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The Risk Management and Payout Design of Reverse Mortgages

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Risk Management and Payout Design of Reverse Mortgages

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Abstract

We analyze the risk and profitability of reverse mortgages with lump-sum or income stream payments from the lender's perspective. Reverse mortgage cash flows and loan balances are modeled in a multi-period stochastic framework that allows for house price risk, interest rate risk and risk of delayed loan termination. A VAR model is used to simulate economic scenarios and to derive stochastic discount factors for pricing the no negative equity guarantee embedded in reverse mortgage contracts. Our results show that lump-sum reverse mortgages are more profitable and require less risk-based capital than income stream reverse mortgages, which explains why this product design dominates in most markets. The loan-to-value ratio, the borrower's age, mortality improvements and the lender's financing structure are shown to be important drivers of the profitability and riskiness of reverse mortgages, but changes in these parameters do not change the main conclusions.

Keywords: Reverse mortgage; Income stream; Equity release; Vector autoregressive model; Stochastic discount factor; Risk-based capital

JEL Classifications: G12; G21; G32

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1 Introduction

Population ageing is a global phenomenon and the question of how to finance the retirement and health care costs of a rapidly growing older population is becoming a major challenge. Households are expected to rely more and more on private savings. A substantial part of household wealth is held as real estate. Homeownership rates are high among the elderly in most developed countries (Chiuri and Jappelli, 2010). Reverse mortgages allow retirees to transform their housing wealth into liquid assets while staying in their home. Reverse mortgages are increasingly used by retirees. The product is available in numerous countries including Australia, Canada, the US, the UK, India and Singapore. The financial crisis has slowed down market growth, especially in the US, but several markets including Australia and the UK have recovered and show strong growth rates (Deloitte and SEQUAL, 2012; Key Retirement Solutions, 2013). From a lender's perspective reverse mortgages differ from (forward) mortgages largely because of the dependence of cash flows on longevity risks, an area attracting increased interest in the banking and finance literature (Horneff *et al.*, 2010, 2009).

Reverse mortgages were initially designed to provide a regular retirement income and/or a line of credit for major expenses such as health care costs or home repairs (Chinloy and Megbolugbe, 1994; Consumer Financial Protection Bureau, 2012; Venti and Wise, 1991). Most markets today are dominated by reverse mortgages with lump-sum payments (Clerc-Renaud *et al.*, 2010; Consumer Financial Protection Bureau, 2012; Deloitte and SEQUAL, 2012). The Consumer Financial Protection Bureau (2012) reports that U.S. borrowers increasingly use their reverse mortgage loans to refinance traditional mortgages. Psychological aspects also play a role: the life-time reverse mortgage income may only moderately increase household income, whereas the equivalent lump-sum payment would increase liquid wealth by a large fraction (Venti and Wise, 1991). Also, 'over-housed' retirees may use the lump sum to reduce their house price risk exposure and to diversify across asset classes (Pelizzon and Weber, 2009). Another important reason is complexity. Reverse mortgages have been criticized as too complex for consumers (see, e.g., Consumer

Financial Protection Bureau, 2012) and this applies particularly to income stream reverse mortgages.

We take a closer look at this growing market and analyze the risk and profitability of reverse mortgage loans with different payout options from the lender's perspective. In particular, we investigate how lump-sum reverse mortgages and income stream reverse mortgages with fixed or inflation-indexed payments are impacted differently by house price risk, interest rate risk and termination risk. Our study extends the growing literature on the pricing and risk management of reverse mortgages (see, e.g., Alai *et al.*, 2013; Chen *et al.*, 2010; Hosty *et al.*, 2008; Shao *et al.*, 2012). Two previous studies have developed pricing frameworks for reverse mortgage loans that provide regular income payments, focusing on valuing the cross-over option (Chinloy and Megbolugbe, 1994) and the fair value of the regular payments (Lee *et al.*, 2012). We assess the lender's net financial position and required risk-based capital for three payout types of reverse mortgages. Our study adds a new contribution to the limited literature on risk-based capital requirements for residential mortgage portfolios (see, e.g., Calem and LaCour-Little, 2004; Qi and Yang, 2009).

We employ a multi-period stochastic framework for modeling and pricing reverse mortgage cash flows that extends the models used in Alai *et al.* (2013) and Shao *et al.* (2012). Loan termination probabilities are derived from a multi-state Markov model. A vector autoregressive model is used to generate economic scenarios and to derive stochastic discount factors that reflect the key risk factors of reverse mortgage cash flows and their dependencies. The stochastic discount factors are used to price the no negative equity guarantee typically embedded in reverse mortgage contracts and to determine the risk premium lenders should charge for this guarantee. We compute risk measures such as the Value-at-Risk (VaR) and the Conditional Value-at-Risk (CVaR) to determine the amount of risk-based capital the lender should set aside for each type of reverse mortgage.

The results of our study show that lump-sum reverse mortgages have lower risks and are more profitable than income stream products. Lump-sum reverse mortgages start with higher loan balances and accumulate to higher levels early on. However, they are also less exposed to longevity risk than income stream products. The effective loan amount for an income stream is determined by the number of payments made, which is subject to longevity risk. When an individual lives longer, the accumulated loan amount of the income steam exceeds the equivalent lump-sum loan at the older ages. This main result is robust to changes in the contract characteristics and to key model assumptions tested in the sensitivity analysis. The result provides an additional explanation as to why lump-sum products are the most popular type of reverse mortgage internationally. The sensitivity analysis confirms the loan-to-value ratio and the borrower's age as important pricing factors for all three reverse mortgages types.

The remainder of this article is organized as follows. Section 2 introduces the different reverse mortgage contracts and describes their cash flows. Section 3 sets out the multi-period stochastic reverse mortgage pricing and risk management framework. Section 4 reports the results and Section 5 concludes.

2 Reverse Mortgage Product Design

Detailed descriptions of existing reverse mortgages products can be found, for example, in Chen *et al.* (2010) for the US market, in Hosty *et al.* (2008) for the UK and in Alai *et al.* (2013) for reverse mortgages in Australia. In the following, stylized products with typical product characteristics are described.

