Australian National University

Choosing and Using Utility Functions in Forming Portfolios

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Overview

How might an industry practitioner use utility functions to form optimal portfolios in applied settings?

Agenda:

- Background
- Characterising and selecting utility functions
- Simplified approach for forming portfolios
- Illustration for four investors

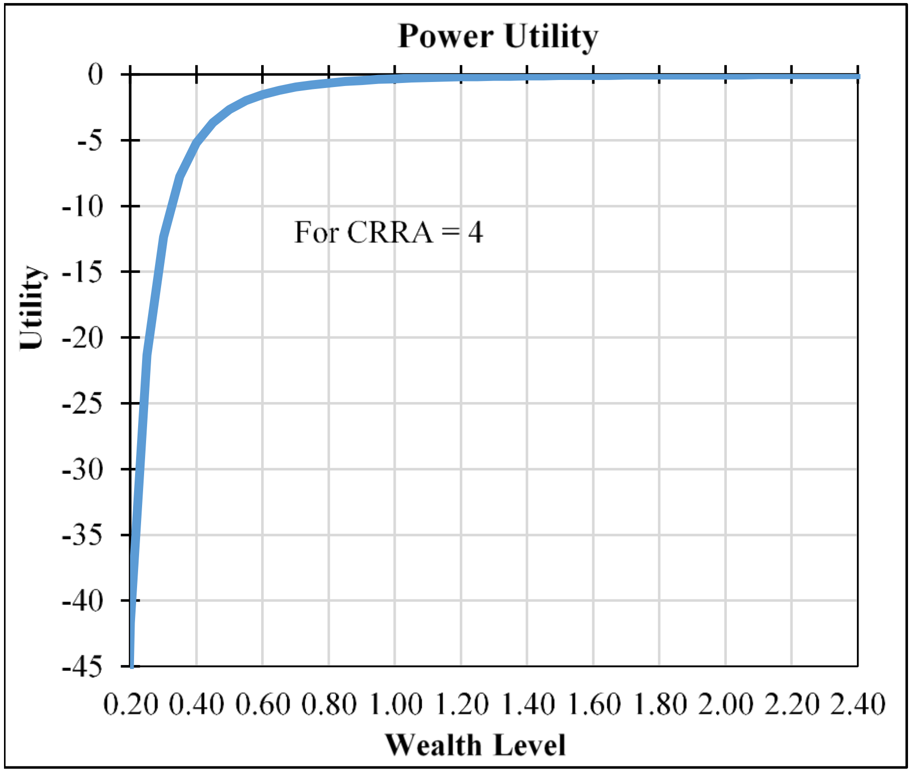
Background

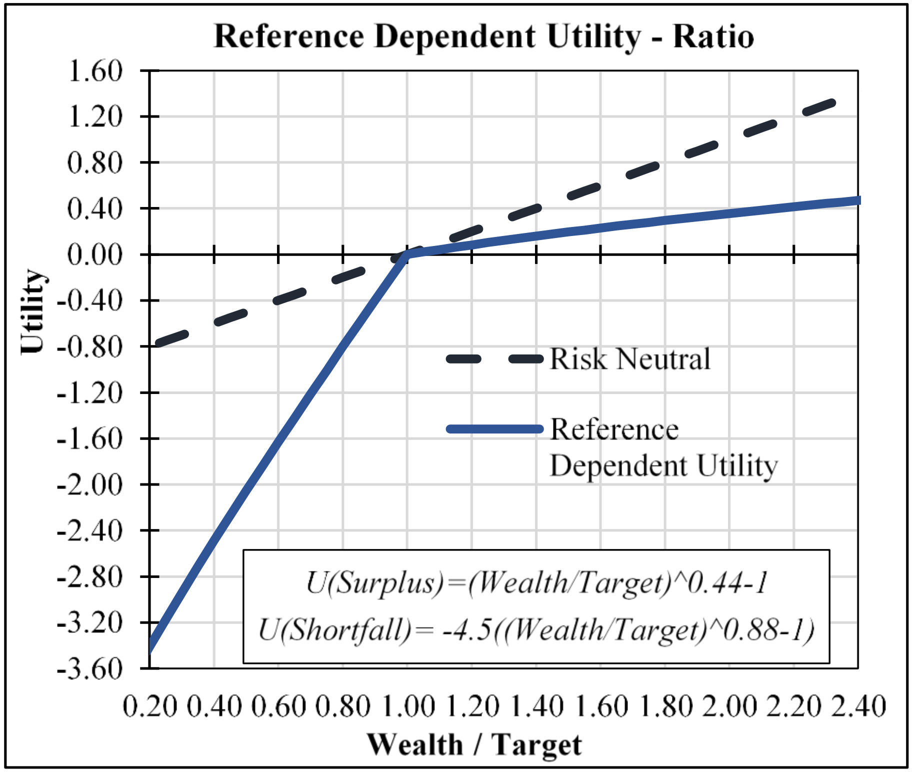
- Mean-variance is widely used in practice, but has shortcomings:
  - Single-period measure, the problem is often multi-period
  - Outcomes are not normal, especially over long horizons
- The academic literature has long-used utility functions. However:
  - Ongoing debate over which utility function is 'best'; as well as how any particular function should be parameterised
  - Utility functions are not 'one-size-fits-all'
- *Why not turn the problem around?* Start with the investor, and choose a utility function to reflect their objectives and preferences.

