CEPAR Colloquium Presentation
A structured investigation of retirement income products

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Introduction

Problem: wide range of retirement income products, difficult to compare guarantee structure and determine value for the policyholder

Research aim

- Modelling: to develop a mathematical framework to represent the guarantee structure in retirement income products
- Evaluation: to comprehensively evaluate such products using utility maximisation and quantitative measures, informed by the behavioural economics literature

Integrated with this aim is the development of a computational framework in `R`. This will enable the framework to be applied to:

- new products which are proposed in the future
- new models for mortality rates or financial returns.

Modelling framework

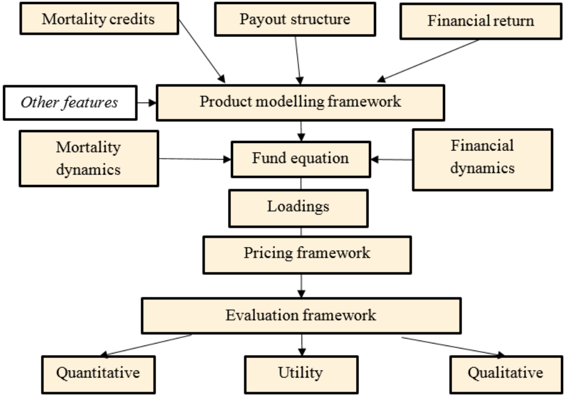
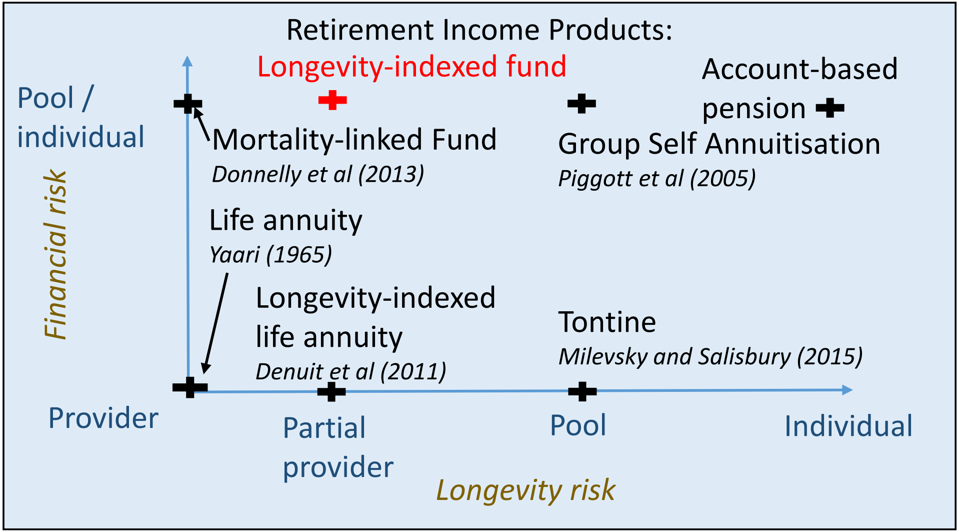


Figure 1: Modelling framework

Annuity product innovation / Product landscape



Focus on guarantee structure

Product	Financial risk	Longevity risk	
		Idiosyncratic	Systematic
Life annuity	Provider	Provider	Provider
Longevity-indexed life annuity	Provider	Provider	Individual
Tontine	Provider	Pool	Pool
Mortality-linked fund	Individual	Provider	Provider
Longevity-indexed fund	Individual	Provider	Individual
Group self annuitisation	Pool	Pool	Pool
Account-based pension	Individual	Individual	Individual

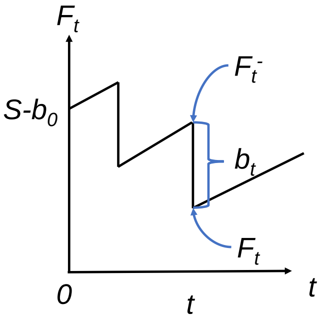
Table 1: Risk-sharing in retirement income products

A unifying framework: The fund equation

The fund equation *for an individual policyholder* is given by:

$$\underbrace{F_t}_{\text{fund value}} = F_{t-1} \underbrace{(1+\Theta_t)}_{\text{mortality credit}} \underbrace{(1+R_t)}_{\text{financial return}} - \underbrace{b_t}_{\text{payout structure}} \qquad F_0 = S - b_0$$

where $b_t = f(F_{t^-}, ...)$ and S is the policyholder's initial investment (Pitacco et al. 2009).