Under a reverse mortgage, the lender advances the borrower cash and takes a mortgage charge over the borrower's property. Borrowers retain the right to stay in their home until they die or sell the property. In either case, the contract is terminated, the property is sold and the loan and the accumulated interest are repaid. Contracts also often allow for refinancing or early repayment. Reverse mortgage loans are typically non-recourse: borrowers are protected from providing assets other than the house by the 'no negative equity guarantee' (NNEG). The maximum loan amount is determined by the age and gender of the borrower and the appraisal value of the property. Reverse mortgages can be issued to couples or single borrowers and can carry fixed or variable interest rates. Reflecting typical loan characteristics, we model reverse mortgages with variable interest rates issued to a single female borrower.

Reverse mortgages expose lenders to house price risk, interest rate risk and the risk of delayed termination (longevity risk). These interrelated risks impact lump-sum and income stream reverse mortgages to different extents. For example, a longer loan duration increases the probability that the loan balance exceeds (*'crosses over'*) the property value at maturity for both reverse mortgage types and results in a more payments to the borrower for income stream products. The following subsections describe the products' cash flows in more detail. The cash flows are modeled on a quarterly basis.

2.1 Lump-Sum Reverse Mortgage

The most common type of reverse mortgage pays out the loan amount as a lump sum at the beginning of the contract. We denote the outstanding loan balance at time t as L_t^{LS} and the value of the property at t as H_t. The lump-sum payment to the borrower is $P^{LS} = L_0^{LS}$.

Each quarter, the loan balance increases by a variable mortgage rate, r_t^{κ} and by the risk-adjusted premium rate for the NNEG, π (see Section 2.3). The loan balance at time t is given by:

$$L_t^{LS} = L_0^{LS} \exp\left\{ \sum_{i=0}^{t} (r_i^{\kappa} + \pi) \right\}, \tag{1}$$

The reverse mortgage lender finances the lump-sum payout with equity and with borrowed capital. The borrowing ratio is denoted by φ. Borrowed capital is assumed to accumulate with the short rate, $r_t^{(1)}$. The financing cost for a single lump-sum reverse mortgage loan at time t is given by:

$$C_t^{LS} = \varphi L_0^{LS} \exp\left\{\sum_{i=0}^{t} r_i^{(1)}\right\} + (1-\varphi) L_0^{LS}. \tag{2}$$

The payoff the lender receives at the date of loan termination, T, is capped by the sale proceeds of the property by the NNEG. We assume that there is a proportional transaction cost, γ, of selling the house. The date of loan termination is random, determined by the borrower's health state and prepayment or refinancing decisions. We model the randomness using the probability of loan termination for a borrower initially aged x, ${}_{t|}q_x^c$. The expected present value of the lender's net payoff from the lump-sum reverse mortgage is given by:

$$EPV^{LS} = \sum_{t=0}^{\omega-x-1} {}_{t|}q_x^c \exp\left\{-\sum_{i=0}^{t} r_i^{(1)}\right\} \left[\min(L_t^{LS}, (1-\gamma)H_t) - C_t^{LS}\right], \tag{3}$$

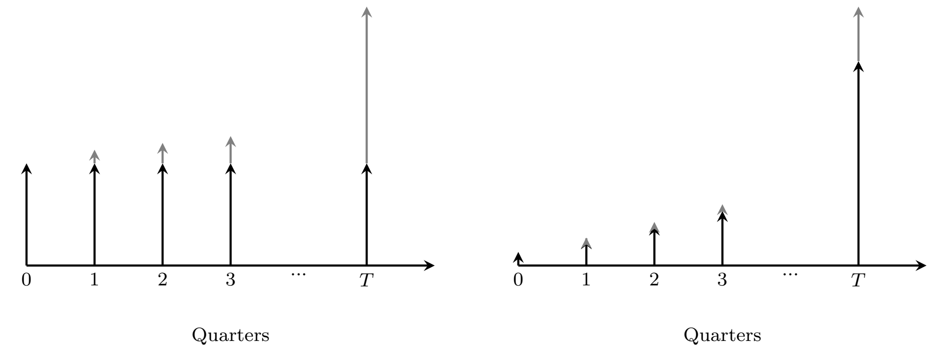
where ω is the maximum attainable age of the borrower.

2.2 Income Stream Reverse Mortgage

Figure 1 compares the development of the loan balance over time for lump-sum and income stream reverse mortgages with fixed payment amounts. Income stream reverse mortgages pay regular payments to the borrower until the contract is terminated. We model two types of payments: fixed and inflation-indexed. In both cases, the loan balance starts very low with the first payment and increases quarterly with the accrued variable mortgage rate, each new payment to the borrower, and the risk-adjusted premium rate for the NNEG, π. Income stream reverse mortgages accumulate interest rates slower, which preserves the equity of the collateralized property and helps mitigate longevity risk as described later.

To make the income stream product comparable with the lump-sum reverse mortgage, we calibrate the quarterly income payments such that the expected present value of

Figure 1: Illustration: Development of the loan balance over time.
(The loan principal is given in black. The accumulated interest and no negative equity guarantee premiums are given in gray.)



(a) Lump-sum reverse mortgage. **(b)** Income stream reverse mortgage.

all payments equals the lump-sum payment, P^{LS}. The quarterly fixed payments are calculated as:

$$P^{LS} = P^{IS} \sum_{t=0}^{\omega-x-1} {}_tp_x^c \exp\left\{-r_0^{(t)} t\right\}, \tag{4}$$

where $r_0^{(t)}$ are quarterly zero-coupon yields for maturity t at time zero and ${}_tp_x^c$ the probability that the reverse mortgage loan is in-force in year t. The quarterly inflation-indexed payments are derived similarly by:

$$P^{LS} = P_t^{IIS} \sum_{t=0}^{\omega-x-1} {}_tp_x^c \exp\left\{-r_0^{(t)} t + \sum_{i=0}^{t} d\ln CPI_i\right\}. \tag{5}$$

where $d\ln CPI_t$ is the quarterly inflation rate and CPI is the consumer price index. Inflation-indexed payments are initially lower than fixed payments and increase quarterly.[1]

The outstanding loan balances of the fixed income stream reverse mortgage at time t,

[1]The payments in Equations (4) and (5) are discounted using zero-coupon yields for maturity t at time zero. Zero-coupon yield data for June 2011 published by the Reserve Bank of Australia ('Zero-coupon Interest Rates - Analytical Series', accessed August 2012) was used. The original data is provided for maturities of up to 10 years. The Nelson-Siegel function (Nelson and Siegel, 1987) was fitted to extrapolate yields for higher maturities. The Nelson-Siegel function is a parsimonious model for yield curves, which was found to provide a very good fit.