Three utility functions *(for demonstration purposes)*

Function	Equation	Inputs
Power utility $U_{PU,t}$	$U_{PU,t} = \dfrac{W_t^{(1-CRRA)}}{1-CRRA}$	W_t = wealth, period t $CRRA$ = relative risk aversion
Reference dependent utility, difference form *(similar to prospect theory value function)* $U_{RDU(D),t}$	$U_{RDU(D),t} = I_{(W_t > W_t^*)}\, \gamma\left((W_t - W_t^*)^{\propto}\right) - I_{(W_t < W_t^*)}\, \lambda\left((W_t^* - W_t)^{\beta}\right) + I_{(W_t = W_t^*)}\, 0$	W_t = wealth, period t W_t* = target wealth, period t I = indicator function (1, 0) α = curvature parameter on gains, $W_t > W_t*$ β = curvature parameter on losses, $W_t < W_t*$ γ = weighting parameter on gains, $W_t > W_t*$ λ = weighting parameter on losses, $W_t < W_t*$
Reference dependent utility, ratio form *(Tarlie, 2017)* $U_{RDU(R),t}$	$U_{RDU(R),t} = I_{\frac{W_t}{W_t^*} \geq 1}\, \gamma\left(\left(\dfrac{W_t}{W_t^*}\right)^{\propto} - 1\right) + I_{\frac{W_t}{W_t^*} < 1}\, \lambda\left(\left(\dfrac{W_t}{W_t^*}\right)^{\beta} - 1\right)$	

