A Value-Based Longevity Index for Hedging Retirement Income Portfolios

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Outline

- **Research Motivation and Background**
- **Value-Based Longevity Index**
  - **Mortality Modelling Framework**
  - **Interest Rate Modelling Framework**
- **Liability Profile**
- **The Hedging Framework**
- **Basis Risk Metrics**
- **Sensitivity Analysis**

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Background and Context

- Retirement income providers are heavily exposed to longevity risk.
- The traditional approach to managing longevity risk has involved insurance or reinsurance-based solutions (Coughlan et al., 2011).
- Reinsurers have a limited appetite and capacity to absorb longevity risk (Wadsworth, 2005).
- Global longevity risk exposure is approaching the limit of the global reinsurance capacity (Cairns and El Boukfaoui, 2018).
- The development of a longevity risk transfer market offers a potential solution (Coughlan, 2009; Xu et al., 2019).
- Investors have the potential to earn a risk premium by diversifying into securities with near zero correlation with traditional asset classes (Anderson and Baxter, 2017).

The Case for Index-Based Longevity Hedging

- There are two broad categories of hedging longevity risk: customised (indemnity-based) hedges and standardised (index-based) hedges.
- To date, customised transactions have dominated the longevity market (Anderson and Baxter, 2017).
- Indemnity-based hedges have drawbacks (Coughlan, 2009):
  - Disclosure of pension fund/annuity book data,
  - Complex for capital markets to analyse transactions and manage risks,
  - Lack of transparency,
  - Discourages investment and market liquidity, and
  - High cost of hedging for retirement income providers.
- Standardised hedges overcome these shortcomings (Villegas et al., 2017).
- However, they are subject to basis risk (Coughlan et al., 2007).

Barriers to Index-Based Longevity Hedging

1. Availability of a **longevity index** that closely tracks the value of longevity-linked liabilities (Sweeting, 2010).
    - Retirement income providers are exposed to longevity risk, interest rate risk and inflation risk (Towers Watson, 2013)
    - Value-based longevity indices offer a potential solution (Sherris 2009; Chang and Sherris, 2018).
2. **Basis risk**. (Li et al., 2017)
    - Materiality of the residual risk exposure.
    - Robust basis risk quantification framework for the proposed longevity index that can be applied to individual retirement income portfolios.

- Research motivation: a framework to facilitate the transition towards index-based longevity hedging by addressing these two issues.

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Value-Based Longevity Index

- We consider a value-based index, $I_{x,t}$, which quantifies the expected present value of a unit of longevity and inflation indexed income paid annually in arrears to a cohort aged, x, at initial time, t.
- The value of the index is represented as

$$I_{x,t} = \sum_{i=1}^{\omega - x} S^R(x, t, t+i) \times P_R(t, t+i),$$

where

- ω is the maximum attainable age,
- $S^R(x, t, t+i)$ denotes the i year survival probability of the population underlying the index, forecast using mortality modelling frameworks, and
- $P_R(t, t+i)$ denotes the time t price of an inflation-indexed zero coupon bond making a single unit payment at time $t+i$, forecast using interest rate modelling frameworks.

Mortality Data Description

- Reference population data: US population level mortality data sourced from the Human Mortality Database (HMD).
- Book population data: US annuitant mortality data sourced from the United States Mortality Database. States in the highest income quintile (Small Area Income and Poverty Estimates Program) are used to approximate annuitant mortality.

Joint Affine Term Structure Model

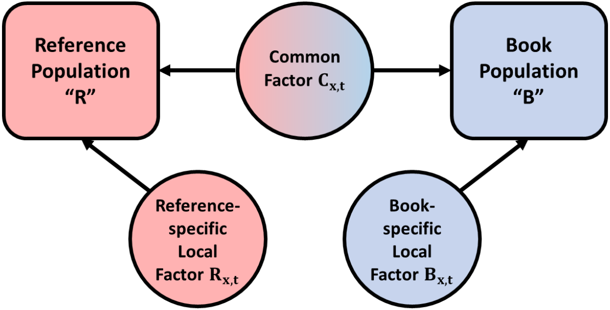


Figure 1: Structure of the joint affine term structure model for mortality.

Joint Affine Term Structure Model cont...

- Common factor captures all the dependence in mortality experience across the two populations arising from their mutual exposure to certain common influences (for example, a strong winter).
- The two local factors facilitate discrepancies in mortality dynamics over time between the two populations owing to differences in their demographic composition.
- The average mortality intensities $\bar{\mu}^R_{x,t}$ and $\bar{\mu}^B_{x,t}$ of the book and reference populations are modelled as

$$\begin{aligned}\bar{\mu}^R_{x,t} &= \delta_{R,0} + \delta_{R,1} C_{x,t} + \delta_{R,2} R_{x,t},\\ \bar{\mu}^B_{x,t} &= \delta_{B,0} + \delta_{B,1} C_{x,t} + \delta_{B,2} B_{x,t}.\end{aligned}$$

Joint Affine Term Structure Model cont...

