Long Term Care and Morbidity Models

Michael Sherris

School of Risk & Actuarial Studies, UNSW Business School, UNSW Sydney
ARC Centre of Excellence in Population Ageing Research (CEPAR)

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Coverage

- Overview of models for Long Term Care and Morbidity
- Highlight key research topics and potential future research
- Overview past research from CEPAR actuarial longevity risk research group
- Key topics
  - Health Status: Frailty models (fixed)
  - Health Status: Multiple State Models (Markov ageing model)
  - Health Status: Risk Factors
  - Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty
  - Models of Functional Disability (Markov models)
  - Models of Functional Disability (Markov model) with Systematic trend and uncertainty (frailty factor)

Overview - Models for Long Term Care and Morbidity

Models to classify individual health, or functional disability, status and model how this develops through time.

Actuarial applications include pricing and reserving for insurance contracts that depend on health or functional disability such as life annuities, long term care insurance, and new product innovations.

Although this status is a continuous process, this may be unobserved or we only have discrete time observations available, which may be cross sectional or sometimes longitudinal.

For more detailed references see:

Presentation by Prof Ermanno Pitacco titled Health Status at the CEPAR Longevity Research Forum UNSW, Kensington Campus, February 16, 2017

Presentation by Prof Annamaria Olivieri titled Frailty and Risk Classification for Life Annuities at the CEPAR Longevity Workshop hosted by PWC, PWC Sydney Office, February 15, 2017.

Books: Haberman and Pitacco (1999), Pitacco (2014).

Health Status Unobservable: Fixed Frailty

Fixed frailty approach (see Pitacco (2017), Haberman and Olivieri (2014) and references).

What follows draws on Su and Sherris (2012).

Individual has a fixed frailty at birth

- Unobserved mortality risk factor fixed at birth, mathematically defined in terms of force of mortality: $\mu(x, z) = z \cdot \mu(x, 1)$
- Examples: standard force of mortality and frailty distribution
  - Standard force of mortality $\mu(x, 1) = \alpha \cdot e^{\beta x}$
  - Frailty distribution
    - Gamma $f_Z(z) = \frac{\lambda^k}{\Gamma(k)} \cdot z^{k-1} \cdot e^{-\lambda \cdot z}$
    - Inverse Gaussian $f_Z(z) = (\frac{\delta}{\pi})^{\frac{1}{2}} \cdot e^{\sqrt{4\delta\theta}} \cdot z^{-\frac{3}{2}} \cdot e^{-\theta z - \frac{\delta}{z}}$

Health Status Unobservable: Fixed Frailty

- Distribution of frailty at age x
  - Gamma distribution

$$f_{Z|X}(z|X=x) = \frac{(\lambda(x))^k}{\Gamma(k)} \cdot z^{k-1} \cdot e^{-\lambda(x)\cdot z}$$

with

$$\lambda(x) = \lambda + H(x,1), \text{ and } E[z] = \frac{k}{\lambda(x)}, \quad Var[z] = \frac{k}{(\lambda(x))^2}$$

  - Inverse Gaussian distribution

$$f_{Z|X}(z|X=x) = (\frac{\delta}{\pi})^{\frac{1}{2}} \cdot e^{\sqrt{4\delta\theta(x)}} \cdot z^{-\frac{3}{2}} \cdot e^{-\theta(x)\cdot z - \frac{\delta}{z}}$$

with

$$\theta(x) = \theta + H(x,1), \text{ and } E[z] = (\frac{\delta}{\theta(x)})^{\frac{1}{2}}, \quad Var[z] = \frac{1}{2}\sqrt{\frac{\delta}{(\theta(x))^3}}$$

Health Status Unobservable: Fixed Frailty

- Estimation using mean frailty approach
  - Assumes the average force of mortality is the cohort force of mortality

$$\bar{\mu}_x = \mu(x,1) \cdot \bar{z}_x$$

  - Number of deaths follows $Poisson(\bar{\mu}_x E_x)$
- Normal approximation for sample mean mortality rates
  - The observed cohort is a sample of size E_x of the population
  - Sample mean force of mortality is approximately normally distributed with
    - Under Gamma distributed frailty

$$E[\hat{\mu}_x] = \mu(x,1) \cdot \frac{k}{\lambda + H(x,1)}, \quad Var[\hat{\mu}_x] = \frac{(\mu(x,1))^2 \cdot k}{E_x \cdot (k + H(x,1))^2}$$

    - Under Inverse Gaussian distributed frailty

$$E[\hat{\mu}_x] = \mu(x,1) \cdot (\frac{\delta}{\theta + H(x,1)})^{\frac{1}{2}}, \quad Var[\hat{\mu}_x] = \frac{(\mu(x,1))^2}{2 \cdot E_x} \sqrt{\frac{\delta}{(\theta + H(x,1))^3}}$$