Functional forms

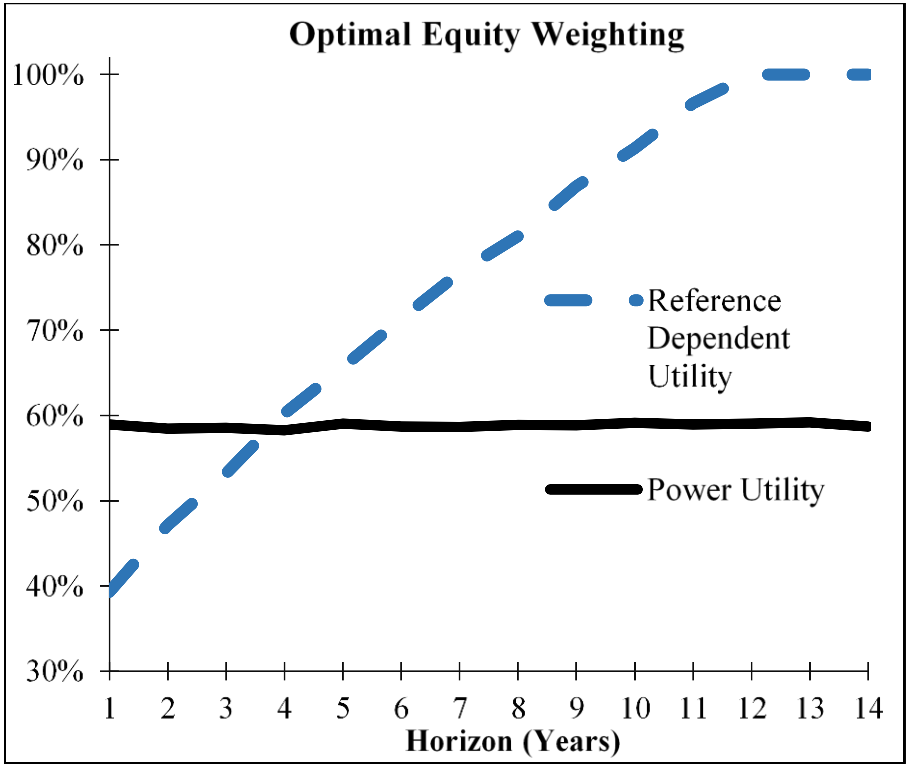


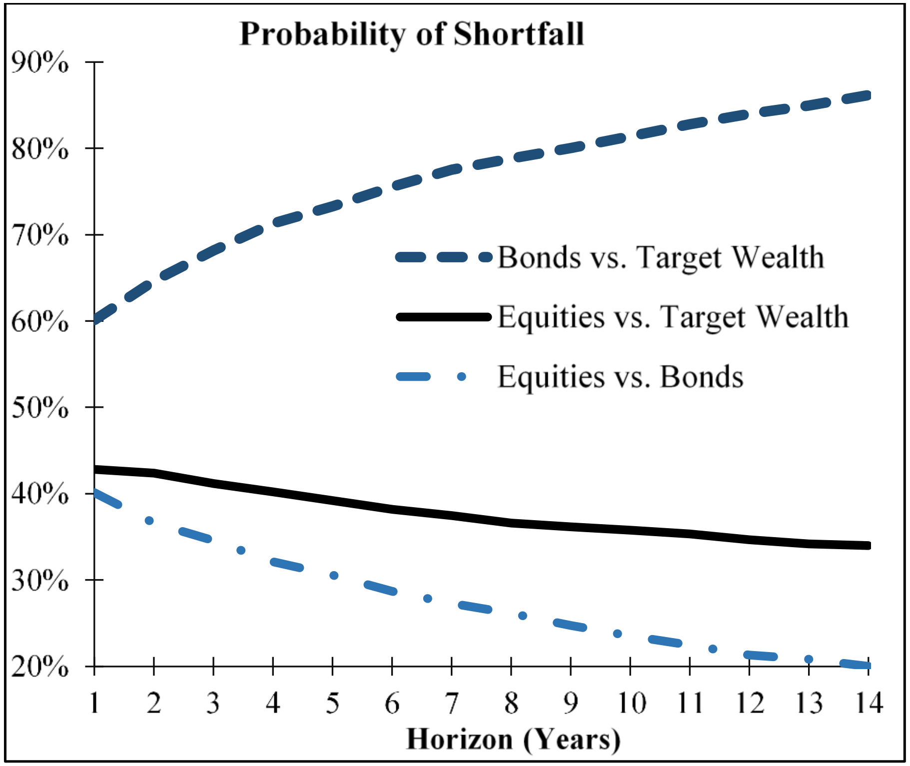


Things to note:

- Relative shape – especially in the lower tail
- Relative flexibility of reference dependent utility functions
- Units of measurement

