Model Risk for Pricing Guaranteed Lifetime Withdrawal Benefits

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UNSW, Nov, 2019

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Outline

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    - Challenges
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    - VA+GMBs Valuation
    - Equity Processes
    - Computational Methods
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    - GMWB pricing results
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Overview

What are GLWBs?

Variable annuities (VAs) with guaranteed lifetime withdrawal benefits (GMB) riders.

- Lifetime version of Guaranteed Minimum Withdrawal Benefits (GMWBs)
- Provide lifetime withdrawals regardless of investment performance
- The highest election: GLB - 72 percent
- GLB: GMAB, GMIB, GMWB, GLWB
- Total GLWBs Assets were $561B as of 2015

Challenges

Traditional Black-Scholes framework:

- Underestimate the tail events
- No presence of jumps or stochastic volatility

Complicated equity models (Lévy processes):

- No analytical formula for option prices
- Pricing via characteristic function

Efficient numerical techniques (COS, SWIFT, etc.):

- Accuracy and efficiency in long term

What do we study?
A comparison on the equity models for GLWBs with stochastic mortality

Research Objective

Objective: Model risk of GLWBs pricing

- different equity models
  (10 models: Jump-diffusion, Lévy, Time-changed Lévy)
- different computational methods
  (ASCOS, ASWIFT)
- stochastic mortality
  (Cohort Affine (Xu et al., 2018))

To show: GMWBs and GLWBs

- Importance of jumps and stochastic volatility
- The efficient ASWIFT method
- Cohort effect on pricing GLWBs

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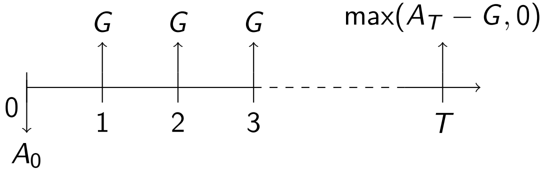
GMWB Valuation

Cash flow for GMWB:



Fund dynamics:

$$dA_t = \frac{dS_t}{S_t} A_t - (mA_t + g)dt,$$

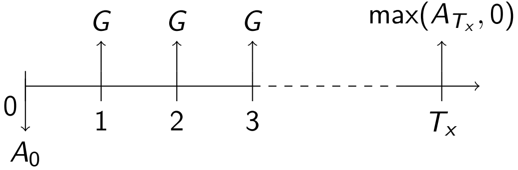
GMWB payoff:

$$V_0 = \frac{1}{T} A_0 \sum_{i=1}^{T} e^{-ir\Delta t} + e^{-rT} E^{\mathbb{Q}}[\max(\tilde{A}_T, 0)] = A_0$$

$E^{\mathbb{Q}}[\max(\tilde{A}_T, 0)]$ as Floating strike Asian call option

GLWB Valuation

Cash flow for GLWB:



GLWB payoff:

$$A_0 = \int_0^{T_x} G e^{-rs} ds + E^{\mathbb{Q}}\left[e^{-rT_x} A_{T_x} \mathbf{1}_{\{A_{T_x}>0\}}\right] \quad (1)$$

Approximated as,

$$A_0 = \sum_{j=1}^{T_{\max}} \left(\mathcal{S}_x(0, j-1) - \mathcal{S}_x(0, j)\right) V_0^{GMWB}(j) \quad (2)$$

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Stochastic processes

- Standard model: the Black-Scholes model;
- Diffusion model: the Heston model;
- Jump-diffusion model: the Merton model, the Bates model;
- Lévy process: the VG model, the NIG model;
- Time-changed Lévy process: VG-OU, NIG-OU, VG-CIR, NIG-CIR.