- The factors are assumed to evolve independently, implying that the common factor does not depend on the local factors.
- This allows the joint ATSM to be decomposed into two single-population term structure mortality models.
- Due to the incompleteness of the longevity market, Xu et al. (2019) define a best-estimate measure $\bar{Q}$, fixed to observed mortality rates. Factor dynamics under $\bar{Q}$ can be represented as

$$\begin{bmatrix} dC_{x,t} \\ dR_{x,t} \\ dB_{x,t} \end{bmatrix} = - \begin{bmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & \phi_3 \end{bmatrix} \begin{bmatrix} C_{x,t} \\ R_{x,t} \\ B_{x,t} \end{bmatrix} dt + \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \begin{bmatrix} dW_t^{\bar{Q},C} \\ dW_t^{\bar{Q},R} \\ dW_t^{\bar{Q},B} \end{bmatrix},$$

where ϕ_1, ϕ_2, ϕ_3, σ_1, σ_2 and σ_3 are constant parameters with $W_t^{\bar{Q},C}$, $W_t^{\bar{Q},R}$ and $W_t^{\bar{Q},B}$ being Wiener processes under the best-estimate measure.

Joint Affine Term Structure Model cont...

- The survival probabilities for the reference and book populations are respectively given by

$$S^R(x,t,T) = e^{B_1(t,T)C_{x,t}+B_2(t,T)R_{x,t}+A^R(t,T)},$$
$$S^B(x,t,T) = e^{B_1(t,T)C_{x,t}+B_3(t,T)B_{x,t}+A^B(t,T)},$$

where

$$B_j(t,T) = -\frac{1-e^{-\phi_j(T-t)}}{\phi_j} \qquad \text{for} \quad j=1,2,3,$$

$$A^R(t,T) = \frac{1}{2}\sum_{j=1,2}\frac{\sigma_j^2}{\phi_j^3}[\frac{1}{2}(1-e^{-2\phi_j(T-t)})-2(1-e^{-\phi_j(T-t)})+\phi_j(T-t)],$$

$$A^B(t,T) = \frac{1}{2}\sum_{j=1,3}\frac{\sigma_j^2}{\phi_j^3}[\frac{1}{2}(1-e^{-2\phi_j(T-t)})-2(1-e^{-\phi_j(T-t)})+\phi_j(T-t)].$$

Joint Affine Term Structure Model cont...

- The average force of mortality of the reference population (R) and book population (B) for a given age x at initial time t are modelled as affine functions of latent time-varying factors.

$$\bar{\mu}^R_{x,t}(T) = \frac{1 - e^{-\phi_1(T-t)}}{\phi_1(T-t)} C_{x,t} + \frac{1 - e^{-\phi_2(T-t)}}{\phi_2(T-t)} R_{x,t} - \frac{A^R_t(t,T)}{(T-t)},$$

$$\bar{\mu}^B_{x,t}(T) = \frac{1 - e^{-\phi_1(T-t)}}{\phi_1(T-t)} C_{x,t} + \frac{1 - e^{-\phi_3(T-t)}}{\phi_3(T-t)} B_{x,t} - \frac{A^B_t(t,T)}{(T-t)}.$$

- The model can be written in state space form and can therefore be estimated using the Kalman filter.
- The state space form consists of
    1. A measurement equation, which specifies the relationship between the average mortality intensities $\bar{\mu}_{x,t}$ and the factors $R_{x,t}$, $B_{x,t}$ and $C_{x,t}$;
    2. A state transition equation which describes the time series dynamics of the latent time-varying factors.

Validation with M7-M5 Mortality Model

- For the discrete-time framework, the M7-M5 model advocated in Li et al. (2017) is adopted.
- Reference population mortality:

$$\text{logit}(q_{x,t}^R) = \kappa_{t,1}^R + (x - \bar{x})\kappa_{t,2}^R + ((x - \bar{x})^2 - \sigma_x^2)\kappa_{t,3}^R + \gamma_{t-x}^R,$$

- Difference between the book and reference mortality rates:

$$\text{logit}(q_{x,t}^B) - \text{logit}(q_{x,t}^R) = \kappa_{t,1}^B + (x - \bar{x})\kappa_{t,2}^B,$$

where

  - $\kappa_{t,1}^R$, $\kappa_{t,2}^R$ and $\kappa_{t,3}^R$ are factors corresponding to the reference mortality curve's level, slope and curvature respectively,
  - $\kappa_{t,1}^B$ and $\kappa_{t,2}^B$ explain the difference in logit mortality rates,
  - γ_{t-x}^R is the cohort effect for those born in year $t - x$ in the reference population, while $\bar{x}$ and σ_x^2 denote the sample age mean and variance.