Interaction with horizon *(iid simulations)*

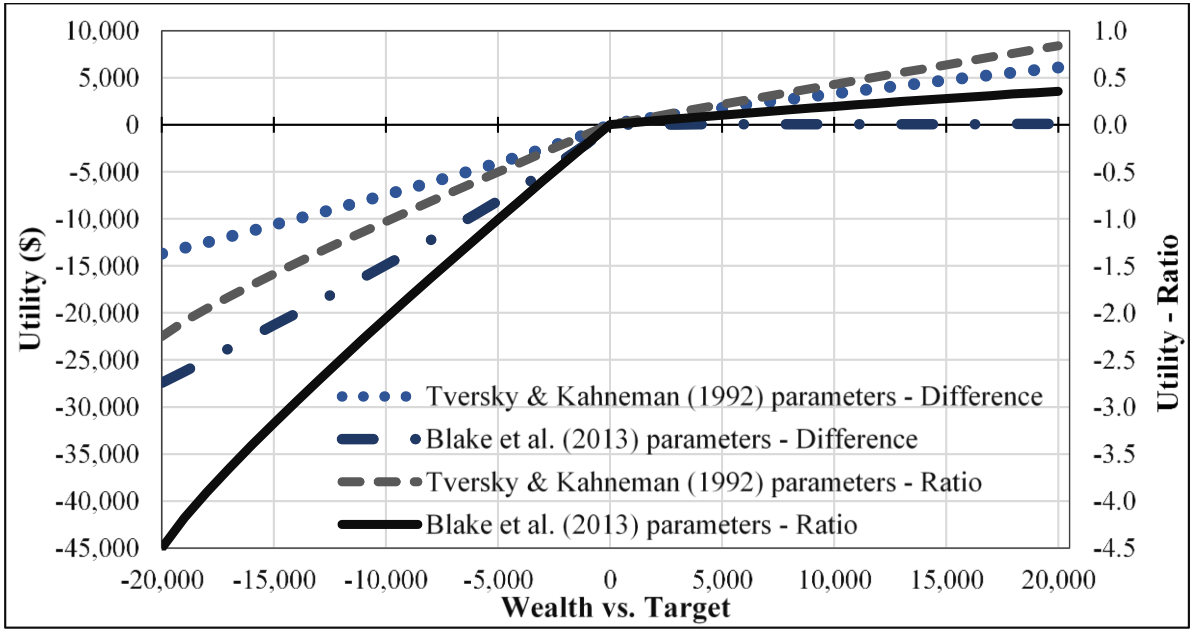




- Power utility results accord with Samuelson and Merton
- Reference dependent results in line with Benartzi and Thaler (1995), Bierman (1998) and Levy and Levy (2017). They are driven by ‘shift’ in the distribution relative to the target as horizon lengthens, due to compound return differences.

Parameterising *(be wary of plugging-in from the literature)*

Comparison of Parameterizations under Reference Dependent Utility



Ratio of Utility Values for Gains vs. Losses * -1

Functional Form:		**Difference**		**Ratio**		**Power Utility**
Parameters:		*Tversky & Kahneman (1992)*	*Blake et al. (2013)*	*Tversky & Kahneman (1992)*	*Blake et al. (2013)*	*CRRA = 4*
Outcomes compared						
+$2,000 vs -$2,000	*±10%*	44.4%	0.8%	43.9%	10.7%	4.56%
+$4,000 vs -$4,000	*±20%*	44.4%	0.6%	43.4%	10.4%	2.47%
+$8,000 vs -$8,000	*±40%*	44.4%	0.4%	42.3%	9.8%	0.66%
+$16,000 vs -$16,000	*±80%*	44.4%	0.3%	39.8%	8.7%	0.01%

Basic approach for forming portfolios in four steps

A. **Choose a utility function** – tailor to investor objectives and preferences

B. **Specify a plan for withdrawal and evaluation**
   - Withdrawals are cash distributed from the portfolio
   - Evaluation is based on withdrawals and/or terminal portfolio value

C. **Project joint distributions of wealth outcomes for the assets**
   - Embed covariance, and allow for reinvestment
   - My paper uses random draws (lognormal) and historical simulation

D. **Solve for optimal portfolio weights**
   - Sequence of portfolio outcomes formed for each path in forward direction, conditional on asset weights, asset returns and withdrawals
   - Outcomes evaluated using the utility function
   - Locate weights that maximise E[U] across paths

Illustration for four investors

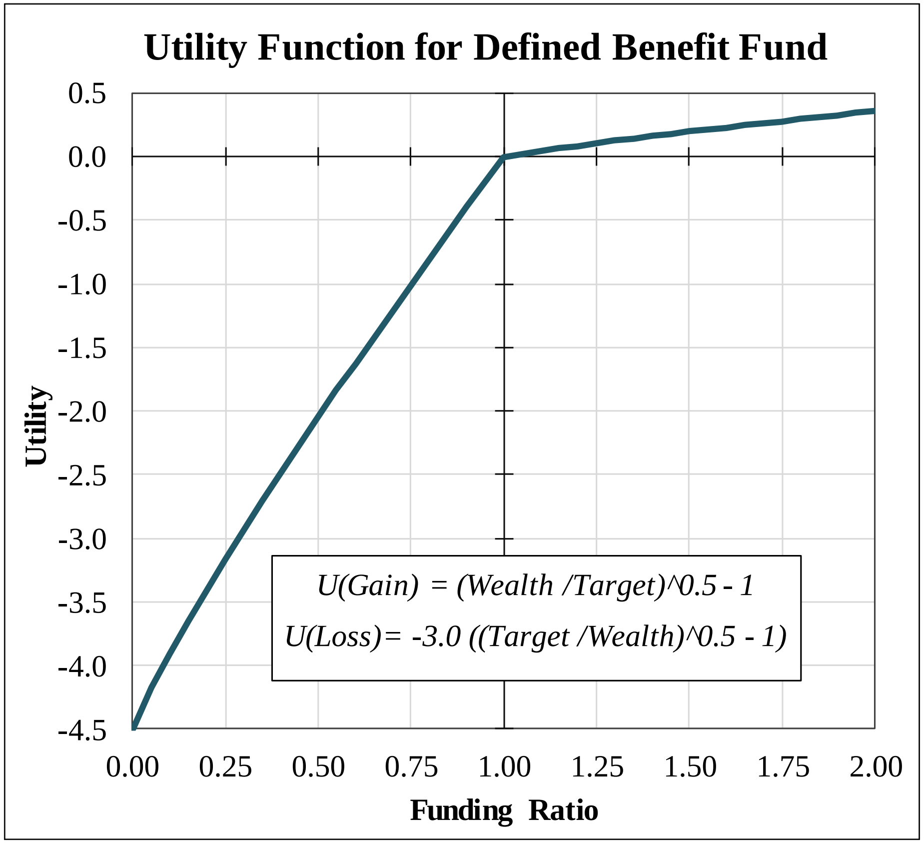
a) **Private investor** *(end-horizon wealth focus)*
- Power utility; CRRA ranging from 1 (log) to 8 examined
- 3- and 10- year horizon

