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Risk-Sensitive Preferences and Age-Dependent Risk Aversion

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Risk-sensitive preferences and age-dependent risk aversion*

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Abstract

People in different age groups have shown to differ in their degrees of risk aversion. This study investigates the macroeconomic implications of population aging when households are assumed to be increasingly risk-averse in future utility when they age. The model incorporates risk-sensitive preferences used by Hansen & Sargent (1995), which are the only recursive preferences that can separate risk aversion and intertemporal elasticity of substitution while being monotonic, into a 16-generation discrete-time OLG model with undiversifiable income risk. Compared to a time-additive counterpart, risk-sensitive preferences capture precautionary saving motives that exacerbate adverse responses of aggregate macroeconomic variables under a population aging scenario through demographic re-weighting and life-cycle redistribution channels. Varying risk aversion also allows households to internalize future uncertainties when evaluating their welfare impacts of demographic change, resulting in non-monotonic welfare dynamics with higher welfare loss under a high-risk environment and vice versa. Risk-sensitive preferences with age-dependent risk aversion can play an important role in optimal policy settings by introducing uncertainties into the welfare impact analysis, while taking into account more realistic risk-taking behavior of different age cohorts.

Keywords: Demographic change, risk-sensitive preferences, overlapping-generation model, precautionary savings, risk aversion

JEL Classification Numbers: D52 , E21, E60

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1 Introduction

Population aging has important implications for the macroeconomy. The common consensus is that when the population ages, it will reduce aggregate output growth as an older population is associated with decreases in labor force participation and productivity. This also leads to changes in price variables, especially a lower interest rate, as the capital-to-labor ratio varies with demographic-driven saving and labor supply decisions. The ability to understand the macroeconomic impact of population aging is crucial for sound policy decisions, both for fiscal policy where a well-planned social security program is needed to maintain fiscal sustainability and for monetary policy where transmission mechanisms, as well as the real interest rate, are influenced by demographic changes.

According to Gagnon et al. (2016), an overlapping generations (OLG) model that incorporates changes in the US population, life expectancy, and labor market activity could largely predict the low real GDP growth and the low real interest rates of the US since the Great Recession. Likewise, Fujita & Fujiwara (2016) show that changes in demographic structure significantly affect per capita consumption growth and the interest rate in the case of Japan. Aging demographics also increases future payroll tax rates as the tax base decreases due to the lower income of working cohorts (Fehr et al., 2013).

The effect of changing demographics is also reflected in price variables. Many studies suggest that aging population leads to the lower real interest rate. For example, Carvalho et al. (2016) suggest that an aging population affects the equilibrium interest rate through an increase in longevity, an increase capital per worker, and a recompositional effect from a higher dependency ratio that, in total, lead to a reduction in global equilibrium real interest rates between 1990 and 2014. This is in line with Kara & von Thadden (2016) who find that the slowdown in population growth and an increase in longevity slowly contribute to a decline in equilibrium interest rate.

Despite extensive research on demographic changes, most of the literature looks at the impacts of demographic changes through adjustments of the effective labor supply and consumption-saving behavior. One aspect that has been surprisingly left out is the variable rates of risk aversion. Indeed, Roalf et al. (2011) find that older adults are less risk-tolerant than younger adults. This is in line with the research by, for example, Bakshi & Chen (1994) and Pålsson (1996) that suggests an increasing age-dependent risk aversion. This risk-aversion pattern could influence decisions on consumption, labor participation, precautionary savings, and ultimately on how policymakers should design macroeconomic policies. In fact, a seminal paper by Carroll & Samwick (1997) finds that a precaution against future income uncertainty is quantitatively the primary motive for savings, which further emphasizes the importance of increasing risk aversion in the old-age population.

Because most of the overlapping generation (OLG) literature does not distinguish between the rate of the intertemporal elasticity of substitution and risk aversion, it is unable to account for varying degrees of risk tolerance when the demographic structure changes. Specifically, most of the literature assumes different specifications of time-additive expected utilities (for instance, Auerbach & Kotlikoff (1987), İmrohoroglu et al. (1995), Huggett & Ventura (1999), De Nardi et al. (1999), Attanasio et al. (2007), and Vogel et al. (2017)), which do not allow the separation of risk aversion from intertemporal substitution. Some of the more recent research including, for example, that conducted by Gottardi et al. (2015), Nishiyama (2015), and Karantounias (2018) use Epstein-Zin-Weil recursive utility (EZW henceforth) that, although able to separate risk aversion from intertemporal elasticity of substitution, may still not be ideal for studying the issue of varying risk aversion due to its non-monotonic property.

Time-additive preferences also have other drawbacks. According to Anderson (2005), Pareto optimal allocation under time-additive preferences is an unrealistic time-invariant function of aggregate consumption. Many studies, for example, those undertaken by Cochrane (1991), Hayashi et al. (1996), and Attanasio & Davis (1996), find that consumption allocations are affected by many factors such as health, employment, and income. Time-additive preferences also require agents to be risk-neutral in future utility, making them indifferent between gambles with the same expected payoffs regardless of the distribution. This coincides with other studies, for example, Miao (2014), Bommier & Grand (2018), and Karantounias (2018), who state that a time-additive expected utility lacks the flexibility to exclusively explore risk aversion as a time discount factor and utility function are pinned down. Such utility also ignores the timing of the resolution of uncertainty, which is crucial for explaining the equity premium puzzle (Miao, 2014) and, therefore, could change the nature of how the economy responds to demographic changes.

To study risk aversion, Epstein & Zin (1989) modified the recursive preferences of Kreps & Porteus (1978) to an infinite time horizon. The commonly used specification was proposed by Epstein & Zin (2013) and Weil (1990) (EZW henceforth), which can disentangle risk aversion from intertemporal substitution while ruling out problems of time-inconsistency. Despite its usefulness in studying risk aversion, the EZW preferences fail to fulfill the natural property of monotonicity (unless it is reduced to standard time-separable preferences), which requires an agent to always take an action that is preferable in all circumstances.[1] In other words, EZW preferences could result in a counter-intuitive outcome. For example, higher risk aversion might lead to lower savings in certain circumstances.

As an alternative to the preferences mentioned above, Anderson (2005) argues in favor of risk-sensitive preferences as they allow for a time-variant Pareto optimal allocation and a separation between risk aversion and elasticity of substitution. This form of risk-sensitive preferences was first used by Weil (1993) to study precautionary savings and by Hansen & Sargent (1995) as a quadratic control model. Tallarini (2000) uses Hansen and Sargent's risk-sensitive linear quadratic control methods to study the business cycle, asset pricing, and welfare effects of increased risk aversion. He suggests that an increased risk aversion, while holding intertemporal substitution constant, could improve asset market predictions as it leads to a higher market price of risk and a lower risk-free rate.

Later studies reiterate the advantages of risk-sensitive preferences. For example, Bommier & Grand (2018) review possible models that are flexible enough to single out the impacts of risk aversion on heterogeneity in saving behavior and conclude that Hansen and Sargent's risk-sensitive preferences are the only Kreps-Porteus recursive preferences that could disentangle risk aversion from intertemporal substitution

[1]EZW preferences are monotonic only when the elasticity of intertemporal substitution is zero. Otherwise, the certainty equivalent I of EZW preferences are not translation-invariant and therefore is not monotonic.

while being monotonic. This monotonicity property offers an intuitive interpretation as the willingness to reduce risk cannot lead to choices that reduce lifetime utility in all states. Without an assumption of monotonicity, it is challenging to interpret comparative statics in certain situations (Bommier et al., 2017). Bommier & Grand (2018) also show that, with risk-sensitive preferences, higher risk aversion leads to a higher propensity to save for precautionary purposes at any date, which also implies higher accumulated wealth. More risk-averse agents will save more in the presence of income uncertainty to reduce the dispersion in lifetime utilities. Agents with risk-sensitive preferences also possess a stationary optimal policy function as shown by Bäuerle & Jaśkiewicz (2018), using a one-sector infinite horizon growth model under mild assumptions on productivity and utility functions.

This paper applies the risk-sensitive preferences in the form of entropic risk measure as developed by Hansen & Sargent (1995) to a 16-generation discrete time OLG model with undiversifiable income risk to introduce a precautionary incentive to different demographic structures. Separately taking age-dependent risk aversion and intertemporal substitution into account, this paper explores macroeconomic implications associated with different demographic scenarios. The demographic impacts on the macroeconomy using risk-sensitive preferences are contrasted with time-additive preferences (i.e., a special case of risk-sensitive preferences where the risk parameter is zero) which are commonly used in the existing literature. This is to better understand channels through which degrees of risk aversion influence economic adjustments and also to understand how future income uncertainty affects precautionary savings.

Indeed, the results generate useful policy implications as economic impacts vary when the model allows agents to become more risk-averse as they age. The discrete-time OLG model also provides insights into both the aggregate macroeconomic effect and the intergenerational redistribution effect of life-cycle variables. This paper shows that age-dependent risk aversion changes the dynamic solution of the OLG model, resulting in a heightening of adverse effects and a dampening of positive deviations of economic variables under a population aging scenario. The result also suggests different welfare impacts as the growing risk-averse scenario takes into account changes in future uncertainties, whereas the time-additive counterpart does not. All in all, this paper shows that age-dependent risk aversion plays an essential role in optimal policy settings as demographic impacts on the macroeconomy and welfare depart noticeably from the case of the time-additive preferences.

This paper contributes to a number of strands in the existing literature. First, it contributes to the OLG literature by introducing the use of the monotonic risk-sensitive preferences to a discrete-time OLG model. Second, it allows the degree of risk-aversion in future utility to vary as Bakshi & Chen (1994) to study the effects of age-dependent risk aversion on the economy. Third, it contributes to the literature on risk aversion and precautionary savings. Unlike most of the existing literature that assumes two-period economies, this study expands the number of cohorts to have a more detailed look at life-cycle distribution.

This paper is organized as follows. Section II explains the model structure and derives optimality conditions of households and firms. Section III describes the equilibrium conditions. Section IV details the calibration process, including demographic scenarios and a derivation of age-dependent risk aversion values. Section V explains the computation process. Section VI presents the main results by using two approaches, one with different demographic assumptions on initial equilibrium and another with a demographic transition under a population aging assumption. Each scenario is contrasted between time-additive and risk-sensitive preferences and look at impacts on aggregate macroeconomic variables, life-cycle allocations, and welfare. Section VII summarises important findings and draw implications for macroeconomic policies.

2 The model

2.1 Demographics

There are $J = 16$ overlapping generations in the economy at each point in time t, with a variable $j = 1, ..., J$ denoting household's age. Life starts at age 1 (corresponding to the actual age of 20-24 years old) and ends at age 16 (corresponding to the actual age of 95-99 years old). Individuals work from $j = 1$ up to the end of the retirement age $J_r = 9$ (actual age of 64 years old).

The population dynamics of cohort j at time t, N_t^j, is explained by fertility rates ,f_t, and survival probabilities from age $j-1$ to age j, ξ^j:

$$N_{t+1}^{j+1} = \begin{cases} \xi^{j+1} N_t^j & \text{if } j > 1 \\ N_t^j (1 + f_{t+1}) & \text{if } j = 1 \end{cases} \tag{1}$$

The total population N_t is the sum of all cohorts alive in period t:

$$N_t = \sum_{j=1}^{J} N_t^j \tag{2}$$

with $n_t = \frac{N_t}{N_{t-1}} - 1$ representing the total population growth rate at time t

The share of each age cohort j at time t, m_t^j, can be written as

$$m_t^j = \frac{N_t^j}{N_t}. \tag{3}$$

To disentangle the impacts of structural demographic changes and total population changes, I normalize m_t^j such that

$$\sum_{j=1}^{J} m_t^j = J. \tag{4}$$

This allows comparison between economies with different demographic structures, while holding the total population constant. Alternatively, this can be seen as comparing the per-capita values of economic variables.