The equity process as exponential of L_t,

$$S_t = S_0 \exp\left(L_t\right).$$

Equivalent Martingale Measure

In the risk neutral setting,

$$\tilde{S}_t = \exp\left(-(r-q)t\right) S_t.$$

Mean-correcting adjustments from $\mathbb{P}$ to $\mathbb{Q}$,

- Additive

$$\tilde{S}_t = S_0 \exp\left((r-q)t + \tilde{L}_t - \omega_t^L\right).$$

- Multiplicative

$$\tilde{S}_t = \frac{S_0 \exp\left((r-q)t + L_t\right)}{\omega^L},$$

where

$$\omega_t^L = E[L_t] = \log\left(\phi_{L_t}(-i)\right). \tag{3}$$

Characteristic functions

1. Black-Scholes

$$\phi_t(u) = \exp\left(iu\mu t - \frac{u^2\sigma^2}{2}t\right)$$

2. Variance-Gamma (VG)

$$\phi_t(u) = \left(\frac{GM}{GM + (M-G)iu + u^2}\right)^{Ct}$$

3. Normal Inverse Gaussian (NIG)

$$\phi_t(u) = \exp\left(-\delta t(\sqrt{\alpha^2 - (\beta + iu)^2} - \sqrt{\alpha^2 - \beta^2})\right)$$

4. Merton

$$\phi_t(u) = \exp\left(iu\mu_J t - \frac{\sigma^2 u}{2}t + \lambda t\left(\exp(i\mu_J u - \frac{1}{2\sigma_J^2 u^2} - 1)\right)\right)$$

Stochastic volatility processes

Two ways to describe stochastic volatility:

- Another process for volatility
- Time-changed Lévy processes

5. Heston model: $\phi(u)$ available in closed form by the affine property

$$\frac{dS_t}{S_t} = (r-q)dt + \sqrt{v_t}dW_t$$
$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}d\tilde{W}_t,$$

6. Bates model: Heston + Merton jump diffusions (MJD)

$$\phi_L(u) = \phi_{Heston}(u)\phi_{Merton}(u)$$

Time-changed Lévy processes

The time-changed Lévy process $Y_t \equiv L_{T_t}$ is obtained by evaluating the Lévy process L at random time T

$$\phi_{Y_t}(u) = E[e^{-T_t \psi(u)}] = \mathcal{L}_{T_t}(\psi(u))$$

If tractable, the characteristic function:

$$\phi_{Y_t}(u) = \exp(iu(r-q)t) \frac{\phi_T(-i\psi_L(u); t, T_0)}{\phi_T(-i\psi_L(-i); t, T_0)^{iu}}$$

Common stochastic clock:

- Cox-Ingersoll-Ross (CIR)
- Ornstein-Uhlenbeck (OU)

Integrated processes assumed in this research:

- VG-CIR
- NIG-CIR
- VG-Γ-OU
- NIG-Γ-OU

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Computational Methods

Numerically compute the financial derivatives in the Fourier domain:

1. ASCOS (Zhang and Oosterlee, 2013)
   Extend the Fourier-Cosine (COS) method (Fang and Oosterlee, 2008)
2. ASWIFT (Leitao et al., 2018)
   Extend the Shannon wavelet inverse Fourier technique (SWIFT) (Ortiz-Gracia and Oosterlee, 2016)

COS - Fourier Cosine expansions

Recall for European options, $x = \ln(S_0/K), y = \ln(S_T/K)$,

$$v(x,t) = e^{-(T-t)} \int_{-\infty}^{\infty} v(y,T) f(y|x) dy,$$

$$f(y|x) \approx \frac{2}{b-a} \sum_{k=0}^{N-1}{}' \Re\left\{ \phi(\frac{k\pi}{b-a}; x) \exp(-i\frac{ka\pi}{b-a}) \right\} \cos\left(k\pi \frac{x-a}{b-a} \right)$$

where pay-off coefficients V_k in closed form,

$$v(x,t) \approx \sum_{k=0}^{N-1}{}' \Re\left\{ \phi(\frac{k\pi}{b-a}; x) \exp(-ik\pi \frac{a}{b-a}) \right\} V_k$$

Asian option

Define new series R_j for monitoring dates $[1, M]$,

$$R_j = \log \frac{S_j}{S_{j-1}}, \quad j = 1, \ldots, M$$

Define Y_j where $Y_1 = R_M$ and $Y_j = R_{M+1-j} + Z_{j-1}$

$$Y_j = \log\left(\frac{S_{M-j-1}}{S_{M-j}} + \frac{S_{M-j-2}}{S_{M-j}} \cdots + \frac{S_M}{S_{M-j}}\right).$$

$$\frac{1}{M}\sum_{j=1}^{n} S_j = \frac{\exp(Y_M)S_0}{M}$$

The arithmatic Asian option therefore can be valued by,

$$v(x_0, t_0) = e^{-r\Delta t}\int_{-\infty}^{\infty} v(y, T) f_{Y_{M(y)}} dy$$

Asian option - Cont'd

Density recoveries:

$$\phi_{Y_j}(u) = \phi_{R_{M+1-j}}(u)\phi_{Z_{j-1}}(u) = \phi_{R_M}(u)\phi_{Z_{j-1}}(u)$$