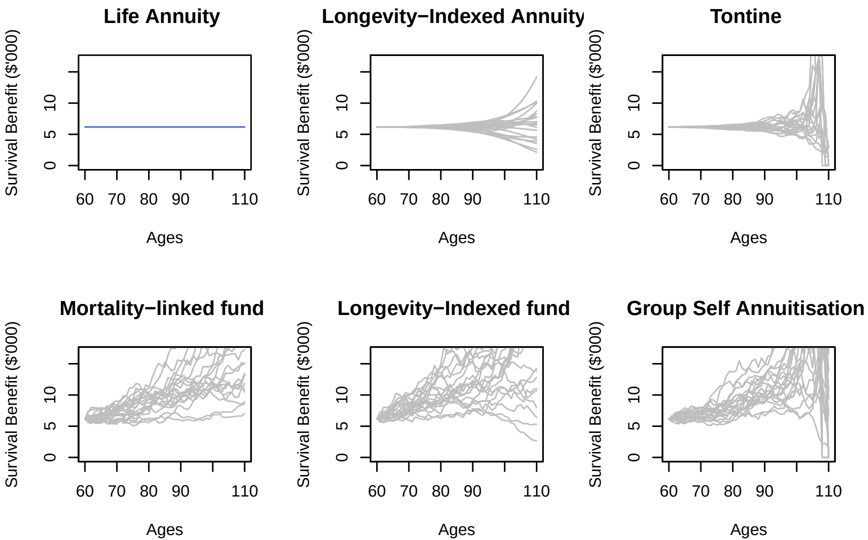


Modelling assumptions

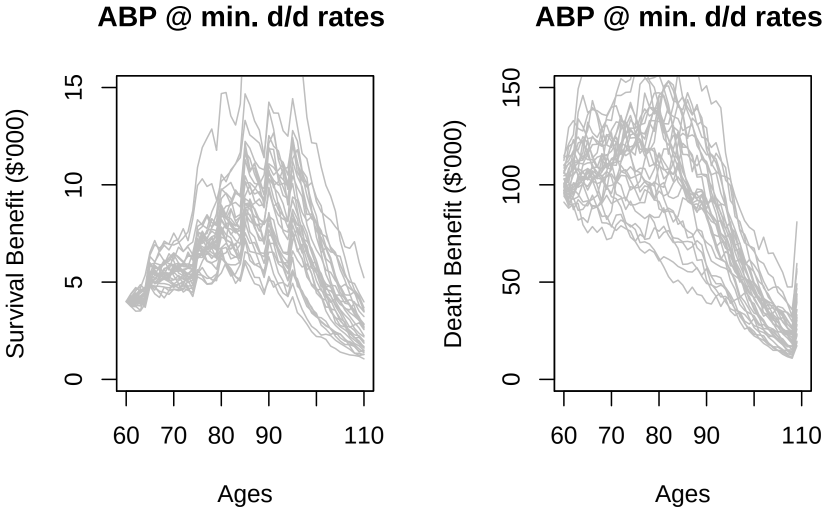
- Mortality environment: Lee-Carter model fitted to Australian Male data from 1967–2016 with simulated idiosyncratic risk
- Financial environment:
  - Risk-free asset which is assumed to return 4%
  - Stock follows a GBM with $\mu = 0.11$ and $\sigma = 0.17$
  - GBM is calibrated to Australian All Ordinaries (Accumulation) data from 1980–2018
- Initial capital for each individual: $100,000
- Pool characteristics: 1000 lives initially all at age 60
- Number of simulations: 5000

For products where the financial risk is taken by the individual, we assume all individuals follow the same strategy of investing 30% in stocks, and 70% in the risk-free asset. For other products, the investment strategy is decided by the provider.

