Fundamentals of variable annuity market, product designs and latest technical developments

CEPAR Longevity Risk Workshop 2019

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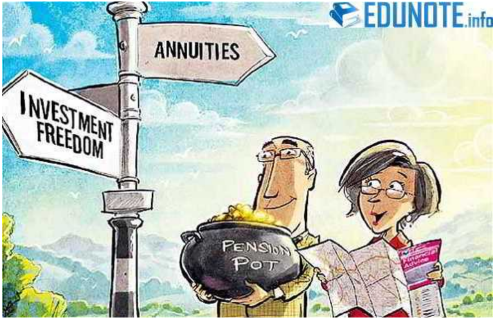
November 29, 2019

Introduction to Equity-Linked Insurance
- Variable annuities
- Equity indexed annuities

Fundamental Principles

Latest development

Retirement planning



Social security, 401(k), IRA, retirement investment

Equity linked insurance products

- Indexed universal life
- Fixed indexed annuities
- Variable annuities
- Hybrid annuities

Equity linked insurance products

- Indexed universal life
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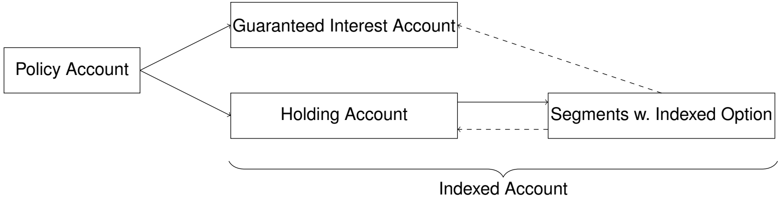


Abbildung: the flow of the net premium

- Variable annuities

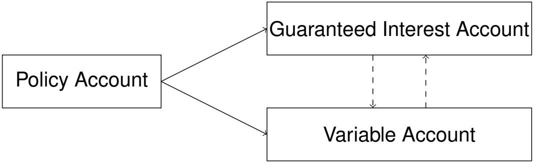


Abbildung: the flow of the net premium

Variable annuity product from a policyholder's perspective

- Arguably the most complex equity-based guarantee available to individual investors;
- Policyholders make contributions (called **purchase payments**) into subaccounts; typically single purchase payment at the inception;
- The term of the product can be broken down into two parts:
  - Accumulation phase; (resembles mutual funds)
  - Income phrase. (various types of guaranteed benefits)

Product design

- Accumulation phase
  - (Accumulation unit) The policyholder's account is credited with a number of accumulation units based on the policyholder's net asset value.
  - (Accumulation unit value) The initial accumulation unit value for each sub-account is often set arbitrarily. Each sub-account's accumulation unit value is then adjusted each business day to reflect income and capital gains of underlying equity index/fund, and the deduction of M&E fees, etc.

Accumulation phase

- Example: If you make an initial purchase payment of $\$1,000$ on Monday and the accumulation unit value at the end of day is \$10, then your account is credited with

$$\$1,000 \div \$10 = 100(\text{units}).$$

If you take a withdrawal of \$100 before the end of Tuesday, then the number of accumulation units reduces to

$$100 - \$100 \div \$10 = 90(\text{units}).$$

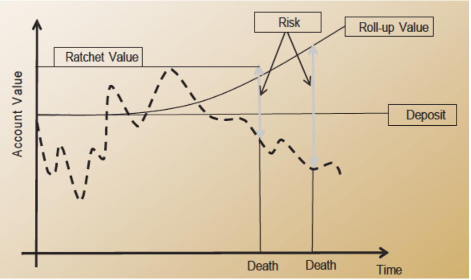
After the stock exchange closes on Tuesday, it is determined that each unit value increases from \$10 to \$10.25 for your selection of investment fund. Then your account is worth on Tuesday night

$$\$10.25 \times 90 = \$922.5.$$

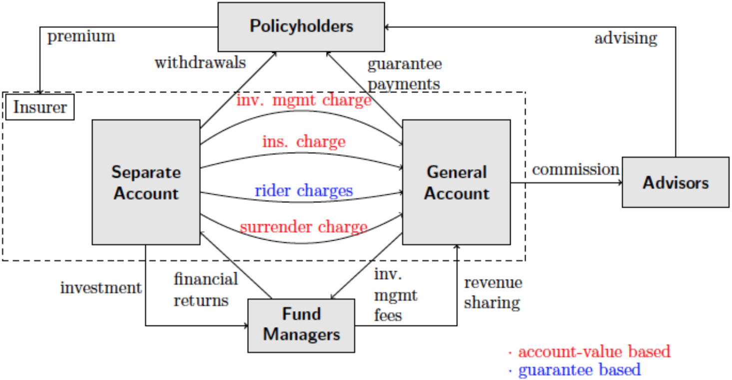
Accumulation phase

- Without any guarantee, equity participation involves no risk to the variable annuity writer, who merely acts as a steward of the policyholders' funds.

