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Means Testing of Australia's Age Pension: A Numerical Analysis with an OLG Model

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Abstract

The Australian government has recently strengthened the age pension means test by raising the income taper and also introduced labour earnings exemptions from the means testing to encourage labour supply of older Australians. This paper assesses economy-wide implications of further hypothetical policy changes to the means testing of the age pension. To this end, we apply an extension of the overlapping generations (OLG) model developed by Kudrna and Woodland (2011a, b), with the capacity to investigate changes in the taper rate and labour earnings exemptions. The simulation results indicate that further increases in the taper combined with lower income tax rates lead to higher per capita labour supply and assets, as well as to welfare gains in the long run, while labour earnings exemptions have largely positive effects on average labour supply at older ages. Further increases in the taper are also shown to generate significant reductions in overall government spending on the age pension and, therefore, could be used as an alternative to increasing the age pension access age.

1 Introduction

The age pension, which represents the first safety-net pillar of Australia's retirement income policy, is currently a major income source for most Australian retirees. The pension is non-contributory, funded through general tax revenues and means tested against private resources of pensioners. The means test has been an important component of the age pension since its introduction more than a century ago. The Australian government has recently implemented several changes in the income test of the age pension, with aims to better target the pension payments to those in need and to encourage labour supply of older Australians. These changes include an increase in the taper rate from 0.4 to 0.5 in 2009 and an exemption of up to $6,500 of annual labour earnings from the means testing.[1]

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[1]The increase in the income taper was part of the 2009 age pension reform that also included (i) a 10 percent increase in the maximum pension for single pensioners, (ii) gradual increases in the age pension

In this paper, we assess economy-wide implications of further hypothetical policy changes that both relax and strengthen the means testing of the age pension.[2] Given the recently adopted changes in the income test, we consider the following two sets of policy experiments: (i) changes in the income taper rate (from the current rate of 0.5 to zero, 0.25, 0.75 and one) and (ii) changes in labour earnings exemptions (from the current exemption of up to $6,500 per year to 100% and 0%).

The purpose of this study is to explore the implications of these policy changes for incentives of individuals to work and save, for macroeconomic aggregates and for individual welfare. One specific aim is to determine whether the policy changes encourage labour supply of older Australians. While it is well known that public pensions may discourage labour supply and saving over a life-cycle as they act as a substitute for private income in retirement, the effects of the means testing are not a clear-cut. On the one hand, means tests generate high effective marginal tax rates (EMTRs), which have negative implications for labour supply and saving of older people. On the other hand, means tests reduce public pensions, thus resulting in higher life-cycle labour supply. Other aims of the paper are to determine distributional welfare effects and to draw out budgetary implications for the government.

To undertake this task, we use an extension of the overlapping generations (OLG) model developed by Kudrna and Woodland (2011a, b), with a detailed disaggregation of households into income quintiles and an updated calibration to recent Australian data. Our methodology has a range of features that make it particularly appropriate for the analysis of the pension means testing. First, the model employs life-cycle utility maximisation with endogenous retirement and a broader pension means test imposed on both assets income and labour earnings, allowing different means test treatments of the two sources of private income. Note that most analyses of the means testing of public pensions use models with exogenous retirement and thus assess only assets income under the pension means test (e.g., Sefton et al. (2008), Kumru and Piggott (2009, 2012) and Cho and Sane, 2013). Second, we incorporate inter- and intra-generational heterogeneity among households into the model, which allows us to evaluate policy impacts upon different household types. Third, the model includes a detailed model-equivalent representation of Australia's age pension, superannuation and income tax policy settings and hence captures important interactions between household behaviour and these policy settings. Another important contribution of our analysis to existing literature on means testing, which has so far focused on the long term effects (see, for example, Määttänen and Poutvaara (2007) and Tran and Woodland, 2011), is that we investigate the implications of the policy changes in the pension income test upon impact, over the transition and in the long term.

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access age to 67 years and (iii) a new work bonus with only half of the first $13,000 of annual labour earnings assessed under the income test (see Kudrna and Woodland (2011b) for the analysis of the 2009 age pension reform). In 2011, the work bonus was enhanced such that the labour earnings exemption from the means testing of the age pension now applies up to the first $6,500 per year.

[2]This paper focuses on the income test of the age pension as it currently affects over 60% of those on part age pension, with less than 40% affected by the assets test. Furthermore, for those pensioners on full age pension (about 50% of all Australians aged 65 years and over), it would be the income test that would bind, if they experienced an increase in their assets holdings. This is because the income test, which includes assets income (i.e., deemed income) from assets holdings, binds for smaller assets amounts.

The simulation results of further increases in the income taper show a significant reduction in the age pension expenditures (by 17.04% for taper increased to one), allowing for lower income tax rates that we adjust to maintain a balanced government budget. Further increases in the income taper combined with lower income tax rates have positive effects on per capita labour supply (0.82% increase), domestic assets (4.28% increase) and consumption (1.63% increase).[3] Interestingly, average labour supply at older ages also improves as most older households see their pensions reduced, with some elderly not qualifying for any pension and, therefore, no longer facing high EMTRs on their earnings. Similarly to Kumru and Piggott (2009) and Tran and Woodland (2011), we find positive effects of a higher taper on average welfare in the long term, caused by large welfare gains to higher income types of households who benefit from reduced income tax rates. However, the short term welfare effects are significantly negative for current pensioners who experience large cuts in their pensions.

The examined policy changes in labour earnings exemptions have much smaller aggregate effects due to the relatively small numbers of people affected and assumed productivity rates of the elderly workforce. However, we find that the labour earnings exemptions from the means testing are important and have largely positive implications for average labour supply of older Australians. This result supports the finding of empirical literature that examined labour supply responses to changes in the earnings tests of social security benefits in other developed countries (see, for example, Baker and Benjamin (1999) for Canada, Disney and Smith (2002) for the UK and Friedberg (2000) for the US).

The rest of this article is organised as follows. In the next section we provide an overview of the simulation model and present the benchmark solutions for key life-cycle profiles and macroeconomic aggregates. Section 3 reports on the simulation results for the examined policy changes in the income taper and labour earnings exemptions in the long run, upon the impact and during the transition. The results are discussed in terms of the disaggregate effects on household life-cycle behaviour as well as of macroeconomic and welfare implications. The final section offers some concluding remarks.

2 The model and benchmark economy

In this section, we first provide a non-technical, brief description of the model that is used to simulate the hypothetical changes to the age pension income test.[4] We then report on benchmark solutions for key variables at both household and aggregate levels and provide comparison with Australian data.

2.1 Model overview

We use an extension of the computable OLG model of the Australian economy developed by Kudrna and Woodland (2011a, b) that includes (i) a more detailed intra-generational

[3] The percentage changes in the brackets show the long run implications of the income taper increased to one, relative to the benchmark scenario with the current taper of 0.5.

[4] A more detailed, algebraic description of the model is relegated to the Appendix.

heterogeneity based on Australian Bureau of Statistics [ABS] (2012a) and (ii) an updated calibration to more recent data with a detailed model-equivalent representation of the age pension policy settings in 2012. The model is a small open economy version of Auerbach and Kotlikoff's (1987) OLG model that consists of household, pension, production, government and foreign sectors.

The household sector is populated with 70 overlapping generations aged 21 to 90 years, with each generation consisting of five income types of households distinguished by their productivity, social welfare and other tax payments. Households face lifespan uncertainty described by survival probabilities and make optimal consumption/saving and leisure/labour supply choices to maximise their inter-temporal utility. Importantly, retirement from work-force is also endogenous, which is also affected by the age pension policy setting.

The pension sector incorporates essential features of the two publically-stipulated pillars of Australia's retirement income policy – the means tested age pension and mandatory superannuation. The production sector contains a large number of perfectly competitive firms. The firms demands capital and labour to produce a single all-purpose output good that can be consumed, invested in production capital or traded internationally. The total wage bill also includes the mandatory superannuation contributions that are required to be made in full by employers.