Health Status Unobservable: Fixed Frailty

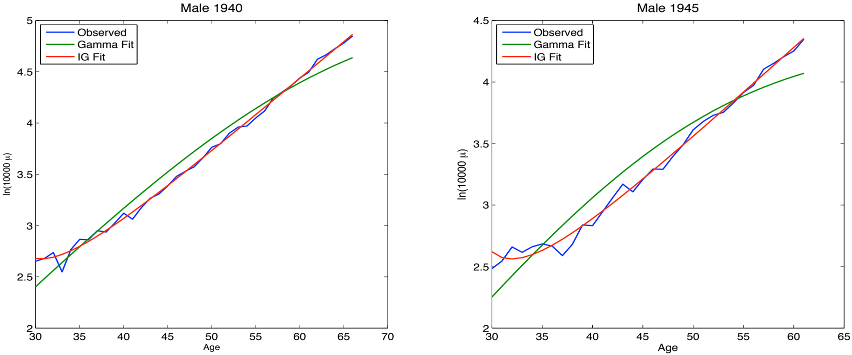


Figure 1: Observed versus Fitted Cohort Average Force of Mortality: Australian Males, Cohorts born 1940 and 1945

Health Status Unobservable: Fixed Frailty

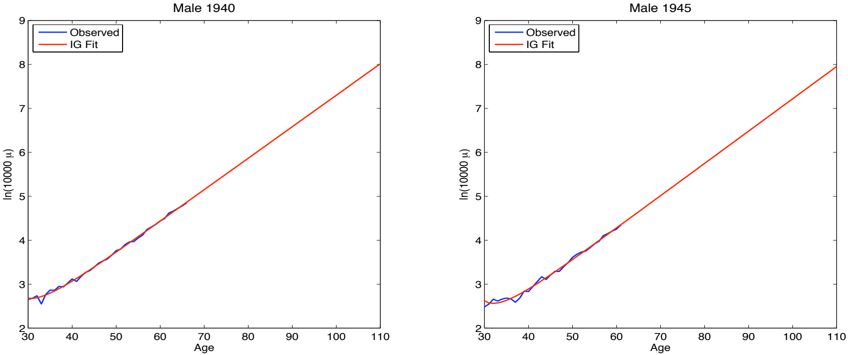


Figure 2: Projected Cohort Average Force of Mortality with IG Frailty: Australian Males, Cohorts born 1940 and 1945

Health Status Unobservable: Fixed Frailty

Some comments:

- Difficult to capture full age range (poorer fit at young ages, accident hump).
- Distribution of frailty reduces with age as more frail lives die (less heterogeneity at older ages).
- Gompertz-Gamma model captures reducing rate of increase in older age mortality (curvature at older ages).
- Need to classify individuals according to frailty (see Olivieri (2017), Olivieri and Pitacco (2016)).
- Ignores the impact of environmental and behavioural factors that impact mortality (only recognises genetic factors at birth).

Health Status Unobservable: Markov Ageing Modell

Initially proposed by LeBras (1976). We follow Lin and Liu (2007).

- Ageing process modeled in terms of changes in physiological functions
- Physiological age: represents the degree of ageing
  - Model based on "physiological age" - an indication of an organism's physiological healthiness
  - For any given age there is a range of physiological ages (representing heterogeneity)
  - Higher mortality rates for higher physiological ages
  - $n = 200$ transient states and 1 absorbing state (death)
  - System starts in state 1, transitions to higher states only.
  - For state i, transition occurs with rate λ_i to the next state, or with rate q_i to the absorbing state.

Health Status Unobservable: Markov Ageing Model

- Transition matrix:

$$\Lambda = \begin{pmatrix} -(\lambda_1 + q_1) & \lambda_1 & 0 & \cdots & 0 \\ 0 & -(\lambda_2 + q_2) & \lambda_2 & \cdots & 0 \\ 0 & 0 & -(\lambda_3 + q_3) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -q_n \end{pmatrix}$$

- λ_i is constant after the fourth state
- $q_i = i^p \cdot q + h_i$, where p and q are constants, h_i has two values: higher during the accident hump ages.
- Time to death follows phase-type distribution with $\hat{S}(t) = \alpha exp(\Lambda t) e$
- $\hat{q}_x = \frac{\hat{S}_x - \hat{S}_{x+1}}{\hat{S}_x}$
- Weighted least squares estimation: $\sum_x (q_x - \hat{q}_x)^2 \cdot w_x$

Health Status Unobservable: Markov Ageing Model

Su and Sherris (2012) modification of Lin and Liu (2007)