- In order to compete with mutual funds, nearly all major variable annuity writers start to offer various types of investment guarantees, which transfer certain financial risks to the insurers. (Liabilities) The insurer pays the difference, should the guaranteed amount exceeds the account value.



Variable annuity product from an insurer's perspective



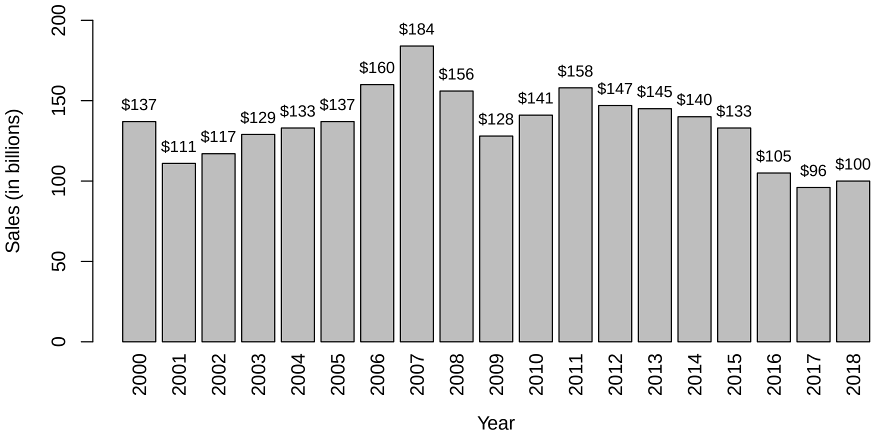
Equity-indexed annuities (a.k.a fixed indexed annuities)

- The rate of credited interest is linked to the returns of a stock index, such as the S&P 500.
- Higher return than fixed annuities.
- Protection against the downside of equity risk.
- Equity-indexed annuities may appeal to moderately conservative investors.

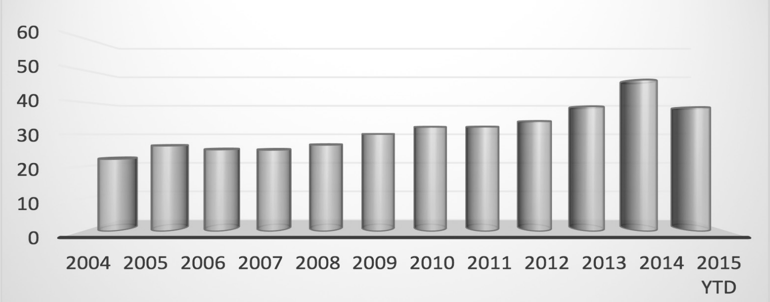
Difference between VA and EIA

	EIA	VA
Cash flows	General account	Separate account
Risk and Reward	Equity risk shared between PH & insurer (cap, floor, participation)	All equity risk passed on to PH (no limit)
Fees	No direct E&M fees (cost control)	E& M fees and rider charges
Regulation	Insurance product (state insurance dept)	Securities (SEC)

Variable annuity sales ($*billion*)



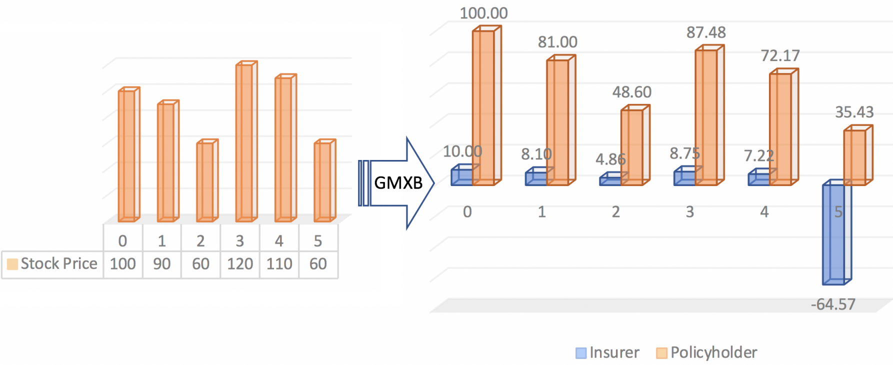
Equity-indexed annuity sales ($*billion*)