L_t^{IS}, and of the inflation-indexed income stream reverse mortgage, L_t^{IIS}, are given by:

$$L_t^{IS} = P^{IS} \sum_{i=0}^{t} \exp\left\{ \sum_{k=0}^{i} r_k^{\kappa} + \pi \right\} \quad (6)$$

$$L_t^{IIS} = P^{IIS} \sum_{i=0}^{t} \exp\left\{ \sum_{k=0}^{i} r_k^{\kappa} + \pi + d \ln CPI_k \right\}. \quad (7)$$

As in the case of the lump-sum reverse mortgage, we assume that the lender finances the payments to the borrower with capital and borrowings/deposits. The lender maintains the required amount of capital and borrowing to meet each payment as it is made. Borrowed capital accumulates with the short rate. The total cost of financing for income stream reverse mortgages is given by:

$$C_t^{IS} = \varphi P^{IS} \sum_{i=0}^{t} \exp\left\{ \sum_{k=0}^{i} r_k^{(1)} \right\} + t(1-\varphi)P^{IS} \quad (8)$$

$$C_t^{IIS} = \varphi P_t^{IIS} \sum_{i=0}^{t} \exp\left\{ \sum_{k=0}^{i} r_k^{(1)} \right\} + t(1-\varphi)P_t^{IIS}. \quad (9)$$

Similar to the lump-sum reverse mortgage, the outstanding loan balance is fully paid only if the outstanding loan amount balance is lower than proceeds from the property less transaction costs. The expected present value of the lender's net payoff from the two income stream reverse mortgages is:

$$EPV^{IS} = \sum_{t=0}^{\omega-x-1} {}_{t|}q_x^c \exp\left\{ -\sum_{i=0}^{t} r_i^{(1)} \right\} [\min(L_t^{LI}, (1-\gamma)H_t) - C_t^{IS}] \quad (10)$$

$$EPV^{IIS} = \sum_{t=0}^{\omega-x-1} {}_{t|}q_x^c \exp\left\{ -\sum_{i=0}^{t} r_i^{(1)} \right\} [\min(L_t^{LI}, (1-\gamma)H_t) - C_t^{IIS}]. \quad (11)$$

2.3 Pricing the No Negative Equity Guarantee

The reverse mortgage contracts considered in this study are non-recourse. The no negative equity guarantee (NNEG) limits the loan repayment to the sale proceeds of the property. The lender's payoff from the NNEG at the time of loan termination, T, is:

$$NNEG_T = \max(L_T - (1-\gamma)H_T, 0), \tag{12}$$

where L_T is the outstanding loan balance at termination, H_T is the value of the property and γ are the proportional sale transaction costs. $NNEG_T$ is the same for lump-sum or income stream reverse mortgages.

The structure of the NNEG is similar to that of a series of European put options with uncertain maturity T (Chen *et al.*, 2010; Chinloy and Megbolugbe, 1994). Previous research has priced the NNEG using the Black-Scholes option pricing framework (Ji, 2011). Li *et al.* (2010) and Chen *et al.* (2010) suggest that the Black-Scholes assumptions are not appropriate for the dynamics of the underlying house price. We adopt the pricing approach used in two recent studies, where risk-adjusted stochastic discount factors are used to discount the cash flows arising from the NNEG (Alai *et al.*, 2013; Shao *et al.*, 2012). Using the same notation, the expected present value of the NNEG is given by:

$$NN = \sum_{t=0}^{\omega-x-1} E\left[\prod_{s=0}^{t}(m_s)\, {}_{t|}q_x^c \max\left(L_t - (1-\gamma)H_t, 0\right)\right], \tag{13}$$

where m_t is the quarterly stochastic discount factor at time t. The estimation of the discount factors, which reflect house price risk, interest rate risk, rental yield risk and inflation risk, is described in Section 3.3.

We assume that costs of providing the NNEG are charged to the borrower in the form of a quarterly premium at a fixed rate, π, applied to the loan amount. The premiums are accumulated and paid at the termination of the contract. The expected present value of all premiums payable throughout the loan duration is given by:

$$MIP = \pi \sum_{t=0}^{\omega-x-1} E\left[\prod_{s=0}^{t}(m_s)\, {}_{t}p_x^c L_t\right]. \tag{14}$$

The fair premium rate, π, is calculated by setting the expected present value of the NNEG equal to the expected present value of the total insurance premium: $NN = MIP$. From the equations above, the value of NNEG depends on how the loan balance accumulates over time. The mortgage insurance premiums for lump-sum and income stream reverse mortgages will differ as a result.

3 The Reverse Mortgage Pricing Framework

This section describes the framework used to simulate reverse mortgage cash flows and to analyze the lender's net financial position. We adopt and extend the pricing method used in two recent studies (Alai *et al.*, 2013; Shao *et al.*, 2012). Australian market and mortality data is used to calibrate the model. The Australian reverse mortgage market has nearly tripled in terms of the total loan book size over the last decade and is expected to continue growing (Deloitte and SEQUAL, 2012).

3.1 The Multi-State Markov Termination Model

The probability of reverse mortgage loan termination for a single female borrower initially aged x is derived from the Markov termination model developed in Alai *et al.* (2013) based on work by Ji (2011). We extend the model by Alai *et al.* (2013) by including prepayment and refinancing as causes of loan termination in addition to death and long-term care move-out. All four causes were also considered by Ji (2011).