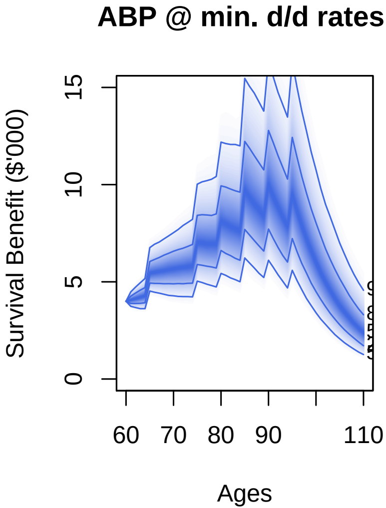
Product benefit profile

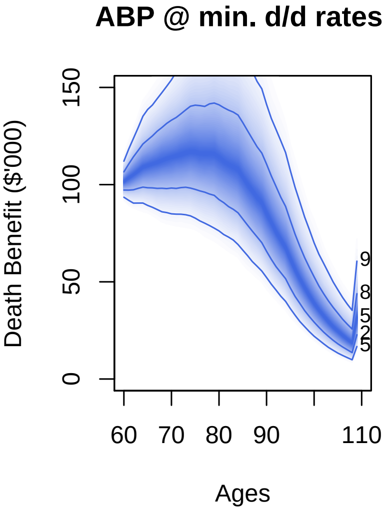


Product benefit profile – Account-based pension



Product benefit profile – Account-based pension





Product riskiness

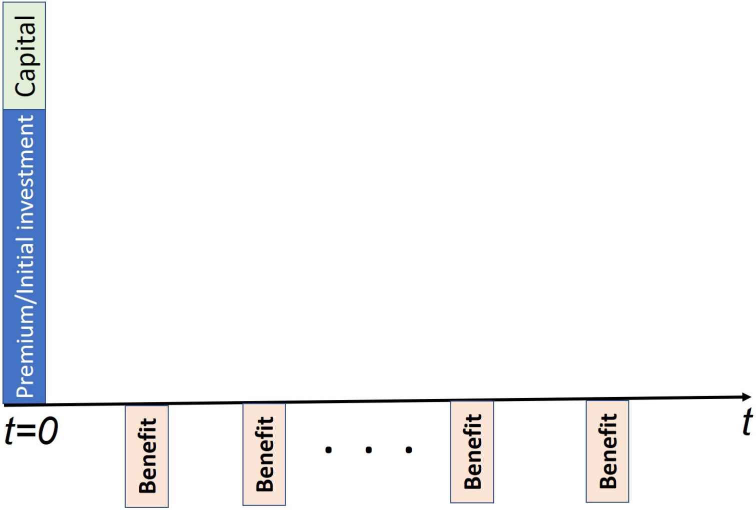


Figure 7: Insurer's cash flows

Capital distribution

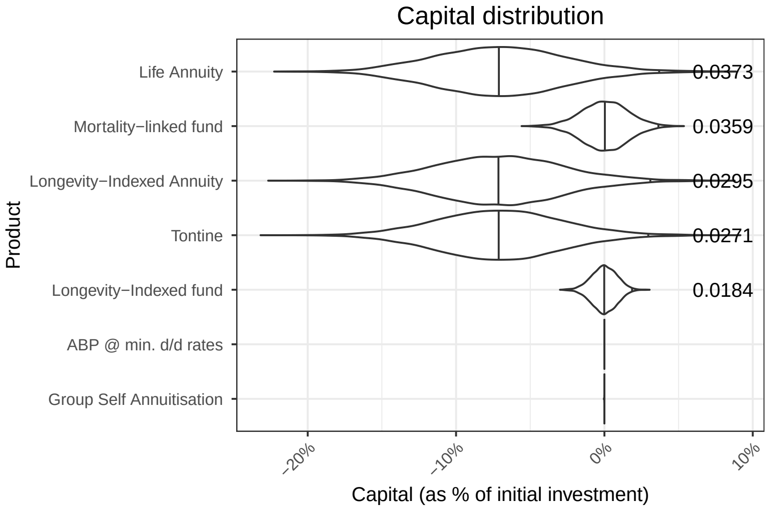


Figure 8: Comparison of simulated capital distribution

Applying loadings to products

- **We need to turn this one-time capital into a price charged to the policyholder.**
- We approximate the required capital each period C_t^* as a constant proportion of the policyholder's fund value at each time
- In each period, the contributor of capital should receive a return equal to $C_t^* \times \text{CoC}$, where the cost of capital CoC is set at 11%.
- We then calculate the NPV discounting using the risk-free rate, and express as a percentage of S.