The government collects tax revenues from households and firms to pay for its general government consumption and transfer payments to households that include the age pension and other social welfare. In the present paper, we assume that the government always maintains a balanced budget by adjusting the progressive income tax schedule, as in Tran and Woodland (2011).

We employ a small open economy framework with an exogenous interest rate since that description best fits the Australian economy. Finally, equilibrium in the model requires all markets to clear. That is, in every time period, (i) the demand for labour from perfectly competitive firms must equal the supply of labour from households; (ii) the value of the capital stock must equal the domestic and foreign assets; and (iii) output is equal to the sum of private and public consumption, investment and trade balance.

2.2 Benchmark results and comparison with data

The benchmark economy is assumed to be in a steady state equilibrium. We calibrate this benchmark economy to key Australian data averaged over the 5-year period ending in June 2012 and assume stationary demographics. The values assigned to the model parameters are taken from related literature, calibrated to key macroeconomic aggregates or exactly matching actual policy settings in 2012.[5]

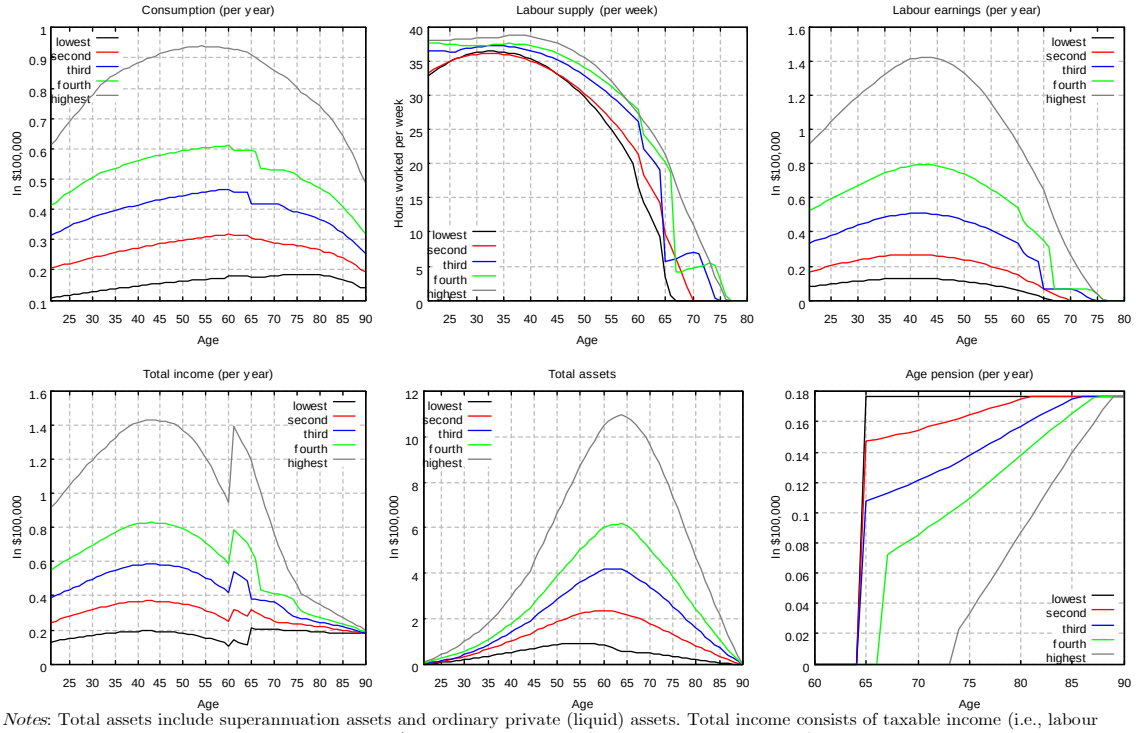
[5] As for the demographics, the age specific survival rates are taken from the 2010-12 life tables (ABS, 2013a) and the annual population growth rate of 1.8% is chosen to generate a realistic old-age dependency ratio of 0.22. The calibrated parameters of the utility function include the subjective rate of time preference and the leisure preference parameter that target the capital to output ratio (=3) and the average fraction of time spent working by those aged 25 to 60 years (=0.33), respectively. Most of the production function parameters are also calibrated to replicate other calibration targets in the 5-year period ending in 2012,

The benchmark steady state solution as well as transition paths for each of the examined policy changes discussed in the next section are obtained via the Gauss-Seidel iterative method, using GAMS software (see Kudrna and Woodland (2011a) for details).

2.2.1 Life-cycle profiles

The benchmark solution for life-cycle profiles of consumption, labour supply, total assets, labour earnings, total income and age pension payments is depicted by Figure 1. The life-cycle profiles of consumption expenditures, labour supply and labour earnings for each income quintile exhibit the standard hump-shape, rising at early ages and then declining. The shapes of these profiles reflect the assumed hump-shaped productivity profile and the increasing mortality risk, while the age profile of total assets reflects the saving decision along with the assumed zero initial and terminal asset holdings by households.[6]

Figure 1: Benchmark steady state solution for life-cycle household variables



Notes: Total assets include superannuation assets and ordinary private (liquid) assets. Total income consists of taxable income (i.e., labour earnings, private asset income and the age pension) and other social transfers, which are income-specific.

Figure 1 also shows sudden reductions in consumption, labour supply and labour earnings for some income quintiles at older ages, which are due to the retirement income policy.

including the investment rate of 0.09 and the foreign debt to capital ratio of 19.5 percent. The wage rate is normalised to one and the exogenous interest rate is set to 5 percent. We also make use of the adjustment parameters to target the ratios of consumption and corporate tax revenues to output and the ratios of public consumption, pension expenditures and other social welfare to output. The tax and pension parameters match actual policy settings in 2012.

[6] Following Gokhale et al. (2001), we assume that all inter-generational transfers are accidental and, hence, that there are no intended bequests. We also assume that the accidental bequests are equally redistributed to all surviving households of the same income type aged between 45 and 65 years.

First, the superannuation savings are assumed to be illiquid until age 60, at which each quintile is assumed to receive accumulated superannuation assets in the form of a lump sum. Subsequently, the payouts have an income effect on labour supply. Note that the drop in labour supply is particularly large for lower income types of households at age 60.[7]

Second, households at age 65 become eligible for the age pension, provided that they satisfy the means test. The graph with the age profiles of pension payments shows that the lowest quintile gets full age pension from age 65 onwards. The second and third quintiles receive part age pension at age 65, while households in the highest quintile do not receive any pension until age 72. The two lowest quintiles reduce their working hours at age 65 as a result of the income effect of the pension payment. The sudden drop in labour supply of the third quintile is due predominantly to the effective means testing with the preferential treatment of labour earnings. In particular, households in the third quintile at early age pension ages reduce their working hours to earn exactly $6,500 per year that is not means tested. The same labour supply behaviour is shown for the fourth quintile at age 67.[8] The effects of the age pension on consumption and labour supply of the highest income quintile are insignificant because the pension is of less importance to them in comparison with lower income quintiles.