- Mode of the distribution of individuals on the state space roughly corresponds to the age of these individuals.
- $n = 100$ transient states
- Transition rates: $\lambda_i = \lambda$ and $q_i = \gamma + \alpha e^{\beta i}$
- 4 developmental periods
- $\lambda_i = \lambda$ for $i = 5, 6, ..., n-1$
- Death rates for $i = 5, 6, ...n$

$$q_i = \begin{cases} \gamma + \gamma_1 + \alpha e^{\beta i} & : \text{ for } i_1 < i < i_2 \\ \gamma + \alpha e^{\beta i} & : \text{ otherwise} \end{cases}$$

Health Status Unobservable: Markov Ageing Model

Transition matrix

$$\boldsymbol{\Lambda} = \begin{pmatrix} -(\lambda_1 + q_1) & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & -(\lambda_k + q_k) & \lambda_k & \cdots & 0 \\ 0 & \cdots & 0 & -(\lambda + \gamma + \alpha e^{\beta(k+1)}) & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & -(\alpha + e^{\beta n}) \end{pmatrix}$$

Fitting and Projection for Markov Ageing Model

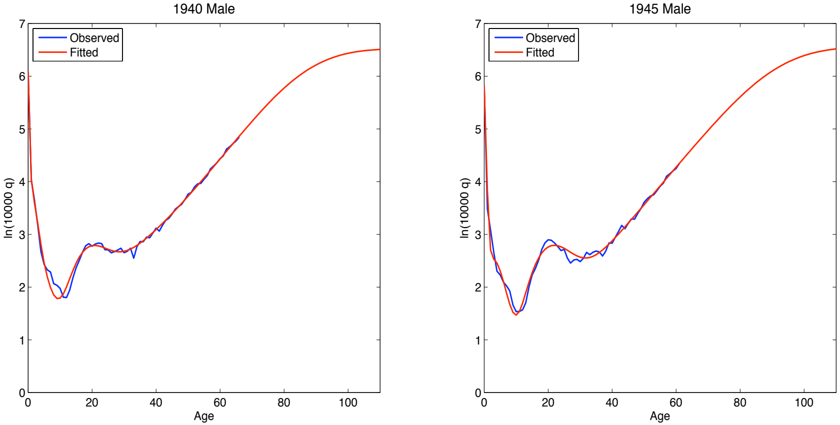


Figure 3: Observed versus Fitted Death Probability with Projection at Higher Ages: Australian Males, Cohorts born 1940 and 1945

Health Status Unobservable: Markov Ageing Model

Distribution of Physiological Age

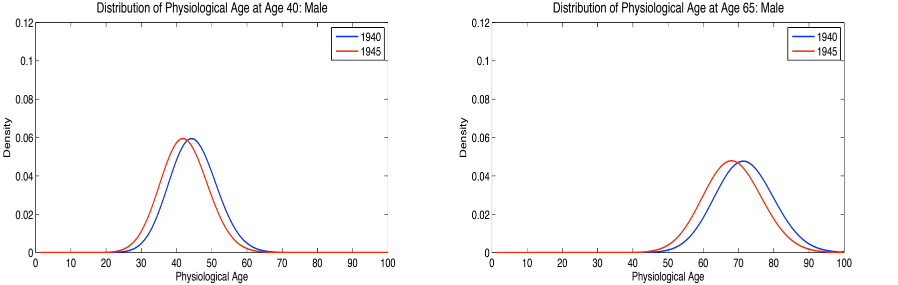


Figure 4: Distribution of Physiological Age: Australian Males, Cohorts born 1940 and 1945

Health Status Unobservable: Markov Ageing Model

Some comments:

- Model can be calibrated to fit full age range (including accident hump at young ages)
- Produces curvature in mortality curve at the older ages
- Heterogeneity increases with age
- Does not incorporate mortality improvement or uncertainty
- Numerical calibration methods

Health Status: Risk Factors

Mortality risk factors are longitudinal with both cross-sectional and temporal or time series data. Variable of interest is survival and risk factors are covariates. We draw on Alai and Sherris (2012).

- Survival Analysis
  i) Measure the response as a numerical survival time.
  ii) Fit the data using survival models, specifically, proportional hazard models.
  iii) Obtain hazard ratios that measure the impact of each covariate.
- Panel Analysis
  i) Measure the response as a successive binary survival condition through time.
  ii) Fit the data using marginal models, specifically, with logistic regression.
  iii) Obtain odds ratios that measure the impact of each covariate.

Health Status: Risk Factors

- Survival Analysis
  - Each individual contributes one observation. No dependence issue.
  - Time-changing covariates can be incorporated, but not as easily and freely.
  - External covariates can not be incorporated in the model.
  - Censored data can be incorporated.
- Panel Analysis
  - Each individual contributes at least one observation. Multiple observations from one individual should be dependent.
  - Time-changing covariates are easily incorporated in the modelling framework.
  - External covariates can be incorporated in the model.
  - Utilize a missing at random mechanism due to the nature of the data.