There is no publicly available data on reverse mortgage terminations in Australia, so we adopt several assumptions made by Ji (2011) for the US and the UK. At-home mortality rates are derived by scaling down the underlying age-specific mortality rates with a factor θ_x to represent the better health of retirees who have not moved into a long-term care facility. The probability of a move into long-term care is derived by multiplying the mortality rate with an age-varying adjustment factor, ρ_x, based on the UK experience reported in Institute of Actuaries UK (2005). Both the probability of prepayment and the probability of refinancing are assumed to depend on the in-force duration (in years) of the reverse mortgage loan (Hosty *et al.*, 2008; Institute of Actuaries UK, 2005). The loan

Table 1: Assumptions on reverse mortgage loan termination based on Ji (2011).

	At-home mortality	LTC incidence	Prepayment		Refinancing	
Age	factor θ_x	factor ρ_x	Duration	Probability	Duration	Probability
65-70	0.950	0.100	1	0.00%	1-2	1.00%
75	0.925	0.150	2	0.00%	3	2.00%
80	0.900	0.200	3	0.15%	4-5	2.50%
85	0.875	0.265	4	0.30%	6-8	2.00%
90	0.850	0.330	5	0.30%	9-10	1.00%
95	0.825	0.395	6+	0.75%	11-20	0.50%
100+	0.800	0.460			21+	0.25%

termination assumptions are summarized in Table 1. Parameters for ages not reported in the table are obtained by linear interpolation.

The underlying age-specific mortality rates are modeled by the Gompertz law of mortality. The model assumes that the force of mortality μ_x of an x-year-old is given by:

$$\mu_x = \alpha \exp\left\{\gamma x\right\}, \tag{15}$$

where α and γ are two parameters. We estimate these parameters based on mortality data for Australian females of ages 50-105 for the period 1950-2009 from the Human Mortality Database (2012). We approximate the instantaneous force of mortality, μ_x, by the death rate, which is calculated as the number of deaths, $D_{x,t}$, in a given year t divided by an estimate of the population exposed to the risk of death, $E_{x,t}$. A Poisson regression was fitted to the natural logarithm of death counts, $D_{x,t}$,:

$$\ln D_{x,t} = \ln E_{x,t} + \beta_0 + \beta_1 x + \epsilon_{x,t}. \tag{16}$$

The estimated Gompertz parameters are $\hat{\alpha} = 0.000014$ and $\hat{\gamma} = 0.103916$. Using the estimated force of mortality, $\hat{\mu}_x$, from the Gompertz model and the annual loan termination assumptions, the probability ${}_tp_x^c$ that the reverse mortgage loan is in in-force in policy year t is given by:

$${}_tp_x^c = \exp\left\{-\int_0^1 (\theta_{x+s} + \rho_{x+s})\hat{\mu}_{x+s}\,ds\right\}(1 - \mathrm{P}(Prepayment))(1 - \mathrm{P}(Refinancing)), \tag{17}$$

where P($Prepayment$) and P($Refinancing$) are the age-specific probabilities that the loan is terminated because of prepayment or refinancing, respectively. The annual probability of loan in-force was converted into a quarterly frequency by cubic spline interpolation. Finally, the quarterly probability of loan termination is calculated as ${}_{t|}q_x^c = {}_{t+1}p_x^c - {}_tp_x^c$. The resulting average contract in-force duration is 16.1, 9.3 and 4.4 years for borrowers initially aged 65, 75 and 85. These durations are slightly shorter than those reported in Alai *et al.* (2013, Table 3), because we include prepayment and refinancing as additional reasons for reverse mortgage termination.

3.2 VAR-Based Economic Scenario Generation

A vector autoregressive (VAR) model is used to jointly model house prices, interest rates and other relevant economic variables, to project economic scenarios and to derive stochastic discount factors using the same data and methodology as described in Alai *et al.* (2013). In addition, we include the consumer price index in the model, as a driver of interest rate and house price dynamics. The estimation results can also be compared to Sherris and Sun (2010), who estimate a VAR model using similar data over a different time period (Mar-1982 - Dec-2008).

Table 2 summarizes the raw data, variable names and data sources. The data was accessed in August 2012. The sample period is Jun-1993 - Jun-2011, the longest period for which all variables are available. Daily and monthly series are converted to quarterly series. The ten-year term spread is calculated as the difference between ten-year and three-month zero-coupon yields: $r^{(40)} - r^{(1)}$. Growth rates of the house price index, of the rental index, of GDP and of CPI are determined by differencing the log series and are denoted as '$d\ln$'.

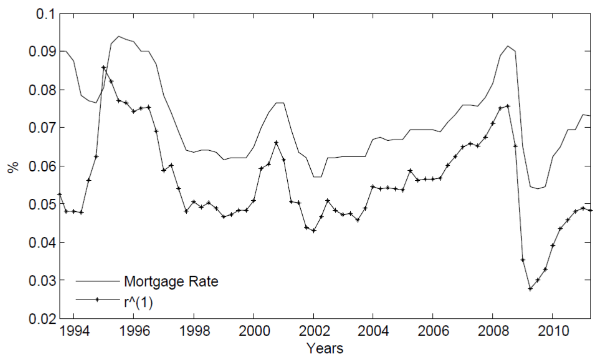
Figure 2 plots the three-month zero-coupon yields $r^{(1)}$ and the variable mortgage rate MR. The two variables are highly correlated, with correlation coefficient of 0.77. To avoid multicollinearity in the VAR model, Alai *et al.* (2013) only include the short rate in the VAR model and derive variable mortgages rates, r_t^κ, by adding a fixed lending

Table 2: Definitions, data sources and frequency.

Variable	Definition	Source	Frequency
$r^{(1)}$	Three-month zero-coupon yield	Reserve Bank of Australia	Daily
$r^{(40)}$	Ten-year zero-coupon yield	Reserve Bank of Australia	Daily
MR	Nominal variable mortgage rate	Reserve Bank of Australia	Monthly
H	Nominal Sydney house price index	Residex Pty Ltd.	Monthly
R	Nominal Sydney rental yield index	Residex Pty Ltd.	Monthly
GDP	Nominal Australian gross domestic product	Australian Bureau of Statistics	Quarterly
CPI	New South Wales consumer price index	Australian Bureau of Statistics	Quarterly

The sample period is Jun-1993 - Jun-2011.