Product	Price (%)
Life annuity	5.13
LLLA	4.07
Tontine	3.74
MLF	6.27
LIF	3.22
GSA	0
ABP	0

Apply loadings to products

The policyholder's initial investment with loadings S^* should equal their investment without loadings S plus the loading charged p, which is a percentage of S, i.e.:

$$S^* = Sp + S$$

Since the fund equation does not assume loadings, we set the same loaded price S^* for all contracts and solve for the equivalent unloaded price $S = \frac{S^*}{1+p}$.

Product comparison from the annuitant perspective

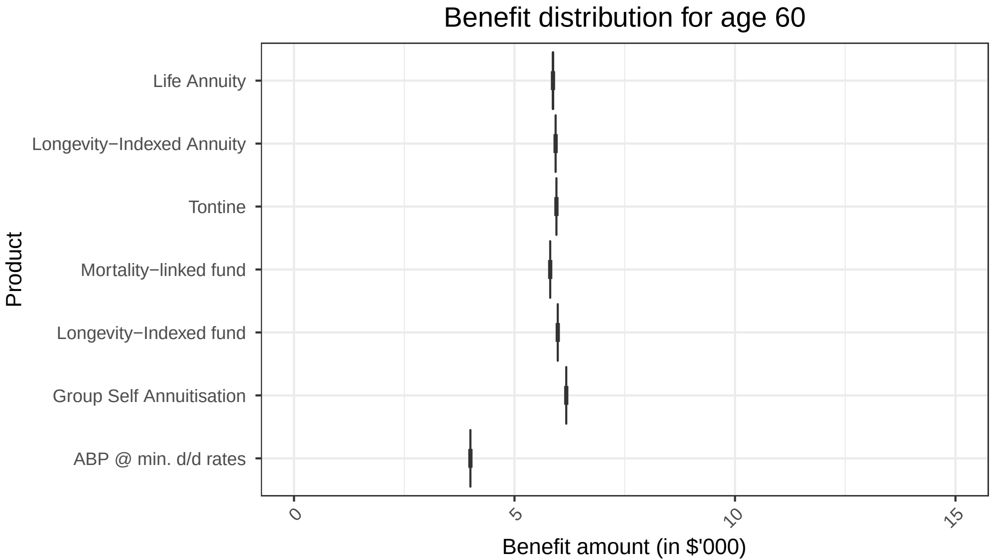
- We re-simulate the products according to the loadings in the previous slide
- **They are now able to be fairly compared, taking into the cost of meeting the financial and longevity guarantees**

We focus on two measures applied to the *loaded benefit payouts*:

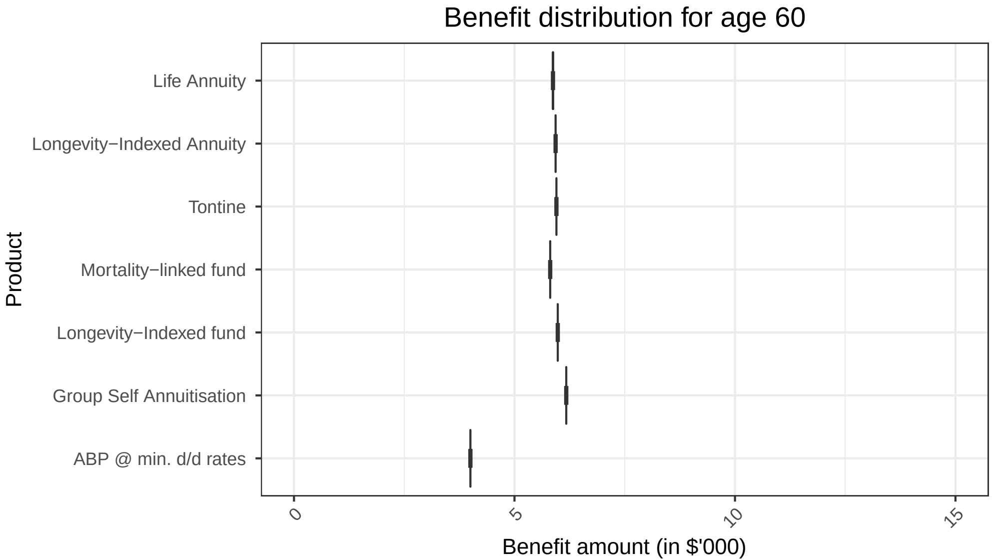
- Risk-based metrics
  - Distribution of benefit payout at certain ages
  - Actuarial present value
  - Australian Government Actuary risk measure
- Utility