Health Status: Risk Factors

Subject ID	Wave	Time	Response	Explanatory Variables
1	w_1	t_1	$Y_{1,1}$	$X_1(t_1)$
1	w_2	t_2	$Y_{1,2}$	$X_1(t_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1	w_M	t_M	$Y_{1,M}$	$X_1(t_M)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	w_j	t_j	$Y_{i,j}$	$X_i(t_j)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N	w_M	t_M	$Y_{N,M}$	$X_N(t_M)$

Figure 5: Panel data.

Subject ID	Survival Time	Death Indicator	Explanatory Variables
1	T_1	d_1	$X_1^H(T_1)$
2	T_2	d_2	$X_2^H(T_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	T_i	d_i	$X_i^H(T_i)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
N	T_N	d_N	$X_N^H(T_N)$

Figure 6: Survival data.

Health and Retirement Study

- 1992-2008 U.S.
- Wave every two years.
- Individuals born 1931-1941.
- Approximately $10,000$ subjects in the study.

Health Status: Risk Factors

The Odds and Hazard Ratios

Marginal Model with Logistic Regression

- For each covariate X_k, obtain an odds ratio o_k.
- The interpretation is a proportional effect on the instantaneous odds $(\pi/(1-\pi))$ of death per unit increase of the covariate.

Proportional Hazard Model

- For each covariate X_k, obtain a hazard ratio h_k.
- The interpretation is a proportional effect on instantaneous probability π of death per unit increase of the covariate.

Health Status: Risk Factors

N	9,761	Mean (St.Dev.)
	Age	55.54 (3.19)
Gender	Male	47.04%
	Female	52.96%
Education	Less than high school	26.72%
	GED, HS or some college	56.68%
	College and above	16.60%
Self-Report Health	Excellent	21.72%
	Very good	27.89%
	Good	27.76%
	Fair	14.41%
	Poor	8.22%
BMI	Underweight	1.36%
	Normal weight	33.70%
	Overweight	41.00%
	Obese	16.88%
	Morbidly obese	7.06%
	Drinks ever	60.34%
	Smokes ever	63.54%
	Smokes now	27.44%
Wealth and Income	Net value of primary residence	61,505 (95,404)
	Total non-housing assets	147,649 (410,983)
	Total houshold income	46,434 (50,784)

Table 1: Descriptive statistics of the HRS dataset used.

Health Status: Risk Factors

		Current Value Covariates			
		-2 LOG L		34,746	
		AIC		34,816	
		SBC		35,013	
		β_k	h_k	St.Dev β_k	
	Age	0.0571	1.0587	0.0072	***
	Male	0.4894	1.6314	0.0504	***
Education Ref: GED,	¡High School	-0.1536	0.8576	0.0510	***
HS or Some Coll.	College +	0.0068	1.0068	0.0767	
Self-Report	Excellent	-0.6658	0.5139	0.1231	***
Health	Very Good	-0.3543	0.7017	0.0801	***
Ref: Good	Fair	0.5389	1.7141	0.0645	***
	Poor	1.1046	3.0181	0.0714	***
BMI	Underweight	0.8746	2.3979	0.1038	***
Ref: Normal	Overweight	-0.3434	0.7094	0.0544	***
Weight	Obese	-0.5474	0.5784	0.0697	***
	Morb. Obese	-0.4199	0.6571	0.0820	***
	Drinks Ever	-0.1214	0.8857	0.0491	**
	Smokes Ever	0.3693	1.4467	0.0592	***
	Smokes Now	0.2728	1.3136	0.0538	***
Health History	High BP	0.2425	1.2744	0.0499	***
	Diabetes	0.5452	1.7250	0.0515	***
	Cancer	0.7984	2.2219	0.0545	***
	Lung Disease	0.3922	1.4803	0.0574	***
	Heart Prob.	0.2906	1.3372	0.0504	***
	Stroke	0.3841	1.4684	0.0639	***
	Psych. Prob.	0.1200	1.1275	0.0588	**
	Arthritis	-0.1556	0.8559	0.0483	***

Table 2: Proportional hazards model.