Implementing the COS method on $\phi_{Z_{j-1}}(u) = E[e^{iu\log(1+\exp(Y_{j-1})}]$

$$\begin{aligned}\hat{\phi}_{Z_{j-1}}(u) &= \int_a^b (e^x+1)^{iu} f_{Y_{j-1}}(x)dx \\ &= \frac{2}{b-a}\sum_{k=0}^{N-1}{}'\Re\{\hat{\phi}_{Y_{j-1}}(\frac{l\pi}{b-a})\exp(-ia)\frac{l\pi}{b-a}) \\ &\int_a^b (e^x+1)^{iu}\cos\left((x-a)\frac{l\pi}{b-a}\right)dx\},\end{aligned}$$

Solved by Clenshaw-Curtis Quadrature $\Phi_{j-1} = M\mathbf{A}_{j-1}$,

The ASCOS Method

The algorithmic summary:

- Calculate $M(k,l)$ by the CC-Quad and set $\phi_{Y_1} = \phi_R$
- Recover ϕ_{Y_M}, for $j = 2, \ldots, M$
  - Compute Φ_{j-1} with $\hat{\phi}_{Z_{j-1}}(u_k)$, for $k = 1, \ldots N$,
  - Recover $\hat{\phi}_{Y_j}(u_k)$, for $k = 1, \ldots N$.
- Compute $\hat{v}(x_0, t_0)$ with $\hat{\phi}_{Y_M}(u_k)$

Only need to calculate Φ_{j-1} once

SWIFT - Shannon wavelet inverse Fourier technique

Recall for European options, $x = \ln(S_0/K), y = \ln(S_T/K)$,

$$v(x,t) = e^{-(T-t)} \int_{-\infty}^{\infty} v(y,T) f(y|x) dy,$$

The density function can be written with Shannon wavelets,

$$f(y|x) \approx f_m(y|x) = \sum_{k=k_1}^{k_2} c_{m,k}(x) \Phi_{m,k}(y),$$

The scaling coefficients,

$$c_{m,k} = 2^{m/2} \int_{\mathbb{R}} f(x) \Phi(2^m x - k) dx \quad \text{as real part of harmonic series}$$

where the payoff coefficients,

$$V^{\alpha}_{m,k} = \int_{\mathcal{I}_m} v(y,T) \Phi_{m,k}(y) dy$$

Asian option - Cont'd

Similar density recoveries:

$$\hat{f}_{Y_1}(\xi) = \hat{f}_R(\xi), \quad \hat{f}_{Y_j}(\xi) = \hat{f}_{R_M}(u)\hat{f}_{Z_{j-1}}(\xi),$$

$$\hat{f}_{Z_{j-1}}(\xi) = E[e^{i\xi \log(1+\exp(Y_{j-1}))}]$$

$$= 2^{\frac{m}{2}} \sum_{k=k_1}^{k_2} c_{m,k}^{j-1} \int_{-\infty}^{\infty} (e^x + 1)^{-i\xi} \operatorname{sinc}(2^m x - k) dx.$$

With the transition matrix

$$M(n,k) = 2^{m/2} \int_{-\infty}^{\infty} (e^x + 1)^{-i\xi_n} \operatorname{sinc}(2^m x - k) dx,$$

Z can be found recursively,

$$\hat{f}_{Z_j}(\xi) = M(n,k) c_{m,k}^{j}.$$

The ASWIFT Method

The algorithmic summary:

- Calculate $M(n,k)$ by direct function and set $\hat{f}_{Y_1} = \hat{f}_R$
- Recover $\hat{f}_{Y_M}$, for $j = 2, \ldots, M$
  - Compute $\hat{f}_{Z_j}(\xi) = M(n,k)c^j_{m,k}$,
  - Recover $\hat{f}_{Y_{j+1}}(\xi) = \hat{f}_R(\xi)\hat{f}_{Z_j}(\xi)$.
- Compute $\hat{v}(x_0, t_0)$ with $\hat{f}_{Y_M}(\xi)$