Note: The mortality assumption for the utility metric ${}_tp_x$ is defined as the simulated pool's mortality, with 1000 lives.

Distribution of benefit payout at certain ages



Distribution of benefit payout at certain ages



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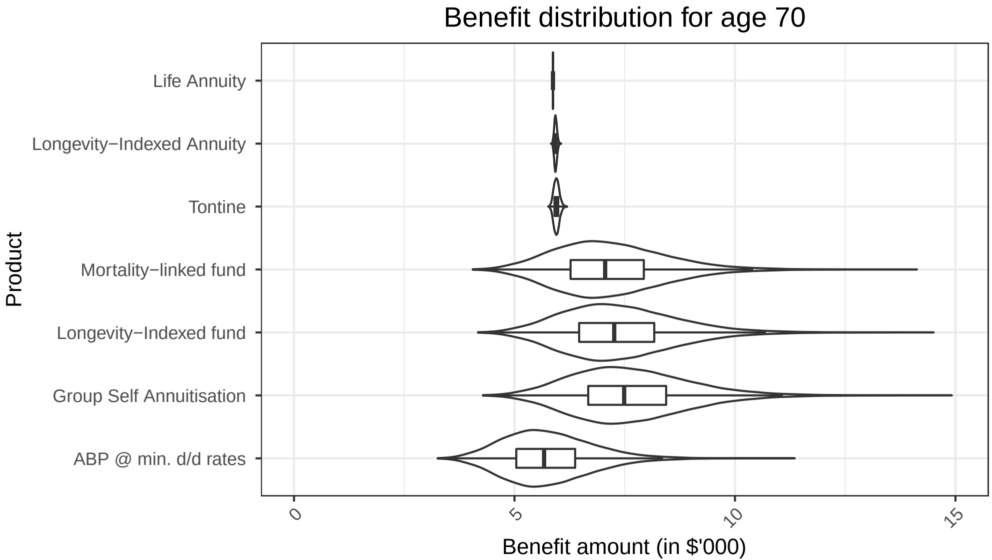
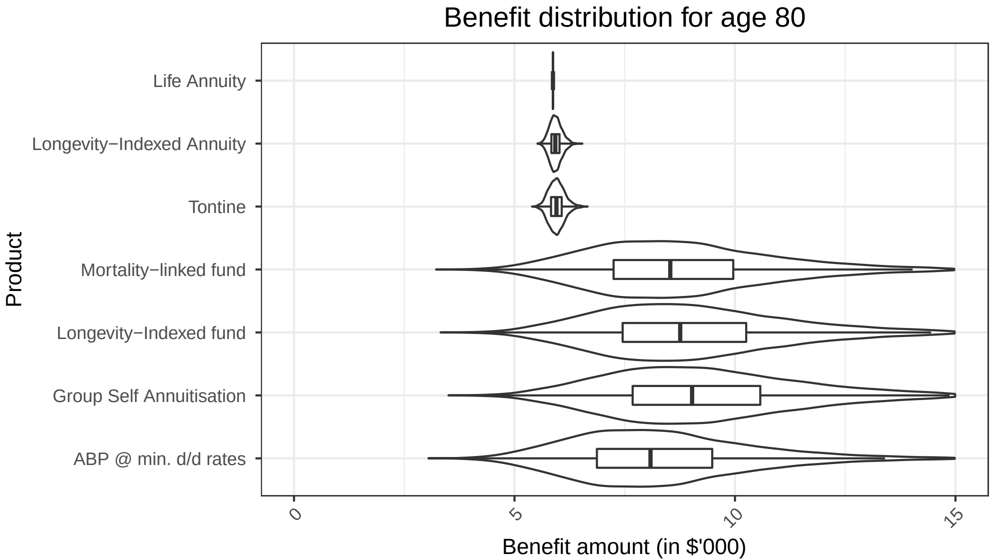
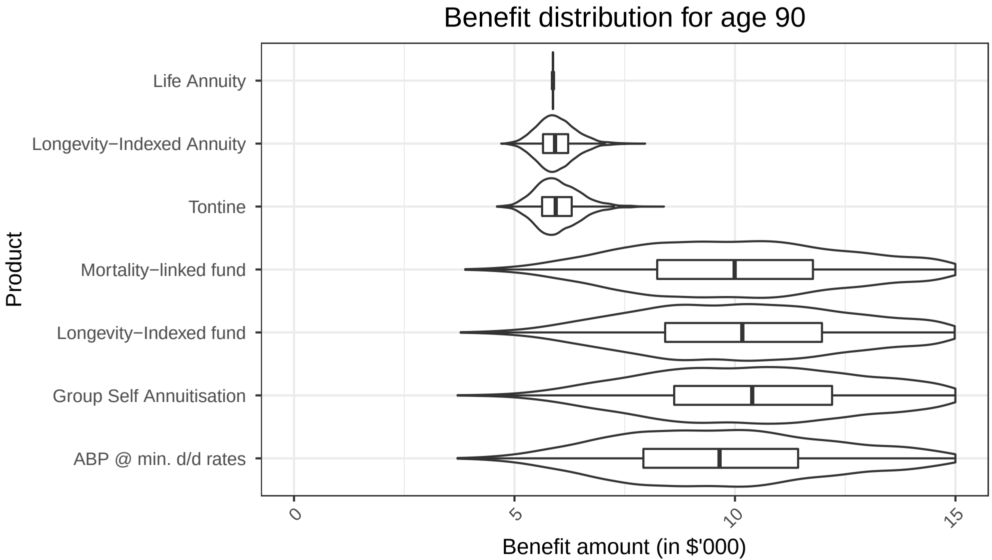