Health Status: Risk Factors

			Categorical Wave			External Socio-Economic		
			AIC	15,783		AIC	15,957	
			SBC	16,190		SBC	16,336	
			β_k	h_k		β_k	h_k	
		Intercept	-7.7105		***	14.2893		***
External	Wave 2	GDP	0.0698	1.0723		0.1243	1.1324	**
	Wave 3	Health Exp.	0.0343	1.0349		-1.0000	0.3679	***
	Wave 4	Unemploy.	0.0676	1.0699		-0.1277	0.8801	***
	Wave 5	Smoking Pre.	0.1934	1.2134	*	-0.7024	0.4954	***
	Wave 6	Inflation	-0.1556	0.8559		-0.1228	0.8844	***
	Wave 7		-0.0572	0.9444				
	Wave 8		-0.2232	0.8000				
	Wave 9		-3.4732	0.0310	***			
		Age	0.0566	1.0582	***	0.0540	1.0555	***
		Male	0.4985	1.6463	***	0.4923	1.6361	***
Education Ref:		¡High School	-0.1687	0.8448	***	-0.1684	0.8450	***
HS or Some Coll.		College +	0.0040	1.0040		0.0044	1.0044	
Self-Report		Excellent	-0.6268	0.5343	***	-0.6164	0.5399	***
Health		Very Good	-0.4017	0.6692	***	-0.3953	0.6735	***
Ref: Good		Fair	0.5312	1.7010	***	0.5334	1.7047	***
		Poor	1.1339	3.1078	***	1.1207	3.0670	***
BMI		Underweight	0.9160	2.4993	***	0.9158	2.4988	***
Ref: Normal		Overweight	-0.3559	0.7005	***	-0.3496	0.7050	***
Weight		Obese	-0.6123	0.5421	***	-0.6036	0.5468	***
		Morb. Obese	-0.4290	0.6512	***	-0.4168	0.6592	***
		Drinks Ever	-0.1447	0.8653	***	-0.1338	0.8748	***
		Smokes Ever	0.3818	1.4649	***	0.3773	1.4583	***
		Smokes Now	0.2444	1.2769	***	0.2490	1.2827	***

Table 3: Marginal models with external covariates.

Health Status: Risk Factors

Some comments:

- Incorporation of a frailty factor in a GLMM (Generalized Linear Mixed Model) with proportional hazards along with covariates using HRS data see Meyricke and Sherris (2013)
- Limitations arise from lack of data availability for long periods of time and for larger populations and privacy issues with individual level government administrative data (Medicare etc)
- Time trends are readily incorporated along with interaction of time and covariates to assess the risk factors driving mortality improvement as in Xu et al. (2018) - see next slide
- How to incorporate risk factors into aggregate mortality models with systematic uncertainty and to calibrate with individual data?
- Explanatory and not predictive - role for techniques from data analytics and predictive models.

Health Status: Risk Factors - with time trends

Variable	Estimate	SE
Intercept	-9.3063***	1.2623
Age	0.0645***	0.0207
Gender		
Male	0.5834***	0.1820
Marital Status (Ref: Married)		
Widowed	0.6629**	0.2676
Self-Report of Health (Ref: Good)		
Fair	0.8648***	0.2415
Poor	1.3258***	0.2716
Body Mass Index (Ref: Normal weight)		
Underweight	0.9870***	0.3789
Obese	-0.4391*	0.2527
Drink/Smoke Status		
Smoked ever	0.4929**	0.2188
Smokes now	0.3144*	0.1871
Health History		
High blood pressure	0.3956**	0.1743
Diabetes	0.8466***	0.1874
Cancer	1.6632***	0.2097
Lung disease	0.5735***	0.2163
Heart problems	0.8010***	0.1785
Cancer $\times$ Time	-0.0883*	0.0478
Heart problems $\times$ Time	-0.0740*	0.0419
Stroke $\times$ Time	0.1350**	0.0647
Stroke $\times$ Time^2	-0.0085***	0.0032
Cognition		
Cognition score	-0.0125**	0.0058

Note: *** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$.

Table 4: Marginal model with time trends - selected variables Xu et al. (2018)

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

Draws on Liu and Lin (2012) and Sherris and Zhou (2014)

- Underlying multi-state model, made stochastic through time-change
- Gamma time-change:
  - Survival probability at time t = survival probability given by underlying model at time γ_t, $S(\gamma_t)$, $\gamma()$ is a Gamma process
- Sherris and Zhou (2014) calibrate to both cross-sectional aggregate health status data and survival probabilities.

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

- Transition rates parameterised by: time (s), cohort (c), and a Gamma time-change
- 5 transient states (health states) with transitions: $\lambda_i(s) = n_i + k \cdot \exp(m \cdot s)$
- In state i, transition to absorbing state at time s: $q_{c,i}(s) = d_i r^c \cdot \exp(b \cdot s)$
  - d_i: proportional relationship between states, $d_1 \leq d_2 \leq d_3 \leq d_4 \leq d_5$
  - r^c: cohort trend: r is a positive constant less than 1, c is cohort number ($c = 1$ for 1935, ... $c = 39$ for 1973)
  - e^{bs}: exponential increase with time s
- Gamma time-change: survival probability at time t = survival probability given by underlying model at time γ_t, which is Gamma distributed with mean t and variance νt.

