^p	q^i		q^p	q^i
r=0.025	7.11	4.80	σ=0.15	0.47	1.74	r=0.025	4.06	5.20	σ=0.15	0.22	1.73
r=0.030	1.97	3.42	σ=0.20	1.97	3.42	r=0.030	1.93	3.51	σ=0.20	1.93	3.51
r=0.035	0.56	2.42	σ=0.25	5.00	5.27	r=0.035	0.56	2.43	σ=0.25	3.91	5.68
	κ=0.005			κ=0.01			κ=0.005			κ=0.01	
	q^p	q^i		q^p	q^i		q^p	q^i		q^p	q^i
τ=0	3.35	3.35	τ=0	2.57	2.57	τ=0	3.35	3.35	τ=0	2.57	2.57
τ=0.225	1.97	3.42	τ=0.225	1.23	2.60	τ=0.225	1.93	3.51	τ=0.225	1.24	2.64

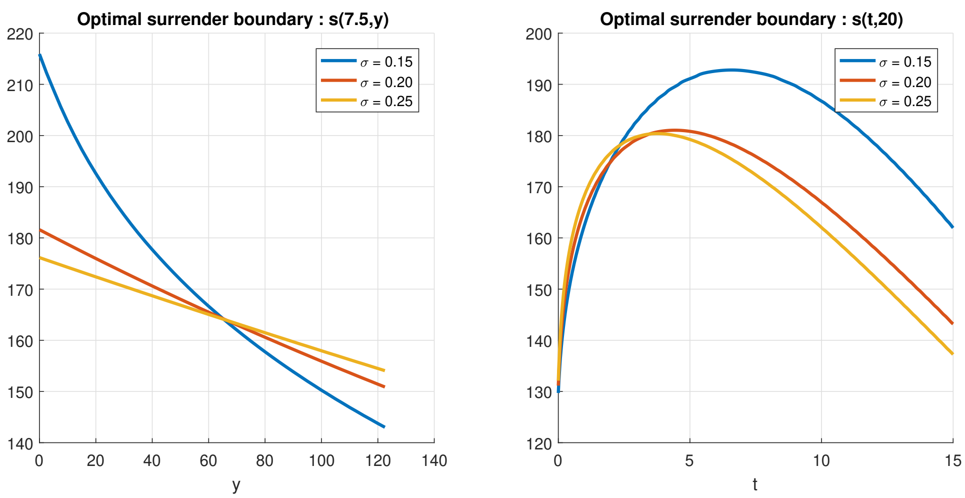
In Table E.1, a higher r is accompanied by a lower fee rate. Since policyholders can obtain a greater return in the risk-free market, they may be less interested to enter the contract for the same level of maturity guarantee. Hence as reflected in Figures E.1(a) the surrender boundary is shifted up implying that the policyholder is still willing to remain invested in the contract in spite of the guarantee being worth less.

The combination of $r = 2.5\%$ and $\sigma = 20\%$ creates the possibility for such a market to exist as $q^p > q^i$. For this parameter choice, any rational policyholder and insurer are willing to enter into a contract because the policyholder receives their fair value while the insurer profits at the expense of the government. The problem of how to divide such profits is similar to the well known 'ultimatum game' problem from game theory (see for example Güth et al. (1982)). In this case, a likely equilibrium solution to this game is one where the insurer charges the highest fee possible and the policyholder does not capture any of the gain. Such a situation could materialise because the insurer is the agent who makes the offer in the game. It is rational for the policyholder to accept any fee rate, no matter how low the offer is since any positive net profit is preferable to rejecting the contract. Therefore, it is rational for the insurer, the agent making the offer, to offer the fee rate that yields to them the highest profit without any regard for the policyholder.

Figure E.1: Impact of interest rates (a) and volatility (b) on optimal surrender at given amounts of cumulative fees paid and times to maturity, with offsets allowed



(a)



(b)

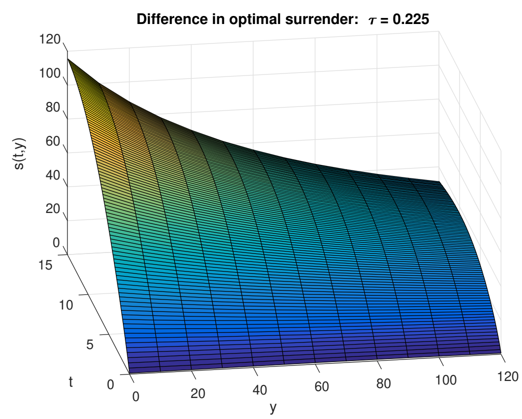
In Table E.1 we note that for higher values of σ, q^p increases faster than q^i. This because the higher fee rate which accompanies greater market volatility corresponds to more savings for the policyholder when losses are allowed to offset gains. In contrast, when only losses cannot be used to reduce tax payable, the policyholder fair fee is lower.

