FORECASTING MORTALITY BY MIXING MORTALITY EXPERIENCES

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Introduction

Focus is on the robustness of the forecasted mortality when dealing with small populations

Due to the small sample size, data are very volatile and yield unreliable estimates

This problem applies in any case to the oldest ages, also when larger populations are referred to

Our proposal: “replicate” the mortality of the small population by mixing appropriately the mortality data from neighboring countries

this way, the mortality forecast is build on a larger sample
$\Rightarrow$ estimates should be less volatile and asymptotically more robust

Some (recent) references

Cairns, Blake, Dowd, Coughlan and Khalaf-Allah (2011) model the joint development over time of mortality rates in a pair of related populations. The framework is designed for large populations coupled with a small sub-population

Fiig Jarner and Masotti Kryger (2009) propose a methodology for robust forecasting based on the existence of a larger reference population sharing the same long-term trend as the population of interest

Li and Lee (2005), Russolillo, Giordano and Haberman (2010) extend the Lee-Carter model in order to account for different populations

The idea

Mortality data from n countries
Further: mortality data from country C, which has a small population

Notation

$m_{x,t}^{[k]}$ central death rate in country k ($k = 1, 2, \ldots, n$ and C), calendar year t, age x

$E_{x,t}^{[k]}$ number of the exposed to risk

$\Delta m_{x,t}^{[k]}$ annual variation of the central death rate, in the period $(t, t+1)$ (i.e.: $\Delta m_{x,t}^{[k]} = m_{x,t+1}^{[k]} - m_{x,t}^{[k]}$)

Definition: Average central death rate within the n countries

$$m_{x,t}^{[\mathrm{AVE}]} = \sum_{k=1}^{n} w_k \, m_{x,t}^{[k]}$$

where the w_k's are appropriate weights (non-negative and summing to 1)

Definition: Average annual variation of the central death rate within the n countries

$$\Delta m_{x,t}^{[\mathrm{AVE}]} = \sum_{k=1}^{n} w_k \, \Delta m_{x,t}^{[k]}$$

with appropriate weights w_k

Remark: reasonably, for any country k, both $m_{x,t}^{[\mathrm{AVE}]}$ and $\Delta m_{x,t}^{[\mathrm{AVE}]}$ are less affected by random fluctuations than the $m_{x,t}^{[k]}$ and $\Delta m_{x,t}^{[k]}$, due to the larger size of the sample

Purpose: replace $m_{x,t}^{[C]}$ with $m_{x,t}^{[\mathrm{AVE}]}$, or $\Delta m_{x,t}^{[C]}$ with $\Delta m_{x,t}^{[\mathrm{AVE}]}$, so to reduce the effect of random fluctuations when projecting mortality for population C

Task: selection of the weights w_k

The optimization problem

To find the weights w_k, we follow the WLS approach

If reference is to the central death rates, the objective function is

$$\sum_{x=x_0}^{x_1} \sum_{t=t_0}^{t_1} \eta_{x,t} \left(m_{x,t}^{[C]} - m_{x,t}^{[\mathrm{AVE}]} - \mathrm{const}\right)^2$$

where

$\eta_{x,t} = \frac{E_{x,t}^{[C]}}{m_{x,t}^{[C]}}$: coefficient accounting for the volatility of data (due to random fluctuations)

$x_0, x_0 + 1, \ldots, x_1$: reference range of ages

$t_0, t_0 + 1, \ldots, t_1$: calendar time of reference

const: a parameter of the optimization problem

If reference is to the annual variation of the central death rate, the objective function is

$$\sum_{x=x_0}^{x_1} \sum_{t=t_0}^{t_1-1} \eta_{x,t} \left(\Delta m_{x,t}^{[C]} - \Delta m_{x,t}^{[\mathrm{AVE}]}\right)^2$$

Selection of the weights w_k

Reasonable requirements: the weight w_k should account for

- the size of the population in country k
- the similarity of the populations of countries k and C

Further: $w_k \geq 0$ and $\sum_{k=1}^{n} w_k = 1$

Splitting of the weight w_k

$$w_k = \underbrace{w_k^{(1)}}_{\text{ensuring best fit}} \times \underbrace{w_k^{(2)}}_{\text{accounting for population size}} \times \underbrace{w_k^{(3)}}_{\text{accounting for similarities}}$$

Definition of $w_k^{(2)}$

Let $\bar{E}^{[k]} = \frac{\sum_{x=x_0}^{x_1} \sum_{t=t_0}^{t_1} E_{x,t}^{[k]}}{(x_1 - x_0 + 1)\,(t_1 - t_0 + 1)}$ be the average number of the exposed to risk in country k. Then

$$w_k^{(2)} = \frac{\bar{E}^{[k]}}{\sum_{j=1}^{n} \bar{E}^{[j]}}$$

Definition of $w_k^{(3)}$

Work in progress. Some measure of distance. In particular, if reference is to the $\Delta m_{x,t}^{[k]}$, distance between the mortality profiles