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

- For each cohort c, the survival probability in t year's time is:

$$S_c(t) = \pi_{c,0} \exp\left(\sum_{s=0}^{t-1} \Lambda_c(s)\right) \mathbf{1}$$

- Density function of life time distribution of cohort c, at time t:

$$f_c(t) = \pi_{c,0} \exp\left(\sum_{s=0}^{t-1} \Lambda_c(s)\right) \cdot (-\Lambda_c(t-1) \cdot \mathbf{1})$$

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

- The expected value of $S_c(t)$ is:

$$\mathrm{E}\left(S_c(t)\right) = \pi_{c,0} \exp\left(\sum_{s=0}^{t-1} \tilde{\Lambda}_c(s)\right) \mathbf{1}$$

where

$$\tilde{\Lambda}_c(s) = \sum_{i=1}^{5} \left[\frac{1}{\nu} \ln\left(1 + \nu\left(-q_{c,i}(s) - \lambda_i(s)\right)\right)\right] \mathbf{h_i v_i}$$

$\mathbf{h_i}$ and $\mathbf{v_i}$ are the right and left eigenvectors of the transition matrix Λ_c and such that $v_i h_i = 1$

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

Sherris and Zhou (2014) calibrate the model to survival and health data for Australian cohorts 1934-1973 (male and female combined) using data sources:

- cohort death rates: Human Mortality Database
- prevalence of health conditions: National Health Survey 2007-2008, Ritchie (1992) estimated average dementia prevalence, Australian Cancer Incidence and Mortality Books 2008
- death by cause: WHO mortality database, for Australia 2006 (number of deaths by age), Australian Bureau of Statistics Causes of Death 2008 (aggregate of all ages)
- population size (number of persons aged x in a specific year): Human Mortality Database

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

- The health conditions were ranked and divided into five groups according to their probability of causing death for 65-74 year-old individuals within 1 year.
  - Mortality by condition: number of deaths caused divided by prevalence of the condition.
  - Deaths by cause data (of 2006) was scaled by the ratio of total number of deaths to match the prevalence data (of 2008).
- Long term conditions are assumed to be independent and for a person affected by more than one condition, the highest death rate among all of the conditions was used for the death rate.

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

- State 1 includes - hypotension, mood disorder.
- State 2 includes - asthma, hypertensive disorders, endocrine disorders.
- State 3 includes - dementia, diabetes, substance abuse.
- State 4 includes - breast cancer, hodgkins disease, prostate cancer.
- State 5 includes - bladder cancer, kidney cancer, liver cancer, lung cancer.
    - Prevalence varies by age group
    - Mortality rates similar across ages by health condition

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

Model calibration

- Parameters were estimated by minimizing the difference between observed and model values of
  1. expected cohort survival probabilities,
  2. health state-distribution in 2008 and
  3. probability of death within a year for each state in 2008.
- Expected health state distribution and expected mortality rates ${}_1q_x$ for persons aged 35 to 75 in 2008 calibrated to the average health state distribution and average ${}_1q_x$ in the data for the expected values for persons aged: 29.5, 39.5, 49.5, 59.5
- Values for individual ages are estimated using interpolation. Calibration was for both the parameters in the transition matrix and the initial state distribution of each cohort at age 30.

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

Increased proportion in better health states for later cohorts

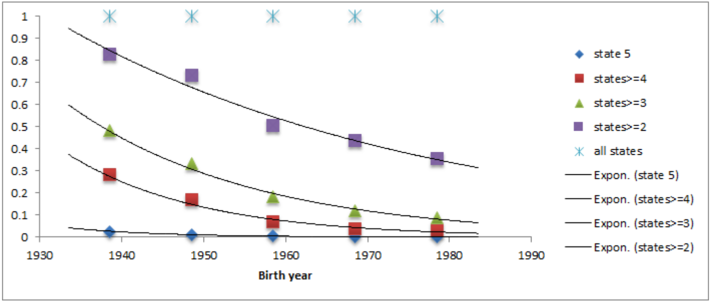


Figure 7: Health state distribution: y values are percentage of people in a specified health state or worse. Smoothed using an exponential trend line of the form $y = a \cdot \exp(bx)$

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

Improvement in mortality rates for less healthy states for later cohorts

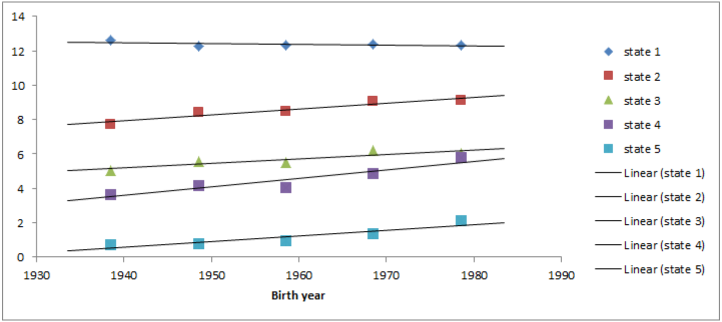


Figure 8: Mortality rates ${}_1q_x$ by cohort: y values are: $-\ln({}_1q_x)$. Interpolated using linear trend line of the form $y = ax + b$