At the initial steady state, the model assumes a constant fertility rate of f_0, and cohort shares can be written as

$$m_0^j = \frac{(1 + f_0)^{1-j} \prod_{j+1}^{J} \xi^j}{J}. \tag{5}$$

2.2 Households

2.2.1 Preferences

The model considers an economy in a finite discrete time horizon in which households have a recursive non-expected utility value function as in Epstein & Zin (1989) and Weil (1990)(EZW),

$$W(U(c_t^j, 1-l_t^j), I) = \begin{cases} \left[(1-\beta)U(c_t^j, 1-l_t^j)^{1-\rho} + \beta I^{1-\rho}\right]^{\frac{1}{1-\rho}} & \text{if } 0 < \rho \neq 1 \\ U(c_t^j, 1-l_t^j)^{1-\beta} I^{\beta} & \text{if } \rho = 1, \end{cases} \tag{6}$$

where W is a time aggregator function, $1/\rho$ captures an intertemporal elasticity of subsitution, $\beta \in (0,1)$ is a constant discount factor, and I is a certainty equivalent of the next-period random value function, V_{t+1}, contingent on current states of labor productivity shock η_t^j:

$$I(V_{t+1}) = \mathbb{E}_t(V_{t+1}^{1-\gamma^j} \,|\, \eta_t^j)^{\frac{1}{1-\gamma^j}}. \tag{7}$$

The coefficients of relative risk aversion of age-j household are represented by γ^j. When $\rho = \gamma$, EZW preferences reduce to standard time-additive expected utility.

Consumers derive utility from the composite good between consumption, c_t^j and leisure $(1 - l_t^j)$ which is expressed in the form of Cobb-Douglass utility function

$$U(c_t^j, 1-l_t^j) = (c_t^j)^{\nu}(1-l_t^j)^{1-\nu}, \tag{8}$$

where ν is the taste parameter of consumption.

To convert EZW preferences into risk-sensitive preferences, this study follows the approach of Tallarini (2000), which considers a special case of EZW preferences where $\rho = 1$. The recursive utility becomes

$$V_t = \left[(c_t^j)^{\nu}(1-l_t^j)^{1-\nu}\right]^{1-\beta} \left[\mathbb{E}_t(V_{t+1}^{1-\gamma^j} \,|\, \eta_t^j)^{\frac{1}{1-\gamma^j}}\right]^{\beta}. \tag{9}$$

Taking logs and rearrange gives

$$\frac{\log V_t}{1-\beta} = \left(\nu \log c_t^j + (1-\nu)\log(1-l_t^j)\right) + \frac{\beta}{(1-\gamma^j)(1-\beta)} \log \mathbb{E}_t(V_{t+1}^{1-\gamma^j} \,|\, \eta_t^j). \tag{10}$$

By transforming $\tilde{V} = \frac{\log V_t}{(1-\beta)}$ and $\psi^j = -(1-\beta)(1-\gamma^j)$, the above equation can be re-written as

$$\tilde{V} = \left(\nu \log c_t^j + (1-\nu)\log(1-l_t^j)\right) - \frac{\beta}{\psi^j} \log \mathbb{E}_t(e^{-\psi^j \tilde{V}_{t+1}} \,|\, \eta_t^j) \tag{11}$$

which resembles risk-sensitive preferences of Hansen & Sargent (1995) and Weil (1993) where the risk-sensitivity parameter ψ^j measures degrees of risk aversion toward future utility of age j agents. Unlike EZW, this particular form of preferences satisfies translation-invariant property, making it monotonic and suitable for the study of additive risk such as labor income risk

It is important to distinguish between ψ^j which denotes the agent's risk attitude toward future utility and the variable γ^j in (8), which represents agent's risk aversion in future consumption (Bäuerle & Jaśkiewicz (2018), Anderson (2005)). Agents are risk-averse toward future utilities when $\psi^j > 0$ and are risk-loving when $\psi^j < 0$. When $\psi \to 0$, risk-sensitive preferences in (11) convert into von Neumann-Morgenstern expected utility, in which case the risk aversion in future consumption has a direct relationship with the intertemporal elasticity of substitution. Further interpretation of risk aversion parameter is discussed in section 6.1.

2.2.2 Budget constraints

The working-age individuals earn labor income according to their effective labor supply, $h_t^j l_t^j$, and a net wage, $w_t^n = w_t(1-\tau_t^w-\tau_t^p)$, where τ_t^w and τ_t^p are a labor-income tax rate and a pension contribution rate respectively. Labor productivity, h_t^j, depends on age-earning profile, e_j, inherent fixed productivity, θ, and autoregressive productivity, η_t^j, that evolves according to a stochastic Markov process and can be expressed as

$$\begin{aligned} h_t^j &= \begin{cases} e_j \cdot exp[\theta + \eta_t^j] & \text{for } j \leq J_r \\ 0 & \text{for } j \geq J_r \end{cases} \\ \eta_{t+1}^{j+1} &= \eta_t^j + \epsilon_{t+1}^{j+1} \quad \text{with} \quad \epsilon_{t+1}^{j+1} \sim N(0, \sigma_\epsilon^2) \end{aligned} \tag{12}$$

Labor productivity drops to zero at the end of the retirement age, J_r, after which individuals receive pensions, s_t^j, equal to a fixed replacement rate, κ, of their last-period average working income. Individuals also receive saving income determined by an asset holding at the beginning of the period, a_t^j, and a net interest rate, $r_t^n = r_t(1-\tau_t^r)$, where τ_t^r is a tax rate on interest income. Without annuity markets, the assets of the deceased are equally distributed among living individuals as accidental bequests, b_t. Households' budget constraints can be written as

$$a_{t+1}^{j+1} = (1+r_t^n)a_t^j + (1-\tau_t^w-\tau_t^p)w_t^n h_t^j l_t^j + s_t^j - p_t c_t^j + b_t \tag{13}$$

where a_{t+1}^{j+1} is a saving at the beginning of next period and c_t^j is a consumption of household j at time t. Households cannot have negative savings, i.e., $a_t^j \geq 0$. Consumption price is $p_t = 1 + \tau_t^c$ where τ_t^c is a consumption tax.

2.2.3 Dynamic programming problem

Each household in every age cohorts chooses a stream of consumption, labor supplies, and next-period savings to solve the dynamic programming problem. The Bellman equation can be written as

$$\begin{aligned} V_t^j(z_t) &= \max_{\{c_t^j, l_t^j, a_{t+1}^{j+1}\}} \left\{ \left(\nu \log c_t^j + (1-\nu)\log(1-l_t^j)\right) - \frac{\beta\xi^{j+1}}{\psi^j} \log \mathbb{E}_t(e^{-\psi^j V_{t+1}} \,|\, \eta_t^j) \right\} \\ \text{s.t.} \quad a_{t+1}^{j+1} + p_t c_t^j &= (1+r_t^n)a_t^j + (1-\tau_t^w-\tau_t^p)w_t^n h_t^j l_t^j + s_t^j + b_t \\ h_t^j &= \begin{cases} e_j \cdot exp[\theta + \eta_t^j] & \text{for } j \leq J_r \\ 0 & \text{for } j \geq J_r \end{cases} \\ \eta_{t+1}^{j+1} &= \eta_t^j + \epsilon_{t+1}^{j+1} \quad \text{with} \quad \epsilon_{t+1}^{j+1} \sim N(0, \sigma_\epsilon^2) \end{aligned} \tag{14}$$

where $V_t^j(z)$ is the value function of a household of age j at time t, and $z = (j, a, \theta, \eta)$ is a vector of state variables .

2.2.4 First-order conditions

The first-order conditions of the Bellman equation with respect to current-period consumption and labor, and the next-period savings are

$$\frac{\nu}{c_t^j} = \lambda p_t \tag{15}$$

$$\frac{1-\nu}{1-l_t^j} = (1-\tau_t^w-\tau_t^p)w_t^j h_t^j \lambda \tag{16}$$

$$\beta \xi^{j+1} \frac{1}{\int_{\mathbb{R}^+} e^{-\psi^j V_{t+1}^{j+1}} v(dz_{t+1}|\eta_t)} \int_{\mathbb{R}^+} e^{-\psi^j V_{t+1}^{j+1}} \frac{\partial V_{t+1}^{j+1}}{\partial a_{t+1}^{j+1}} v(dz_{t+1}|\eta_t) = \lambda \tag{17}$$

We can transform the derivative term in (17) by appealing to the envelope theorem on the Bellman equation to obtain

$$\frac{\partial V_{t+1}^{j+1}}{\partial a_{t+1}^{j+1}} = (1+r_t)\frac{\nu}{p_{t+1}c_{t+1}^{j+1}} \tag{18}$$

Substituting (17) and (18) into (15) yields

$$\frac{\nu}{p_t c_t^j} = (1+r_t)\beta \xi^{j+1} \frac{1}{\int_{\mathbb{R}^+} e^{-\psi^j V_{t+1}^{j+1}} v(dz_{t+1}|\eta_t)} \int_{\mathbb{R}^+} e^{-\psi^j V_{t+1}^{j+1}} \frac{\nu}{p_{t+1}c_{t+1}^{j+1}} v(dz_{t+1}|\eta_t) \tag{19}$$

Combining (15) and (16) yields the intratemporal optimal condition between consumption and labor for $j = 1, ..., J_r - 1$ as

$$c_t^j = \frac{\nu}{p_t(1-\nu)}(1-\tau_t^w-\tau_t^p)w_t^j h_t^j (1-l_t^j) \tag{20}$$

By substituting (20) into the budget constraint, l_t^j can be expressed as a function of a_{t+1}^{j+1} as

$$l_t^j = l_t^j(a_{t+1}^{j+1}) = min\left\{ max\left\{ \left(\nu + \frac{1-\nu}{(1-\tau_t^w-\tau_t^p)w_t^j h_t^j}(a_{t+1}^{j+1} - (1+r_t)a_t^j - s_t^j - b_t) \right), 0 \right\}, 1 \right\}. \tag{21}$$

Minimum and maximum operators constrain labor supplies to be positive and less than the time endowment of one. Likewise, we can also express c_t^j as

$$c_t^j = c_t^j(a_{t+1}^{j+1}) = \frac{1}{p_t}\left((1+r_t)a_t^j + (1-\tau_t^w-\tau_t^p)w_t h_t^j l_t^j + s_t^j + b_t - a_{t+1}^{j+1} \right). \tag{22}$$

Equations (21) and (22) reduce the choice variables down to only a_{t+1}^{j+1}. Hence, we can then rewrite the optimality condition (19) as

$$\frac{\nu}{p_t c_t^j(a_{t+1}^{j+1})} = (1+r_t)\beta \xi^{j+1} \frac{1}{\int_{\mathbb{R}^+} e^{-\psi^j V_{t+1}^{j+1}} v(dz_{t+1}|\eta_t)} \int_{\mathbb{R}^+} e^{-\psi^j V_{t+1}^{j+1}} \frac{\nu}{p_{t+1}c_{t+1}^{j+1}(a_{t+2}^{j+2})} v(dz_{t+1}|\eta_t). \tag{23}$$

Each household of every age j at every time period t choose policy functions $a_{t+1}^{j+1}(z_t)$, and corresponding policy functions $c_t^j(a_{t+1}^{j+1})$ and $l_t^j(a_{t+1}^{j+1})$ to solve the Euler equation (23).

2.3 Firms

The model assumes an infinite number of identical firms that hire capital K_t and labor L_t to produce output Y_t with a Cobb-Douglas production function

$$Y_t = \Omega_t K_t^{\alpha} L_t^{1-\alpha}, \tag{24}$$

where Ω_t represents a fixed technology level, and α and $1-\alpha$ are the output elasticity of capital and labor respectively.

The law of motion for capital is characterized by

$$(1+n_t)K_{t+1} = (1-\delta)K_t + I_t, \tag{25}$$

where capital depreciate at rate δ and I_t is the amount of investment in period t.

The firms maximize their profit choosing K_t and L_t, while taken an interest rate r_t and a wage w_t as given. The firms' profit maximization problem is

$$\max_{\{K_t, L_t\}} \Omega_t K_t^{\alpha} L_t^{1-\alpha} - (r_t + \delta)K_t - w_t L_t, \tag{26}$$

and the profit maximization conditions are

$$(1-\alpha)\Omega_t \left(\frac{K_t}{L_t}\right)^{\alpha} = w_t, \tag{27}$$

$$\alpha\Omega_t \left(\frac{L_t}{K_t}\right)^{1-\alpha} = r_t. \tag{28}$$

The factor market clearing conditions require $K_t = K_t^s$ and $L_t = L_t^s$.