Investment Guarantee as a Functional of Equity Value Process

To practitioners the relationship is often represented by a spreadsheet. We are interested in the functional relationship.

$$(\{F_t, t \geq 0\}, T_x) \Longrightarrow \{L_t, t \geq 0\} \text{ or } L$$



Introduction to Equity-Linked Insurance

Variable annuities

Equity indexed annuities

Fundamental Principles

Latest development

Modeling of investement guarantees

- Consider a discrete time model with a valuation period of $1/n$ of a time unit, i.e. $t = 1/n, 2/n, \cdots, k/n, \cdots, T$. The equity-linked mechanism for variable annuity dictates that at the end of each trading day, the account value fluctuates in proportion to the value of equity fund in which it invests and deducted by account-value-based fees.

$$F_{k/n} = F_0 \frac{S_{k/n}}{S_0}\left(1 - \frac{m}{n}\right)^k, \qquad k = 1, 2, \cdots, nT,$$

  where m is the annual rate of total charge compounded n times per year, and all charges are made at the beginning of each valuation period.
  - F_t – total value of subaccounts at time t;
  - S_t – value of equity-index at time t;

- M&E charge, rider charge are made on a daily basis as a certain percentage of subaccount values. (Assets) The present value of fee incomes, also called margin offset, up to the k-th valuation period is given by

$$M_{k/n} = \sum_{j=1}^{k} e^{-r(j-1)/n} \left(\frac{m_e}{n} \right) F_{(j-1)/n}.$$

Guaranteed minimum maturity benefit (GMMB)

The GMMB guarantees the policyholder a minimum monetary amount G at the maturity T. The present value of the gross liability to the insurer is

$$e^{-rT}(G - F_T)_+ I(T_x > T),$$

where $(x)_+ = \max\{x, 0\}$. Consider the *individual net liability* of the guaranteed benefits from the insurer's perspective, which is the gross liability of guaranteed benefits less the fee incomes. The present value of the GMMB net liability is given by

$$L_e^{(n)}(T_x) \quad := \quad e^{-rT}(G - F_T)_+ I(T_x > T) - M_{T \wedge T_x},$$

where $x \wedge y = \min\{x, y\}$.

We shrink the valuation period to zero by taking n to ∞, thereby reaching the limiting continuous-time model. Recall that

$$\lim_{n\to\infty}\left(1-\frac{m}{n}\right)^n = e^{-m},$$

where m in this case should be interpreted as the continuously compounded annual rate of total charges. As a result,

$$F_t = \lim_{n\to\infty} F_{\frac{\lceil nt\rceil}{n}} = \frac{F_0}{S_0}\lim_{n\to\infty} S_{\frac{\lceil nt\rceil}{n}}\left[\left(1-\frac{m}{n}\right)^n\right]^{\frac{\lceil nt\rceil}{n}} = F_0\frac{S_t}{S_0}e^{-mt}.$$

Observe that the limit of the margin offset is given by

$$M_t = \lim_{n\to\infty} M_{\frac{\lceil nt\rceil}{n}} = \lim_{n\to\infty}\sum_{j=1}^{\lceil nt\rceil}\frac{1}{n}e^{-r(j-1)/n}m_e F_{(j-1)/n} = \int_0^t e^{-rs}m_e F_s\,\mathrm{d}s$$

where m_e is interpreted as the continuously compounded annual rate of rider charge allocated to the GMMB rider.

Continuous time model

The limit of L leads to a continuous time model. In the case of the GMMB,

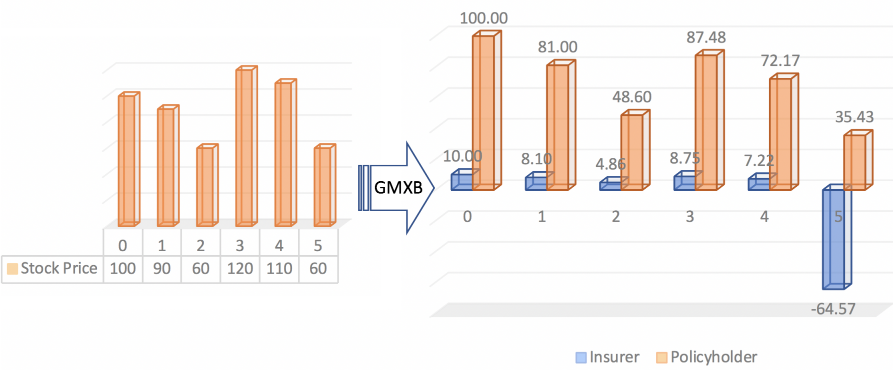
$$L_e^{(\infty)}(T_x) \;=\; e^{-rT}(G - F_T)_+ I(T_x > T) - \int_0^{T \wedge T_x} e^{-rs} m_e F_s \, \mathrm{d}s.$$

- The net liability L should be negative with a sufficiently high probability, as the products are designed to be profitable. However, in adverse scenarios, the net liability can be positive.
- The equity risk on the asset side is often overlooked in the literature.
- Connection to exponential functionals.