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

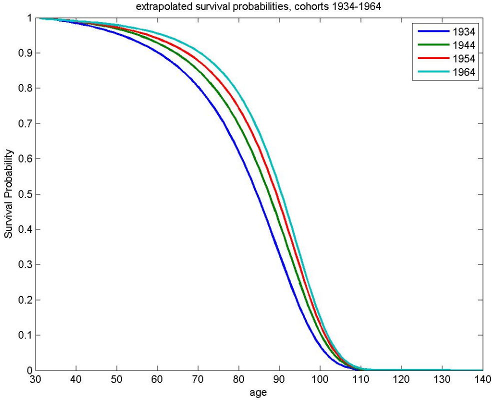


Figure 9: Survival probabilities - showing improvment trend

Health Status: Multiple State Models (Markov ageing model) with Systematic trend and uncertainty

Some comments:

- Require cross sectional data for health status and cohort survival curves
- Number of parameters and calibration to data
- Need to reduce number of states by grouping similar health states
- More efficent numerical methods for calibrating parameters
- Incorporation of systematic population level stochastic mortality models

Models of Functional Disability (Markov models)

Multi-state models in insurance have a long history Janssen (1966), Hoem (1969), Hoem (1988), Haberman and Pitacco (1999), Pitacco (2014)

Multi-state models focussing in long term care applications

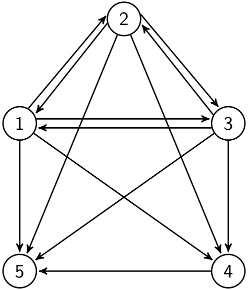
Markov models include Leung (2004) Pritchard (2006) Brown and Warshawsky (2013) Fong et al. (2015)

Semi-Markov model for LTC Biessy (2015)

Models with improvement trend Planchet and Tomas (2016), Li et al. (2017)

Models of Functional Disability (Markov models)

- 1 - Healthy (difficulty in no ADLs)
- 2 - Mildly disabled (difficulty in 1 ADL) and staying at home
- 3 - Severely disabled (difficulty in 2+ ADLs) and staying at home
- 4 - Institutionalized
- 5 - Dead



Models of Functional Disability (Markov model) with Systematic trend and uncertainty (frailty factor)

Following Li et al. (2017)

- One level of disability

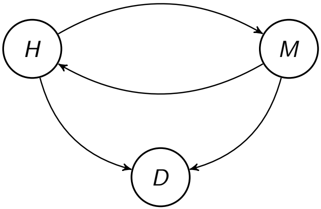


Figure 10: Simple three-state LTC transition model allowing for recovery; H = Health, M = Disability and D = Death.

- The latent systematic factor evolves roughly every two years

Models of Functional Disability (Markov model) with Systematic trend and uncertainty (frailty factor)

No Frailty Model

$$\ln\{\lambda_{skx}\} = \beta_s + \gamma_s^{age} \cdot x + \gamma_s^{female} \cdot F,$$

No Frailty Model with Time Trend

$$\ln\{\lambda_{skx}(t)\} = \beta_s + \gamma_s^{age} \cdot x_t + \gamma_s^{female} \cdot F + \phi_s t,$$

Frailty Model

$$\ln\{\lambda_{sk}(t)\} = \beta_s + \gamma_s^{age} \cdot x_t + \gamma_s^{female} \cdot F + \phi_s t + \alpha_s \psi(t) + \ln\{H_{sk}(t)\},$$

$$\psi(t) = \psi(t-1) + \epsilon_t, \qquad \epsilon_t \sim NIID(0, \sigma_t = 1).$$

In case of Markov assumption, $H_{sk}(t) = 1$
Health transition rates/probabilities estimated using GLM Data: Health and Retirement Study (HRS) - Individual survey data for waves 1998 to 2010

Models of Functional Disability (Markov model) with Systematic trend and uncertainty (frailty factor)

Transition Type $s =$ $N = 1000$	H - M 1	M - H 2	H - D 3	M - D 4
β_s	-7.9237*** *(0.1085)*	0.9163*** *(0.1268)*	-9.8886*** *(0.1157)*	-6.1963*** *(0.1484)*
γ_s^{age}	0.0682*** *(0.0014)*	-0.0318*** *(0.0017)*	0.1015*** *(0.0014)*	0.0648*** *(0.0018)*
γ_s^{gender}	0.2896*** *(0.0284)*	0.0481 *(0.04)*	-0.4568*** *(0.0271)*	-0.378*** *(0.0357)*
ϕ_s	-0.0117 *(0.0075)*	0.0243** *(0.0102)*	-0.0777*** *(0.0076)*	0.0036 *(0.0094)*
α_s	0.0173 *(0.0202)*	0.2065*** *(0.0304)*	-0.0664*** *(0.0204)*	0.0449 *(0.0253)*
Log Likelihood	**-51,697**			

Table 5: Frailty Model: parameter estimates (Monte Carlo MLE)