Only need to calculate $\hat{f}_{Z_j}(\xi)$ once

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Stochastic Mortality

The T-year survival probability for an x-year-old entered PH at time t,

$$\mathcal{S}_x(t,T) = E^{\mathbb{Q}}\left[\exp - \int_t^T \mu_x^i(s)ds\right]$$
$$= \exp\left(\mathcal{B}_1(t,T)X_1(t) + \mathcal{B}_2(t,T)X_2(t) + \mathcal{B}_3^i(t,T)Z^i(t) + \mathcal{A}^i(t,T)\right)$$

where $\mathcal{B}(t,T) = \frac{1-e^{-\delta(T-t)}}{\delta}$.

The coefficients $\mathcal{A}, \mathcal{B}$ are available in closed form for affine property.

Estimations are based on US Male cohort-based mortality (Huang et al., 2018).

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Calibration Result

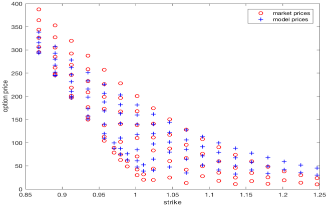
Data: 95 S&P 500 call option prices on Dec. 8, 2016

Minimizing the root-mean-square error (RMSE) $:= \sqrt{\frac{\sum(O_l^{market} - O_l^{model})^2}{N}}$

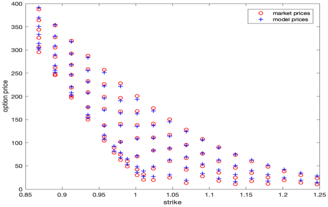
Model	Calibration Result				
BS	σ	RMSE			
	0.1644	19.1726			
VG	C	G	M	RMSE	
	0.5351	3.4765	12.2341	5.7170	
NIG	α	β	δ	RMSE	
	5.9368	-4.6975	0.0906	4.7740	
MJD	σ	μ_J	σ_J	λ	RMSE
	0.1100	-1.0000	0.0020	0.0629	4.5798
Heston	v	θ	κ	σ	ρ	RMSE
	0.0250	0.1100	0.5499	0.7456	-0.7000	5.8816
Bates	v	θ	κ	σ	ρ	
	0.0250	0.1192	0.6227	0.9499	-0.7000	
	λ	μ_J	σ_J	RMSE		
	0.0103	0.1000	0.0484	5.9087		
VG-CIR	C	G	M			
	1.5890	8.4303	18.8006			
	κ	η	λ	RMSE		
	0.7679	1.3298	1.9957	2.4795		
NIG-CIR	α	β	δ			
	11.0675	-6.2444	0.1705			
	κ	η	λ	RMSE		
	0.8902	1.2112	1.9815	2.1050		

Calibration plot for selected models

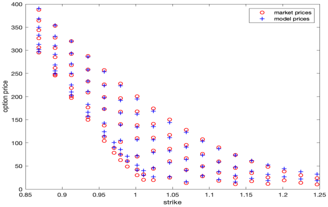
Black-Scholes



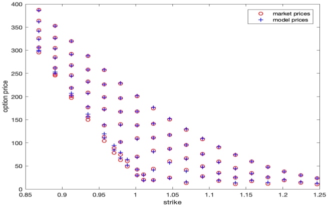
Merton Jump Diffusion



Bates

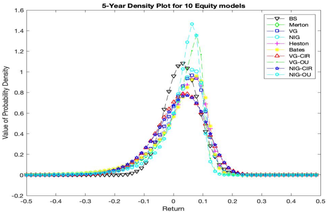


VG-CIR

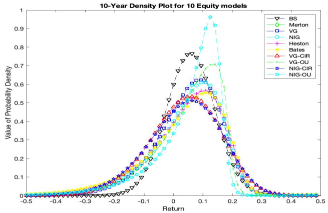


Risk-neutral density plot (r = 0.05)