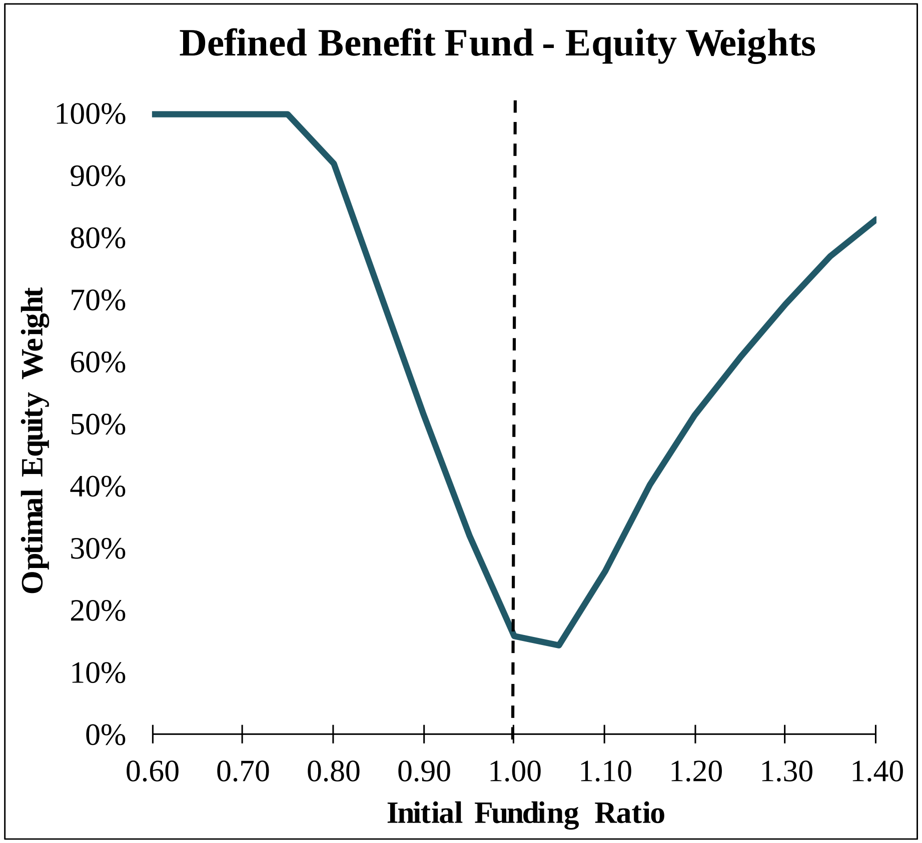
b) **Endowment fund** *(long-term philanthropy)*
- Reference dependent utility; moderate loss aversion
- Target of maintaining distributions and corpus in real terms over 10-years

c) **Defined benefit pension fund** *(the sponsor's perspective)*
- Reference dependent utility over funding ratio in year 3; high loss aversion
- Range of initial funding ratios examined