2.4 Government

The government keeps separate balanced budgets regarding the tax system and the pension system. The model assumes that government spending and public debt are constant shares of GDP in each period, i.e., $G_t = g_y Y_t$ and $B_t = b_y Y_t$. In every periods, the government maintains a budget balance

$$\tau_t^c C_t + \tau_t^w w_t L_t + \tau_t^r r_t A_t + (1+n_t)B_{t+1} = G_t + (1+r_t)B_t. \tag{29}$$

On the revenue side, government taxes consumption, labor income, and returns on assets. In addition, it obtains revenue by issuing new debt, which grows at the population growth rate in each period, n_t. On the expenditure side, the government needs to repay debt with interest rate $(1+r_t)B_t$ in addition to its spending G_t.

The government also operates a pay-as-you-go pension system. It collects contribution, τ_t^p, as a percentage of labor income and redistributes it as pension to retirees. The pension amount is a fixed replacement rate, κ, of average working income in the previous period

$$pen_t = \kappa_t \cdot \frac{w_{t-1} L_{t-1}^s}{N_t^L} \tag{30}$$

where L^s_t is a time-t aggregate effective labor supply and $N^L_t = \sum_{j=1}^{J_r-1} m^j_t$ is the aggregate size of working cohorts in period t. The government balances the pension system in each period by choosing the contribution rate τ^p_t that satisfies

$$\tau^p_t w_t L^s_t = pen_t \cdot N^R_t \tag{31}$$

where $N^R_t = \sum_{j=J_r}^{J} m^j_t$ is a number of total retirees in period t.

Another role of the government is to equally allocate accidental bequests to all living individuals according to

$$b_t \sum_{j=1}^{J} m^j_t = \sum_{j=1}^{J} \left[(1-\xi^{j+1}_{t+1}) a^{j+1}_{t+1} m^j_t\right]. \tag{32}$$

3 Equilibrium conditions

With an assumption of perfect foresight in the intertemporal dynamic OLG model, the equilibrium condition requires consistencies such that optimal choices of households, firms, and the government are in accordant with the entire future path of price variables which depend on future conditions. In particular, given the path of exogeneous variables $\{G_t, B_t, \tau^c_t, \tau^w_t, \tau^r_t, \tau^p_t, \kappa_t\}_{t=0}^T$ and initial conditions of the economy, the recursive equilibrium satisfies

1. Aggregate and inividual variables are consistent.

$$C_t = \sum_{j=1}^{J} m^j_t c^j_t \tag{33}$$

$$L_t = \sum_{j=1}^{J} m^j_t l^j \tag{34}$$

$$A_t = \sum_{j=1}^{J} m^j_t a^j_t \tag{35}$$

2. Given prices $\{w_t, r_t\}_{t=0}^T$, households' policy functions $\{a^{t+1}_{t+1}(z_t), c^j_t(z_t), l^j_t(z_t)\}_{t=0}^T$ solve the Bellman equation (14).

3. Given prices $\{w_t, r_t\}_{t=0}^T$, firm factor input choices K_t, L_t solve firm's profit optimization (26)

4. The government's budget balances, (29) and (31), are satisfied, and accidental bequests are allocated according to (32).

5. All markets clear

   - Good market:

$$Y_t = C_t + ((1+n_{t+1})K_{t+1} - (1-\delta)K_t) \tag{36}$$

   - Factor market for capital:

$$K_t = K^s_t, K_t + B_t = A_t \tag{37}$$

   - Factor market for labor:

$$L_t = L^s_t \tag{38}$$

4 Calibration

The model uses year 2015 as a base year with associated survival probabilities from the United Nations life table to compute the initial steady state. The values of parameters are either taken from related literature or calibrated to match certain targets in 2015. This section examines in detail demographic scenarios and how to obtain age-dependent risk aversion in future utilities as well as explains the calibration strategy for other parameters.

4.1 Demographics scenarios

This paper separately examines static equilibrium and transition dynamics of different demographic structures. The first part contrasts economic impacts between households with time-additive (TA) expected preferences and risk-sensitive (RS) non-expected preferences. These two preference types are applied to two demographic scenarios (Figure 1), one with a constant fertility rate (baseline) and another with a fertility rate dropping at 1% per year (aging). Survival probabilities are assumed to be constant at the year 2015's value.

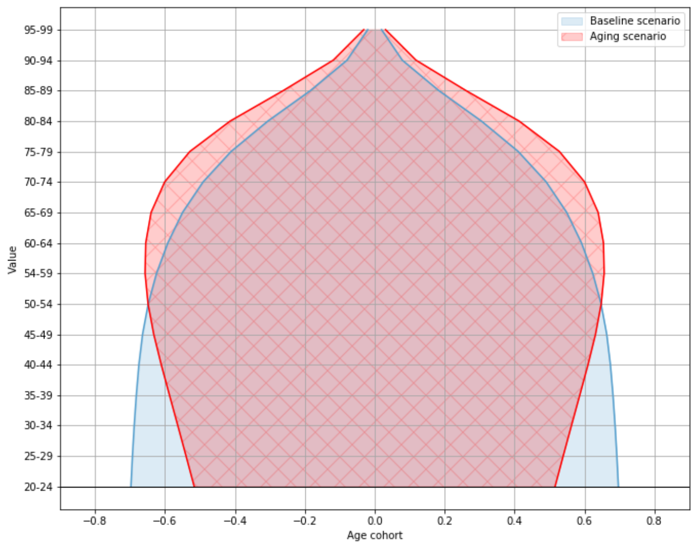


Figure 1: Population pyramids

The second part of the analysis examines the transition dynamics of the economy when faced with demographic shocks. Again, I assume counterfactual demographics between 2015 and 2200 to compare transition dynamics between TA and RS cases. The baseline case assumes zero fertility rate and the aging case assumes fertility rate at -0.5% per year from 2020-2100 and 0% afterward until year 2200.

4.2 Age-dependent risk aversion in future utilities

The model assumes an age-dependent increasing risk aversion in future utility. To get the risk aversion parameter, ψ^j, this paper obtains the estimates of age-dependent relative risk aversion (RRA) from Bakshi & Chen (1994) and transforms them into risk aversion in future utility.

Bakshi & Chen (1994) assume RRA to be linear in age, given by $RRA^{age} = -24.61 + 0.65 \cdot age$, which suggests that people turn from being risk-loving when they are young to being risk averse when they are old. The transformation to risk aversion in future utility is in accordance with the derivation of risk-sensitive preferences in section 2.2.1 which gives [2]

$$\psi^j = -(1-\beta)(1-RRA^j) \tag{39}$$

where RRA^j is the average value of RRA for the corresponding actual age of cohort j. The value of β corresponds to the time period used which is five year in this case. Figure 2 depicts the values of risk aversion in future utility of each age cohort.

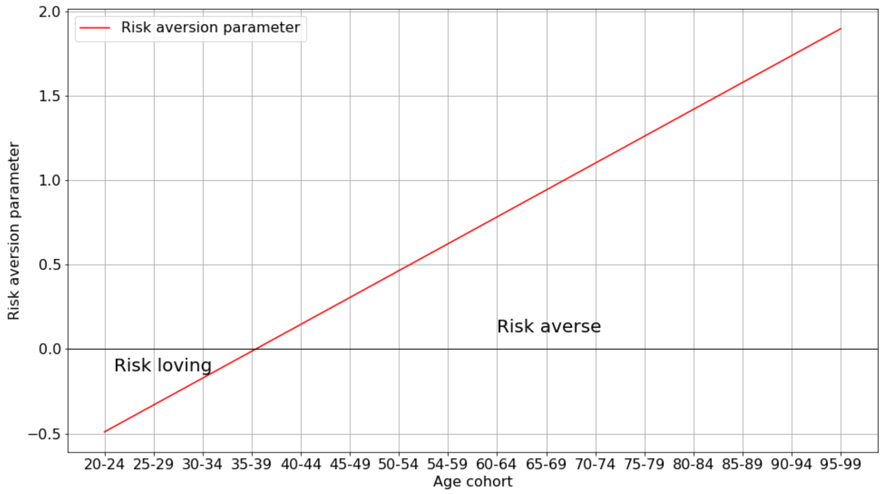


Figure 2: Risk aversion toward future utility

4.3 Other parameters

Calibration strategies as well as values of most parameters on households' preferences, households' risk process, and production are taken from Fehr & Kindermann (2018) which are standard in the literature (Table 1).

On the production side, the productivity parameter, Ω is calibrated so that the wage is normalized to one which gives the value of 1.6. Depreciation rate, δ is calculated from the relationship $\frac{I}{Y} = (n_p + \delta) \cdot \frac{K}{Y}$, where investment-to-GDP ratio is 24 per cent, five-year capital-to-output ratio is 0.6, and n_p is the actual population growth rate of the year 2015 which is 0.8 per cent per annum (or 4.06 percent over five years). This gives the five-year value of δ of 0.36 or an annual value of 8.52 per cent. Age-earning profile, e_j, is normalized to the value of 1 for $j = 1$ and peaks at age 45-55 at $e_j = 1.97$ before slightly decreases toward the retirement as in Fehr & Kindermann (2018).

On the demand side, the taste parameter, ν, is calibrated to be 0.353 to have the average share of working time equals 33 per cent of total time endowment. The intertemporal discount factor β is calibrated so that the annual capital-to-output ratio

[2]In Tallarini (2000), ψ is defined as $\psi = 2(1-\beta)(1-RRA)$ to relate to Hansen & Sargent (1995)'s risk aversion which interpreted negative value of ψ as increasing risk aversion. In this paper, higher ψ means higher risk aversion in future utility.

matches the empirical data of 3[3]. Government spending is exogenous at 19 per cent of GDP and debt-to-GDP ratio is assume to be 60 per cent.

Table 1: Parameter values

Parameter		value
Household preferences		
- Taste parameter of consumption	ν	0.353
- Discount factor	β	0.9875^5
Household risk process		
- Variance of fixed productivity	σ_θ^2	0.23
- Variance of idiosyncratic risk of autoregressive productivity shock	σ_ϵ^2	0.05
- Autocorrelation of autoregressive productivity shock	ρ	0.98^5
Production parameters		
- Capital share in production	α	0.36
- Depreciation rate	δ	$1-(1-0.0852)^5$
- Technology level	Ω	1.6
Government parameters		
- Consumption tax rate	τ_c	0.075
- Replacement rate	κ	0.5
- Government spending as % of GDP	g_y	0.19
- Bond as % of GDP	b_y	0.12

5 Computation

The solution method follows a Gauss-Seidel procedure by Auerbach & Kotlikoff (1987) to solve for a macroeconomic solution and uses microeconomic numerical solutions including a Newton method and interpolation algorithms to solve the household problem. The economy is discretized over state space $\{a, \theta, \eta\}$ to simplify the complex household dynamic programming problem. A brief overview of the solution can be explained as follows.[4]

The code first initializes parameters and discretizes state space. It then calculates price variables according to (27) and (28). After that, it uses a numerical maximization and interpolation algorithm to solve for policy functions $a_{t+1}^{t+1}(z_t), c_t^j(a_{t+1}^{t+1})$, and $l_t^j(a_{t+1}^{t+1})$ that satisfy the household optimal decision represented in (23). Consequent policy functions are then used to solve for household distributions over state space, which are then aggregated into age-cohort specific variables and aggregate variables accordingly. The code then updates tax rates to achieve the government budget balance and updates price variables and iterates the process until all markets clear.

To compute transition dynamics, the code follows the same steps as above with addition time variables that take into account changes in demographic assumptions. Policy functions, prices variables, and other economic variables are computed for each time t such that equilibrium conditions are met.

[3]average data from 2000-2017 obtained from the national income and product account (NIPA)

[4]See detailed computation of the solution method as well as the Hicksian Equivalent Variation in the appendix.

6 Results and analysis

6.1 Interpreting risk aversion values

Central to this study is the role of risk aversion parameter, ψ, in the household risk-sensitive preferences. The functional form of the certainty equivalent (CE) of the risk-sensitive preferences in (11) can also be called the entropic risk measure which represents the relative entropy (i.e., the statistical proximity between different conditional densities) of the next-period value functions.