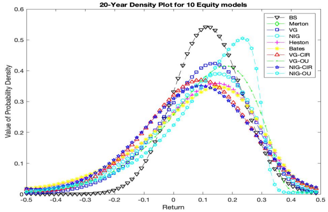
5-year



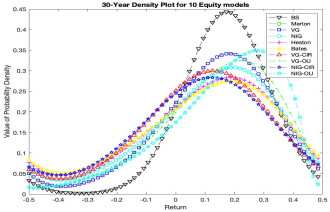
10-year



20-year



30-year



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GMWB calculations - no mortality

Black-Scholes framework: $r = 0.05, \sigma = 0.2$

Comparison with Liu (Liu, 2010) and Luo (Luo and Shevchenko, 2015)

Table: GMWB yearly fee

GMWB	Maturity	h=1	Liu	ASCOS	ASWIFT
g	T	annuity	q(bps)	q(bps)	q(bps)
0.05	20	61.64	27.65	27.65	27.65
0.1	10	76.74	92.41	92.41	92.41

Table: GMWB quarterly fee

rate, g	T	bps (Luo)	bps (ASCOS)	bps (ASWIFT)
4%	25	17.69	17.69	17.69
5%	20	28.33	28.33	28.33
10%	10	95.81	95.81	95.81

GMWB results - 10 year

All equity models: $r = 0.05$ with calibrated parameters

Table: 10-year GMWB fee

T = 10	L=12	ASWIFT (m=6)		ASCOS (nq = 400)	
Model	Fee (bps)	Option	GMWB	Option	GMWB
BS	56.21	23.257	100.000	23.257	100.000
VG	60.54	23.254	99.997	23.254	99.997
NIG	62.31	23.257	100.000	23.257	100.000
MJD	68.04	23.257	100.000	23.257	100.000
Heston	60.61	23.256	99.999	23.260	100.003
Bates	63.61	23.255	99.998	23.274	100.017
VG-CIR	72.44	23.257	100.000	23.257	100.000
VG-OU	59.11	23.258	100.001	23.258	100.001
NIG-CIR	69.82	23.257	100.000	23.257	100.000
NIG-OU	62.16	23.257	99.999	23.257	99.999

GMWB results - 20 year

All equity models: $r = 0.05$ with calibrated parameters

Table: 20-year GMWB fee

T = 20	L=12	ASWIFT (m=6)		ASCOS (nq = 400)	
Model	Fee (bps)	Option	GMWB	Option	GMWB
BS	14.79	38.355	100.000	38.355	100.000
VG	16.45	38.357	100.002	38.355	99.999
NIG	17.25	38.357	100.002	38.355	100.000
MJD	19.84	38.357	100.002	38.355	100.000
Heston	16.77	38.356	100.001	38.355	100.000
Bates	18.14	38.356	100.001	38.355	100.000
VG-CIR	20.37	38.354	99.999	38.355	99.999
VG-OU	15.80	38.354	99.999	38.355	100.000
NIG-CIR	19.43	38.355	100.000	38.355	100.000
NIG-OU	16.93	38.355	100.000	38.355	100.000

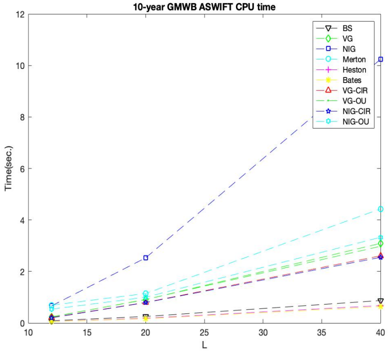
GMWB results - error

Table: 10-year GMWB CPU time ($L = 20$)