Finding the minimum

$$\min_{w_k^{(1)}} \quad \sum_{x=x_0}^{x_1} \sum_{t=t_0}^{t_1} \left(m_{x,t}^{[C]} - \sum_{k=1}^{n} w_k^{(1)} \frac{\bar{E}^{[k]}}{\sum_{j=1}^{n} \bar{E}^{[j]}} w_k^{(3)} m_{x,t}^{[k]} - \text{const} \right)^2$$

$$\text{s.t.} \quad \sum_{k=1}^{n} w_k^{(1)} \frac{\bar{E}^{[k]}}{\sum_{j=1}^{n} \bar{E}^{[j]}} w_k^{(3)} = 1$$

$$w_k^{(1)} \geq 0$$

Solution with standard R package lsei, after rewriting the problem as a quadratic programming problem of the form $\min ||Ax - b||^2$

If reference is to the $\Delta m_{x,t}^{[k]}$'s, the objective function is as above, but with $\Delta m_{x,t}^{[C]}$ and $\Delta m_{x,t}^{[k]}$ instead of $m_{x,t}^{[C]}$ and $m_{x,t}^{[k]}$

In the following: $w_k^{(3)} = 1$. Data from HMD

Some numerical findings

Country $C =$ Slovenia

Calendar time interval: $t_0 = 1972$, $t_1 = 1990$

Reference to the central death rates, $m_{x,t}^{[k]}$

Weights w_k

Country	males		females	
	ages 60–80	ages 70–80	ages 60–80	ages 70–80
Denmark	0.0000	0.0000	0.0000	0.0000
Bulgaria	0.1693	0.1113	0.0536	0.0154
Czech Republic	0.0782	0.0743	0.2489	0.3200
Portugal	0.0748	0.2610	0.3422	0.3341
Norway	0.0000	0.0000	0.0000	0.0000
Belgium	0.0000	0.4740	0.0000	0.0000
Slovakia	0.0061	0.0000	0.0000	0.0119
Estonia	0.0000	0.0000	0.0000	0.0000
Hungary	0.1495	0.0795	0.0532	0.0000
Switzerland	0.0000	0.0000	0.0000	0.0000
Austria	0.3117	0.0000	0.1133	0.0000
Germany	0.2104	0.0000	0.0000	0.0000
Nothern Irland	0.0000	0.0000	0.0503	0.0000
Britain	0.0000	0.0000	0.0000	0.0000
Scotland	0.0000	0.0000	0.0000	0.0000
Italy	0.0000	0.0000	0.1384	0.3185
France	0.0000	0.0000	0.0000	0.0000
Spain	0.0000	0.0000	0.0000	0.0000
Sweden	0.0000	0.0000	0.0000	0.0000
minimum	682.7905	393.9918	754.1315	431.8223

Country $C =$ Austria

Calendar time interval: $t_0 = 1972$, $t_1 = 1990$

Reference to the central death rates, $m_{x,t}^{[k]}$

Weights w_k

Country	males		females	
	ages 60–80	ages 70–80	ages 60–80	ages 70–80
Slovenia	0.0502	0.0039	0.0163	0.0000
Denmark	0.0000	0.0000	0.0000	0.0000
Bulgaria	0.0000	0.0000	0.0783	0.1043
CzechRepublic	0.0000	0.0243	0.0587	0.0921
Portugal	0.0362	0.0159	0.0339	0.0000
Norway	0.0000	0.0000	0.0000	0.0000
Belgium	0.0000	0.0000	0.3341	0.2814
Slovakia	0.0000	0.0000	0.0000	0.0000
Estonia	0.0000	0.0000	0.0000	0.0000
Hungary	0.0177	0.0000	0.0000	0.0014
Switzerland	0.0000	0.0000	0.0000	0.0000
Germany	0.8366	0.9559	0.4113	0.5207
Nothern Irland	0.0000	0.0000	0.0000	0.0000
Britain	0.0000	0.0000	0.0000	0.0000
Scotland	0.0000	0.0000	0.0000	0.0000
Italy	0.0000	0.0000	0.0674	0.0000
France	0.0592	0.0000	0.0000	0.0000
Spain	0.0000	0.0000	0.0000	0.0000
Sweden	0.0000	0.0000	0.0000	0.0000
minimum	644.0759	376.8077	643.8202	354.4234

Remark: dependence on range of ages and calendar time

Country $C =$ Slovenia

Calendar time interval: $t_0 = 1972$, $t_1 = 1990$

Reference to the annual variation of the central death rates, $\Delta m_{x,t}^{[k]}$