Figure 2: Three-month zero-coupon yields, $r^{(1)}$, and variable mortgage rates, Jun-1993 - Jun-2011.



margin, κ, to the short rate:

$$r_t^{\kappa} = r_t^{(1)} + \kappa. \tag{18}$$

We follow this approach and estimate the lending margin as the average difference $MR - r^{(1)} = 1.65\%$ over the sample period. Assuming continuous compounding, the quarterly lending margin is calculated as $\kappa = 0.41\%$.

The time series were tested for stationarity using the augmented Dickey-Fuller test and the Phillips-Perron test. Both tests correct for possible serial correlation in the error terms of the test equation. The Phillips-Perron test is also robust to unspecified heteroscedasticity in the error terms. The results of these tests, given in Table 3, indicate that all variables except the rental yield growth rates $d\ln R$ are stationary at a 10% significance level. We include $d\ln R$ in the VAR model to avoid over-differencing.

To determine the optimal lag length, we estimate the VAR model for different lag lengths

Table 3: Results of the stationary tests.

	ADF test		PP test	
Variable	t-statistic	p-value	t-statistic	p-value
$r^{(1)}$	-3.308	0.018	-2.621	0.094
$r^{(40)} - r^{(1)}$	-2.951	0.045	-2.739	0.073
$d\ln H$	-3.136	0.028	-6.412	0.000
$d\ln R$	-1.298	0.626	-1.218	0.662
$d\ln GDP$	-3.507	0.011	-2.729	0.074
$d\ln CPI$	-6.430	0.000	-6.430	0.000

ADF denotes the augmented Dickey-Fuller test. PP denotes the Phillips-Perron test.

Table 4: Model selection criteria and residual analysis for VAR models with different lag lengths.

	Model Selection Criteria			Autocorrelation	Heteroscedasticity	Normality
Lag length	AIC	BIC	HQC	p-value	p-value	p-value
1	-0.011	**1.371**	0.536	0.030	0.119	0.000
2	-1.150	1.417	**-0.134**	0.565	0.019	0.000
3	**-1.384**	2.367	0.100	**0.746**	0.132	0.000
4	-0.998	3.938	-0.955	0.354	**0.390**	0.003

The model selection criteria are denotes as: AIC-Akaike Information Criterion, BIC-Schwarz's Bayesian Information Criterion and HQC-Hannan-Quinn Criterion. The model residuals are tested for serial correlation using the multivariate Lagrange Multiplier test, for heteroscedasticity using the White test and for normality using the multivariate Jarque-Bera test.

and compare three commonly used model selection criteria (Akaike Information Criterion, Schwarz's Bayesian Information Criterion and Hannan-Quinn Criterion). To support the model choice we also analyze the estimated residuals of each VAR model. We test for serial correlation using the multivariate Lagrange Multiplier test, for heteroscedasticity using the White test and for normality using the multivariate Jarque-Bera test. Based on the test results, which are reported in Table 4, and in accordance with previous literature using similar data (Alai *et al.*, 2013; Shao *et al.*, 2012; Sherris and Sun, 2010), we choose a VAR(2) model. The model is given by:

$$\mathbf{z_t} = \mathbf{c} + \boldsymbol{\phi_1} \mathbf{z_{t-1}} + \boldsymbol{\phi_2} \mathbf{z_{t-2}} + \boldsymbol{\epsilon_t}, \tag{19}$$

where $\mathbf{z_t}$ denotes the vector of the economic variables listed in Table 3, $\mathbf{c}, \boldsymbol{\phi_1}$ and $\boldsymbol{\phi_2}$ are parameter vectors and matrices and $\boldsymbol{\epsilon_t}$ is a vector of multivariate normally distributed error terms with $\boldsymbol{\epsilon_t} \sim \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma})$.

Table 5: Estimated parameters of the VAR(2) model.

$\mathbf{z_t}$	$\mathbf{c}$ (6×1)	$\phi_\mathbf{1}$ (6×6)									
$r^{(1)}$	0.090	1.072	0.341	0.003	0.465	0.081	0.068				
$r^{(40)} - r^{(1)}$	0.117	-0.203	0.702	-0.001	0.319	-0.046	-0.013				
$d \ln H$	2.405	-0.482	1.323	-0.067	3.025	0.242	-0.881				
$d \ln R$	-0.024	0.059	-0.009	-0.007	1.008	0.008	0.009				
$d \ln GDP$	1.236	0.525	-0.014	0.015	0.765	1.228	0.053				
$d \ln CPI$	0.853	0.674	-0.652	0.087	1.415	-0.262	0.304				
$\phi_\mathbf{2}$ (6×6)						$\mathbf{\Sigma}$ (6×6)					
-0.175	-0.046	-0.006	-0.572	-0.019	-0.024	0.012	-0.007	0.001	0.000	0.012	0.014
0.055	-0.082	-0.004	-0.093	-0.001	-0.043		0.018	0.029	0.000	-0.003	-0.004
-1.961	-4.381	0.496	0.362	-1.008	0.264			3.403	-0.018	0.022	-0.193
-0.051	0.007	-0.004	-0.004	0.009	-0.019				0.001	0.000	0.004
-0.327	-0.010	0.001	-1.134	-0.888	-0.041					0.049	0.037
-0.706	1.040	-0.055	-1.688	0.194	0.007						0.296

The model equation is given by: $\mathbf{z_t} = \mathbf{c} + \phi_\mathbf{1}\mathbf{z_{t-1}} + \phi_\mathbf{2}\mathbf{z_{t-2}} + \epsilon_\mathbf{t}$ with $\epsilon_\mathbf{t} \sim \mathbf{N}(\mathbf{0}, \mathbf{\Sigma})$.

The VAR(2) model was estimated using SAS's *varmax* procedure. The estimated parameters are given in Table 5. The model exhibits a good fit with an average R^2 of 72.5% across the six equations in the VAR system. There are several significant dependencies between the economic variables. The growth rate of the house price index is very volatile, with an estimated variance of 3.403. The results are comparable with those in Alai *et al.* (2013), where a VAR(2) is estimated for the same variables excluding CPI.