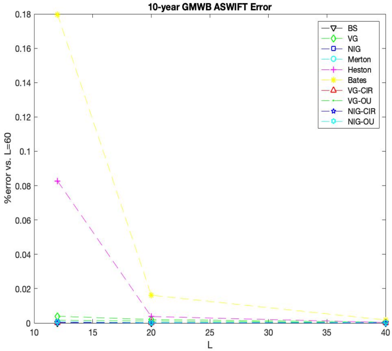
CPUtime Model	ASWIFT Price	m=4 Time (s)	ASCOS Price	nq = 200 Time (s)	ASCOS Price	nq = 400 Time (s)
BS	100.00	**0.25**	100.00	**5.50**	100.00	**10.65**
VG	100.00	0.91	98.55	5.35	100.00	10.29
NIG	100.00	**2.53**	96.94	5.23	100.00	10.27
MJD	100.00	1.14	100.58	5.19	100.00	**10.11**
Heston	100.00	0.18	100.00	5.45	100.00	10.51
Bates	100.00	**0.17**	100.01	5.46	100.02	10.55
VG-CIR	100.00	0.80	100.01	5.38	100.00	10.39
VG-OU	100.00	0.91	98.87	5.30	100.00	10.38
NIG-OU	100.00	1.00	98.64	5.29	100.00	10.34
NIG-CIR	100.00	0.79	100.02	5.31	100.00	10.42

GMWB results - L convergence for ASWIFT

ASWIFT computational time for a 10-year GMWB



ASWIFT error for a 10-year GMWB



The maximum error is observed within 0.2% for L = 12.

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GLWB calculations

Scenario 1: Mortality assumption - Static

The Gompertz-Makeham law (Feng and Jing, 2017).

The survival probability is given by,

$$ {}_t p_x = \exp\left(\mu(x+t)\right) = \exp\left(-\mathcal{A}t - \frac{\mathcal{B}c^x(c^t - 1)}{\ln c}\right), $$

where $\mathcal{A} = 7 \times 10^{-4}, \mathcal{B} = 5 \times 10^{-5}, c = 10^{1/25}$.

Interest rate assumption: $r = 5\%$

For a 60-year-old male policyholder, the GLWB fees are compared.

GLWB calculations

Table: Continuous GLWB fee in Feng (Analytical)

BS	g = 0.05	g = 0.06	g = 0.07	g = 0.08
$\sigma = 0.2$	0.27%	0.65%	1.40%	3.08%
$\sigma = 0.3$	0.64%	1.22%	2.24%	4.31%

Table: Discrete GLWB fee by ASWIFT

BS	g = 0.05	g = 0.06	g = 0.07	g = 0.08
$\sigma = 0.2$	0.2709%	0.6185%	1.3063%	2.7908%
$\sigma = 0.3$	0.6450%	1.2014%	2.1603%	4.0034%

Small differences arises from the continuous vs. discrete setting.

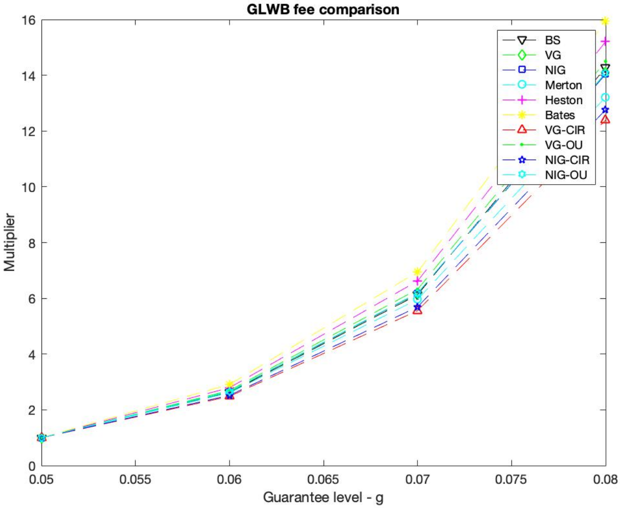
GLWB results - static mortality

Table: GLWB fees for 10 equity models

GLWB	g = 0.05	g = 0.06	g = 0.07	g = 0.08
Model	Fee(bps)	Fee(bps)	Fee(bps)	Fee(bps)
BS	16.50	43.19	101.07	235.53
VG	19.14	50.69	118.09	269.67
NIG	20.14	53.50	124.44	282.75
MJD	23.00	60.05	137.02	303.99
Heston	17.95	50.13	118.82	272.98
Bates	17.52	50.96	121.66	279.11
VG-CIR	21.56	53.53	119.81	267.16
VG-OU	18.51	49.82	116.99	268.35
NIG-CIR	20.87	52.57	118.72	266.35
NIG-OU	19.64	52.24	121.54	276.43

Fair fee charges increase with the level of guarantee.