2.2.2 Data comparison

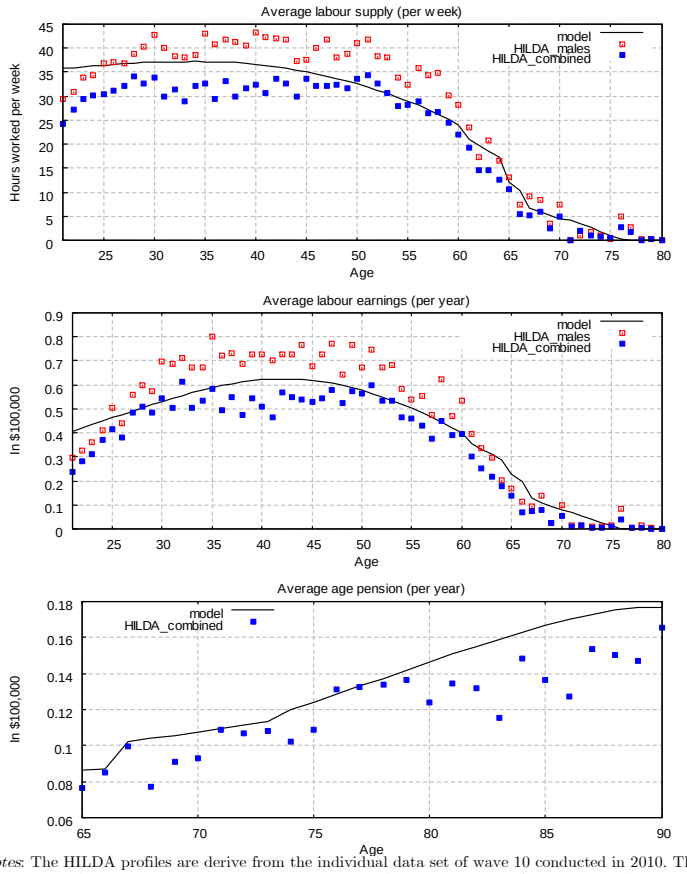
We now compare some of the life-cycle profiles and the main macroeconomic solutions generated by the benchmark steady state model with Australian data. The model-generated profiles for labour supply, labour earnings and pension payments averaged across the quintiles and the cross-section data derived from HILDA surveys are plotted in Figure 2. The comparison reveals similar shapes as well as levels of the model-generated and data-based profiles for the three selected household variables.[9]

[7] As the superannuation guarantee legislation prohibits from borrowing against superannuation assets, we impose the non-negative assets constraint to prevent younger households from borrowing against their future superannuation payouts. This constraint binds for lower income types prior to reaching age 60. The availability of their superannuation savings at age 60 increases their consumption as well as demand for leisure.

[8] Note that older households in the fourth quintile work less than households of the same ages in the third quintile because they are assumed to earn a higher effective wage.

[9] The reason for somewhat higher average pension payments obtained from the model for households aged 80 years and over is the model requirement of zero terminal assets. This requirement means that even households in the highest income quintile eventually qualify for the maximum pension as they draw down their assets and their assets income (subject to the pension income test) declines rapidly at very old ages.

Figure 2: Comparison of the selected average life-cycle profiles with actual data



Notes: The HILDA profiles are derive from the individual data set of wave 10 conducted in 2010. The combined profiles relate to the average across males and females. The HILDA 2010 values for labour income and age pension are inflated at the wage inflation rate of 3.5% to 2012.

Table 1 compares main aggregate solutions with actual values averaged over five years ending in June 2012 and taken from ABS (2012b, 2013b, 2013c). As shown, the benchmark model solution replicates the Australian economy fairly well. The model-generated components of aggregate demand presented in percent of gross domestic product are very close to their actual values, except for the trade balance, which is positive and implied by the targeted foreign debt to capital ratio. Similar conclusions can be drawn for government indicators, some of which are used as the calibration targets. In more detail, we calculate adjustment factors for the pension expenditures, the consumption tax (GST) revenue, the corporation tax revenue and several other government expenditures to match exactly the targeted ratio of each indicator to output.[10] The model overestimates the tax revenues from superannuation as it assumes 40 years of superannuation accumulations with 9% compulsory contributions, whereas the superannuation guarantee was introduced only in 1992 with 3%

[10]The adjustment factor for the pension expenditures is 0.9, which means that the pension payments in Figure 1 are scaled down to account for the maximum pension rate for single pensioners used in the model that is higher than the maximum payment to couple pensioners. The statutory consumption tax rate of 10% and the adjustment parameter of 0.65 imply the effective consumption tax of 6.5%, accounting for the fact that GST of 10% is being imposed on about 65% of all consumption goods in Australia. Finally, the implied effective corporation tax rate is about 25% in the benchmark steady state (i.e. the product of the corporate tax adjustment factor and the statutory rate of 30%).

minimum contributions initially. The personal income tax revenue is also larger in the model as our approximation tax function abstracts from any income tax offsets. Finally, we include other taxes and calculate the other tax revenue as a residual that balance the government budget with the targeted government consumption to output ratio. These other taxes are income-specific derived from the ABS (2012a) data and collected in a lump-sum manner in the model.

Table 1: Comparison of the model solution for 2012 with Australian data

Variable	Model	Australia [a]
Expenditures on GDP (percent of GDP)		
Private consumption	52.96	54.75
Investment	27.06	27.60
Government consumption	18.10	18.10
Trade balance	1.88	-0.54
Government indicators (percent of GDP)		
Age pension expenditure	2.80	2.80
Other social transfers [b]	4.20	4.20
Personal income taxes	12.92	11.50
Corporation taxes	5.10	5.10
Superannuation taxes	1.34	0.75
Consumption taxes (GST only)	3.50	3.50
Other taxes	2.44	2.84
Calibration targets		
Capital-output ratio	3	3
Investment-capital ratio	0.09	0.09
Foreign debt-capital ratio	0.195	0.195
Average hours worked	0.33	0.33

Notes: Actual data are taken from ABS (2012b, 2013b, 2013c) and all are averages over 2008-12; [b] These are social security payments excluding payments to the aged (e.g., disability pensions and family benefits).

The model also does a reasonably good job in matching the net income shares of each income quintile and the Gini coefficient in net income with the actual ABS (2013d) data on income distribution. Details of this comparison are available from the author.