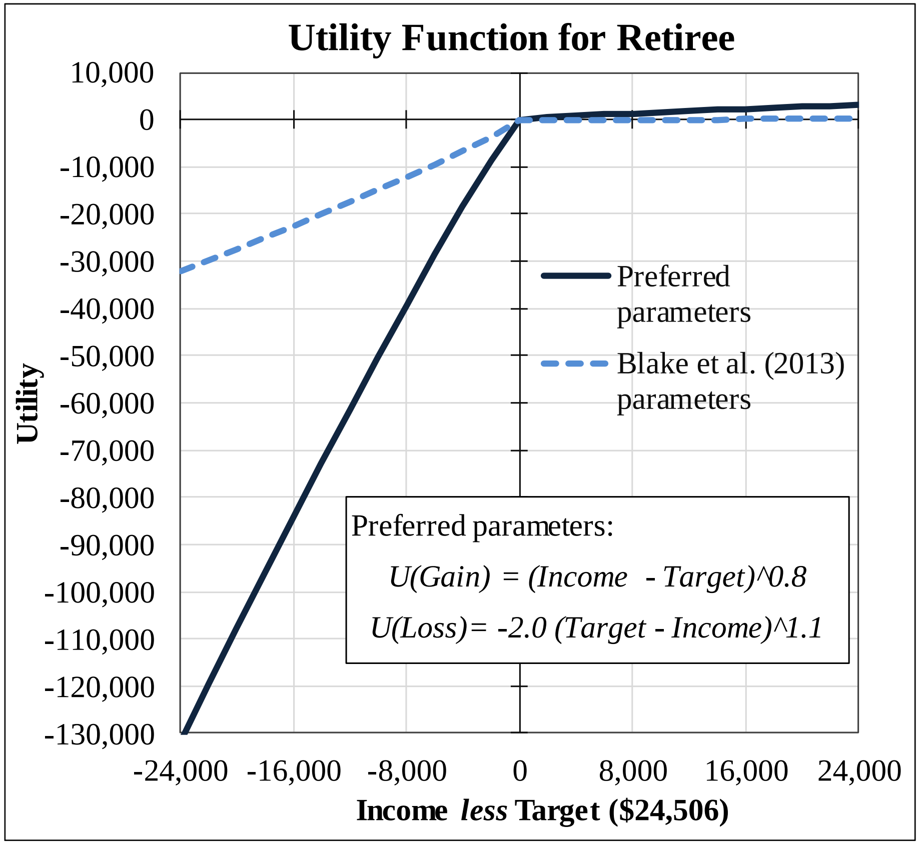
d) **Australian retiree** *(draws income from account based pension)*
- Reference dependent utility with ASFA modest target
- Two sets of utility parameters examined; both high loss aversion
- Simple structure (30-years of income, no pension)