Based on the VAR(2) model, 10,000 simulation paths of the economic variables over 40 years were generated with the MATLAB procedure *vgxsim*. The distribution of the simulated variables closely matches the empirical distribution of historic data, as shown in Figure 3. The distribution functions were smoothed with the MATLAB package *ksdensity*, a kernel smoothing procedure. Figure 4 plots the historical paths of the economic variables and the simulated mean values together with 90% confidence intervals. The graphs are similar to those in Alai *et al.* (2013) based on a VAR(2) model without inflation. House price growth is the most volatile of the economic variables.

3.3 Deriving Stochastic Discount Factors

Building on previous work by Ang and Piazzesi (2003) and Ang *et al.* (2006), Alai *et al.* (2013) develop a VAR-based method to derive stochastic discount factors for pricing reverse mortgages. The key idea of the method is that the discount factors should reflect

Figure 3: Probability density function of the economic variables: historical data (lines with dots) and simulated data (solid lines).

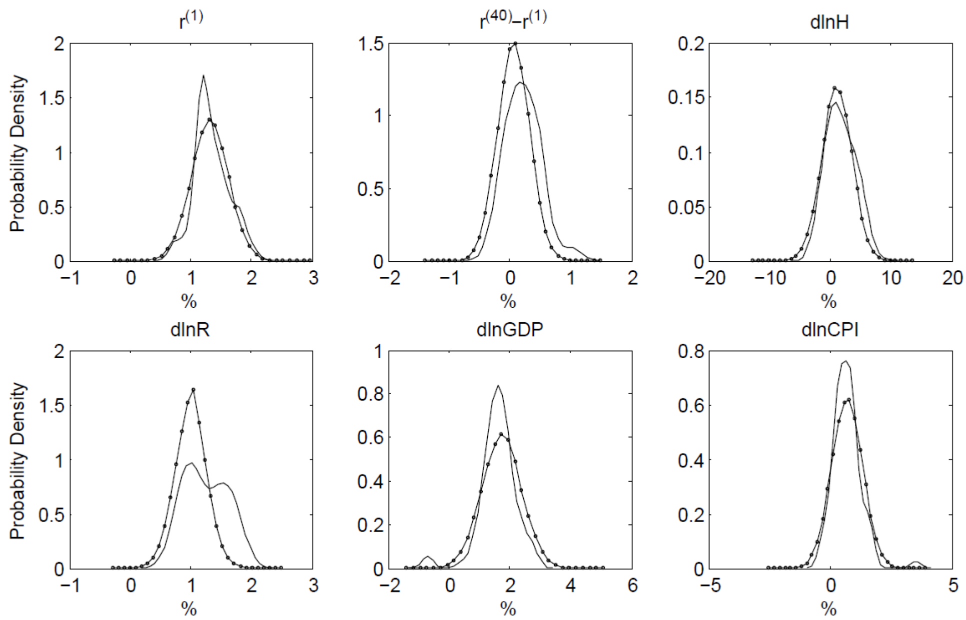
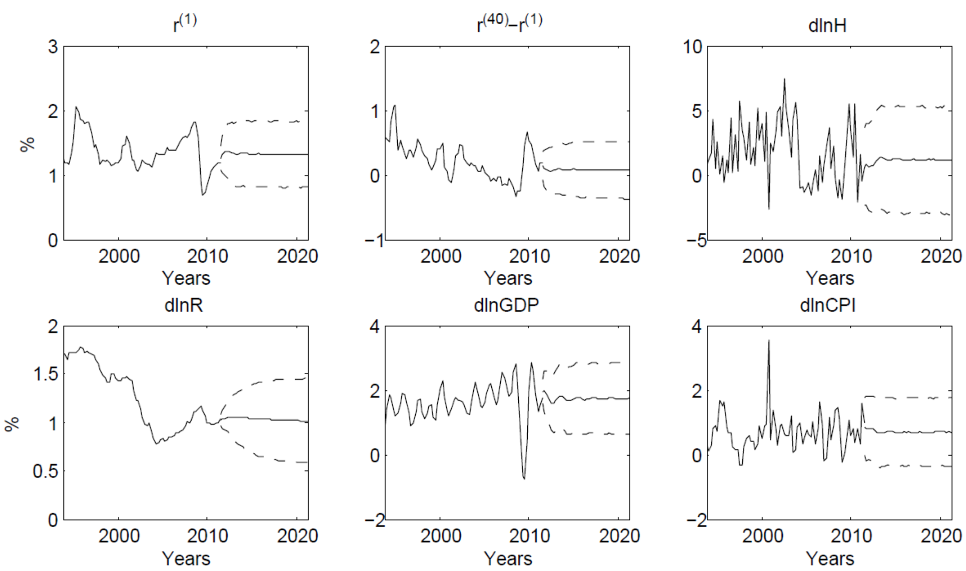


Figure 4: Historical paths of the economic variables and simulated mean values with 90% confidence intervals (dashed).



the main drivers of reverse mortgage cash flows and should account for the risk factors' interdependencies. This is realized by deriving stochastic risk factors from the VAR model used to project the economic variables. There is no allowance for longevity risk or other components of termination risk in the pricing framework. Idiosyncratic longevity risk is assumed to be fully diversifiable and systematic longevity risk is assumed to be hedgeable through reinsurance or securitization. A calibration procedure for the stochastic discount factor model was developed in Shao *et al.* (2012).

We denote with ζ_{t+1} the Radon-Nikodym derivative that converts between the real-world probability measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{Q}$. That is, for any variable X_{t+1} at time $t+1$:

$$\mathbb{E}_t^{\mathbb{Q}}\left[X_{t+1}\right] = \mathbb{E}_t\left[\zeta_{t+1} X_{t+1}\right]/\zeta_t. \tag{20}$$

ζ_t is assumed to follow a log-normal process:

$$\zeta_{t+1} = \zeta_t \exp\left\{-\frac{1}{2}\lambda_t^{'}\lambda_t - \lambda_t^{'}\epsilon_{t+1}\right\}, \tag{21}$$

where λ_t are time-varying market prices of risk associated with the random shocks, ϵ_{t+1}, to the economic variables in the VAR model.