Dynamic Nelson Siegel Model

- We use the Dynamic Nelson Siegel (DNS) interest rate model developed in Diebold and Li (2006).
- The yield function of the model is:

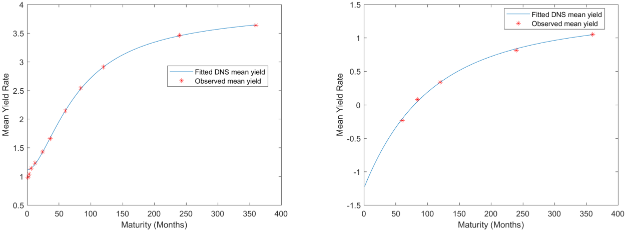
$$y_t(\tau) = L_t + S_t(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}) + C_t(\frac{1 - e^{-\lambda\tau}}{\lambda t} - e^{-\lambda\tau}),$$

where λ is the Nelsen Siegel parameter and

$$\begin{bmatrix} dL_t^N \\ dS_t^N \\ dC_t^N \end{bmatrix} = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & \lambda^N & -\lambda^N \\ 0 & 0 & \lambda^N \end{bmatrix} \begin{bmatrix} L_t^N \\ S_t^N \\ C_t^N \end{bmatrix} dt + \begin{bmatrix} \sigma_1^N & 0 & 0 \\ 0 & \sigma_2^N & 0 \\ 0 & 0 & \sigma_3^N \end{bmatrix} \begin{bmatrix} dW_t^{Q,L^N} \\ dW_t^{Q,S^N} \\ dW_t^{Q,C^N} \end{bmatrix},$$

Dynamic Nelson Siegel Model cont...

- The nominal (N) interest rate model is calibrated using US Treasury security yields with maturities ranging from 1 month to 30 years.
- The real (R) interest rate model is calibrated using US Treasury Inflation Protected Security (TIPS) yields with maturities of ranging from 5 years to 30 years.



(a) Nominal (b) Real

Figure 2: Nominal & Real US bond yields from Oct 2006 to May 2018

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Liability Profile

- We consider a closed annuity pool comprising of individuals from a single cohort initially aged x in year t who are promised \$1 of inflation-indexed income per year upon survival from ages $x+1$ to the maximum attainable age, ω.
- The present value of the retirement income portfolio liability is

$$PV(\text{Unhedged Portfolio}) = \sum_{i=1}^{\omega-x} l^B_{x+i,t+i} \times P_R(t, t+i),$$

where $l^B_{x+i,t+i}$ is the number of surviving annuitants (aged $x+i$ at time $t+i$) and this is dependent on the simulated book population mortality dynamics generated by the mortality model.
- Binomial sampling of deaths used to reflect the sampling variability in a finite book size: $D^B_{x,t} \sim \text{Bin}(E^B_{x,t}, q^B_{x,t})$ where $q^B_{x,t}$ is simulated for each path.

Liability Profile cont...

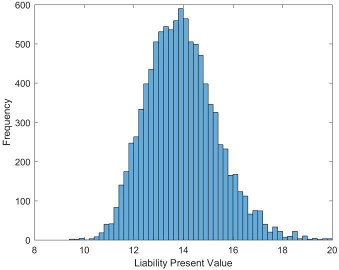

Figure 3: Liability present value histogram for the book population cohort initially aged 65 (joint ATSM, 10,000 simulations, 100,000 lives).

- A degree of positive skewness is apparent, with the simulated distribution exhibiting a heavier right tail.
- This highlights the importance of effectively hedging against more extreme outcomes in pension liabilities resulting from unexpected mortality or financial market experience.

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Index Swap Instrument

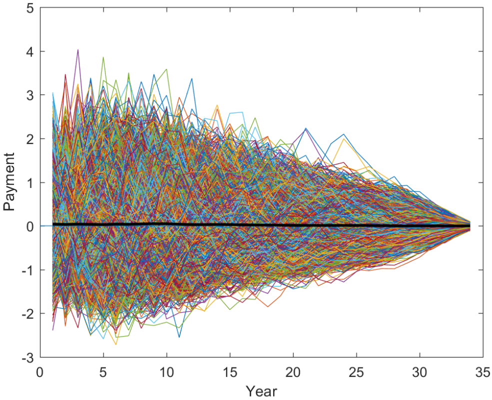
- An annually-settled index swap trades in the longevity risk transfer market where at time $t + i$, the fixed leg pays the i year forward index value $I^f_{x+i,t+i}$ while the floating leg pays the realised index value $I_{x+i,t+i}$.
- The random present value of the swap for the payer of the fixed leg (e.g., a pension fund looking to hedge) is:

$$PV(\text{Index Swap}) = \sum_{i=1}^{\omega-x-1} (I_{x+i,t+i} - I^f_{x+i,t+i}) \times P_N(t, t+i),$$

where the forward values $I^f_{x+i,t+i}$ are computed from central forecasts, while the realised index values $I_{x+i,t+i}$ are simulated.