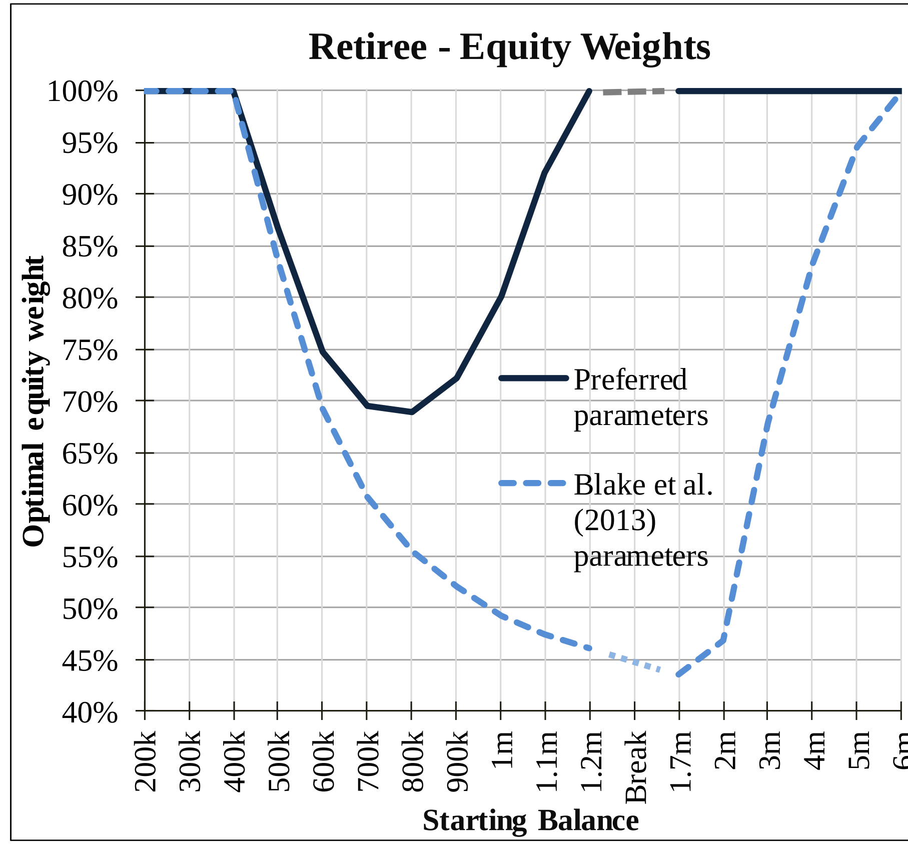
Defined benefit pension fund





Australian retiree





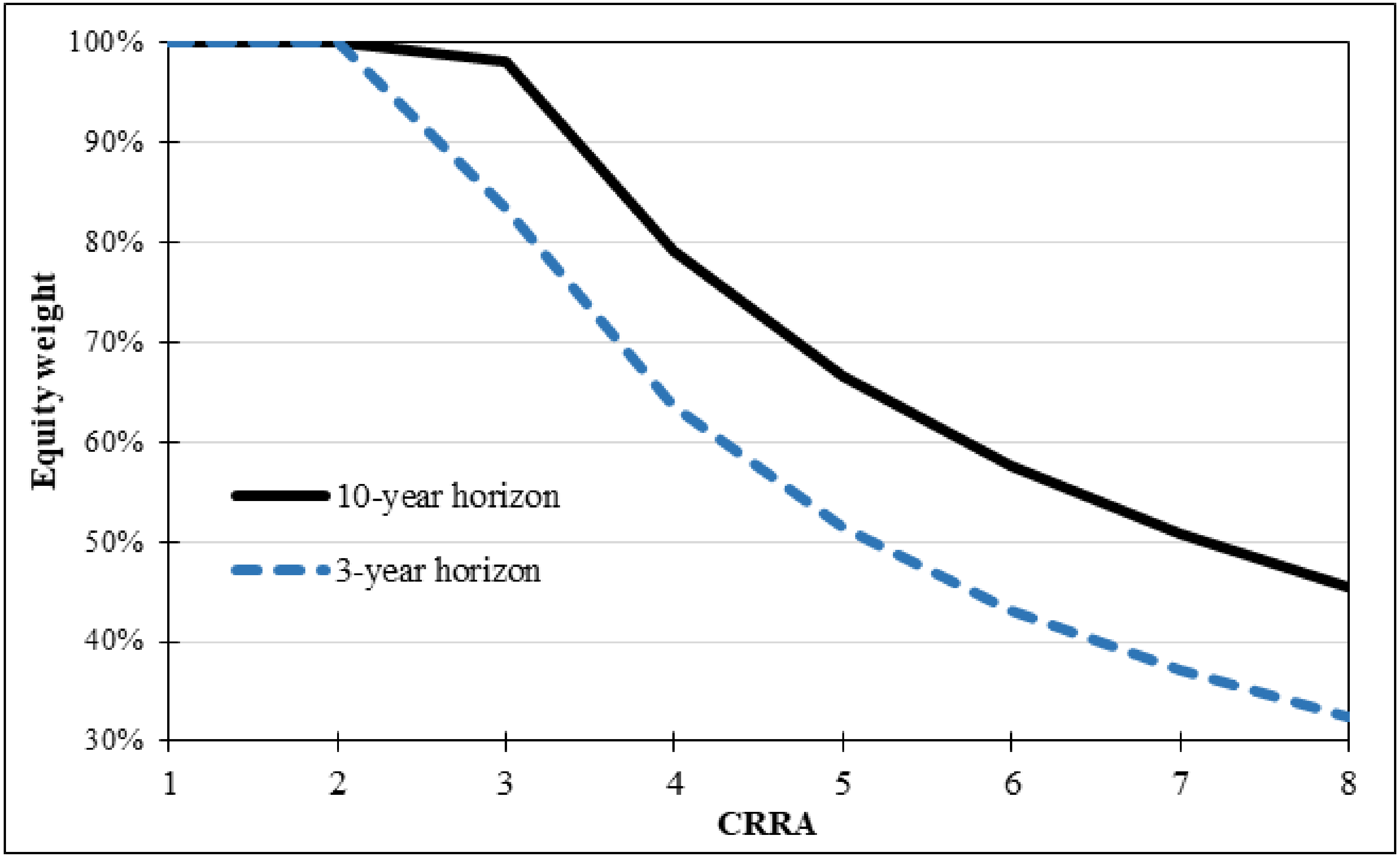
Concluding points

- Utility functions provide an effective and flexible means for forming optimal portfolios across a wide range of circumstances
  - My results were plausible, and sensible
- Tailoring the utility function to the investor is better than drawing directly from the literature, notwithstanding loss of 'validation'
- Optimal portfolios may be formed in practice by conditioning on a predetermined withdrawal plan, and modeling in a forward direction
  - *OK* … a fully optimal joint solution for the withdrawal and investment strategy requires dynamic programming