The vector of the market prices of risk, λ_t, is modeled as a linear function of the economic state variables in the VAR model:

$$\lambda_t = \lambda_0 + \lambda_1 z_t, \tag{22}$$

where λ_0 is a 6-dimensional vector and λ_1 is a 6×6 matrix.

The pricing kernel (or stochastic discount factor), m_t, is given by:

$$m_{t+1} = \exp\left\{-r_t\right\}\zeta_{t+1}/\zeta_t = \exp\left\{-e_1^{'} z_t - \frac{1}{2}\lambda_t^{'}\lambda_t - \lambda_t^{'}\epsilon_{t+1}\right\}, \tag{23}$$

whith $e_1' = (1, 0, 0, 0, 0, 0)$. Using the stochastic discount factors, the price P_t of an asset with a payoff X_{t+1} at time $t+1$ is given by:

$$P_t = \mathbb{E}_t \left[m_{t+1} X_{t+1} \right]. \tag{24}$$

The time t price of an n-period nominal bond can be derived using the following recursive formula:

$$P_t^{(n)} = \mathbb{E}_t \left[m_{t+1} P_{t+1}^{(n-1)} \right], \tag{25}$$

with the initial condition $P_t^{(0)} = 1$. The bond price can be written as an exponential linear function of the state variables in the VAR model:

$$P_t^{(n)} = \exp \left\{ A_n + B_n' z_t + C_n' z_{t-1} \right\}, \tag{26}$$

where A_n, B_n and C_n are given by the difference equations:

$$\begin{aligned} A_{n+1} &= A_n + B_n'(c - \Sigma^{\frac{1}{2}} \lambda_0) + \frac{1}{2} B_n' \Sigma B_n, \\ B_{n+1} &= -\delta_1 + (\phi_1 - \Sigma^{\frac{1}{2}})' B_n + C_n, \\ C_{n+1} &= \phi_2' B_n, \end{aligned} \tag{27}$$

with initial estimates of $A_1 = 0$, $B_1 = -\delta_1$ and $C_1 = \mathbf{0}$ (for the proof see Shao *et al.*, 2012).

The continuously compounded yield $r_t^{(n)}$ on an n-period zero-coupon bond is given by:

$$r_t^{(n)} = -\frac{\log P_t^{(n)}}{n} = -\frac{A_n}{n} - \frac{B_n'}{n} z_t - \frac{C_n'}{n} z_{t-1}, \tag{28}$$

In order to derive the stochastic discount factors, the market prices of risk, λ_t, need to be estimated. λ_t follows the recursive formula given in Equation (22). The starting values λ_0 and λ_1 are estimated by minimizing the squared deviations of the fitted bond yields

Table 6: Fitted values of the market prices of risk λ_0 and λ_1.

	λ_0 (6×1)	λ_1 (6×6)					
$r^{(1)}$	0.242	0.619	-0.153	-0.196	0.017	2.658	0.785
$r^{(40)} - r^{(1)}$	-0.619	1.168	-0.375	-0.435	0.780	1.536	0.663
$d\ln H$	0.097	-0.637	0.290	0.541	0.011	-0.624	-0.760
$d\ln R$	-0.652	0.883	-0.452	-0.896	0.497	1.484	0.794
$d\ln GDP$	0.939	-1.552	0.369	2.027	-0.876	1.739	-1.444
$d\ln CPI$	0.106	0.603	-0.166	-0.335	0.045	1.019	0.422

Table 7: Correlation between stochastic discount factors and economic variables.

Variable	$r^{(1)}$	$r^{(40)} - r^{(1)}$	$d\ln H$	$d\ln R$	$d\ln GDP$	$d\ln CPI$
m_t	-0.723	0.325	0.257	0.032	-0.579	-0.442

from the observed yields:

$$\min_{\lambda_0,\lambda_1} \sum_{t=1}^{T} \sum_{n=1}^{N} \left(\hat{r}_t^{(n)} - r_t^{(n)} \right)^2 . \tag{29}$$

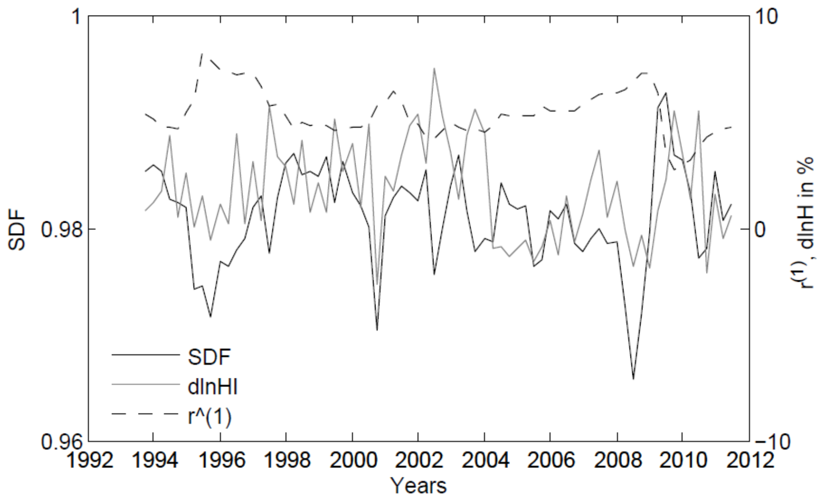
The model is calibrated using zero-coupon yield for four maturities: three months, one year, five years and ten years, i.e. $N = 4$. The calibrated values of λ_0 and λ_1 are given in Table 6.

Figure 5 plots the fitted stochastic discount factors together with historical house price growth rates and three-month zero-coupon yields. The stochastic discount factor is negatively correlated with the short rate, GDP growth and with inflation. It is positively correlated with house price growth and the term spread (see Table 7).