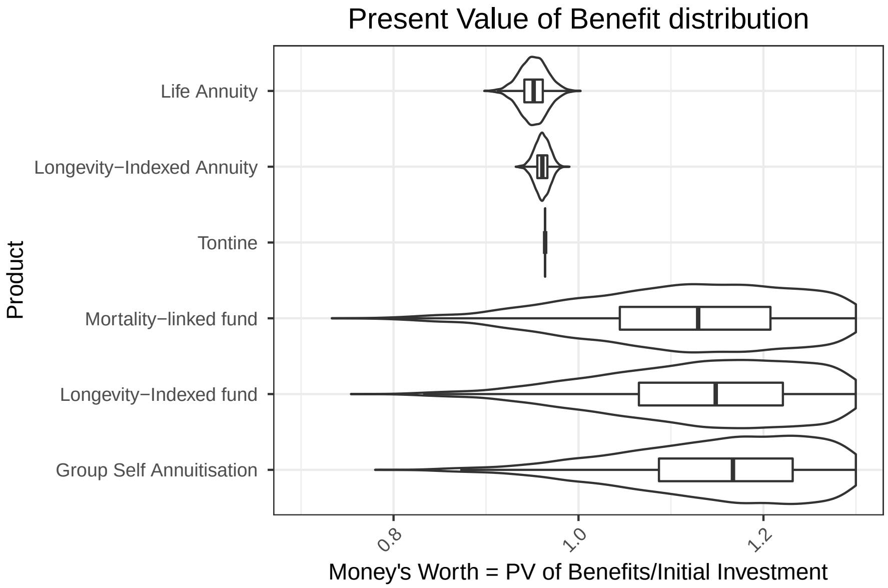
Distribution of benefit payout at certain ages



Distribution of benefit payout at certain ages



Actuarial present value (Money's worth)



Results - AGA risk measure

- We calculate the truncated semi-deviation with reference to the initial benefit.
- Average across all simulations
- We calculate the measure based on nominal payments.