In Figure E.1(b), for small values of t a higher volatility causes the surrender boundary to shift outwards. This is because the guarantee is more attractive in a highly volatile, uncertain market. However early on in the contract, for larger time to maturity t, the surrender boundary is shifted down due to the high fair fee rate which accompanies the increase in σ. For a particular time to maturity, we note that the surrender boundary continues to have a linear dependency on y due to the marginal benefit of τy associated with each additional dollar of fee paid.

In Table E.1 as κ increases, q^i decreases because the insurer is also able to collect more from the larger surrender penalty they impose. The same is true for q^p as observed in Table E.1. From the insurer's perspective, the higher surrender penalty also ensures the policyholder stays in the contract for longer. Thus they can reduce the fee rate that they charge during the life of the contract. Note that in the first row of Table E.1 that examines the sensitivity to κ, $q^{i,*} = q^{p,*}$ are identical since $\tau = 0$.

We also show the impact on optimal surrender after increasing κ. As shown in Figure E.2, a higher surrender penalty shifts the surrender boundary upwards, since an increase in surrender penalty is also accompanied by a decrease in the fair fee rate. This reduces the policyholder's incentive to surrender and as expected the difference is always positive. The surrender penalty is effective for all values of y and t. Since the surrender boundary is higher, the policyholder will require a higher fund value to trigger optimal surrender.

Figure E.2: Optimal surrender when $\kappa = 0.010$ minus when $\kappa = 0.005$



We conclude our sensitivity analysis by summarising our findings as follows;

(i) a higher value of r implies a lower fee rate and an increase in revenue to the government,

(ii) a higher value of σ implies a higher fee rate and a decrease in revenue to the government,

(iii) a higher value of r pushes up the policyholder surrender boundary for all values of y and t,

(iv) a higher value of σ will pull the surrender boundary down during early stages of the contract due to a high fee rate, but pushes it up later on in the life of the contract since the guarantee appears more attractive in a highly volatile and more uncertain market.

F Sensitivity analysis for the Profit and Loss analysis

Each of the tables below are constructed assuming that the policyholder fair fee q^p is charged for that respective parameter setting.

Table F.2: Varying interest rates

	r=0.025, tax free ($q^p = 5.04\%$)			r=0.025, no offset ($q^p = 4.06\%$)			r=0.025, offset ($q^p = 7.11\%$)		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	0	5.156	0	1.1e-4	6.4556	0	0	4.4805
Guarantee cost	56.83	18.11	0	45.86	1.8445	0	74.8896	46.4382	0
Mgmt fees	63.05	78.475	48.8593	54.35	68.023	27.64	77.0478	94.7953	60.2695
Surrender Rate	0%	0%	49%	0%	0.69%	100%	0%	0%	100%
Avg time elapsed	15	15	8.0138	15	14.9998	5.769	15	15	7.6986
Net profit	6.221	60.363	54.02	8.49	66.1755	21.18	2.16	48.357	55.789
(BASE Parameters)	r=0.03, tax free			r=0.03, no offset			r=0.03, offset		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	0.3831	6.8623	5.06E-06	2.3287	7.4236	0	1.8858	7.3462
Guarantee cost	29.998	0	0	7.2757	0	0	7.9411	0	0
Mgmt fees	48.917	59.245	24.8215	31.1907	30.5879	15.2211	31.73	32.9456	16.405
Surrender Rate	0%	100%	100%	0.0002	1	1	0	1	1
Avg time elapsed	15	14.497	6.0104	15	12.2795	6.1216	15	12.8382	6.3975
Net profit	18.92	59.63	31.68	23.915	28.259	7.8	23.79	34.8314	23.7512
	r=0.035, tax free ($q^p = 2.30\%$)			r=0.035, no offset ($q^p = 0.56\%$)			r=0.035, offset ($q^p = 0.56\%$)		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	2.3864	7.551	0.274	3.4169	8.0565	0.2741	3.419	8.0574
Guarantee cost	5.58	0	0	0	0	0	0	0	0
Mgmt fees	38	37.11	17.02	9.873	9.1068	6.0718	9.8725	9.1043	6.0709
Surrender Rate	0%	100%	100%	1	1	1	1	1	1
Avg time elapsed	15	12.25	5.737	14.65	11.6981	7.5543	14.65	12.8382	7.5535
Net profit	32.42	39.496	24.57	10.14	12.523	14.128	10.15	12.524	14.128