To interpret how the risk aversion parameter affects the value of the CE, we can draw implication by using the Taylor expansions. Let $\rho^{ent}(X)$ represent the entropic risk measure which is written as

$$\rho^{ent}(V_{t+1}^{j+1}) = \frac{1}{\psi}\ln(\mathbb{E}(e^{-\psi V_{t+1}^{j+1}})). \tag{40}$$

Applying the Taylor expansions gives

$$\rho^{ent}(V_{t+1}^{j+1}) = \mathbb{E}(V_{t+1}^{j+1}) - \frac{\psi}{2}Var(V_{t+1}^{j+1}) \tag{41}$$

which suggests that households think in term of both the expected value of next period value function as well as its variance. The second term could be interpreted as a penalty function on randomness of V_{t+1}^{j+1}, with a degree of penalty depending on the value of parameter ψ. Figure 3 visualizes how CE reacts to different assumptions of $\psi's$ and standard deviations. More detailed analysis on entropic risk can be found in Föllmer & Schied (2008)

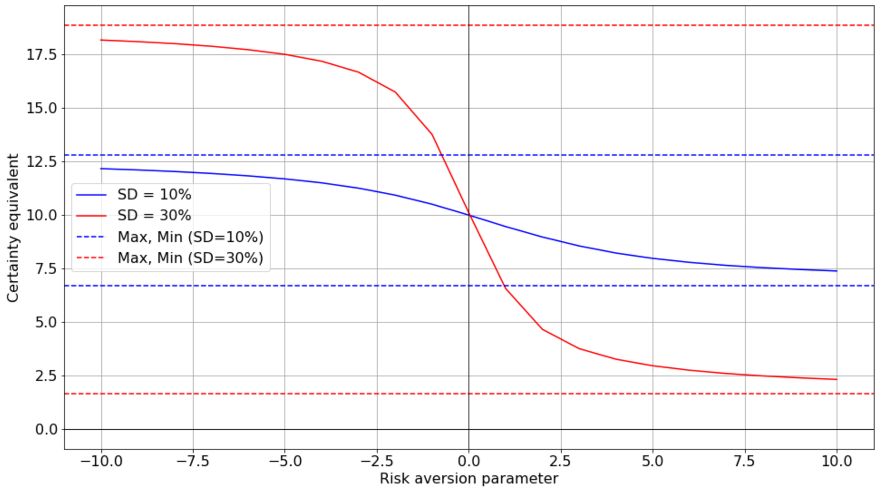


Figure 3: Values of certainty equivalent under different values of risk aversion parameter. Next-period value functions, V_{t+1}^{j+1}, are assumed to have mean value of 10 and normally distributed with standard deviation of 10 and 30 per cent.

6.2 Initial equilibrium

To understand how macroeconomic and life-cycle variables respond to demographic changes as well as changes in risk sensitivities, equilibriums across two demographic scenarios, n_0 equals 0 and -1, and two types of households' preferences, time-additive preferences (TA) and risk-sensitive preferences with age-dependent risk aversion (RS), are compared.

6.2.1 Constant population ($n = 0$)

This section first looks at common patterns of life-cycle variables distributions of both RS and TA households. Once these common results are accounted for, behavioral differences between the two preferences are then clarified to explain discrepancies in aggregate macroeconomic variables.

In equilibrium, each age cohort j decides how much to save, and implicitly consume and work, by optimally trading off instantaneous utility received from consumption and leisure today against long-term utility which increases with savings and working income such that marginal utility in the current period equals discounted marginal utility of next-period certainly equivalent.

Households' optimal decisions lead to a humped-shape life-cycle saving pattern which is mainly driven by a life-cycle saving motive that leads households to increase their savings toward retirement and successively expend it to finance after-retirement consumption before depleting it at the end of life (Figure 4).

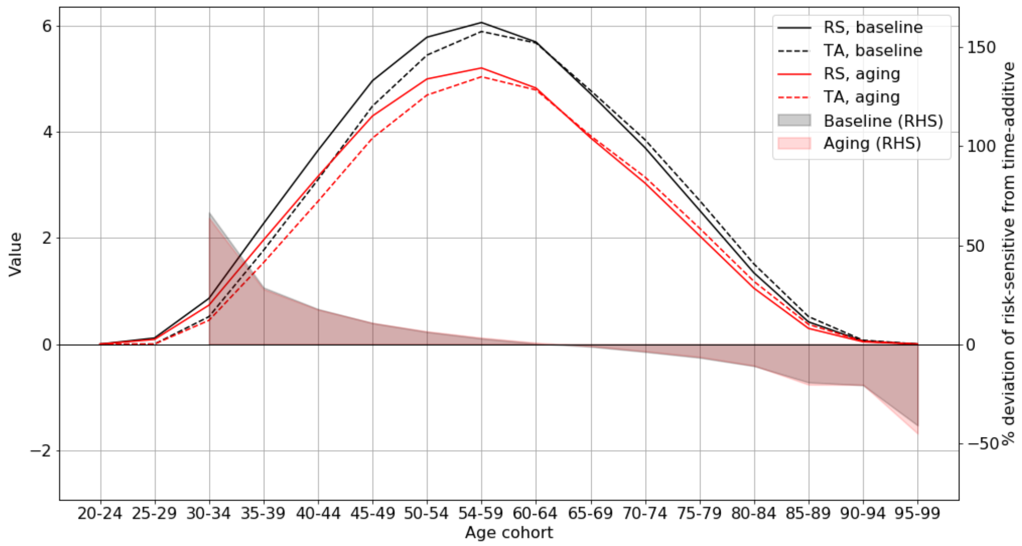


Figure 4: Life-cycle assets distribution

The model also gives a humped-shape distribution of the labor supply (Figure 5). When young, households increasingly supply more labor along with a rapid increase in labor productivity until the age of 35. After which, they begin to favor more leisure over supplying labor, a behavior which can be explained by two reasons. The first is the impact from a positive spread between the rate of return and the time discount factor, causing workers to be patient and wait so they can enjoy more leisure when they are old. Another reason is due to a wealth effect. As households accumulate more wealth, they are less likely to work as being more well-off allows them to work less and enjoy more leisure as well as consumption.

Households' patience to wait also leads to consumption that increases with age (Figure 6). However, decreases in survival rates among older cohorts induces impatience and reverses the consumption pattern. The consumption behavior also reflects

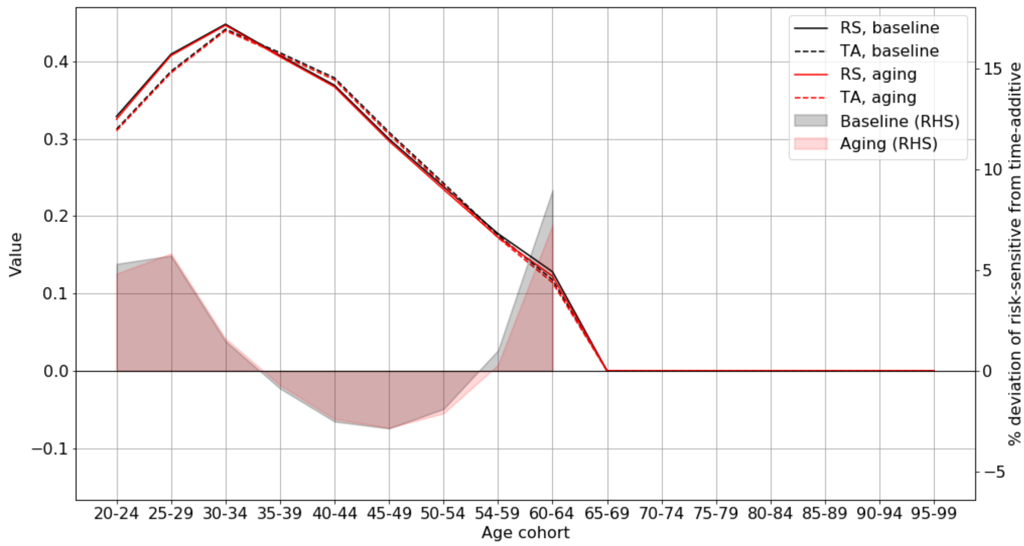


Figure 5: Life-cycle labor distribution

the increasing purchasing power due the a rise in income of the young and middle age cohorts (Figure 7) as well as the role of savings that optimally smooths consumption over a lifetime.

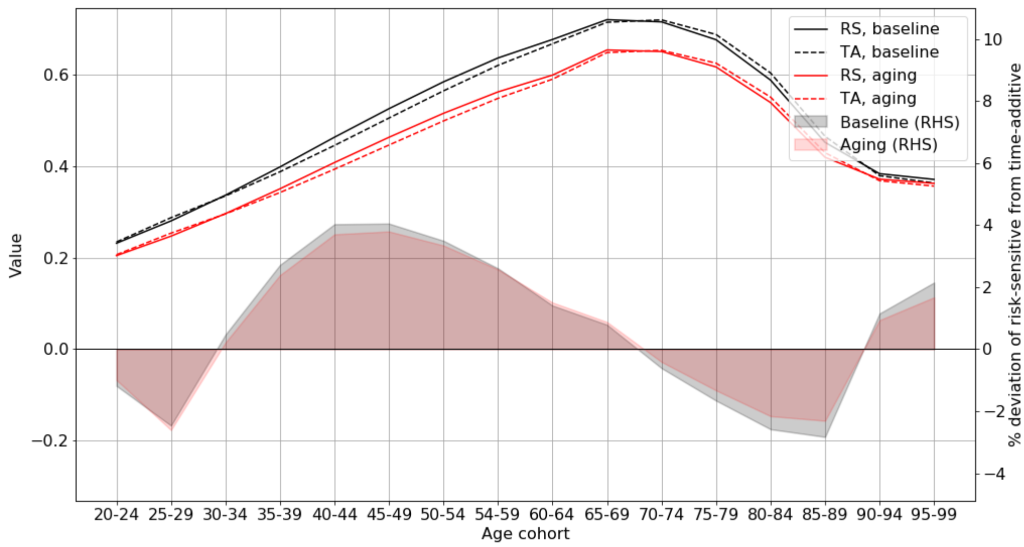


Figure 6: Life-cycle consumption distribution

Comparing RS with TA households

A central factor that sets two types of households apart is the risk aversion in future utility that penalizes future uncertainties when RS households make their decisions to save, consume, and work. This part of the analysis tackles differences in their life-cycle decisions, starting from saving, before moving on to labor supply and consumption.

Disregarded in the TA counterpart, the precautionary saving motive causes RS households to save more during employment and less in retirement (Figure 4). The RS households' disutility from future randomness leads them to increase precautionary savings when they are young to safeguard against uncertain future utilities, which

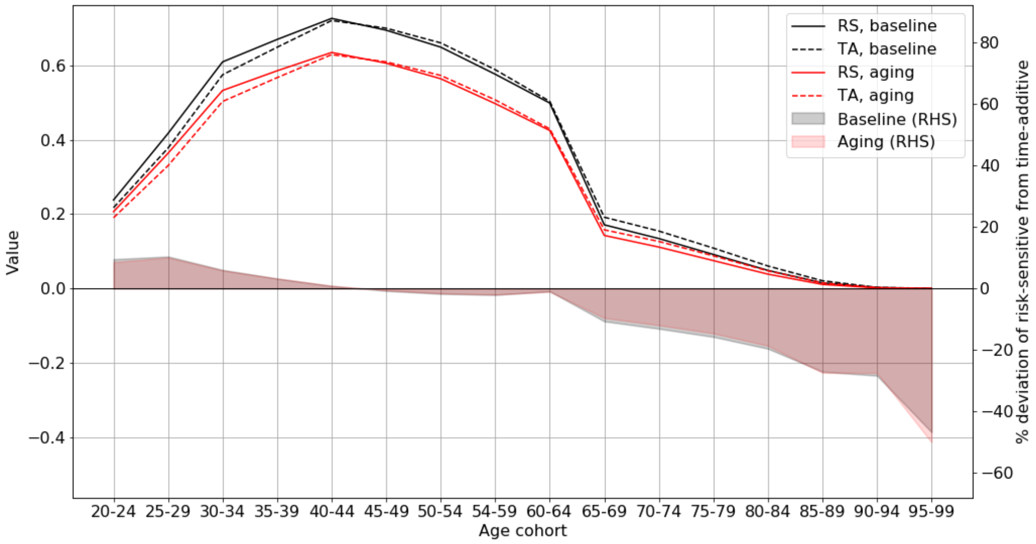


Figure 7: Life-cycle income distribution

increases with an uninsurable stochastic labor productivity. This is true even for the risk-loving young workers as they foresee themselves becoming more risk-averse when old[5]. On the right-axis, the result suggests that precautionary savings motive causes young RS cohorts' savings to increase by about 50 per cent when compared to the TA. However, relative saving behavior between the two types of households changes after retirement. Without labor productivity risk, saving behavior is determined solely by relative interest rates. In the RS case where the capital intensity is higher, households will save less as saving do not yield return as high as in the TA case.