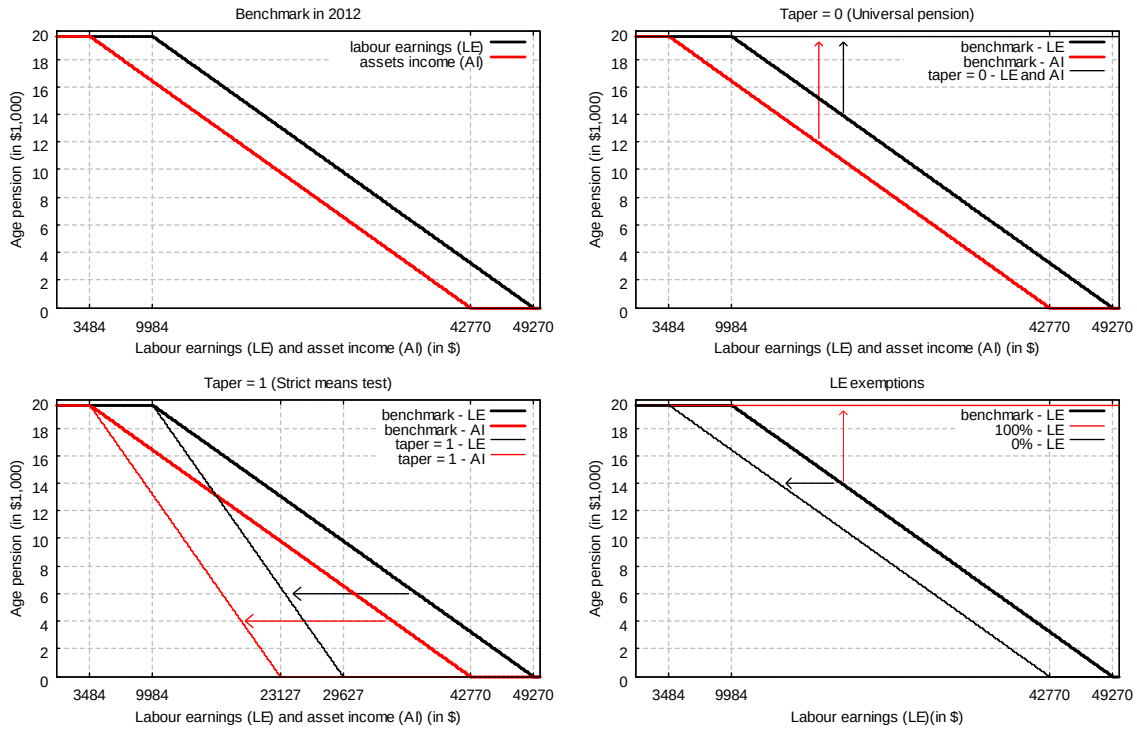
3 Policy simulations and analysis

We now use the model described in the previous section to simulate hypothetical policy changes in (i) the income taper rate to zero, 0.25, 0.75 and one; and (ii) labour earnings exemptions to 100% and 0%. The primary objective is to assess further increases in the income taper and higher concessions to labour earnings in the pension means test as extensions of the 2009 age pension reform. The examination of the policy changes that relax the means test by reducing the taper is motivated by the fact that many countries do not have targeted public pension (e.g. New Zealand). We assume that each of the hypothetical policy changes

is implemented in 2012.[11]

The associations between the age pension and the two sources of private income in the benchmark setting and under the selected hypothetical reforms are depicted by Figure 3. As shown, setting the taper to zero represents a shift to the universal pension (or demogrant) that the government pays to all individuals of the age pension age regardless of their investment income and/or labour earnings. In contrast, setting the taper to one represents a strict income test policy that almost halves the maximum private incomes of pensioners to qualify for any pension. As for the two examined changes in labour earnings exemptions, the figure only shows the association between the age pension and labour earnings because of the unchanged taper of 0.5, which implies the same age pension schedule for assets income as in the benchmark. In the case of 100% labour earnings exemptions, only the assets income is means tested, while the 0% labour earnings exemptions policy treats the two sources of private income in the same way as in the benchmark for assets income.

Figure 3: Association between age pension and private incomes



Notes: The benchmark assumes pension policy settings for single pensioners in 2012, with the income taper of 0.5 and the current labour earnings exemptions of up to $6,500 per year. The arrows show the effects of selected policy changes on the association between age pension and private income.

The changes in the age pension schedule reported in Figure 3 are expected to have direct effects upon life-cycle behaviour of households and also indirect (or general equilibrium) effects from the assumed, budget-equilibrating adjustments in the income tax schedule (i.e.,

[11]Note that our analysis abstracts from any other policy changes that may affect pension payments and total government spending on the age pension, including the already legislated increases in the age pension access age to 67 and in the superannuation guarantee rate to 12% of gross wages that are to be phased in gradually in the near future.

proportional change to average/marginal income tax rates). In this section, we present and discuss disaggregate behavioural effects, as well as the macroeconomic and welfare implications. We start with long run steady state implications and then proceed to short term and transitional implications of the investigated policy changes.

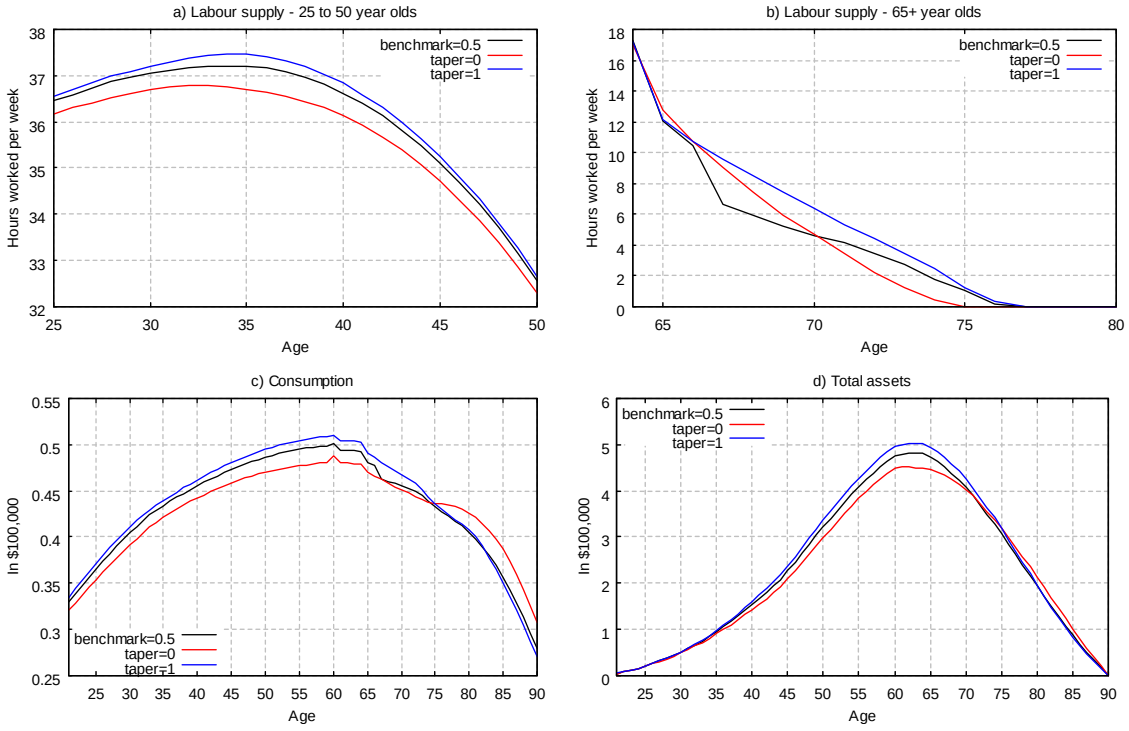
3.1 Long run implications

The long run steady state implications apply if we assume that there has been sufficient time for the economy to adjust completely to the new policy settings. In this case, households of different generations, but of the same income type, face exactly the same economic environments (though at different calendar times) and so behave in exactly the same way. Below we discuss the long run implications separately for the taper rate changes and the change in labour earnings exemptions.

3.1.1 Taper rate changes

The long run effects of the taper rate changes on average life-cycle labour supply, consumption and total assets are presented in Figure 4. For ease of exposition, each graph compares the benchmark steady state profile, which is averaged across five income types of households, only with the average profiles obtained from the two extreme changes in the taper to zero (i.e., universal pension) and to one (i.e., strict means test). Similarly to Kumru and Piggott (2009), the life-cycle results indicate that the high taper rate policy leads to less consumption smoothing, but larger assets accumulations for most of the life-cycle with steeper assets withdrawals at older ages. Furthermore, as the increased taper lowers average pension payments to elderly households, the associated disincentive of the pension to work declines, partly explaining increased labour supply of young and middle age cohorts (Figure 4a). The indirect effect of reduced income tax rates resulting from the strict means test policy also encourages higher average labour supply. The results for the shift to universal pension payments with the taper set to zero show the opposite behavioural effects, compared to those outlined above for the strict means test policy change.

Figure 4: The long run steady state effects of taper rate changes on average life-cycle profiles



The effects on average labour supply of the high taper rate policy are not only positive for young and middle age cohorts but also for older households aged 65 years and over, as shown in Figure 4b. Table 2 with the disaggregate effects on average labour supply for the 25 to 55 and 65 plus year olds shows that under the high taper policy change, many income quintiles aged 65 years and over work longer hours, with average labour supply of 65 plus year olds increasing by 13.43% relative the benchmark. While the labour supply of older households in the lowest quintile who receive the maximum pension regardless of the income taper increases only marginally, the second, third and fourth quintiles at older ages experience significantly higher labour supply. Although the elderly in these quintiles work more to offset reduced age pension payments, the labour supply effects differ among the three income groups. Specifically, households in the second quintile work and earn more but the EMTRs on their labour income are not affected by the increased taper because they do not exceed the maximum labour earnings exemption. The average labour supply of 65 plus year olds in the third quintile also increases but due to an increased retirement age. Note that these households supply the same working hours at early age pension ages as in the benchmark, in order to avoid high EMTRs on their labour income that they would pay if their labour earnings exceeded the maximum exemption. Finally, households in the fourth quintile no longer qualify for any pension at early age pension ages as a result of the increased taper. These elderly households no longer face any labour supply distortions arising from the means testing and, therefore, increase their labour supply to work similar hours as the highest quintile.

