Longevity Risk Products

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CEPAR Workshop – Longevity and Long-Term Care Risks and Products UNSW Sydney

19 July 2018

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Longevity risk

From the perspective of the individual

Risk of outliving his own resources \Rightarrow Focus: Post-retirement income Possible individual targets:

- Lifelong payment
- Fixed or minimum annual amount
- ⇒ Longevity guarantee
- ⇒ Financial guarantee

From the perspective of the provider of a longevity guarantee

The "insurer" has to pay lifelong benefits, whatever

- The individual lifetime
- And the average lifetime of the population
- ⇒ Individual (or Idiosyncratic) longevity risk
- \Rightarrow (Aggregate) Longevity risk

Longevity risk affected also by the benefit amount

Life annuities

Traditional guarantees

- Lifelong payment
- Fixed annual amount or annual revaluation (participating, with-profit or inflation-linked annuities)

The longevity guarantee (& the financial guarantee) are embedded

- In the annuity rate $AR = \frac{1}{a_x}$ (at age *x*)
- As well as in the participating rule (to the investment return or inflation rate)); For example: $b_t = b_{t-1} \cdot \left(1 + \max\left\{\frac{\eta_t g_t i_{(0)}}{1 + i_{(0)}}, 0\right\}\right)$

To avoid high loadings \Rightarrow Innovations in product design

Possibly aimed at:

- Reducing the size of the longevity guarantee
- Delaying the longevity guarantee

Setting the longevity guarantee

In the following, with reference to an individual:

- x Initial age, at time 0 [0, r] "Accumulation" period r Retirement time $[r, \infty]$ Post-retirement period
- x + r Age at retirement



The annuity rate can be set:

- At retirement time
- Before retirement time
- After retirement time

 \Rightarrow Impact on the time-profile of the longevity guarantee, and the location of the longevity risk

Post-retirement income products/arrangements

- Income drawdown
- Traditional life annuities, immediate or deferred
- Late life annuities: Advanced Life Delayed Annuity (ALDA), Ruin Contingent Life Annuity (RCLA)
- Variable annuities

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- Group Self-Annuitization (GSA), Tontine annuities, other pooled annuities
- Mortality/longevity-linked life annuities

Income drawdown (Withdrawal plan)

Given the amount S available at retirement time r, the individual cashes a post-retirement income so long as money is available, choosing the annual amount, the investment profile, and so on

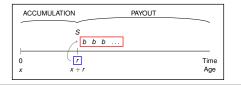
ACCUM	JLATION	WITHDRAWALS		
0 x	r x+r		T	Time Age

- *T* is random, depending on the investment performance, lifetime of the individual and annual amounts
- Longevity risk fully retained by the individual

CAR immediate life annuity

CAR: Current Annuity Rate, set at time r

Fixed benefit or asset-linked benefit

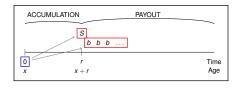


Solution So

- Annual payout
- Technical provision
- Required capital

GAR deferred life annuity – I

GAR: Guaranteed Annuity Rate, set before time r



so Longevity risk on the provider in the time-interval $[0,\infty)$, impacting on:

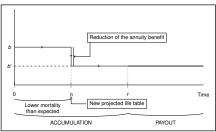
- The technical provision and the required capital for the whole period
- The annual payout starting from time r

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GAR deferred life annuity – II

Conditional GAR

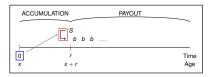
- GAR set at time 0
- In case of unanticipated mortality reduction, after time 0 but before time r the GAR is updated ⇒ The benefit amount is decreased



- A form of risk sharing
- Possible constraints: measure of the mortality reduction; frequency of updates; maximum total benefit reduction; payments to which the update is applied

GAO deferred life annuity - I

GAO: Guaranteed Annuitization Option



Deferred life annuity, providing the following options at retirement:

- Lump sum
- Annuitization at CAR
- Annuitization at GAR

Thus:

$$b = S \cdot \max\left\{\frac{1}{a_{x+r}^{[CAR]}}, \frac{1}{a_{x+r}^{[GAR]}}\right\}$$

In case of annuitization

GAO deferred life annuity – II

Value of the option affected by:

- Individual preferences (lump sum vs annuity)
- Mortality rates
- Interest rates

so Longevity risk on the provider in the time-interval $[0,\infty)$, impacting on:

- The reserve and the required capital in [0, r] and in case of annuitization in $[r, \infty)$
- The annual payments starting from time r, in case of annuitization