Looking at the labor supply in Figure 5, the model suggests that the young RS households work more to increase precautionary savings. Once more savings are accumulated, the volatility in future earnings will decrease, allowing them to work less and enjoy more leisure in comparison with the TA households. Despite less working hours during the middle to late career, the total working income of the RS households is nearly the same as that of the TA's due to a higher wage rate in a more capital-intensive economy. The difference in labor supplies between RS and TA households diminishes before reversing as people move toward retirement. The lower interest rate in the RS case means that the RS households will increase their leisure by less amount over time, causing labor supply to become higher than the TA case right before retirement. This coincides with the period where RS's savings decreases, hence lowers the risk-free source of available resources.

As a result of high precautionary savings, the RS households initially consume less compared to the TA (Figure 6). However, higher income gradually supports the RS households to consumer more as they progress into middle age. This higher total income (Figure 7) comes from higher earnings (Figure 8) as people work more and also earn a higher wage rate. In contrast, the RS households receive a lower total return on savings because of a lower interest rate, despite a higher amount of savings. After retirement, RS consumption becomes lower again because of (i) a faster rundown in savings and (ii) a lower interest rate which causes them to relatively favor immediate consumption over future consumption in comparison to the TA case.

[5]Younger cohorts would have saved even more if there are not risk-loving.

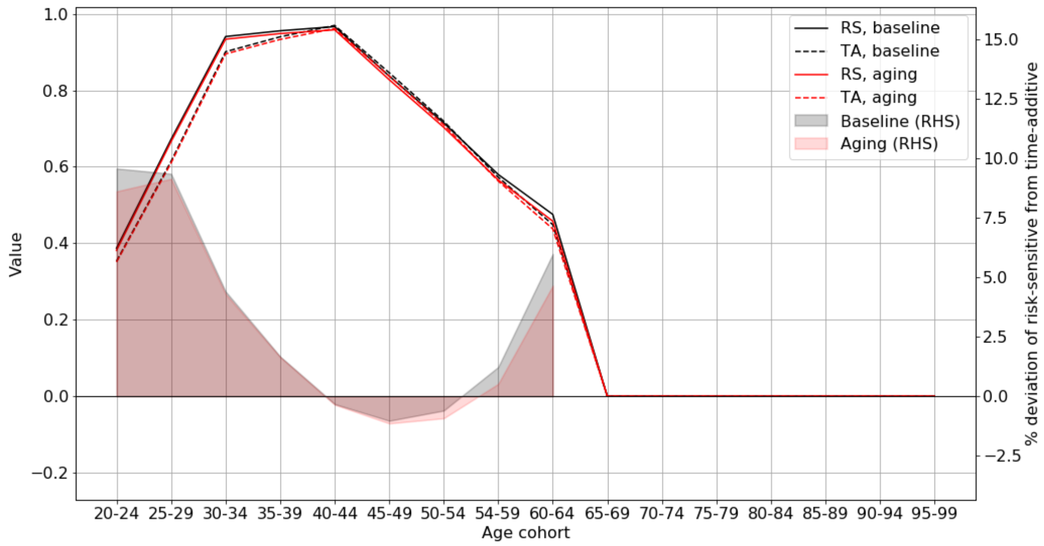


Figure 8: Life-cycle earnings distribution

Aggregate macroeconomic variables

Looking at macroeconomic variables, higher savings in the RS case leads to more investments and capital accumulation which, together with higher labor supply, leads to more output (Table 2). The increase in the capital stock that exceeds the rise in labor supply also contributes to higher capital intensity, driving down the interest rate while driving up the wage rate. Higher aggregate income and savings in RS case allows households to consume more. Regarding the government side, less tax rate is required to keep the government budget balance because of the higher income base in the RS economy.

Table 2: Aggregate variable

	Variable	Baseline		Aging	
		RS	TA	RS	TA
Good market	Y	13.59	13.24	12.11	11.86
	C	8.03	7.94	7.53	7.46
		(59.08%)	(59.95%)	(62.20%)	(62.88%)
	I	2.98	2.79	2.28	2.15
		(21.77%)	(21.05%)	(18.80%)	(18.12%)
	G	2.58	2.52	2.30	2.25
		(19%)	(19%)	(19%)	(19%)
Capital market	A	9.92	9.35	8.79	8.35
	K	8.29	7.77	7.33	6.92
	r (%pa)	4.24	4.65	4.31	4.68
Labor market	L	8.61	8.58	7.70	7.70
	w	1.010	0.9873	1.006	0.9856
Government	τ^C (%)	7.5	7.5	7.5	7.5
	p(%)	17.10	17.10	23.97	23.97
	τ^W, τ^r (%)	21.45	21.42	21.89	21.78

6.2.2 Aging population scenario

After comparing the differences between RS and TA households under a constant population assumption above, this section contrasts macroeconomic and life-cycle variables with the population aging scenario where a fertility rate decreases at the rate of 1 per cent per annum.

To interpret the result, it is essential to understand two main channels through which a change in population growth rate affects the economy. First is the cohort re-weighting which arises from the change in demographic structure and gives different weights to each cohort-specific behavior. Second is the redistribution of life-cycle variables in response to changes in price variables. Any change in a population growth rate would transmit through both channels, with their interactions jointly determine the ultimate outcome of the economy.

For the re-weighting effect, it can be easily seen in Figure 1 that the baseline scenario gives more weight to the younger generations and the aging scenario gives more weights to the older generations, hence different emphasis on life-cycle behavior. The interactions between the two channels are analyzed in the following section.

On the supply side, aging means that the economy is more densely populated by older cohorts who supply less labor compared to the younger cohort. Together with the older cohorts tendency to save more, the consequent increase in capital intensity put a downward pressure on the interest rate which discourages people in every age groups from saving and thus shifts the asset distribution downward (Figure 4). The negative redistribution effect dominates the positive re-weighting that emphasize the old cohorts' saving behavior, resulting in a decrease in the aggregate asset.

Aging impacts on labor supply passes through different channels, each with different directions and magnitudes, which can be inferred from equation (21). First, a lower interest rate is a disincentive for households to work toward more savings as it yields less return. Second, a lower interest rate also leads to lower wealth because household receive less return from any given amount of savings, i.e. $(1+r)a_t^j$ decrease. Lower wealth motivates them to work harder to improve their well-being. Third, aging population leads to a lower amount of bequest which makes households more likely to work to accumulate more wealth. Fourth, an amount of labor supplied has a negative relationship with the net wage because, in this particular model, the income effect exceeds substitution effect causing households to prefer consuming more leisure than increasing working hour when the real wage increases. When real wage decreases due to higher pension contribution under an aging scenario, households prefer working more than increasing their leisure. Looking at the life-cycle labor distribution (Figure 5), the result suggests that the negative effect is more prominent during young age, both cancel out during the mid-career, and positive effects become pronounced during the late-career. When jointly considered with the re-weighting effect, a lower proportion of young worker decreases the aggregate labor supply.

On the demand side, the economy requires less investment to sustain the capital level needed because of the population contraction (Table 4). Experiencing a higher dependency ratio, households face a higher pension contribution rate. Working as well as interest income taxes are also higher as the government can issue less bond to finance its expenditure when population declines. These increases in tax rates together with a lower interest rate and lower savings result in lower households net income and impede every age cohorts consumption (Figure 6). However, when looking at the aggregate level, demographic re-weighting emphasizes the behavior of older cohorts who consume more, and therefore, increases the total consumption share in the GDP.

The same narratives to explain the difference between RS and TA households behavior still hold. However, the deviation of aggregate variables of aging from the baseline are different between the two preferences types.The RS households amplify the negative effect of aging because of a different degree of interactions between re-weighting and redistribution effects. Consider an example of the total asset; the negative redistribution effect, where each age cohort readjusts by lowering their asset holding, overtakes the positive re-weighting effect, where more weights are given to asset-rich older people, and therefore, result in a lower total asset. However, there is less benefit from the re-weighting in the RS case as there will be less weights on the younger cohorts who relatively save more compared to the TA case to guard against future uncertainties. Next section will analyze interaction of the two effects in more detail.

6.3 Transition dynamics

When considering transition dynamics, population aging refers to a process when the economy transitions from high to low population. During which time, it will create a temporary dip in population growth rate, causing the demographic re-weighting to have a higher proportion of the risk-averse older cohorts relative to risk-loving younger cohorts. In the long run, however, demographic re-weighting disappears when an amount of the newborn exactly replaces that of the deceased.

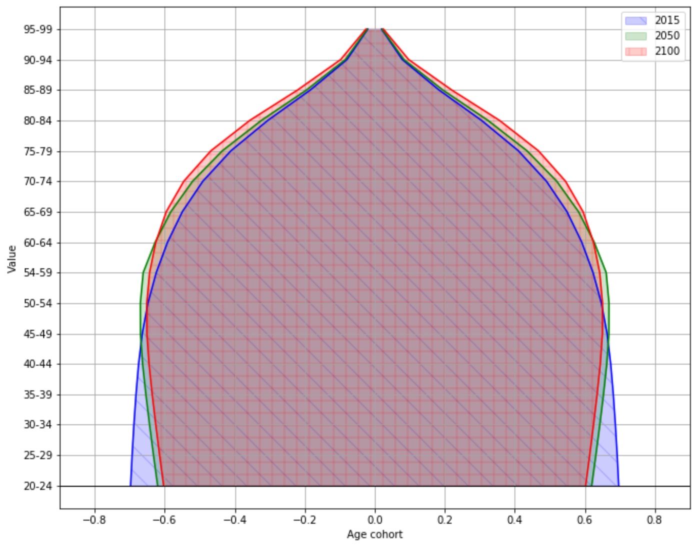


Figure 9: Population pyramids during demographic transitions

This section only looks at transition dynamics of a counter-factual population aging scenario. From Figure 9, the expansion of the old cohort continues and peaks in the year 2100 until it reverts to initial equilibrium with zero fertility rate in the long run. The total amount of population in the population pyramid is normalized to the initial equilibrium to allow a separation of economic impacts between a structural demographic shift and a change in the total population. The population growth scenario can be inferred as a flip-side of the aging scenario and therefore is left out in the analysis.

The analysis that follows will look at transition dynamics in two ways. First is a deviation from the initial equilibrium when keeping the total population constant, i.e. only analyze the impacts from demographic structural change. Second is a deviation when incorporated the changes in the total population. The deviations are explained by breaking down the demographic re-weighting effect and the life-cycle redistribution effect that occur along the transition path. The analysis looks at one variable at a time, and contrast between RS and TA households behavior.

Asset

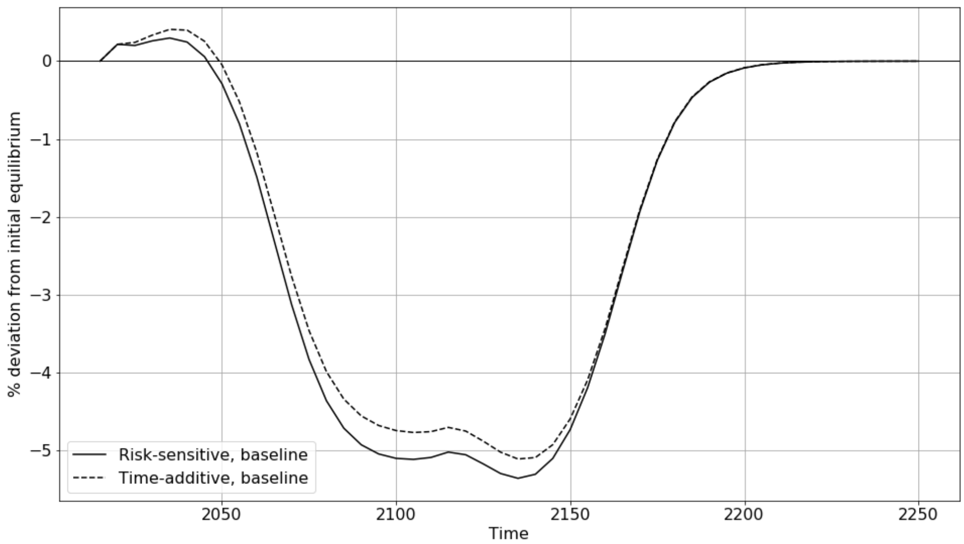


Figure 10: Transition dynamics of total assets

When population aging begins, there is an initial increase in the total asset as the beneficial demographic re-weighting is more pronounced in the early stage of the transition before it gives in to the adverse redistribution effect at a later date (Figure 10).In particular, an increasing share of the older population heightens their influence toward the total asset in the economy. Holding relatively more savings, these older adults initially contribute to the higher aggregate asset. As the transition proceeds, however, adverse distribution effect become more prominent. Increasing share of older cohorts, each of whom holds a lot of asset while supplies little to no labor, puts downward pressure on the interest rate and subsequently discourage every household from saving. Such adverse redistribution effect eventually subdues the beneficial demographic re-weighting, causing the aggregate asset to be below the initial equilibrium. In the long run, the aggregate asset gradually reverts to the initial equilibrium as fertility rate returns to zero.[6]

[6]The sharp drop of asset around the year 2115 is due to the reversal of population structure to long-run equilibrium. During this period, the adverse effect from asset redistribution is still high which, when coupled with diminishing positive demographic re-weighting, lead to a temporary drop in the asset. The total asset only starts to increase when the reversal in asset redistribution is sufficient. This reflects lagging adjustments of redistribution effect.