  - *But* … solving forward may suffice where the withdrawal plan is known, or is limited to a few obvious choices. This is often the case.

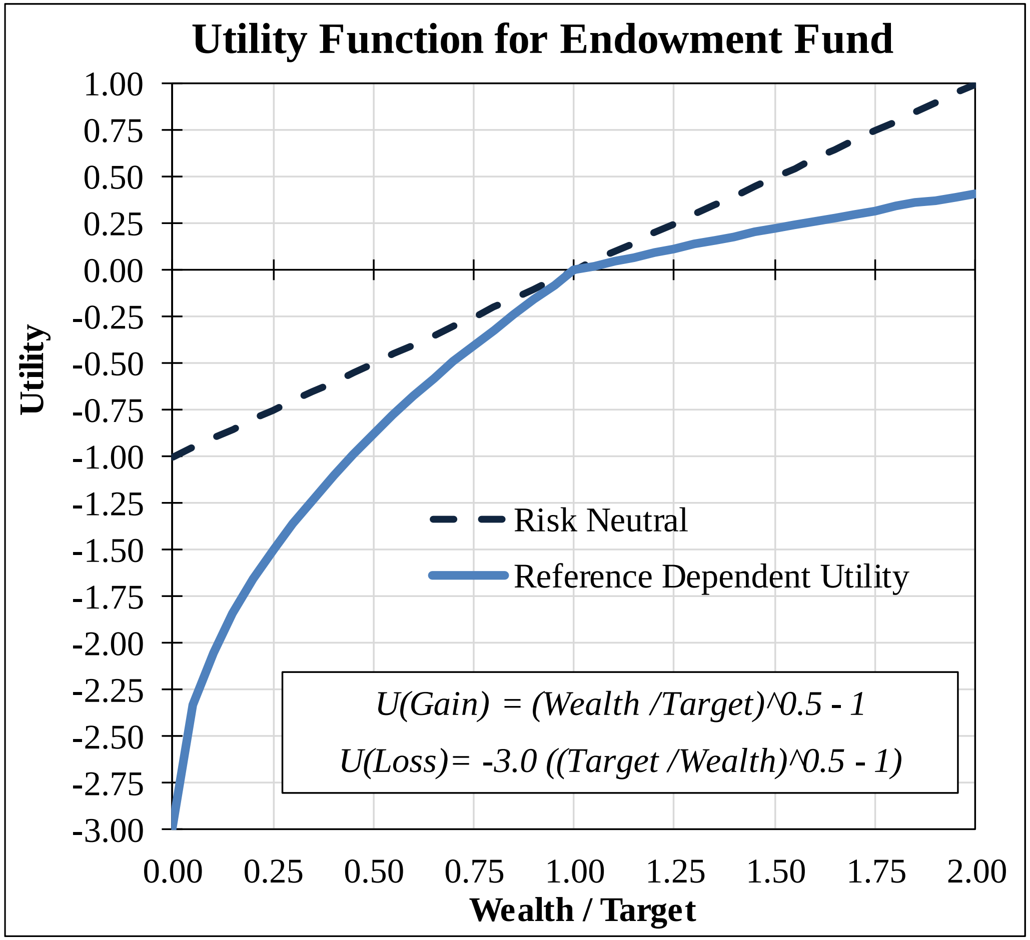
Questions?

Discussion?

Private investor - Optimal equity weights



Endowment fund



- Objective to maintain real value of withdrawals in perpetuity implies reference dependent utility function (ratio form)
- Utility parameters reflect lack of hard commitments, and willingness to place some value on gains
- Evaluates withdrawals over 10-years and year-10 terminal portfolio value, vs. target of maintained in real terms
- Aggregation across time by weighting by level of targets
- 100% equities optimal, unless aversion to losses cranked up

Utility densities under lognormal distribution