GLWB results - fee comparison with base (g=0.05)



Raising the guarantee level will significantly increase the fair fee.

Stochastic Mortality- Affine

Scenario 2: Cohort-based stochastic mortality (Xu et al., 2018).
The T-year survival probability for an x-year-old individual -

$$\mathcal{S}_x(t,T) = E^{\mathbb{Q}}\left[\exp - \int_t^T \mu_x^i(s)ds\right]$$
$$= \exp\left(\mathcal{B}_1(t,T)X_1(t) + \mathcal{B}_2(t,T)X_2(t) + \mathcal{B}_3^i(t,T)Z^i(t) + \mathcal{A}^i(t,T)\right)$$

where

$$\mathcal{B}_j(t,T) = \frac{1-e^{-\delta_j(T-t)}}{\delta_j}$$

$$\mathcal{A}^i(t,T) = \sum_{j=1}^{3}\frac{\sigma_j^2}{2\delta_j^3}\left(\frac{1}{2}\left(1-e^{-2\delta_j(T-t)}\right) - 2\left(1-e^{-\delta_j(T-t)}\right) + \delta_j(T-t)\right)$$

Stochastic Mortality - Parameters

Parameters calibrated with the U.S. male mortality data from the HMD Cohort of 1910s and 1915s specifically (Huang et al., 2018)

Table: Parameters fitted

Parameters	Estimates
δ_{11}	-0.0702
δ_{22}	-0.1860
δ_{33}^{1910}	0.0509
δ_{33}^{1915}	0.1053
σ_{11}	0.0001
σ_{22}	0.0000
σ_{33}^{1910}	0.0020
σ_{33}^{1915}	0.0036

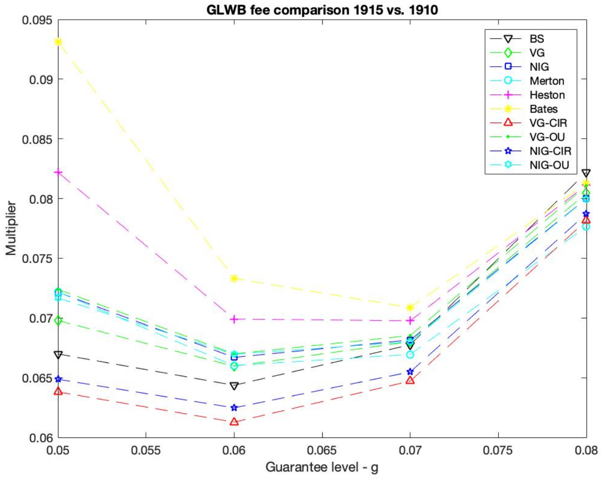
Cohort-based GLWB fees

Table: GLWB Fair fees (bps)

GLWB	Cohort-1910		Cohort-1915	
Model	g = 0.06	g = 0.07	g = 0.06	g = 0.07
BS	37.91	86.64	40.35	92.51
VG	44.27	100.71	47.19	107.56
NIG	46.63	105.92	49.74	113.14
MJD	52.26	116.51	55.71	124.31
Heston	43.34	100.77	46.37	107.8
Bates	43.65	102.72	46.85	110
VG-CIR	47.00	102.76	49.88	109.41
VG-OU	43.44	99.67	46.35	106.5
NIG-CIR	46.10	101.71	48.98	108.37
NIG-OU	45.55	103.53	48.6	110.57

Later cohorts suggest higher guarantee fees (6%-7%).

GLWB results - fee comparison 1915 vs. 1910 cohorts



Different levels of increase in fair fees with different equity models

1 Introduction
- Overview
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- VA+GMBs Valuation
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- Computational Methods
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Conclusions

This paper develops GLWB pricing using the efficient SWIFT method under different equity processes.

- The SWIFT is robustly efficient in GLWBs/static-GMWBs pricing compared to the COS method.
- Black-Scholes assumptions tend to result in lower GMWB/GLWB fees.
- Consideration of cohort-based stochastic mortality is important.
- Using multiple equity models may reduce extrapolation risk.

Further Research

The further research covers:

- Dynamic withdrawal patterns
- Hedging analysis assuming different equity models
- Cost of Capital impact on VA+GMBs pricing

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