For the valuation of the option, addressing stochastic mortality, see:

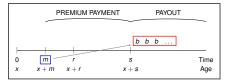
[Ballotta and Haberman, 2006], [Biffis and Millossovich, 2006], [Kling et al., 2014b]

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ALDA deferred life annuity – I

ALDA: Advanced Life Delayed Annuity

- The payout period starts after retirement time (age 80 or 85, say), at time s > r (→ Late life annuities)
- In the period [r, s]: Income drawdown
- A GAR is set, at time *m*, 0 < *m* < *s*



• The premium payment may go beyond retirement time

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ALDA deferred life annuity – II

- Longevity guarantees are provided, starting from time *m*, for deferred payments
- In comparison to traditional products, guarantees are postponed to older ages
 - \Rightarrow The actuarial value of the annuity is reduced

See:

[Milevsky, 2005b], [Gong and Webb, 2010]

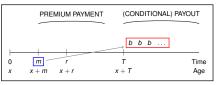
RCLA deferred life annuity - I

RCLA: Ruin Contingent Life Annuity

- The payment of the annuity is contingent on the realization of an adverse (financial and longevity) scenario
- Appropriate index for defining the scenario (~> critical choice)
- Assumed correlation between the scenario and the individual position
- In the meantime (for a random duration): Income drawdown

RCLA deferred life annuity – II

• Starting from time *T* (random), a life annuity is paid conditional on the occurrence of the adverse scenario



Given the presence of the trigger, the cost of the life annuity is reduced

See:

[Huang et al., 2014]

For a general description ...

... of the evolving structures of the longevity guarantees in life annuities, and for further references, see: [Pitacco, 2016]

Basic topics to investigate

- Premium loadings
- Risk margins in technical provisions and required capital
- ⇒ Stochastic mortality model
 - Individual preferences

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Annuitization strategies - I

Common design in the ALDA and RCLA: Income drawdown + Annuity at older ages

A similar design in individual strategies for the post-retirement income

- Optimal annuitization time, partial annuitization, delayed annuitization, staggered annuitization, phased withdrawal, ...
- Problem: When and how much to annuitize
 - If annuitization is postponed: Some mortality credit is lost, but individual funds are retained, and invested with higher flexibility
 - Staggerered (or progressive) annuitization: Progressive annuitization of the individual funds
- Balance between the (lost) mortality credit and the (higher) return on investments
 - \Rightarrow Optimal asset allocation

Annuitization strategies - II

See:

[Milevsky and Robinson, 2000], [Milevsky, 2001], [Milevsky, 2005a], [Milevsky and Young, 2007a], [Milevsky and Young, 2007b], [Gerrard et al., 2012], [Brown, 2001] [Davidoff et al., 2005] [Dus et al., 2005] [Schmeiser and Post, 2005] [Milevsky and Young, 2007a] [Milevsky and Young, 2007b] [Horneff et al., 2008] [Bayraktar and Young, 2009] [Horneff et al., 2010] [Bruhn and Steffensen, 2011] [Hanewald et al., 2013] [Maurer et al., 2013] [Kling et al., 2014a] [Maurer et al., 2016] [Delong and Chen, 2017]

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Variable annuities

GMIB: Guaranteed Minimum Income Benefit

- Lifelong post-retirement income
 - · Guarantee on the amount at retirement time, annuitized at the CAR
 - GAO possibly included

GMWB: Guaranteed Minimum Withdrawal Benefit

- Income drawdown, with a guaranteed duration
 - Fixed duration
 - Fixed duration, provided that the retiree is alive
 - Lifelong duration (logical structure of the RCLA)
- Longevity risk: In some respect, similar to life annuities, but for a shorter duration (unless a lifelong duration is guaranteed)

See: Presentation by Jonathan Ziveyi Annamaria Olivieri (UniPR) Longevity Risk Products Sydney, 19 July 2018 19 / 40

Combining death with annuity benefits - I

From the point of view of the insurer:

- A (natural) hedging effect can be gained
- In particular:
 - In case of a long lifetime, life annuity costs increase while death benefit costs decrease (and vice versa)
 - · Combining life annuity with death benefits should reduce the longevity risk
 - Reason: Less mutuality is required
- \blacksquare Clearly, if less mutuality is required \Rightarrow The mutuality credits are lower \Rightarrow Less favourable annuity rates for the annuitant

From the point of view of the individual

Bequest needs

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Combining death with annuity benefits - II