Simulated Swap Payments

Figure 4: Simulated swap payments for the reference population cohort initially aged 65 (joint ATSM, 10,000 simulations)



Hedge Construction

- The random present value of the annuity provider's aggregate portfolio can therefore be expressed as:

$$PV(\text{Hedged Portfolio}) = PV(\text{Unhedged Portfolio}) + PV(\text{Swap}),$$

$$= \sum_{i=1}^{\omega-x} l^{B}_{x+i,t+i} P_R(t, t+i) + w_0 \sum_{i=1}^{\omega-x} (l_{x+i,t+i} - l^{f}_{x+i,t+i}) P_N(t, t+i),$$

where w_0 refers to the notional amount of the longevity swap which is estimated using numerical optimisation with an objective to minimise the variance of the hedged portfolio's present value as in Li et al. (2017).

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Assessing Risk Reduction

- The survival index value is represented as

$$I^0_{x,t} = \sum_{i=1}^{\omega-x} S^R(x,t,t+i).$$

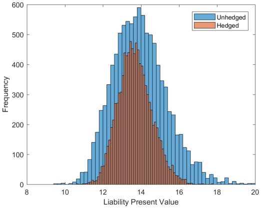
- Nominal-linked index value is represented as

$$I^1_{x,t} = \sum_{i=1}^{\omega-x} S^R(x,t,t+i) \times P_N(t,t+i).$$

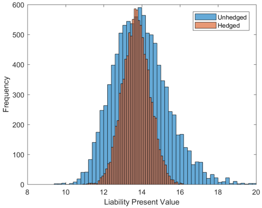
- Risk reduction achieved by hedging the retirement income portfolio using $I^0_{x,t}$ represents the impact of longevity risk.
- Additional risk reduction achieved by hedging using $I^1_{x,t}$ represents the impact of interest rate risk.
- Additional risk reduction achieved by hedging the retirement income portfolio using $I_{x,t}$ a represents the impact of inflation risk.

Liability present value distributions by hedging index

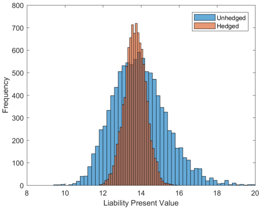
(a) Survival index



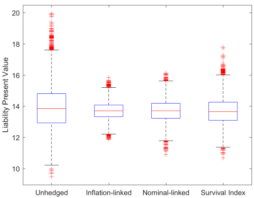
(b) Nominal-linked index



(c) Inflation-linked index



(d) Box and whisker plot



Longevity Risk Reduction

- We define our Longevity Risk Reduction metric as

$$(1 - \frac{\rho(\text{Hedged Portfolio})}{\rho(\text{Unhedged Portfolio})}) \times 100\%,$$

where the risk measures ρ is set to the portfolio variance as in Cairns et al., (2014).

Table 1: Longevity risk reduction: percentage reduction in variance showing the greater effectiveness of the inflation-linked value-based longevity index relative to alternate indices (joint ATSM)

Hedging Index	Book Size		
	1,000	10,000	100,000
Survival index $I^0_{x,t}$	31.52	54.07	58.71
Nominal-linked value index $I^1_{x,t}$	37.82	67.24	74.07
Inflation-linked value index $I_{x,t}$	42.67	77.43	84.58

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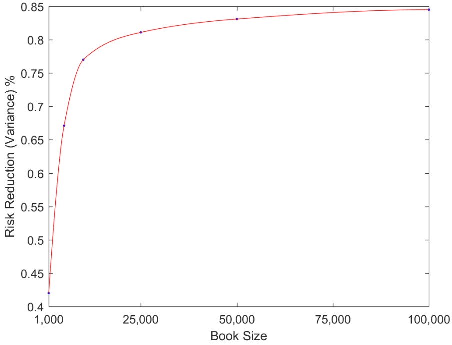
Book Size

Figure 6: Hedge efficiency by book size indicating the diminishing marginal benefit of increasing book size (joint ATSM)



Mortality Model Comparison

Table 2: Inflation-linked value-based longevity index: model hedge effective comparison indicating similar overall outcomes across the two mortality modelling frameworks (percentage reduction in variance, 100,000 lives)

	Joint ATSM	M7-M5 model
w_0 calibrated by same model	84.58%	85.51%
w_0 calibrated by alternate model	84.27%	85.11%

Limitations and Scope for Future Research

- Book population data: older ages and real annuitant mortality
- Application to realistic retirement income portfolios consisting of open-ended pension funds with multiple cohorts
- Dynamic hedging

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Questions and Comments?

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