H = "healthy/non-disabled", M = "morbid/disabled", and D = "dead" *

Models of Functional Disability (Markov model) with Systematic trend and uncertainty (frailty factor)

Simulation used to estimate expected lifetimes with uncertainty

- Three estimated models are used for the simulation: the no-frailty model, the no-frailty model with linear time trend, and the frailty model.
- The life path of a healthy individual aged x up until age 100 is simulated, for age $x = 50, 55, 65, 70$ and 75, and for both genders.
- For the no-frailty models, with and without time trend, 10,000 homogeneous lives are simulated for each starting age and gender, whereas for the frailty model, 10,000 paths of the latent frailty factor are simulated and 10,000 homogeneous lives are simulated for each path.

Models of Functional Disability (Markov model) with Systematic trend and uncertainty (frailty factor)

	Age	No Frailty	No Frailty w/Time	Frailty
		Number of Years		
Males	50	29	33.2	34.2 *(30.7, 37.5)*
	55	24.9	27.7	28.5 *(26.0, 31.1)*
	60	20.6	22.4	23.1 *(21.1, 25.1)*
	65	16.8	17.9	18.4 *(17.0, 20.1)*
	70	13.4	13.9	14.1 *(13.0, 15.4)*
	75	10.3	10.4	10.6 *(10.0, 11.4)*
Females	50	32.2	36.5	37.2 *(34.2, 40.5)*
	55	27.8	30.8	31.7 *(29.5, 34.2)*
	60	23.5	25.7	26.2 *(24.5, 28.5)*
	65	19.6	20.9	21.3 *(19.9, 23.1)*
	70	15.9	16.5	16.8 *(15.8, 18.2)*
	75	12.6	12.8	12.9 *(12.3, 13.7)*

Table 6: Simulated Expected Lifetime

Models of Functional Disability (Markov model) with Systematic trend and uncertainty (frailty factor)

		Number of Years		
	Age	**No Frailty**	**No Frailty w/Time**	**Frailty**
Males	50	27.2	31.1	32.2 *(28.5, 36.9)*
	55	23.1	25.7	26.7 *(23.9, 29.8)*
	60	18.9	20.5	21.4 *(19.3, 24.1)*
	65	15.3	16.3	16.8 *(15.4, 18.8)*
	70	12.0	12.5	12.7 *(11.6, 14.4)*
	75	9.2	9.2	9.5 *(8.7, 10.4)*
Females	50	29.1	32.7	33.9 *(30.1, 39.2)*
	55	24.7	27.2	28.4 *(25.3, 32.1)*
	60	20.5	22.4	23.1 *(20.9, 26.3)*
	65	16.8	18.0	18.5 *(16.8, 20.9)*
	70	13.4	13.9	14.2 *(12.8, 16.2)*
	75	10.3	10.5	10.7 *(9.8, 11.8)*

Table 7: Simulated Expected Lifetime in Healthy State

Models of Functional Disability (Markov model) with Systematic trend and uncertainty (frailty factor)

	Age	No Frailty	No Frailty w/Time	Frailty
Males	50	0.938	0.936	0.941 *(0.917, 0.967)*
	55	0.929	0.928	0.934 *(0.910, 0.955)*
	60	0.920	0.916	0.925 *(0.903, 0.948)*
	65	0.911	0.909	0.915 *(0.896, 0.938)*
	70	0.900	0.897	0.903 *(0.884, 0.928)*
	75	0.889	0.891	0.892 *(0.874, 0.913)*
Females	50	0.901	0.897	0.908 *(0.866, 0.951)*
	55	0.887	0.884	0.897 *(0.859, 0.934)*
	60	0.874	0.873	0.882 *(0.848, 0.922)*
	65	0.859	0.859	0.867 *(0.834, 0.907)*
	70	0.841	0.840	0.847 *(0.809, 0.888)*
	75	0.822	0.825	0.828 *(0.796, 0.861)*

Table 8: HLE/TLE

HLE = “healthy life expectancy”, and TLE = “total life expectancy”

Models of Functional Disability (Markov model) with Systematic trend and uncertainty (frailty factor)

Some comments:

- Morbidity compression, expansion or equliibrium has significant implications for both private long term care insurance and government budgets
- Relationship between increasing life expectancy and healthy life expectancy discussed in Robin and Jagger (2005)
- Impact of different definitions of healthy and disabled (health status, functional disability) including cognitive decline
- Maximum likelihood estimation with frailty and numerical methods with multi-dimensional, and relatively flat likelihood surfaces.

Summary and Conclusions

Some of the challenges

- Incorporating both health status and functional disability in multiple state models
- Semi-Markov models
- Incorporating systematic trend and uncertainty in transition rates in multiple state models
- Developing models consistent with dynamic stochastic mortalty models
- Applications in developing new insurance products and financing long term government health and pension expenditures

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