Table F.3: Varying volatility

	$\sigma = 0.15$, tax free ($q^p = 1.68\%$)			$\sigma = 0.15$, no offset ($q^p = 0.22\%$)			$\sigma = 0.15$, offset ($q^p = 0.47\%$)		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0.0022	2.6831	6.8103	0.5526	2.9691	6.5024	0.5526	2.9691	6.5024
Guarantee cost	0.1094	0	0	0	0	0	0	0	0
Mgmt fees	29.1212	25.3768	13.3015	3.6325	3.3717	2.6709	3.6325	3.3717	2.6709
Surrender Rate	12.43%	100%	100%	100%	100%	100%	100%	100%	100%
Avg time elapsed	14.9966	11.6862	6.2089	14.31	11.947	8.9255	14.31	11.947	8.9255
Net profit	29.01	28.0599	20.1118	4.1851	6.3408	9.1733	4.1851	6.3408	9.1733
	$\sigma = 0.25$, tax free			$\sigma = 0.25$, no offset			$\sigma = 0.25$, offset		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	0	6.21	0	0.0564	7.2973	0	3.90E-06	6.3151
Guarantee cost	61.46	13.47	0	45.6133	2.09E-05	0	56.7	5.3872	0
Mgmt fees	65.17	85.62	47.16	52.4865	69.3425	26.4297	61.4143	80.9474	45.5969
Surrender Rate	0%	0%	100%	0%	97.15%	100%	0%	0.03%	100%
Avg time elapsed	15	15	7.06	15	14.92	5.4782	15	15	7.296
Net profit	3.72	72.15	53.37	6.8733	69.3989	33.727	4.7143	75.5603	51.912
(BASE PARAMETERS)	σ =base, tax free			σ =base, no offset			σ =base, offset		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	0.38	6.862	5.06E-06	2.3287	7.4236	0	1.8858	7.3462
Guarantee cost	30	0	0	7.2757	0	0	7.9411	0	0
Mgmt fees	48.92	59.24	24.82	31.1907	30.5879	15.2211	31.73	32.9456	16.405
Surrender Rate	0%	100%	100%	0.02%	100%	100%	0%	100%	100%
Avg time elapsed	15	14.50	6.011	15	12.2795	6.1216	15	12.8382	6.3975
Net profit	18.91	59.62	31.69	23.915	28.259	7.8	23.79	34.8314	23.7512

Table F.4: Varying taxation rate

	$\tau = 0$			$\tau = 0.175$, no offset			$\tau = 0.175$, offset		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	0.384	6.8598	0	1.9357	7.2912	0	1.303	7.1701
Guarantee cost	30.01	0	0	14.75	0	0	16.61	0	0
Mgmt fees	48.92	59.24	24.8405	37.08	37.9061	17.5272	38.46	42.4385	19.5754
Surrender Rate	0%	100%	100%	0%	100%	100%	0%	100%	100%
Avg time elapsed	15	14.49	6.0143	15	12.6502	5.875	15	13.44	6.2121
Net profit	18.91	59.62	31.7	22.33	39.84	24.82	21.85	43.74	26.74
	$\tau = 0$, tax free			$\tau = 0.275$, no offset			$\tau = 0.275$, offset		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	0.384	6.8598	0.003	2.5692	7.5862	0.0029	2.566	7.59
Guarantee cost	30.01	0	0	0.0802	0	0	0.0774	0	0
Mgmt fees	48.92	59.24	24.8405	21.1409	20.469	11.5236	21.1409	20.48	11.52
Surrender Rate	0%	100%	100%	16.10%	100%	100%	15.90%	100%	100%
Avg time elapsed	15	14.49	6.0143	14.99	12.23	6.8181	14.9958	12.23	6.815
Net profit	18.91	59.62	31.7	21.06	23.03	19.11	21.07	23.05	19.1

Table F.5: Varying roll-up rates

	$\delta = 0.01$, tax free			$\delta = 0.01$, no offset			$\delta = 0.01$, offset		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	2.23	7.3382	0.155	3.27	7.738	0.1577	3.27	7.738
Guarantee cost	5.4402	0	0	0	0	0	0	0	0
Mgmt fees	36.703	36.08	16.7845	14.965	13.49	8.3758	14.88	13.49	8.3758
Surrender Rate	0%	100%	100%	100%	100%	100%	100%	100%	100%
Avg time elapsed	15	12.2622	5.7263	14.77	11.49	6.9513	14.77	11.49	6.9513
Net profit	31.2627	38.3153	24.1227	15.12	16.75	16.11	15.03	16.75	16.11
	$\delta = 0.02$, tax free			$\delta = 0.02$, no offset			$\delta = 0.02$, offset		
	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45	SR=0.10	SR=0.25	SR=0.45
Surrender fee	0	0	5.3834	0	0.0051	6.6672	0	0	5.54
Guarantee cost	60.8298	18.8222	0	45.71	0.1564	0	55.99	11.3	0
Mgmt fees	65.0776	81.2308	49.6514	53.62	67.3915	27.8996	61.46	76.9	46.515
Surrender Rate	0%	0.00%	100%	0%	21.74%	100%	0%	0%	100%
Avg time elapsed	15	15	8.0086	15	14.993	6.042	15	15	8.04
Net profit	4.248	62.4086	55.0348	7.91	67.24	34.56	5.47	65.6	52.06