Investment Guarantee as a Functional of Equity Value Process

To practitioners the relationship is often represented by a spreadsheet. We are interested in the functional relationship.

$$(\{F_t, t \geq 0\}, T_x) \Longrightarrow \{L_t, t \geq 0\} \text{ or } L$$



Resets, roll-ups and ratchets

Reset option is typically associated with the automatic renewal of variable annuity contracts with fixed terms. It is intended to allow a policyholder to lock in investment returns. When the contract is renewed, the new guarantee base is reset at the level which is the greater of the guarantee base from the previous term and the account value at the renewal date. Let $\{T_1, T_2, \cdots,\}$ be a sequence of renewal dates with the understanding that $T_0 = 0$. With a reset option, the guarantee base at time T_k is given by

$$G_{T_k} = \max\{G_{T_{k-1}}, F_{T_k}\}, \qquad k = 1, 2, \cdots$$

Resets, roll-ups and ratchets

Roll-up option allows the guarantee base to accrue interest throughout the term of the policy. For example, if the roll-up rate ρ is a nominal rate payable n times per year, then the guarantee base is determined by

$$G_{(k+1)/n} = G_{k/n}\left(1+\frac{\rho}{n}\right), \qquad \text{for } k = 0, 1, \cdots .$$

Note that this recursive relation implies that

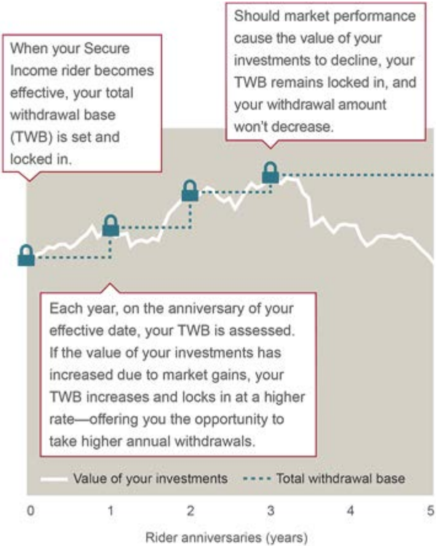
$$G_{k/n} = G_0\left(1+\frac{\rho}{n}\right)^k .$$

In continuous time,

$$G_t = \lim_{n\to\infty} G_{\frac{\lceil nt\rceil}{n}} = G_0 e^{\rho t},$$

where $\lceil x\rceil$ is the integer ceiling of x.

Vanguard – step-up option



Lifetime high step-up option

If the current account value exceeds the guarantee base from the previous period, then the guarantee is reset to the current account value. Otherwise, the guarantee base remains the same.

$$G_{(k+1)/n} = \max\left\{G_{k/n}, F_{(k+1)/n}\right\}, \qquad \text{for } k = 0, 1, \cdots .$$

Observe that this recursive relation leads to the representation

$$G_{k/n} = \max_{j=0,1,\cdots,k} \left\{F_{j/n}\right\}.$$

In continuous time,

$$G_t = \sup_{0 \le s \le t} \{F_s\},$$

where $\sup$ is the supremum.

Combination of step-up and roll-up options

Sometimes these options are combined to offer guaranteed compound growth on the guarantee base and to allow the guarantee base to “lock in”gains from the policyholder’s designated investment:

$$G_{(k+1)/n} = \max\left\{G_{k/n}\left(1+\frac{\rho}{n}\right), F_{(k+1)/n}\right\}, \qquad \text{for } k = 0, \cdots .$$

Using mathematical induction, one can show that this option also has a representation

$$G_{k/n} = \left(1+\frac{\rho}{n}\right)^{k} \max_{j=0,\cdots,k}\left\{\left(1+\frac{\rho}{n}\right)^{-j} F_{j/n}\right\}.$$

In continuous time,

$$G_t = e^{\rho t} \sup_{0\le s\le t}\{e^{-\rho s}F_s\}.$$

Guaranteed minimal death benefit (GMDB)