When taking into account the total population growth, the transition results in a permanent change in the asset level. As opposed to looking only at the demographic-structural impacts, Figure 19a suggests no short-run positive deviation of the asset as it is canceled out by a decrease in population level.

Comparing between RS and TA households, the former has a more severe impact on assets. The reason is twofold. First is because RS households have more precautionary savings, especially when young, which exacerbates the adverse effect of demographic re-weighting in comparison to the TA case. The second is because of a more significant negative redistribution effect as there is a slightly more reduction in total asset holdings across all cohorts in the RS case.

In summary, the transition dynamics of the aging scenario leads to lower savings when households have RS preferences as the decrease in population share is on the younger population, each of whom saves more than TA counterpart as a precaution against their future productivity shock.

Labor

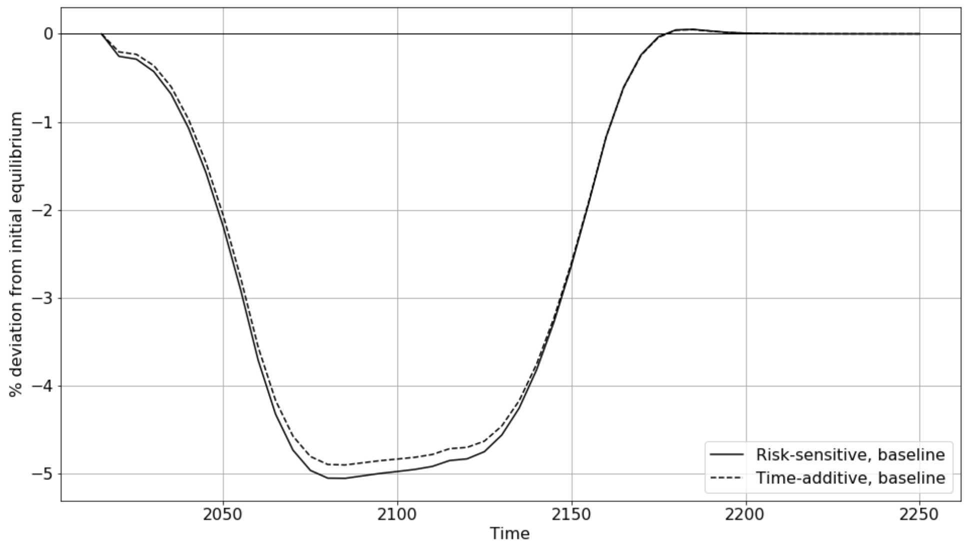


Figure 11: Transition dynamics of total labor

Aging population leads to lower aggregate labor supply (Figure 11). These transition dynamics are mainly driven by the effect of the demographic re-weighting as there is less share of younger cohorts who are the primary workforce. Comparing between RS and TA households, the result suggests that the former leads to larger negative deviation from the initial equilibrium because they spend more time working early in their life, when income risk is low, to save up and guard against future income risk.

Output

Considering both transition dynamics of labor and capital (savings less bonds, which behave in the same manner as the aggregate savings, see Figure 12), we can explain the deviation in economic output (Figure 13). There is not much change in output during the first few years after population aging occurs because the temporary

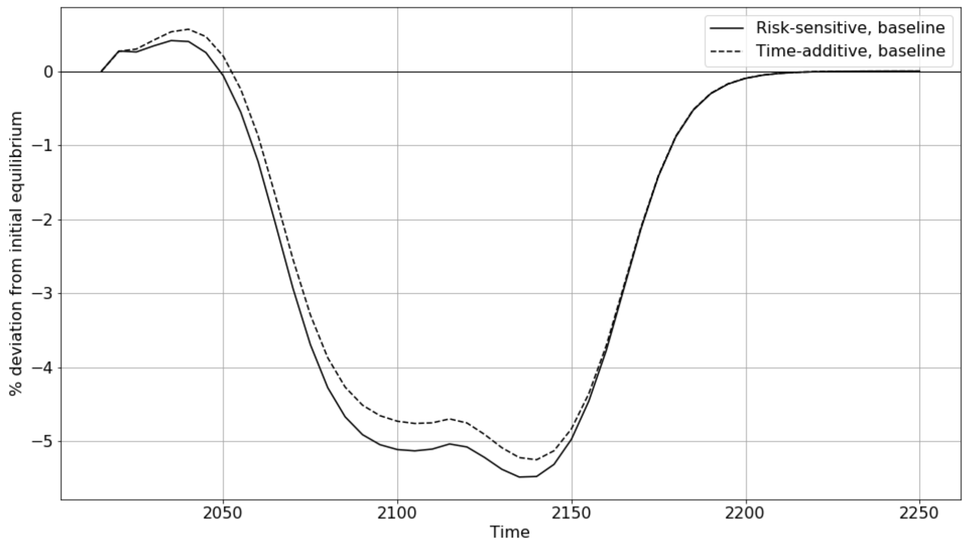


Figure 12: Transition dynamics of total capital

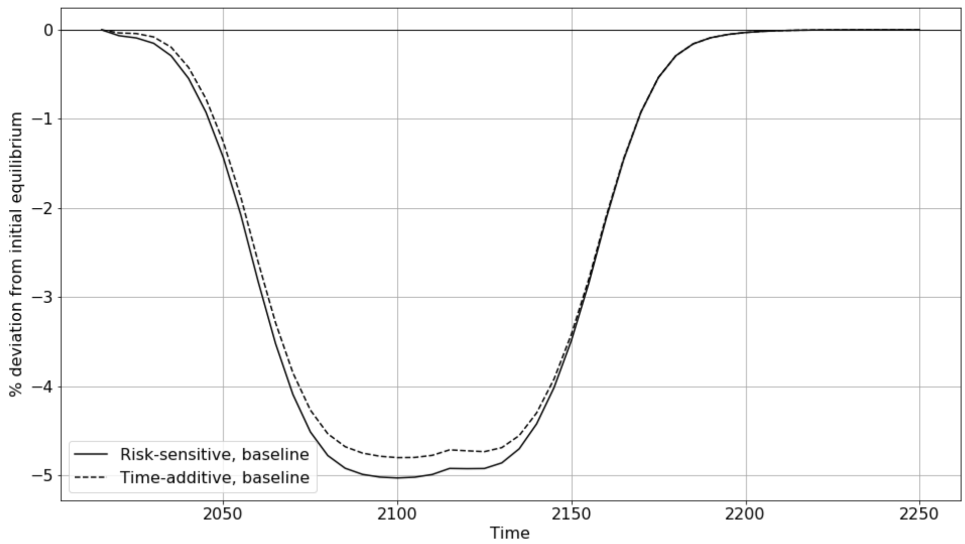


Figure 13: Transition dynamics of total output

increase in capital partly compensates for a drop in labor supply. Thereafter, the output decreases along with both factors of production, with the RS case decreasing more, which coincides with the larger negative responses as explained above. Long term dynamics then revert back to the initial equilibrium when both factors of production return to their initial levels. When the change in the total population is incorporated, the output transitions to a permanently lower equilibrium (Figure 19e).

Consumption

During the period of population aging, consumption initially increases (Figure 14) due to the demographic re-weighting that gives more weights to the older cohorts who consume relatively more. As the transition progresses, and people start to save less and earn less working income, each cohort is gradually able to consume less.

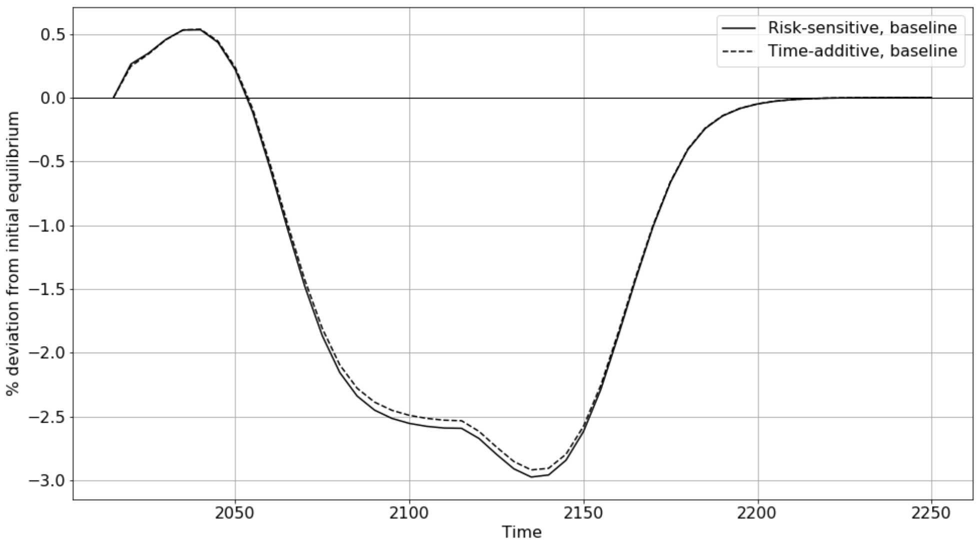


Figure 14: Transition dynamics of total consumption

This adverse redistribution eventually dominates the positive re-weighting, causing aggregate consumption to decrease compared to the initial equilibrium.

Comparing between RS and TA economies, the former is slightly better off. RS households receive more re-weighting effect because middle to older age cohorts consumer more on average due to their higher savings. When taken into account the change in the total population, the consumption dynamics are monotonic as impacts from the change in total population more than cancels out the initial increase in consumption (Figure 19d).

Investment

During a period of population aging, less investment is required to sustain a path of capital, which is declining due to changes in capital level as well as the population growth rate. However, RS investment deviates lower following a lower deviation of the capital level.

Price variables

Transition dynamics in price variables, namely interest rates and wage rates, are determined by dynamics of capital intensity. From Figure 11, labor supply monotonically decreases along with the population decline. However, it is xnot the case for the capital where adjustments are initially in the opposite direction. Such uncoordinated transitions result in fluctuating capital intensity and thus price variables (Figure 16).

In particular, capital intensity in the aging scenario deviates higher from the initial equilibrium from the year 2020 to around the year 2100. As explained earlier, a decrease in population growth rate immediately leads to a demographic re-weighting that decreases total labor supply and increases total capital. However, the life-cycle redistribution effect gradually strengthens and reverses the direction of the aggregate capital deviation, although it does not catch up with labor until the year 2100 when labor starts to revert to long term equilibrium. This consequent increase in capital

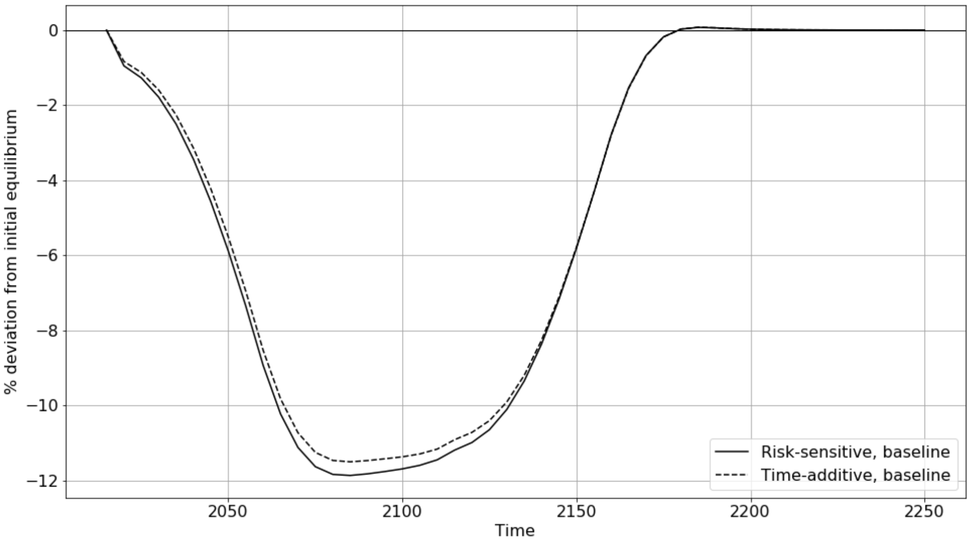


Figure 15: Transition dynamics of total investment

intensity from the year 2020-2100 puts upward pressure on the interest rate and downward pressure on wage. Comparing the two preferences variants, the change in capital intensity in the RS case is slightly lower during this period as there is more negative effect from a demographic re-weighting due to RS households precautionary savings.