Table 2: Long run effects of taper rate changes on household labour supply
(Percentage changes in hours worked per week relative to benchmark in 2012)

Income quintile	Taper = 0		Taper = 1	
	25-50	65+	25-55	65+
- Lowest	-0.21	-21.64	0.09	2.03
- Second	-0.36	-29.39	0.04	36.04
- Third	-1.20	39.99	0.40	21.96
- Fourth	-1.93	24.77	0.93	54.65
- Highest	-1.50	-14.73	0.69	-4.56
Average	-1.41	-0.68	0.62	13.43

Note: The results relate to average labour supply for 25-55 and 65 plus year olds.

Table 3 reports on the long run macroeconomic implications of the examined taper rate changes as percentage changes in the selected per capita variables relative to the benchmark in 2012. The simulation results of hypothetical increases in the income taper show positive long run effects on most macroeconomic variables, including labour supply, assets and consumption as well as reduced age pension expenditures to the government.[12] In particular, the taper increased to one generates 0.82% increase in labour supply, 4.28% increase in domestic assets, 1.63% increase in per capita consumption (a measure of living standards) and 17.04% reduction in age pension expenditures. The positive effects on per capita labour supply are shown to be driven by higher average labour supply of young and middle age households. In contrast with the tougher means test policy changes, we find that lowering the current taper rate of 0.5 has negative macroeconomic and fiscal implications in the long term. For example, the results for the removal of the income test with the taper set to zero show a significant increase in the age pension expenditures by almost 42% from current 2.8% of GDP to over 4% of GDP, requiring over 11% income tax hike.

Table 3: Macroeocnomic effects of taper rate changes in the long run
(Percentage changes in selected variables relative to benchmark in 2012)

Variables	Taper rate changes			
	0	0.25	0.75	1
Labour supply	-1.38	-1.06	0.40	0.82
- 25-55 year olds	-1.41	-0.59	0.34	0.62
- 65+ year olds	-0.68	-24.06	4.97	13.43
Domestic assets	-4.41	-2.94	1.98	4.28
Consumption	-2.30	-1.69	0.78	1.63
Age pension expenditures	41.66	18.23	-9.89	-17.04
Income tax rates [a]	11.16	6.28	-3.19	-6.01

Notes: [a] Adjustments to income taxes assumed to balance government budget.

[12] In our small open economy framework, the capital labour ratio as well as the marginal products of capital and labour and the wage rate faced by the firms are all determined by the exogenously given interest rate in the long run. To keep the capital labour ratio unchanged in the long run, the percentage changes in the per capita labour supply have to be matched by the percentage changes in the capital stock. The long run changes in average labour supply also determine the percentage changes in the output per capita because of the constant return to scale property of the production function.

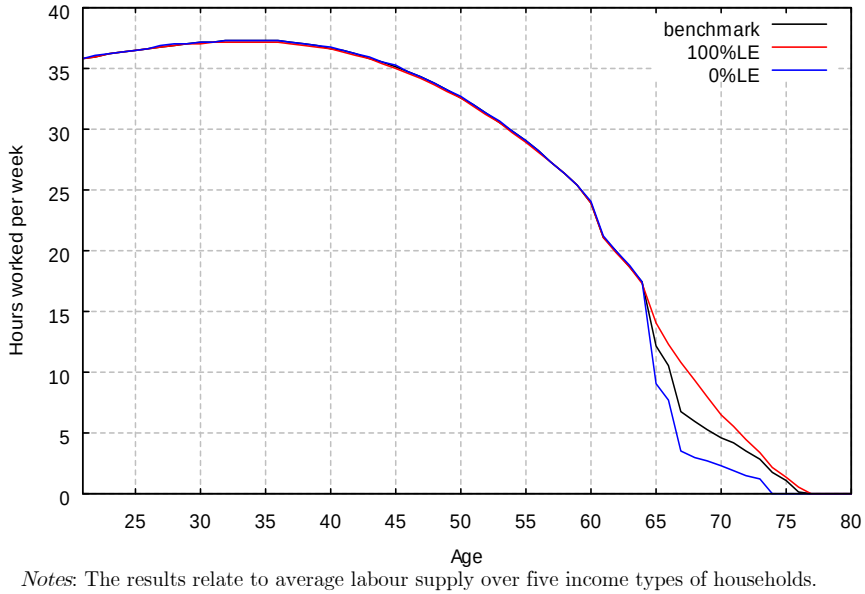
Kudrna and Woodland (2011a) also examined the implications of the means test removal and found positive effects on per capita labour supply during the transition and in the long run. The difference from the effects of the income test removal with the zero taper on per capita labour supply reported above can be explained by (i) a more detailed income distribution based on the ABS (2012a) data with equal population shares of 20% for each income quintile and (ii) budget-equilibrating adjustments to progressive income taxation considered in the present paper. Note that Kudrna and Woodland (2011a) assumed that 30% of each generations belonged to the low income class, 60% to the middle income class and the remaining 10% to the high income class, as in Fehr et al. (2008). Therefore, their positive labour supply effects of the means test removal were largely driven by increased working hours of middle income households eligible for the age pension. That paper also assumed the consumption tax rate to adjust to maintain a balanced government budget. Using the current model and simulating the zero taper rate policy combined with the budget-equilibrating change in the consumption tax rate generates only a 0.35% decrease in per capita labour supply in the long run. This comparison indicates that the economic implications for policy changes in the income taper rate are highly sensitive to the choice of a budget-neutralising policy instrument.

3.1.2 Changes to labour earnings exemptions

Here we discuss the simulation results for the hypothetical changes in the labour earnings (LE) exemptions from the income test to 100% and to 0%. The main objective of these two simulations is to examine the effect of a preferential means test treatment of labour earnings on labour supply of older Australians.

Figure 5 compares the life-cycle labour supply in the benchmark averaged over 5 income types of households with the average labour supply profiles obtained from the two policy changes. The differences among the three profiles are significant for older households aged 65 years and over. While the hypothetical removal of the current labour earnings exemptions reduces labour supply at older ages, the 100% exemption of labour earnings from the means testing increases average labour supply of older households in comparison with the benchmark labour supply. Under the 100% labour earnings exemptions, the increased labour supply of older households is also shown to decline gradually with age. This is because elderly households in the third and fourth quintiles no longer face high EMTRs on their labour income as they did in the benchmark case. Recall that in the benchmark with the current labour earnings exemptions, the working hours of the two quintiles drop suddenly at early age pension ages (see the life-cycle labour supply in Figure 1).

Figure 5: Long run labour supply effects of changes in LE exemptions



Notes: The results relate to average labour supply over five income types of households.

The long run macroeconomic effects of the two policy changes in labour earnings exemptions are provided in Table 4. Compared to the examined taper rate changes, the changes in labour earnings exemptions have much smaller aggregate effects, which is due to low productivity, labour supply and earnings at older ages. Importantly, labour earnings exemptions have significant and positive effects on average labour supply of older Australians. The results for the 100% labour earnings exemptions show a 24.64% long run increase in average labour supply of households aged 65 years and over, which is almost a double of the long run increase in the labour supply of the elderly resulted from the strict means test policy with the taper increased to one.