Product	Initial benefit ($)	AGA (%)	Rank
Life annuity	5872	0	1
LLLA	5932	5.99	5
Tontine	5951	10.31	7
MLF	5810	1.08	2
LIF	5981	1.34	3
GSA	6174	2.61	4
ABP	4000	9.93	6

Utility - CRRA

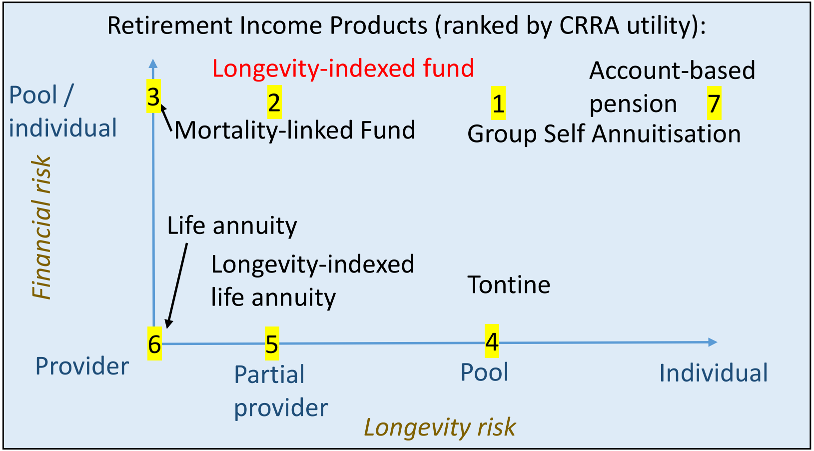
We now present results under the CRRA utility framework.

ABP ignores the bequest component.

- CRRA: $U(c_t) = \frac{c_t^{1-\rho}-1}{1-\rho}$
- Set $\rho = 2$ as the relative risk aversion parameter (Hanewald, Piggott, and Sherris 2013).
- Discounted utility: $U_0 = E_0\left[\sum_{t=0}^{\omega-x} {}_tp_x \beta^t U(c_t)\right]$
- Set $\beta = 0.98$ as the time preference parameter

Product	Certainty equivalent	Ranking
Life annuity	5872.46	6
LLLA	5921.34	5
Tontine	5937.25	4
MLF	7217.9	3
LIF	7422	2
GSA	7658.78	1
ABP	5689.27	7

Ranking of products



Conclusions

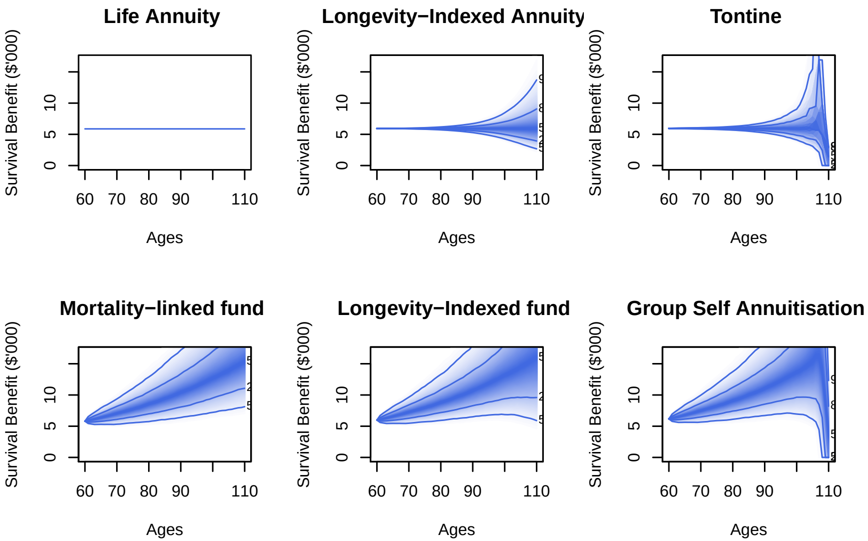
- We have demonstrated a framework to model and evaluate retirement income products
- This framework takes into account the differences in guarantee structure, and hence, the riskiness of the products
- Financial risk appears to be more of a concern throught the lifetime
- Longevity risk is acute at very old ages
- The code is modular and can be easily extended to new products using R
- The ranking of products differs according to the evaluation metric chosen