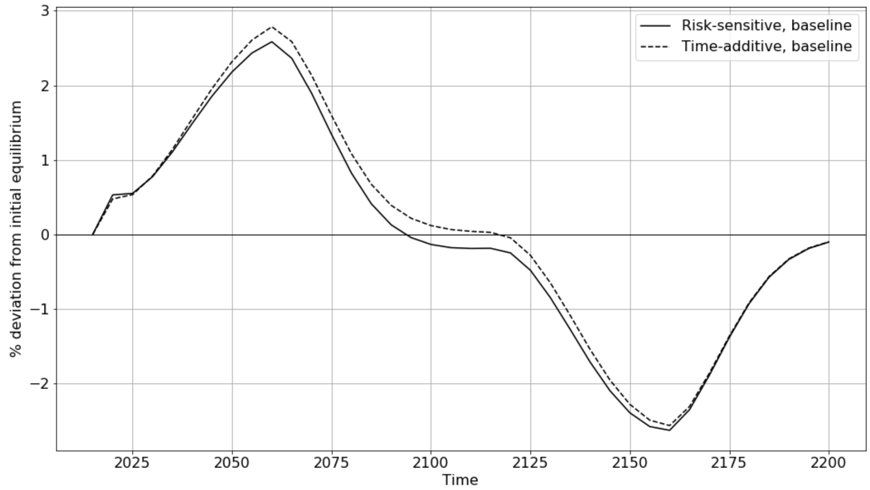
After the year 2100, the capital-labor ratio adjusts in the opposite direction. When labor gradually increases back to the long-run equilibrium, capital takes more time to adjust because of the persistent redistribution effect that temporarily further decreases the aggregate capital. As a result, interest rate overshoots and wage rate undershoots before returning to the equilibrium in the long run.

6.4 Welfare analysis

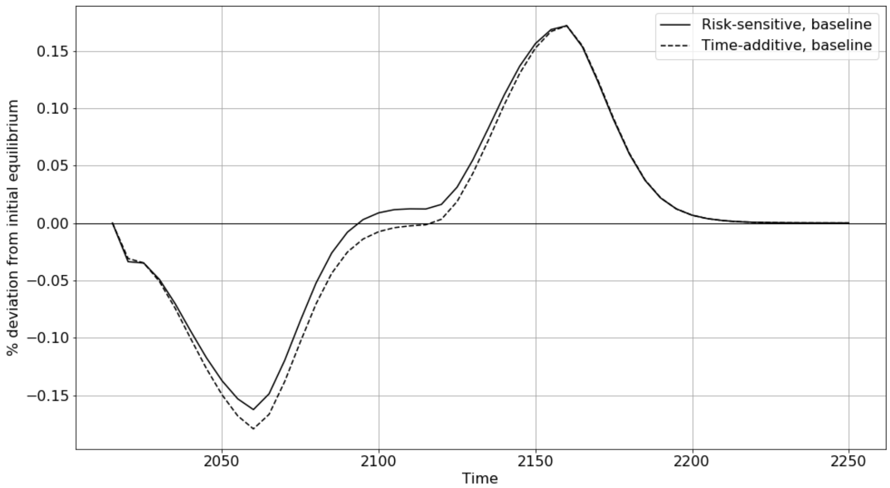
This section examines how the households welfare is influenced when faced with demographic aging. To evaluate cohort-specific welfare impacts, I employ a Hicksian Equivalent Variation (HEV), which represents a uniform proportional change, Δ_t, in consumption and leisure across every initial state z that is required to make households in the initial equilibrium equally well off as households born in time t after a population aging has occurred. For example, $\Delta_{2050} = -1.56$ means that a newborn cohort in an initial equilibrium needs to lower their consumption and leisure 1.56 per cent from her initial equilibrium values to be as well off as a person born in the year 2050. (see calculation detail in Appendix A).

To understand the difference in transition dynamics of the welfare between two types of households, we first look at what causes the welfare to change. Common determinants of changes in the welfare of each cohort are redistributions of leisure and consumption within a households lifetime (16 periods from t to $t+15$). Because there is a minimal change in leisure time, consumption redistribution is the main source of welfare change.

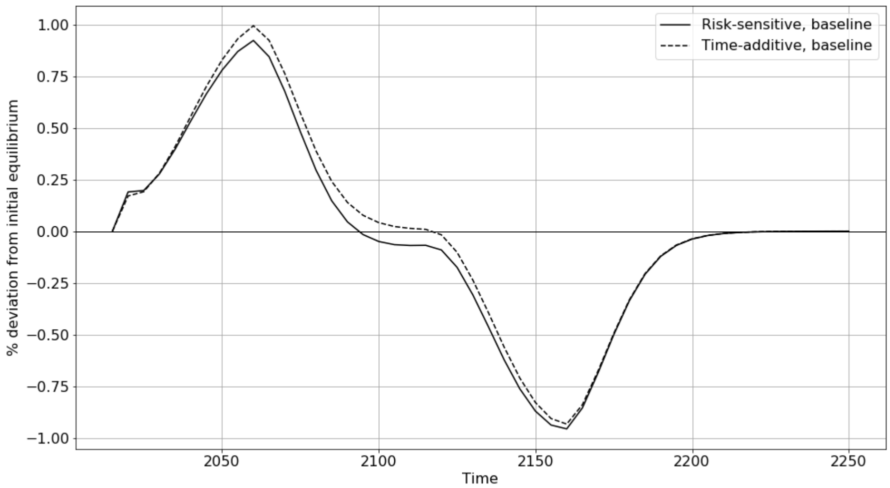
However, the RS households also have another welfare determinant which is the changes in future uncertainties during the next 15 periods. A few factors collectively determine future uncertainties. First is a shift in asset redistribution, where higher



(a) Capital to labor ratio



(b) Interest rate (% pa)



(c) Wage rate

Figure 16: Transition dynamics of capital to labor ratio and price variables

assets, which is assumed to be risk-free, lowers the level of uncertainties. Second are the changes in price variables, including risk-free interest rate which augments the benefit of assets holding, and risky wage which exacerbates stochastic working income. Taken all these factors into account can help explain the difference in welfare dynamics of two preferences types.

The HEV shows that people are worse off when the total population is aging compared to an initial equilibrium. Such welfare loss comes about as population aging creates economic conditions that decrease average savings and wage, hence consumption. The younger the households when demographic shock takes place, the worse off they are, as younger households will face lower consumption for a more extended part of their life.

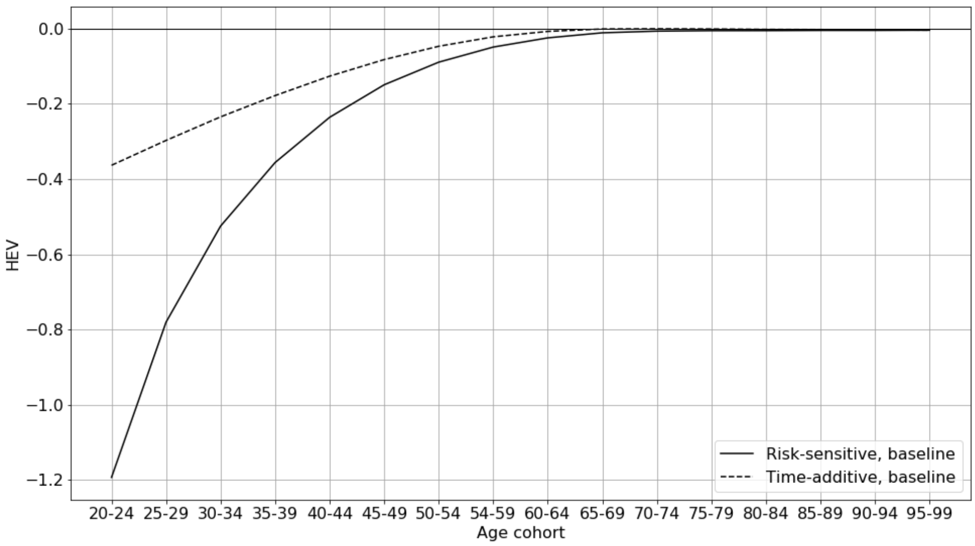


Figure 17: Welfare impact of different age-cohort at time t=1 represented by HEVs.

Comparing the two types of households, the model suggests that the RS households welfare reduces more significantly (17). As RS households savings decreases, their dependence on uncertain working income increases, which heightens uncertainties in their future utilities. The welfare is further worsened with an initial increase in the risky wage rate and a temporary decline in the risk-free rate of return. Elevated uncertainties translate into lower certainty equivalence, and therefore, lower welfare.

Beyond the first period, the result suggests lower RS welfare for cohorts born during the year 2015-2120 (Figure 18). However, a TA welfare loss exceeds that of RS after the year 2065. The RS welfare starts to revert at the year 2045 due to a reversion in future prices where risky wage starts to fall and the risk-free interest rate starts to increase, which together reduce uncertainties in the next 15 periods. In contrast, TA households welfare is not affected by changes in uncertainties and therefore continue to drop along adverse consumption redistributions. For the RS case, there is also a short period where households welfare improves compared to the initial equilibrium which is because of reduced uncertainties when risky wage undershoots and risk-free interest rate overshoots before reverting to equilibrium.

These differences in welfare impacts of population aging have important policy implications, for example, for finding an optimal fiscal policy under different demographic assumptions, which is a subject for future research.

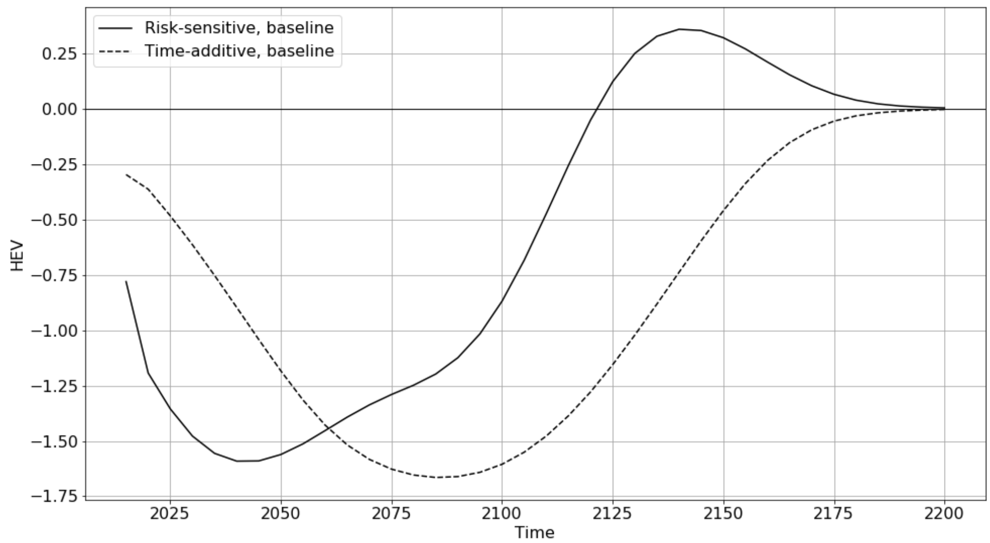


Figure 18: Transition dynamics of welfare impact of different age cohorts born in time $t \geqslant 1$ represented by HEVs.