Table 4: Macroeocnomic effects of labour earnings exemptions in the long run
(Percentage changes in selected variables relative to benchmark in 2012)

Variables	Changes in labour earnings exemptions to	
	100%	0%
Labour supply	0.30	-0.48
- 25-55 year olds	-0.34	0.12
- 65+ year olds	24.64	-24.47
Domestic assets	-2.94	1.33
Consumption	-0.07	-0.38
Age pension expenditures	2.64	-0.22
Income tax rates [a]	0.88	0.35

Notes: [a] Adjustments to income taxes assumed to balance government budget.

Table 4 also shows increased age pension expenditures as a result of the 100% labour earnings exemptions, which calls for higher income tax rates that are assumed to maintain a balanced government budget. Higher income tax rates together with increased age pension

payments lead to smaller assets accumulations. As mentioned, the magnitude of these aggregate effects is much smaller relative to the macroeconomic implications of the taper rate changes (see Table 3 for comparison).

3.2 Short-term and transitional results

The simulation results in the long run presented above established that only the investigate changes in the income taper rate had significant effects on the Australian economy. In this sub-section, we therefore focus on the transitional implications of the taper rate policy changes for the key macroeconomic variables and for welfare of different households.

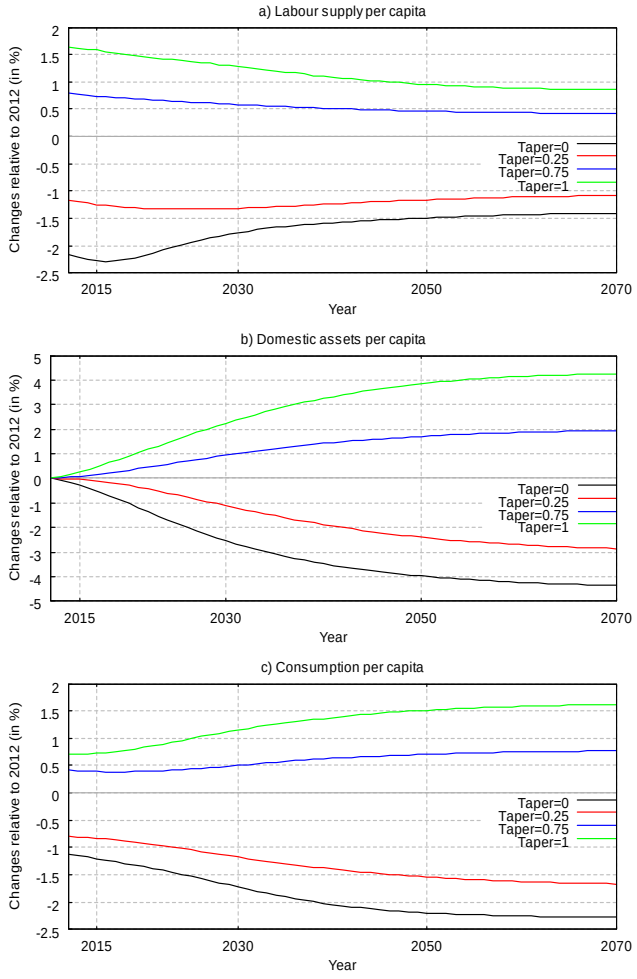
3.2.1 Macroeconomic implications

The macroeconomic effects of the taper rate changes on labour supply, domestic assets and consumption (all measured in per capita terms) upon the impact in 2012 and over the transition are depicted by Figure 6. These effects are presented as percentage changes in the selected variables relative to their benchmark steady state values.[13]

Several observations can be drawn from Figure 6. First, the examined increases in the taper from the benchmark rate of 0.5 (combined with the budget-equilibrating reductions in income tax rates) lead to higher per capita labour supply, assets and consumptions during the transition as well as in the long run. Second, the short run effects on per capital labour supply are larger than the long run implications. This is because of higher labour supply of currently older households who work more to partly offset large cuts in their pensions. The transitional decreases in per capita labour supply relative the impact effect are due to greater assets accumulations by young and future born households, which have some income effect on their working hours. However, under the strict means test policy with the increased taper to one, the long run labour supply is still more than 0.8% higher than in the benchmark. Third, the effects of the two examined reductions in the income taper are almost symmetrically opposite to the higher taper rate changes. For example, the shift to universal pension payments is shown to reduce per capita labour supply more in the short run than in the long run. Older households significantly reduce their working hours because of higher (full) pensions, while future born generations accumulate smaller assets due to increased income tax rates. As a result, per capita labour supply improves but per capita consumption further worsens in the subsequent years of the transition.

[13] The results for year 2070 approximate the long run effects presented above.

Figure 6: Macroeconomic effects of taper rate changes over the transition



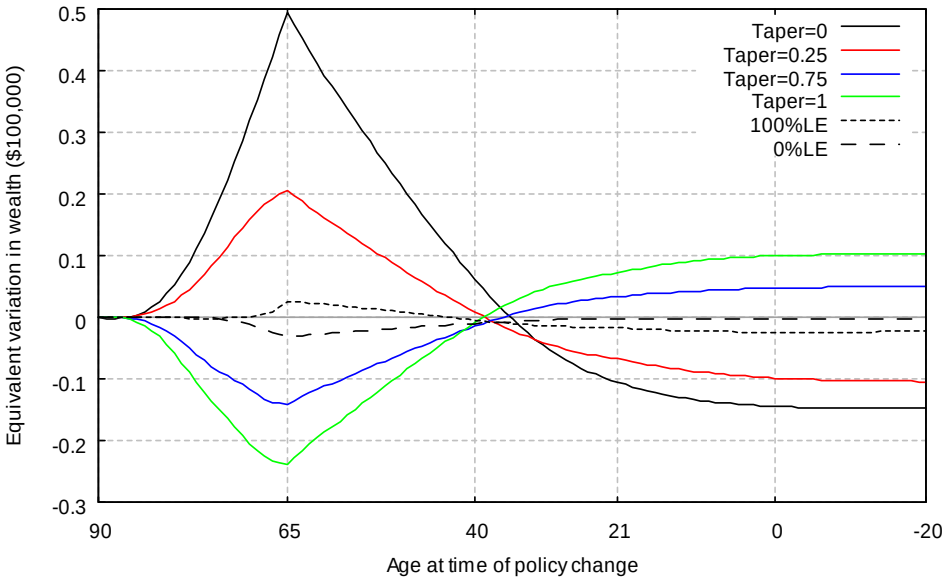
The transitional effects of the taper rate changes on the age pension expenditures and the budget-equilibrating income tax rates (not presented) are similar to the long run effects (see Table 3). Specifically, the zero taper policy change increases the age pension expenditures by 41.66% upon the impact and in the long run as we assume stationary demographics. The examined increases in the income taper reduce the age pension expenditures significantly in the short run, allowing for an immediate income tax cut. Over time, the pension expenditures to the government (and thus income tax rates) decline further because of larger assets accumulated by future-born households that generate higher assets income assessed under the pension means test.

3.2.2 Welfare effects

The welfare effects are assessed on the basis of standard equivalent variations. Following Nishiyama and Smetters (2007), we calculate the change in initial wealth/assets for each generation needed in the benchmark to produce remaining lifetime utility obtained under the policy change. The average welfare effects of the examined policy change (i.e., average welfare across the five income groups) as a function of cohort's age at the time of the policy

change are plotted in Figure 7. Recall that each of the hypothetical changes is assumed to be adopted in 2012, with the cohort aged 21 years being the youngest alive at the time of the policy implementation.

Figure 7: Average welfare effects of the policy changes in income test
(Equivalent variations of one-time wealth transfers at time of policy change)



Notes: The presented welfare results for each cohort show an average over the five income groups.