Limitations and extensions

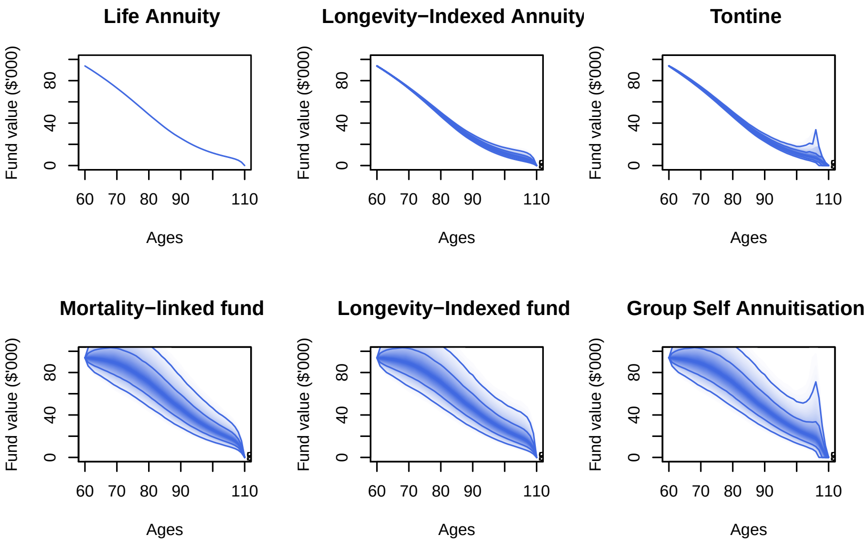
- We need to further analyse the impact of different financial and mortality models on our results
- We can incoporate more sophistcated methods for the calculation of the capital and prices
- We can extend this framework to incorporate other features and hybrid products

Thank you!

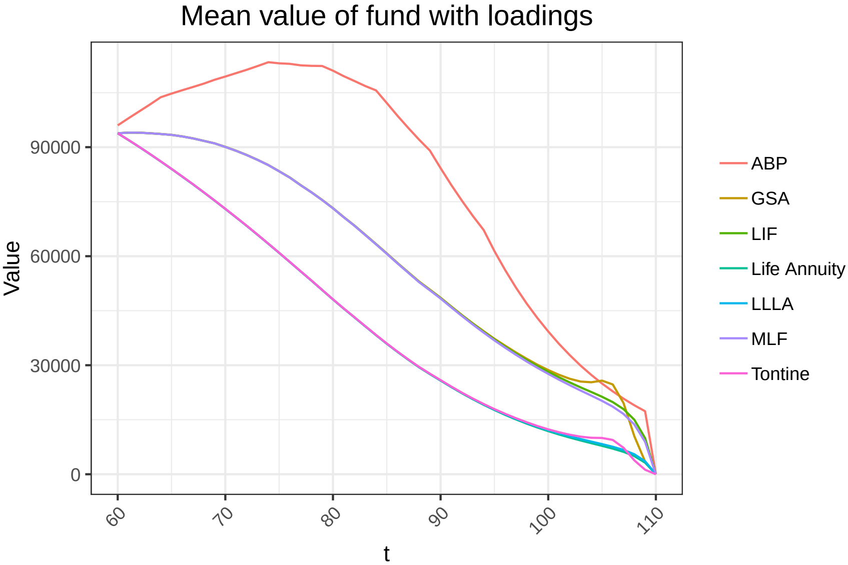
Appendix - benefit payout with loadings



Appendix - policyholder's fund value with loadings



Appendix - comparison of fund value



References I

Hanewald, Katja, John Piggott, and Michael Sherris. 2013. "Individual post-retirement longevity risk management under systematic mortality risk." *Insurance: Mathematics and Economics* 52 (1). Elsevier B.V.: 87–97. doi:10.1016/j.insmatheco.2012.11.002.

Pitacco, Ermanno, Michel Denuit, Steven Haberman, and Annamaria Olivieri. 2009. *Modelling longevity dynamics for pensions and annuity business*. Oxford: Oxford University Press.