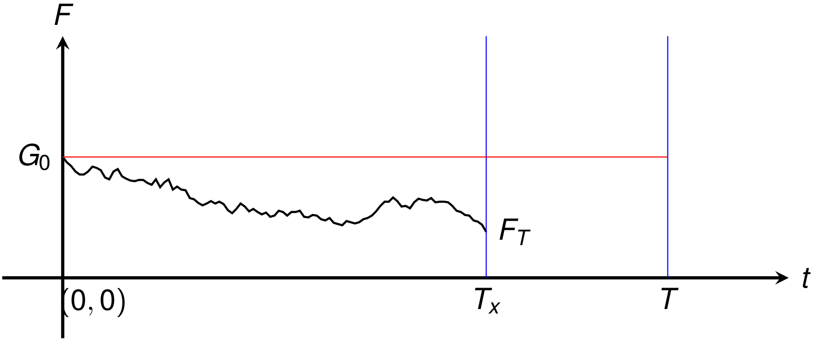


Abbildung: GMDB gross liability

Guaranteed minimal death benefit (GMDB)

The net liability of GMDB with a roll-up feature in a continuous time model

$$L_d^{(\infty)}(T_x) = e^{-rT_x}(Ge^{\rho T_x} - F_{T_x})_+ I(T_x \leq T) - \int_0^{T \wedge T_x} e^{-rs} m_d F_s \, \mathrm{d}s.$$

- Interaction of equity risk and mortality risk.

Guaranteed minimal withdrawal benefit (GMWB)

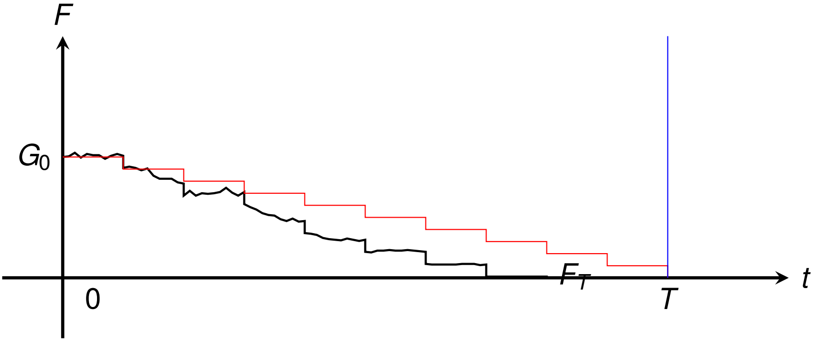


Abbildung: GMWB gross liability

GMWB

In practice, most policyholders choose to withdraw the maximum amount without penalty, denoted by w, which is typically a fixed percentage of the initial premium. Then it takes $T = F_0/w$ periods before the initial premium is fully refunded, at which point the GMWB rider expires. We denote this fixed amount per time unit by w, the total fees per time unit by m. The margin offset per time unit used to fund the GMWB rider is denoted by m_w.

$$\mathrm{d}F_t = \frac{F_t}{S_t}\,\mathrm{d}S_t - mF_t\,\mathrm{d}t - w\,\mathrm{d}t$$

GMWB

From a policyholder's point of view, we obtain the total worth of investment with the variable annuity contract with the GMWB rider in continuous-time

$$\int_0^T e^{-rt} w \,\mathrm{d}t + e^{-rT} F_T I(F_T > 0).$$

Similarly, the continuous-time individual net liability of a GMWB rider from an insurer's point of view is given by

$$L_w^{(\infty)} := \int_\tau^{\tau \vee T} e^{-rt} w \,\mathrm{d}t - \int_0^{\tau \wedge T} e^{-rt} m_w F_t \,\mathrm{d}t,$$

where the ruin time is defined by

$$\tau := \inf\{t > 0 : F_t \leq 0\}.$$

Guaranteed lifetime withdrawal benefit (GLWB)

- Systematic withdrawals is typically determined by a pre-specified percentage h of a guarantee base G_t.
- The guarantee base is a nominal account which only serves as a base to determine withdrawal amounts.
- The actual withdrawals are taken out of the policyholder's own investment account (not the guarantee base!).

GLWB

The dynamics of fund value and guarantee base are given by

$$\mathrm{d}F_t = \frac{F_t}{S_t}\,\mathrm{d}S_t - (m_w + h)G_t\,\mathrm{d}t - mF_t\,\mathrm{d}t.$$

The insurer's net liability for the GLWB rider in the continuous time model is given by

$$L_{lw}^{(\infty)} := \int_{\tau}^{\tau \vee T_x} e^{-rt} G_t h\,\mathrm{d}t - \int_{0}^{\tau \wedge T_x} e^{-rt} G_t m_w\,\mathrm{d}t.$$

Guaranteed minimum surrender benefit (GMSB)

- The rider specifies the minimum amount that a policyholder is guaranteed to receive upon surrender after the application of surrender charges and market value adjustments.
- Let γ_s be the surrender charge at time s and η be the percentage for minimal benefit.
- The policyholder is entitled to

$$\max\{\eta F_0, (1-\gamma_{\tau^l})F_{\tau^l}\},$$

where τ^l is the first jump time of a time-inhomogeneous Poisson process with intensity rate λ_t which represents the lapse rate at time t.