Weights w_k

Country	males		females	
	ages 60–80	ages 70–80	ages 60–80	ages 70–80
Denmark	0.1613	0.3606	0.0000	0.0000
Bulgaria	0.0000	0.0000	0.0807	0.0000
CzechRepublic	0.0000	0.0000	0.0000	0.0000
Portugal	0.0398	0.0000	0.0351	0.0000
Norway	0.0498	0.0000	0.0000	0.0000
Belgium	0.0000	0.0000	0.5319	0.0000
Slovakia	0.0000	0.0000	0.0000	0.0000
Estonia	0.0043	0.1628	0.0000	0.0000
Hungary	0.0000	0.0000	0.0000	0.0000
Switzerland	0.0000	0.0000	0.0000	0.0000
Austria	0.1489	0.0000	0.0450	0.0000
Germany	0.0000	0.0000	0.0000	0.0000
Nothern Irland	0.0000	0.0000	0.0000	0.0000
Britain	0.5196	0.3966	0.0000	0.0000
Scotland	0.0763	0.0800	0.0000	0.0000
Italy	0.0000	0.0000	0.0000	0.1738
France	0.0000	0.0000	0.0000	0.0000
Spain	0.0000	0.0000	0.0000	0.0000
Sweden	0.0000	0.0000	0.3074	0.8262
minimum	105.2262	68.9709	91.2330	23.3408

Country $C =$ Austria

Calendar time interval: $t_0 = 1972$, $t_1 = 1990$

Reference to the annual variation of the central death rates, $\Delta m_{x,t}^{[k]}$

Weights w_k

Country	males		females	
	ages 60–80	ages 70–80	ages 60–80	ages 70–80
Slovenia	0.0534	0.0457	0.0618	0.0000
Denmark	0.0000	0.0000	0.1554	0.4362
Bulgaria	0.1402	0.2050	0.0337	0.0288
CzechRepublic	0.0000	0.0000	0.0000	0.0000
Portugal	0.0034	0.0000	0.0000	0.0000
Norway	0.0000	0.0000	0.0000	0.0000
Belgium	0.1524	0.1857	0.1802	0.1931
Slovakia	0.0294	0.0000	0.0000	0.0000
Estonia	0.0000	0.0124	0.0000	0.0000
Hungary	0.0000	0.0000	0.0000	0.0000
Switzerland	0.0000	0.0000	0.0000	0.0000
Germany	0.0000	0.0000	0.1180	0.0000
Nothern Irland	0.0000	0.1568	0.0527	0.0322
Britain	0.5223	0.3487	0.0805	0.0044
Scotland	0.0989	0.0455	0.3176	0.3053
Italy	0.0000	0.0000	0.0000	0.0000
France	0.0000	0.0000	0.0000	0.0000
Spain	0.0000	0.0000	0.0000	0.0000
Sweden	0.0000	0.0000	0.0000	0.0000
minimum	54.6246	20.8770	139.9233	92.4429

Goodness of fit

We forecast mortality using Lee-Carter on original data and on the replicated mortality, for a given range of ages (say 80–100) and a given calendar time period (say 1972–1990)

The goodness of fit is measured with the MSE among the original and the forecasted data (say in time period 1991–2008)

For simplification, all ages above 100 are grouped into the age group 100

The missing $m_{x,t}^{[k]}$'s are reconstructed through interpolation

Numerical findings

Reduction of the MSE when comparing forecasts based on replicated data with those based on original data

	reference: $m_{x,t}^{[k]}$		reference: $\Delta m_{x,t}^{[k]}$	
country	males	females	males	females
Slovenia	74.3%	89.6%	77.8%	71.0%
Austria	53.7%	21.4%	62.9%	34.3%
Italy	9.7%	3.2%	41.0%	20.2%
Switzerland	6.3%	1.3%	20.3%	22.4%
Hungary	1.9%	18.6%	19.4%	-9.2%
Estonia	47.6%	44.2%	–46.6%	–76.2%
Germany	10.9%	2.2%	10.2%	39.8%
Belgium	–17.7%	7.3%	7.6%	1.7%
Norway	28.0%	13.4%	–4.8%	31.8%
Denmark	–17.9%	–10.6%	–40.5%	21.9%
Portugal	–1.3%	11.0%	–41.6%	–14.4%

Remark: the improvement depends on the size of the population (in general, stronger for small populations) and the quality of data. In general, better fitting when reference is to the $\Delta m_{x,t}^{[k]}$'s

Final remarks

This is a work in progress

Results from backtesting are encouraging

The choice of the reference range of ages and calendar time requires further investigation

Next steps:

- definition of the factors $w_k^{(3)}$
- implementation of the Log-Poisson model
- construction of priors and posterior distributions of mortality within Bayesian inference procedures
- ...

References

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