7 Conclusions

This research has investigated how age-dependent increasing risk aversion in future utilities alters the impacts of demographic changes on the economy, both in terms of the aggregate macroeconomy and intergenerational redistributions of life-cycle variables. Two types of household preferences were compared. The first was the standard time-additive preferences that are risk-neutral in future utilities. The second was the risk-sensitive preferences by Hansen & Sargent (1995), which are the only form of recursive preferences that can separate risk aversion and intertemporal elasticity of substitution while being monotonic.

Age-dependent risk aversion emphasizes the role of precautionary savings. Foreseeing themselves becoming more risk-averse as they get older, young households save more to safeguard against future uncertainties by increasing their labor supply while sacrificing some consumption. The precautionary savings subsequently enable higher consumption and leisure. At the aggregate level, higher savings allows more investment and capital accumulation that, together with higher labor supply, lead to more aggregate output.

When faced with population aging, distinctive life-cycle variable distributions of the two preferences types result in different interactions between the effects of cohort re-weighting and life-cycle redistribution. Compared to the time-additive case, the risk-sensitive preferences exacerbate adverse responses of aggregate macroeconomic variables including savings, consumption, labor supply, and ultimately output. However, wage and interest rate in the risk-sensitive case adjust by a lesser amount because the increase in capital intensity is hindered by a reduction in aggregate precautionary savings from a lower share of young cohorts.

Using risk-sensitive preferences changes the way we analyze welfare impacts. Unlike the time-additive variant, risk-sensitive preferences allow households to factor in possible uncertainties that arise as a result of demographic aging in their lifetime. The results suggest that welfare deteriorates more under population aging because future utilities become more uncertain through decreases in risk-free savings and interest rate and increases in the stochastic wage. There were also welfare improvements during the period when risky wage undershoots and risk-free interest rate overshoots before reverting to long-run equilibrium, a behavior that cannot be captured with time-additive preferences.

This paper has shown that risk-sensitive preferences with age-dependent risk aversion can play an important role in optimal policy settings as demographic impacts on the macroeconomy and welfare depart noticeably from the case of the time-additive preferences. Further studies can incorporate risk-sensitive preferences with a more realistic demographic assumption, risk-aversion parameters and tax system, for instance, to revisit optimal fiscal policies when the economy experiences population aging.

Appendices

A HEV with a risk-sensitive preferences

This appendix explains the process of finding Hisksian Equivalent Variation (HEV) with risk-sensitive preferences to measure the welfare effect of demographic changes. Recall that the household's value function can be written as

$$V_t^j(z_t) = \max\left\{\left(\nu \log c_t^j + (1-\nu)\log(1-l_t^j)\right) - \frac{\beta\xi^{j+1}}{\psi^j}\log \mathbb{E}_t(e^{-\psi^j V_{t+1}}\big|\,\eta_t^j)\right\}. \quad (42)$$

The HEV gives an amount of consumption and leisure that households need in addition to their initial equilibrium so that they are as well-off as in the new equilibrium after a shock has occurred. Let $\Delta_t(\tilde{z})$ be a factor by which consumption and leisure change. The HEV finds $\Delta_t(\tilde{z})$ such that

$$\begin{aligned} V_t^j(\tilde{z}) &= \max\Bigg\{\Big(\nu \log\big((1+\Delta_t(\tilde{z}))(c_0^j(z))\big) + (1-\nu)\log\big((1+\Delta_t(\tilde{z}))(1-l_0^j(z))\big)\Big) \\ &\quad - \frac{\beta\xi^{j+1}}{\psi^j}\log \mathbb{E}_t(e^{-\psi^j V_{t+1}(z_{t+1},\Delta_t(\tilde{z}))}\big|\,\eta_t^j)\Bigg\} \\ &= V_t^j(z,\Delta_t(\tilde{z})) \end{aligned} \quad (43)$$

where z is the initial state vector and $\tilde{z}$ is the state vector after a shock has occurred. I omitted the time-subscript on z to simplify the equation.

To solve for $\Delta_t(\tilde{z})$, the routine solves the problem backward, starting from $j = J$

$$V_{t+J}^J(z,\Delta_t(\tilde{z})) = \max\Bigg\{\Big(\nu \log\big((1+\Delta_t(\tilde{z}))(c_0^J(z))\big) + (1-\nu)\log\big((1+\Delta_t(\tilde{z}))(1-l_0^J(z))\big)\Big)\Bigg\}. \quad (44)$$

Next, it substitutes $V_{t+J}^J(z,\Delta_t(\tilde{z}))$ into the value function of age cohort $j = J-1$, i.e. $V_{t+J-1}^{J-1}(z,\Delta_t(\tilde{z}))$, as

$$\begin{aligned} V_{t+J-1}^{J-1}(z,\Delta_t(\tilde{z})) &= \max\Bigg\{\Big(\nu \log\big((1+\Delta_t(\tilde{z}))(c_0^{J-1}(z))\big) + (1-\nu)\log\big((1+\Delta_t(\tilde{z}))(1-l_0^{J-1})(z)\big)\Big) \\ &\quad - \frac{\beta\xi^{J}}{\psi^{J-1}}\log \mathbb{E}_t(e^{-\psi^{J-1} V_{t+J}^J(z_{t+J},\Delta_t(\tilde{z}))}\big|\,\eta_{t+J-1}^{J-1})\Bigg\} \end{aligned} \quad (45)$$

The value function, $V_{t+J-1}^{J-1}(z,\Delta_t(\tilde{z}))$, depends on the term $\mathbb{E}_t(e^{-\psi^{J-1}V_{t+1}(z_{t+1},\Delta_t(\tilde{z}))}\big|\eta_{t+J-1}^{J-1})$ which can be evaluated using transition probabilities between current and next-period Markov states as

$$\mathbb{E}\left(e^{-\psi V^J_{t+J}(\tilde{z}_{t+J},\Delta_t(z))}\Big|\eta_{t+J-1}\right) = \sum_{IS^+=1}^{5} \phi(IS, IS^+) \cdot e^{-\psi V^J_{t+J}(\eta_{t+J},\Delta_t(z))} \tag{46}$$

where $\phi(IS, IS^+)$ is the transition probability from state IS to IS^+ (discretized to consist of five states) and $V^J_{t+J}(\eta_{t+J}, \Delta_t(z))$ is the average value function when η_{t+J} is in state IS^+.

Once we have the value function of cohort $j = J - 1$, we iterate the same process backward until we have the current-period value function $V^1_t(z, \Delta_t(\tilde{z}))$. We then check the equality between $V^1_t(z, \Delta_t(\tilde{z}))$ and $V^1_t(\tilde{z})$. If the equality doesn't hold, we update the value of $\Delta_t(z)$ and repeat the process until they converge. Given the monotonicity of risk sensitive preferences and utility function, we could apply a Newton method to solve for the values of $\Delta_t(z)$.

B Computational appendix

The general computation method follows that of Fehr & Kindermann (2018), i.e., using a microeconomic solution method to solve the household problem while using the macroeconomic solution method to solve equilibrium prices and quantities. The algorithm starts from solving the initial equilibrium and then the transition dynamics. A brief step-by-step description of the algorithm is as follows:

1. Initialization: Calculate the demographic structure in the initial equilibrium including the amount of population and population share in each cohort as well as total population. Assign values of age-dependent risk parameter. Discretize the Markov process of labor productivity and other state variables. Initialize labor fixed effect. Set up initial values of aggregate variables $\{K_t, B_t, L_t^D, L_t^S, I_t, Y_t\}$ and tax and transfers.

2. Prices: Calculate price variables including $r_t, w_t, r_t^n, w_t^n, p_t$

3. Solve household: Compute for policy functions $a_{t+1}^{j+1}(z_t)$, and corresponding policy functions $c_t^j(a_{t+1}^{j+1})$ and $l_t^j(a_{t+1}^{j+1})$ that satisfy the Euler equation (23) using a Newton method and a linear interpolation on a growing grid. The computation iterates backward from age J and calculates policy function for every possible state $z_t = j, a, \theta, \eta$

4. Distribute a mass of population into different states z_t represented by ϕ_{z_t} using the policy function $a_{t+1}^{j+1}(z_t)$ starting from age 1 until J.

5. Aggregate state-specific variables into life-cycle variables according to ϕ_{z_t} from the previous step. Update aggregate quantities $\{C_t, L_t, A_t, K_t, I_t, Y_t\}$ using the relative population shares m^j and cohort average variables according to a Gauss-Seidel procedure and calculate the difference between supply and demand in the goods market. Stop when the goods market clears.

6. Calculate budget-balancing tax rates and a social security tax

7. Repeat from step 2

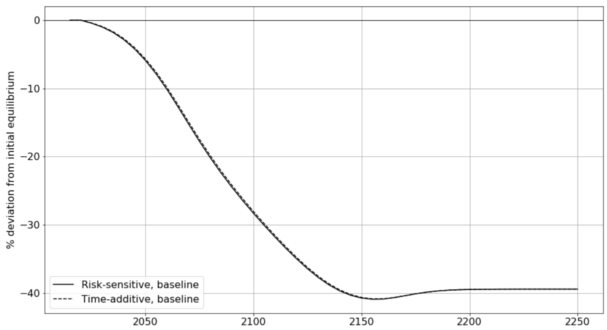
8. Transition dynamics

    - Initialization: Initialize all periods' variables equal that of initial equilibrium. Demographic structure is calculated as in section 4.1.
    - Repeat step 2 to 6 with until the goods market is clear for all $t = 1, ..., T + J - 1$.

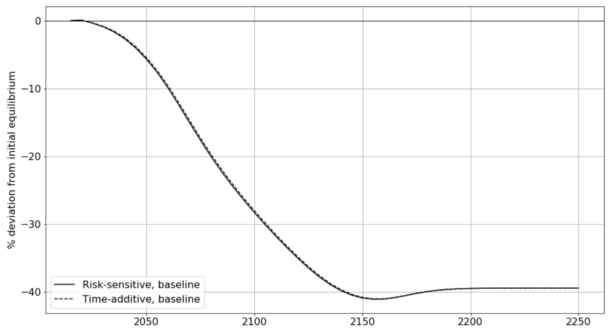
9. Hicksian Equivalent Variation (HEV)

    - Solve for HEV for all age cohorts born in time $t = -(J-1), ..., T$ by comparing all value functions at every states z_t with the value function in the initial equilibrium following the process explained in appendix A.

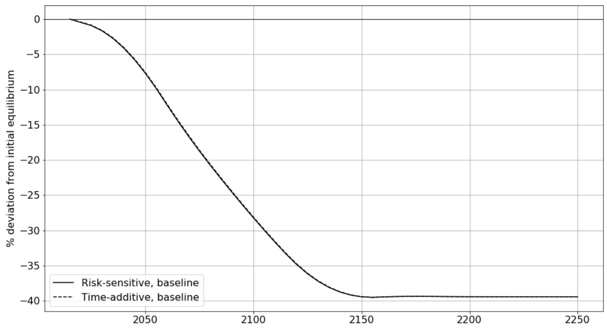
C Other graphs



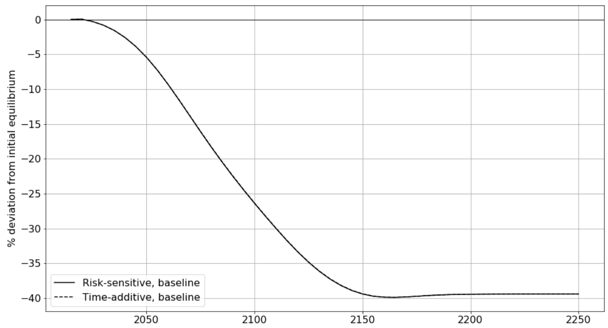
(a) Total asset



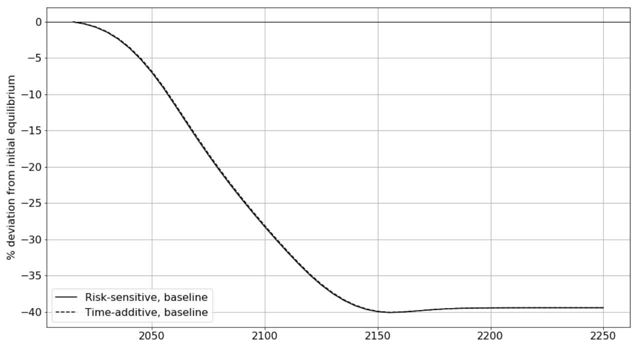
(b) Total capital



(c) Total labor



(d) Total consumption



(e) Total output

Figure 19: Transition dynamics of aggregate macroeconomic variables when changes in total population are taken into account

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