GMSB

- The insurer's GMSB gross liability is given by

$$((1-\gamma_{\tau^l})F_{\tau^l} - \eta F_0)_+ \,,$$

where $\mathbb{P}(\tau^l > t) = \exp\left\{-\int_0^t \lambda_s \, \mathrm{d}s\right\}.$

- The PV of the insurer's GMSB net liability is given by

$$e^{-r\tau^l}((1-\gamma_{\tau^l})F_{\tau^l} - \eta F_0)_+ - \int_0^{\tau^l \wedge \tau \wedge T} e^{-rs} m_s F_s \, \mathrm{d}s.$$

Guaranteed minimum income benefit (GMIB)

- GMIB rider is designed to provide the investor with a base amount of lifetime income when they retire regardless of how the investments have performed.
- Should the policyholder chooses to annuitize, payments are based on the amount invested, credited with an interest rate g.
- The benefit base of a variable annuity with the GMIB at the date of annuitization τ^a is

$$G_{\tau^a} = \max\left(F_0(1+g)^{\tau^a}, F_{\tau^a}\right).$$

GMIB

- The insurer's GMIB gross liability

$$\left(F_0(1+g)^{\tau^a} - F_{\tau^a}\right)_+ .$$

- The PV of the insurer's GMIB net liability is given by

$$e^{-r\tau^a}\left(F_0(1+g)^{\tau^a} - F_{\tau^a}\right)_+ - \int_0^{\tau^a \wedge \tau \wedge T} e^{-rs} m_i F_s \, \mathrm{d}s.$$

Equity-indexed annuities (point-to-point)

- Given a single premium P, the product often offers a guaranteed minimum annual continuously compounding interest rate g on the premium up to maturity T. For example, the guarantee base accumulates at maturity T to

$$G_T := Pe^{gT}.$$

- The contract typically specifies a participation rate $\alpha \in (0, 1)$, which determines the proportion of financial returns on equity index to be received by the policyholder.

Equity-indexed annuities (point-to-point)

- From a policyholder's point of view, the benefit from the equity-indexed annuity is given by

$$\max\left(P\left(1+\alpha\frac{S_T-S_0}{S_0}\right), G_T\right).$$

- The insurer's net liability

$$e^{-rT}\left(P\left(1+\alpha\frac{S_T-S_0}{S_0}\right)-G_T\right)_+ - P.$$

Equity-indexed annuities (Cliquet)

Suppose that an equity-indexed annuity with an annual cliquet design allows a participation rate $\alpha \in (0, 1)$ and the guaranteed minimum rate $g \in (0, 1)$. The value of the policy at maturity T is given by

$$P \prod_{k=1}^{T} \max\left(1 + \alpha \frac{S_k - S_{k-1}}{S_{k-1}}, e^g\right).$$

In some design, the contract caps the annual rate at a maximum rate c where $g < c < 1$. The value of such a policy at maturity T is given by

$$P \prod_{k=1}^{T} \max\left(\min\left(1 + \alpha \frac{S_k - S_{k-1}}{S_{k-1}}, e^c\right), e^g\right).$$

Introduction to Equity-Linked Insurance

Fundamental Principles

Latest development

Risk Management Methods

Risk reduction

- The quintessence of an insurance business is the pooling of funds from a large number of policyholders to pay for losses incurred by a few.

Theorem (Strong law of large numbers)

Let $X_1, X_2, \cdots, X_n$ be an infinite sequence of i.i.d. random variables with $E(X_1) = E(X_2) = \cdots = \mu$. Then

$$\overline{X}_n := \frac{1}{n}(X_1 + \cdots + X_n) \to \mu, \qquad n \to \infty.$$

Does equivalence principle apply to equity-linked insurance?