Life annuity with a guarantee period

Temporary annuity-certain + Deferred life annuity

Again, a design similar to RCLA and ALDA

Value-protected life annuity *or* Life annuity with capital protection *or* Money-back annuities

In case of death of the annuitant prior to a given age:

Unused capital is returned to the beneficiaries

Unused capital = Difference, if positive, between the initial capital S and the total amount of benefits

In both cases

The death benefit is provided in a range of ages in which the mortality level is low \Rightarrow The hedging effect is poor

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Combining death with annuity benefits - III

For a general description, and further references, see: [Pitacco, 2016]

Combining LTC with annuity benefits

See:

Presentation by Ermanno Pitacco

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Special rates (or underwritten) annuities

Annuity rates are differentiated in relation to the individual health condition (at policy issue)

Types

- Lifestyle annuities
- Enhanced annuities
- Impaired annuities
- Care annuities (LTC)

With assumed decreasing life expectancy

For more details, and references, see:

Presentation by Ermanno Pitacco

Sharing the longevity risk

Idea

To remove or reduce the longevity guarantee concerning

- The benefit amount
- Or the mortality credit assigned to the individual account value
- \Rightarrow The longevity risk after retirement is (partially) retained by the individual \Rightarrow Lower loadings/fees

Two approaches: Based on a given mortality experience

- The benefit amount is updated (either increased or decreased)
- The mortality credit assigned to the individual funds corresponds to the individual funds actually released by the deceased

Insured or self-insured arrangements

Self-insured arrangements

- Based on pooling arguments
 - The longevity risk is offset thanks to the pool size
 - However: Only the idiosyncratic longevity risk
- No guarantee ⇒ Lower fees
- Participation both to losses and profits
- Examples: Group Self-Annuitization (GSA), Pooled Annuity Funds (PAF), Annuity Overlay Funds (AOF), tontine investments

Insured arrangements

- Partial longevity guarantee
- Possible participation to profits
- Examples: tontine annuities, longevity-linked annuities (or similar labels)

Group Self-Annuitization (GSA), Pooled Annuity Funds (PAF), Annuity Overlay Funds (AOF)

- Self-insured arrangements
- GSA: the benefit amount is updated, based on the ratio between the available assets and the required reserve
- PAF and AOF: the funds released by the deceased are distributed (as a random mortality credit) to the survivors (recorded either at the end or the beginning of the year). Annual benefits (as well as the investment profiles) are (in principle ...) chosen by the individual
- Critical issue: fairness and solidarity, especially when the population is heterogeneous

See:

- GSA: [Piggott et al., 2005], [Valdez et al., 2006], [Bravo et al., 2009], [Qiao and Sherris, 2012], [Boyle et al., 2015]
- PAF and AOF: [Stamos, 2008], [Donnelly et al., 2013], [Donnelly et al., 2014], [Donnelly, 2015]

Tontine arrangements and annuities

- Originally designed as an investment, in which survivors are credited the funds of the deceased participants (or the nominees)
- The funds released upon death can be reinvested or paid as a dividend
- The dividends can be paid on top of an annuity
- Guarantees can be included, or not

See:

[McKeever, 2009], [Baker and Peter Siegelman, 2010], [Sabin, 2010], [Milevsky, 2014], [Milevsky and Salisbury, 2015], [Milevsky and Salisbury, 2016], [Weinert and Gruendl, 2016], [Chen et al., 2018]

Longevity-linked annuities

- Participating structure
- The benefit amount is allowed to fluctuate, depending on a given longevity experience
- Guarantees can be underwritten (for example: a minimum benefit amount)
- Several labels suggested in the literature: Adaptive algorithmic annuities, Longevity-indexed life annuities, Longevity-contingent life annuities, Mortality-linked annuities, ...

See

[Lüthy et al., 2001], [de Melo, 2008], [Denuit et al., 2011], [Richter and Weber, 2011], [Maurer et al., 2013], [Denuit et al., 2015], [Weale and van de Ven, 2016], [Bravo and de Freitas, 2018]

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Interesting topics to investigate

- Risk/return trade-off for the individual and the provider
- Premium loading and size of the longevity guarantee. Pricing of the embedded options
- Benefit volatility and smoothing tricks
- Value based assessment for the provider. Risk margins in technical provisions, required capital
- Longevity index
- Impact of the heterogeneity of the population on sharing arrangements, in particular in the self-insured solutions

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Many thanks for your kind attention!

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