Similarly to the long run macroeconomic effects, Figure 7 indicates that (i) the welfare effects are almost symmetrically opposite for the two increases and the two reductions in the income taper rate, and (ii) the welfare effects of the changes in the labour earnings exemptions much smaller compared to those obtained from the taper rate changes. Under the increased taper policy changes, the elderly population and households approaching retirement in 2012 experience larger welfare losses due to pension cuts, while young and future-born generations, on average, gain in welfare as they benefit from lower income tax rates and increased savings. On the contrary, the investigated reductions in the income taper have significantly positive effects on welfare of the currently old and middle-age households (who all receive maximum pensions) but negative effects on welfare of future-born generations. For instance, consider the generation aged 65 years in 2012. This generations would gain almost \$50,000 in initial resources under the zero taper policy, whereas the same cohort looses, on average, almost \$24,000 in the case of the increased taper to one. In the long run, however, the average welfare is shown to increase by over \$10,000 in the case of the increased taper policy and to decline by about \$15,000 as a result of the zero taper policy. It should be also pointed out that the currently young and future-born generations who gain from the increased taper are larger in size compared to the currently older generations.

Table 5: Distributional welfare effects of changes in income taper rate
(Equivalent variations of one-time wealth transfers at time of policy change)

Policy change	Age in 2012	Household Income Type				
		Lowest	Second	Third	Fourth	Highest
Taper = 0	80	-0.01	-0.01	0.02	0.07	0.19
	65	-0.02	0.12	0.41	0.78	1.19
	40	-0.02	0.05	0.12	0.16	-0.01
	21	-0.01	0.01	-0.01	-0.08	-0.42
	-80	-0.02	-0.01	-0.05	-0.14	-0.52
Taper = 0.25	80	0.00	0.00	0.01	0.03	0.10
	65	-0.01	0.06	0.20	0.38	0.41
	40	-0.01	0.03	0.06	0.07	-0.11
	21	-0.01	0.00	-0.01	-0.05	-0.27
	-80	-0.01	-0.01	-0.03	-0.09	-0.37
Taper = 0.75	80	0.00	0.00	-0.01	-0.04	-0.10
	65	0.01	-0.06	-0.20	-0.28	-0.17
	40	0.00	-0.03	-0.08	-0.08	0.12
	21	0.00	-0.01	-0.02	0.00	0.19
	-80	0.01	-0.01	-0.01	0.02	0.23
Taper = 1	80	0.00	0.00	-0.02	-0.07	-0.21
	65	0.01	-0.11	-0.38	-0.48	-0.23
	40	0.01	-0.06	-0.17	-0.09	0.27
	21	0.01	-0.02	-0.05	0.05	0.37
	-80	0.01	-0.01	-0.03	0.09	0.45

Note: Initial wealth transfers presented in units of $100,000.

In Table 5, we further decompose the average welfare effects of the taper rate changes presented above to show both the inter-generational implications for the selected cohorts and the intra-generational implications for each income quintile. The welfare implications presented as equivalent variations of one-time wealth transfers are larger for higher income quintiles as they hold much larger lifetime wealth compared to lower quintiles. The welfare of households in the lowest quintile is affected only through indirect effects of the budget-equilibrating changes in income tax rates as these households eligible for the age pension receive the maximum payment regardless of the income taper. The inter-generational welfare implications for higher income quintiles are also affected by direct effects of the changes in current or future age pension payments. The examined increases (reductions) in the taper lower (increase) age pension payments, which lead to welfare losses (gains) to currently older generations. For example, the welfare gain for the highest quintile aged 65 years in 2012 is $119,000 in initial wealth in the case of the zero taper. In other words, the initial wealth of this high income household would need to increase by that amount to generate the level of remaining lifetime utility in the benchmark with the taper of 0.5 as under this policy change. In contrast, the future-born generations of the highest income quintile experience large welfare losses ($52,000 in initial wealth) from the zero taper rate policy that increases disincentives to work and save and requires higher income tax rates. Note that lower income tax rates resulting from the increased taper are particularly important for long term welfare

gains to higher income quintiles. Using a different budget-equilibrating policy instrument such as the consumption tax rate is likely to reduce the welfare gains of the increased taper for higher income quintiles as found in Kudrna and Woodland (2011a).

4 Concluding remarks

In this paper, we have examined hypothetical policy changes in the taper rate and labour earnings exemptions applied to the income test of the age pension, using an OLG model stylised to the Australian economy. The primary objective was to assess further increases in the income taper and the labour earnings exemptions as extensions of the 2009 age pension reform. To complete our analysis, we have also considered reductions in the income taper and the removal of current labour earnings exemptions.

On the basis of our simulations, we find that further tightening the taper leads to higher per capita labour supply, assets, consumption and long term welfare gains, but also to significant welfare losses of many currently older generations. These positive macroeconomic implications and long term welfare improvements are to a large extent due to the reduced income taxes needed to support a balanced government budget with reduced spending on the age pension. Similarly to Kumru and Piggott (2009), we show that tightening the taper leads to faster drawdowns of assets in retirement. However, the asset decumulations are not large and only gradual, compared to large adjustments in labour supply of some pensioners who face high EMTRs on their earnings. We also find that while relaxing the income test for earned income has little aggregate impact (including implications for pension expenditures to the government), the policy has important and largely positive effects on labour supply at older ages.

The fiscal effects of further increases in the taper show significant reductions in total government spending on age pension and, therefore, could be used as an alternative policy with potentially more equitable distributional implications to increasing the age pension access age. Furthermore, policy reforms of tightening the taper combined with labour earnings exemptions from the means testing have recently been recommended to advanced economies by the International Monetary Fund [IMF] (2014).

It is important to note that any modelling analysis such as that employed in this paper is subject to qualifications and limitations. First, comparing our results with those obtained by Kudrna and Woodland (2011a) and Määttänen and Poutvaara (2007) suggests that the economic implications of tightening the taper are highly sensitive to the choice of a budget-equilibrating policy instrument, with the improvements in per capita labour supply and long term welfare found in our paper being conditional on reduced income tax rates.[14] Second, relaxing our small open economy assumption and considering imperfect capital mobility with an endogenous domestic interest rate as in Guest (2006) are likely to further strengthen the case for means testing, as potentially reduced interest rates resulting from lower foreign debt

[14]Note that Kudrna and Woodland (2011a) used the consumption tax rate to fund their policy experiment of a shift to universal pension payments and Määttänen and Poutvaara (2007) assumed increases in the maximum pension benefit resulting from their simulation of abolishing the earnings test of social security benefits in the US.

would further encourage investment demand and have positive effects on wages. Third, our analysis abstracts from income uncertainty that was considered by Kumru and Piggott (2009) and Tran and Woodland (2011). In this regard, the means testing of pubic pensions is likely to reduce precautionary savings of young and middle age cohorts and, therefore, mitigate the observed positive effects on asset accumulations documented in this paper. Finally, allowing for non-stationary demographics with an ageing of the Australian population that is projected to accelerate in next few decades would provide a further support for tightening the means test of the Australian age pension.

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Appendix: Technical Description of the Simulation Model

This Appendix provides a technical description of the simulation model. We start with the demographic structure and then proceed to the individual sectors of the model.

Demographics

We consider a model economy that is populated by sequences of generations aged between 21 and 90 years ($a = 21, ..., 90$) at any time t. Each generations consists of five income types i - the lowest, second, third, fourth and highest quintiles, with intra-generational shares given by ω_i. Every year, a new generation aged 21 years enters the model structure and faces random survival with the maximum possible lifespan of 70 years, while the oldest generation aged 90 dies. Lifespan uncertainty is described by the conditional survival probabilities, s_a. We assume stationary demographics with a constant population growth rate, n, which implies time-invariant cohort shares, $\mu_a = [s_a \diagup (1+n)]\, \mu_{a-1}$.