- Gross liability of GMMB: (T is the maturity date)

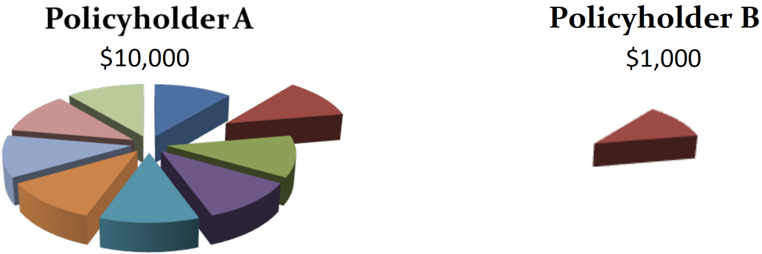
$$e^{-rT}(G - F_T)_+ I(\tau_x > T),$$

  where τ_x is the death time of policyholder aged x at issue, $(x)_+ = \max\{x, 0\}$.
- All accounts are linked to the same equity index.

$$\begin{aligned} F_t^{(1)} &= F_0^{(1)} \frac{S_t}{S_0} e^{-mt}, \\ F_t^{(2)} &= F_0^{(2)} \frac{S_t}{S_0} e^{-mt}, \\ &\cdots \end{aligned}$$

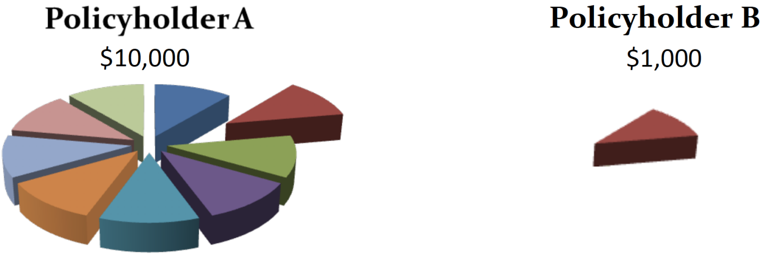
F investment fund value; G guaranteed benefit; T maturity date; r risk-free interest rate.

Does equivalence principle apply to equity-linked insurance?



Answer: NO! Well... not in the classic sense. No standardized contract size and $L^{(i)}$'s are not mutually independent.

Does equivalence principle apply to equity-linked insurance?



Answer: NO! Well... not in the classic sense. No standardized contract size and $L^{(i)}$'s are not mutually independent.
But practitioners have been using the LLN implicitly for equity-linked insurance business.

Are they wrong?

Diversification of mortality risk

- A stand-alone contract (full concentration of mortality risk)

$$L_0^{(i)} = e^{-rT}(G - F_T)_+ I(\tau_x^{(i)} > T) - \int_0^{T \wedge \tau_x^{(i)}} e^{-rs} m_e F_s ds,$$

where $\tau_x^{(i)}$ is the future lifetime of the i-th policyholder.

- Two contracts of equal size (diversification of mortality risk)

$$\frac{1}{2} L_0^{(1)} + \frac{1}{2} L_0^{(2)}$$

- Since the CTE is subadditive (for cont. r.v.'s), then

$$\mathrm{CTE}\left(\frac{1}{2}\sum_{i=1}^{2} L_0^{(i)}\right) \leq \mathrm{CTE}(L_0^{(1)}).$$

Diversification of mortality risk

- A stand-alone contract (full concentration of mortality risk)

$$L_0^{(i)} = e^{-rT}(G - F_T)_+ I(\tau_x^{(i)} > T) - \int_0^{T \wedge \tau_x^{(i)}} e^{-rs} m_e F_s ds,$$

  where $\tau_x^{(i)}$ is the future lifetime of the i-th policyholder.
- n contracts of equal size (diversification of mortality risk)

$$\frac{1}{n} L_0^{(1)} + \frac{1}{n} L_0^{(2)} + \cdots + \frac{1}{n} L_0^{(n)}$$

- Since the CTE is subadditive (for cont. r.v.'s), then

$$\mathrm{CTE}\left(\frac{1}{n}\sum_{i=1}^{n} L_0^{(i)}\right) \leq \mathrm{CTE}(L_0^{(1)}).$$

- What happens as $n \to \infty$?

Law of large numbers for equity-linked insurance

- Individual model:

$$L_0 = e^{-rT}(G - F_T)_+ I(\tau_x > T) - \int_0^{T \wedge \tau_x} e^{-rs} m_e F_s ds.$$

- Aggregate model:

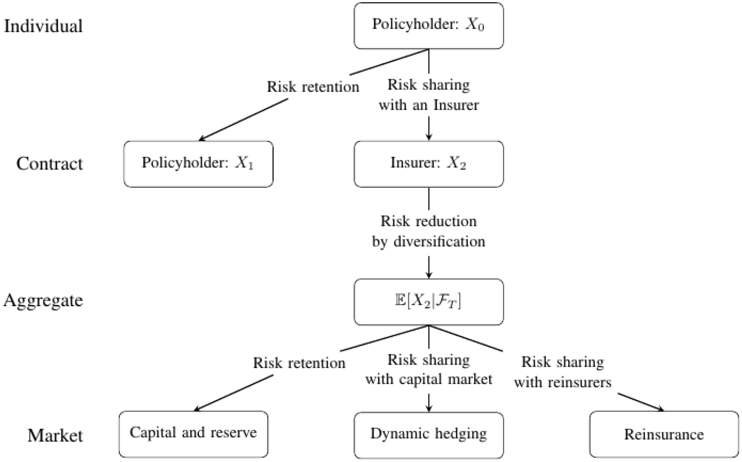
$$L_0^* = \mathbb{E}[L_0 | \mathcal{F}_T] = e^{-rT}(G - F_T)_+ \, {}_T p_x - \int_0^T {}_s p_x e^{-rs} m_e F_s \, \mathrm{d}s.$$

Theorem (Feng & Shimizu (2016))

If all policies are **of equal size** *and future lifetimes of policyholders are mutually independent, then*

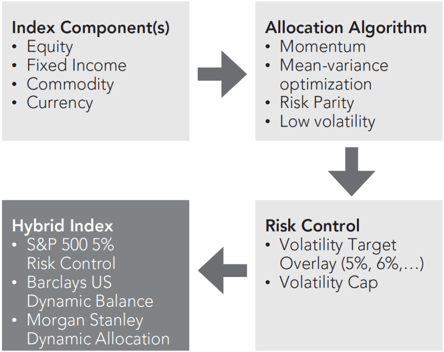
$$\frac{1}{N} \sum_{i=1}^{N} L_0^{(i)} \longrightarrow L_0^*, \qquad \textit{almost surely.}$$

Risk Engineering of Equity-Linked Insurance

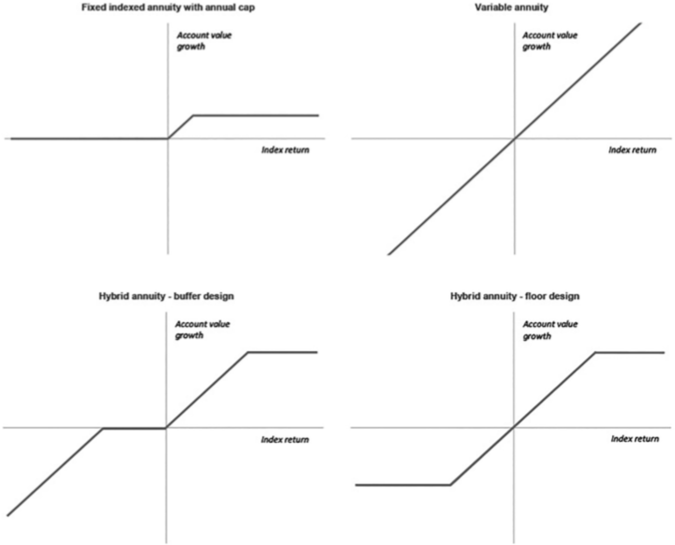


Recent development

- Hybrid indices
  (Overall lower allocation to equities in order to keep guarantees more affordable and manageable)



Hybrid annuity



Hybrid annuity

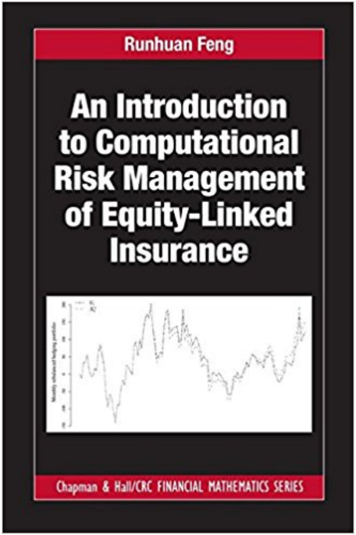
- Indexed variable annuity

$$F_T = F_0 \prod_{k=0}^{T} \left[\max\left(\min\left(\frac{S_k}{S_{k-1}}, 1+c\right), 1-f\right) - m\right]$$

  - Fills the risk spectrum between FIA and VA.
  - FIA: limited exposure to positive market performance
  - VA: full exposure to market performance

Research questions:

  - Discrete model: compound options;
    continuous model: bounded variation
  - Optimal designs?
  - Protection of downside risk versus living benefit?



https://www.crcpress.com/An-Introduction-to-Computational-Risk-Management-of-Equity-Linked-Insurance/Feng/p/book/9781498742160

Thank you very much for your attention!