Households

Each i-type household who begins her economic life at time t is assumed to optimally choose consumption, c, and leisure, l, at each age and the timing of retirement to maximise the expected lifetime utility function given by

$$\max_{\left\{c^i_{t+a-21},\, l^i_{t+a-21}\right\}} \frac{1}{1-1/\gamma} \sum_{a=21}^{90} S_a \beta^{a-21} u(c^i_{t+a-21}, l^i_{t+a-21})^{1-1/\gamma},$$

subject to the per-period budget constraint written as

$$\begin{aligned} A^i_{a,t} \;\; = \;\; & (1+r) A^i_{a-1,t-1} + w_t e^i_a (1 - l^i_{a,t}) + AP^i_{a,t} + SA^i_{60,t} \\ & SP^i_{a,t} + ST^i_a + B^i_{a,t} - T(y^i_{a,t}) - (1+\tau^c)\, c^i_{a,t}, \end{aligned}$$

where the annual utility, $u(c,l) = \left[c^{(1-1/\rho)} + \alpha l^{(1-1/\rho)}\right]^{1/(1-1/\rho)}$, being discounted by the subjective discount factor, β, and the unconditional survival probability, $S_a = \prod_{j=21}^{a} s_{j-1}$. The remaining utility function parameters are the inter- and intra-temporal elasticities of substitution denoted by γ and ρ and the leisure distribution parameter, α.

In the per-period budget constraint, $A^i_{a,t}$ denotes the stock of ordinary private assets held at the end of age a and time t, which equals the assets at the beginning of the period, plus the sum of interest income, $rA^i_{a-1,t-1}$, labour earnings, $w_t e^i_a (1 - l^i_{a,t})$, age pension, $AP^i_{a,t}$, superannuation payouts, $SA^i_{60,t}$ and $SP^i_{a,t}$, social transfer payments, ST^i_a, and accidental bequest receipts, $B^i_{a,t}$, minus the sum of income taxes paid, $T(y^i_{a,t})$, and consumption expenditures, $(1+\tau^c_t)\, c^i_{a,t}$. Labour earnings are the product of labour supply, $1 - l^i_{a,t}$, and the hourly wage, $w_t e^i_a$, where w_t is the market wage rate and e^i_a is the age- and income-specific earnings ability variable. The labour supply is required to be non-negative, $1 - l^i_{a,t} \geq 0$. The income tax is a function of the taxable income, $y^i_{a,t}$, which comprises labour earnings, investment income and the age pension. We also assume that households are born with no wealth and exhaust all wealth at age 90 (i.e., $A^i_{20,t} = A^i_{90,t+70} = 0$) and that they are constrained from borrowing (i.e., $A^i_{a,t} \geq 0$).

The means tested age pension, AP^i_a, that is paid to households aged 65 years and over

can be expressed as

$$AP^i_{t,a} = \max\left\{\min\left\{p, p - \theta\left(\widehat{y}^i_{t,a} - IT\right)\right\}, 0\right\},$$

where the pension parameters include the maximum age pension, p, the income taper rates denoted by θ and the income thresholds given by IT. The assessable income, $\widehat{y}^i_{t,a}$, includes investment earnings and labour income in excess of the first \$6500.

The model also incorporates mandatory superannuation. Each producer is assumed to pay mandatory contributions for households aged 21 to 60 years at the after-tax contribution rate, $(1-\tau^s)\,cr$, from their labour income, $w_t e^i_a(1-l^i_{a,t})$, to the superannuation fund. The contributions are added to superannuation assets, $SA^i_{a,t}$, which earn investment income at the after-tax interest rate, $(1-\tau^r)\,r$. The stock of superannuation assets accumulates in the fund until age 60, when households receive lump-sum payouts, $SA^i_{60,t}$, and the superannuation accumulation ceases. The superannuation asset accumulation during $a \leq 60$ can be expressed as

$$SA^i_{a,t} = \left[1 + (1-\tau^r)\,r\right] SA^i_{a-1,t-1} + \left[(1-\tau^s)\,cr\right] w_t e^i_a (1 - l^i_{a,t}),$$

where τ^r is the earnings tax rate, τ^s denotes the contribution tax rate and cr is the mandatory contribution rate. We further assume that working households aged 60 years and over are paid mandatory contributions directly into their private assets account, denoted by $SP^i_{a,t}$ in the budget constraint.

Firms

The production sector assumes a large number of perfectly competitive firms that demand capital, K_t, labour, L_t, and investment, I_t, to maximise the present value of all future profits subject to the (per capita) capital accumulation equation:

$$\begin{aligned} \max_{\{K_t,\ L_t,\ I_t\}} \quad & \sum_{t=0}^{\infty} D_t \left[\left(1-\tau^f\right)\left(F(K_t, L_t) - C(I_t, K_t) - I_t - (1+cr) w_t L_t\right)\right] \\ \text{s.t.} \quad & (1+n) K_{t+1} = I_t + (1-\delta)\,K_t, \end{aligned}$$

where $D_t = (1+n)^t/(1+r)^t$ accounts for discounting and population growth and τ^f stands for the effective corporation tax rate. The adjustment cost function is taken from Fehr (2000) and given by $C(I_t, K_t) = 0.5\psi\left(I_t/K_t - (n+\delta)\right)^2 K_t$, where ψ is the adjustment cost coefficient and δ denotes the capital depreciation rate. The CES production function is $F(K_t, L_t) = \kappa\left[\varepsilon K_t^{(1-1/\sigma)} + (1-\varepsilon) L_t^{(1-1/\sigma)}\right]^{[1/(1-1/\sigma)]}$, with the productivity constant, κ, the capital intensity parameter, ε, and the elasticity of substitution in production, σ.

Solving the firm's maximisation problem yields the first-order necessary conditions and gives expressions for the equilibrium wage rate, w_t, interest rate, r, and the price of capital, q_t.

Government

The government is assumed to maintain a balanced budget, which can be expressed, in per capita terms, as

$$TR_t^Y + TR_t^C + TR_t^S + TR_t^F = G + ST + AP_t,$$

where the per capita expenditures are government consumption, G, and social transfer payments, ST, which both are assumed constant, and the expenditure on the age pension, AP_t, while TR_t^Y, TR_t^C, TR_t^S and TR_t^F are per capita tax receipts from the taxation of household income, consumption, superannuation and corporate profits, respectively. The consumption tax rate, τ_t^c, that is assumed to adjust endogenously to balance the government budget is given as

$$\tau_t^c = \frac{G + ST + AP_t - \left(TR_t^Y + TR_t^S\right)}{\sum_{i=1}^{5} \omega_i \sum_{a=21}^{90} \mu_a c_{a,t}^i}.$$

Small Open Economy and Market Equilibrium

The model is a small open economy model with the exogenous interest rate, r. When domestic savings fall short of the domestic capital, foreign capital will be employed, which adds to foreign debt. The accumulation of net foreign debt, FD_t, in per capita terms, is

$$(1+n)FD_{t+1} - FD_t = TB_t - rFD_t,$$

where TB_t is the trade balance and rFD_t is the interest payments on net foreign debt.

The endogenous variables in the model are determined such that all agents (i.e., households, firms and the government) make their choices optimally and that all markets clear in every time period. The equilibrium conditions for labour, capital and output markets may be expressed as

$$\begin{aligned} L_t &= \textstyle\sum_{i=1}^{5} \omega_i \sum_{a=21}^{90} e_{a,t}^i (h - l_{a,t}^i)\mu_a, \\ q_t K_t &= \textstyle\sum_{i=1}^{5} \omega_i \sum_{a=21}^{90} \left(A_{a,t}^i + SA_{a,t}^i\right)\mu_a - FD_t, \\ Y_t &= \textstyle\sum_{i=1}^{5} \omega_i \sum_{a=21}^{90} c_{a,t}^i \mu_a + I_t + G_t + TB_t. \end{aligned}$$