4 Reverse Mortgage Risk and Profitability Analysis

To assess how risk and profitability differ for reverse mortgages with different payout designs, quarterly cash flows of reverse mortgage loans are computed based on 10,000 paths of the economic variables simulated from the VAR(2) model over a 40-year period along with the projected probabilities of loan termination from the Markov model. Risk and profitability are assessed on a representative loan basis. The key drivers of the reverse mortgage cash flows, such as the outstanding loan balance and house prices, are

Figure 5: Fitted stochastic discount factors, m_t, house price growth rates, $d\ln H$, and three-month zero-coupon yields, $r^{(1)}$.



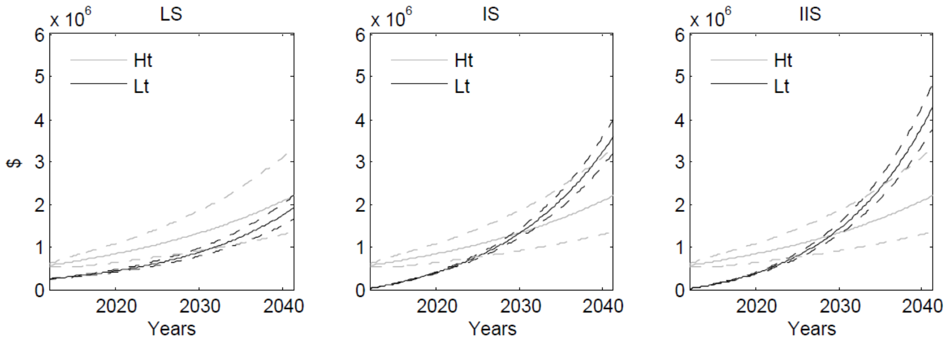
analyzed separately to show how these factors impact the lender's financial position. Two commonly used risk measures, the Value-at-Risk (VaR) and the Conditional Value-at-Risk (CVaR), are computed at the 99.5% level to determine the amount of risk-based capital the lender should set aside for the different types of reverse mortgage. Detailed sensitivity analysis is conducted to test the impact of contract settings including the loan-to-value ratio (LTV), the borrower's age and key model assumptions on the results.

4.1 Base Case Settings

In the base case, we consider a single female borrower aged 75, who is subject to the Australian mortality experience. The borrower's maximum attainable age is $\omega = 105$. The initial house price is set to $H_0 = \$600,000$.[2] The LTV is set to 40%, resulting in an initial loan amount of $L_0 = \$240,000$. In the sensitivity analysis, we consider other borrower ages, other LTVs and also allow for mortality improvements. Sale transaction costs are set to $\gamma = 6\%$ as in Alai *et al.* (2013). The reverse mortgage loan lender is assumed to borrow $\varphi = 92\%$ of the loan principal(s) and to use capital to finance the remainder. Alternative risk-baaed capital ratios are considered in the sensitivity analysis.

[2]The median price of established house transfers in Sydney was $595,000 in the second quarter of 2011 (see Australian Bureau of Statistics, 6416.0 House Price Indexes: Eight Capital Cities).

Figure 6: Average loan balance, L_t, and house price, H_t, over time with 90% confidence intervals for reverse mortgages with lump-sum (LS), fixed income stream (IS) or inflation-indexed income stream (IIS) payments.

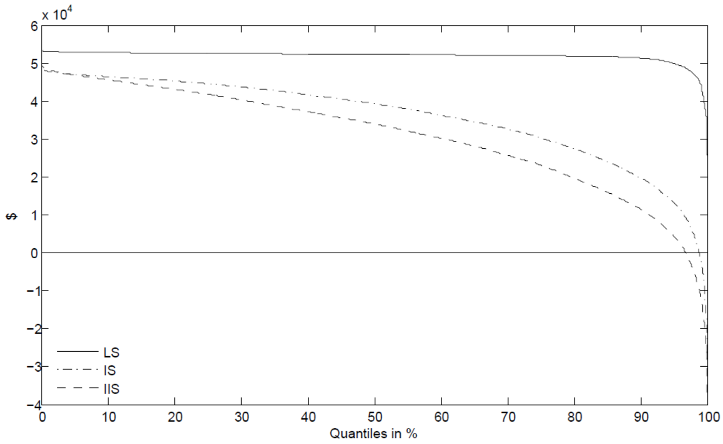


4.2 Base Case Results

The first step of the cash flow analysis is to quantify the value of the no negative equity guarantee (NNEG) and the corresponding insurance premium. Panel A of Table 8 shows that the expected present value of the NNEG is very low for the lump-sum reverse mortgage and significantly higher for the two income stream reverse mortgages. The value of the guarantee is highest for inflation-indexed income stream reverse mortgages. The reason for the different exposures to negative equity risk is that the loan balances of the three reverse mortgage types accumulate differently over time. Figure 6 compares the development of the loan balances with the growth of the house price index. Negative equity arises when the accumulated loan balance *crosses over* the value of the property. Income stream reverse mortgages start with lower loan balances than the lump-sum reverse mortgage, but their outstanding loan balances accumulate faster over time. As a result, income stream reverse mortgages are subject to an earlier crossover point and a higher risk of negative equity than lump-sum reverse mortgages.

The volatility of house price growth is the major contributor to negative equity events. Figure 6 shows that although the lender of the lump-sum reverse mortgage on average does not face negative equity risk, even if the loan is accumulated for 40 years, negative equity events can occur within 30 years of the loan duration in the case of extreme real estate market downturns, which are represented by the lower 5%-quantile of the house price distribution. With an income stream payout structure, negative equity on average

Figure 7: Distribution of the expected present value of the lender's net payoff for reverse mortgages with lump-sum (LS), fixed income stream (IS) or inflation-indexed income stream (IIS) payments



happens after 25 years in the case of sluggish house price growth.

Negative equity is a major risk in reverse mortgages, but the lender does not incur losses at the point of negative equity. Actual losses arise when the total financing costs of the loan exceeds the cash received from selling the collateralized property. The numerical results given in Panel A of Table 8 show that the lender's expected net present payoff in the base case scenario is positive and high for all three reverse mortgage types. The highest net payoff results for lump-sum reverse mortgages. The two risk measures, Value-at-Risk and the Conditional Value-at-Risk, show that the lender faces no financial risks from lump-sum reverse mortgages with a very high probability of 99.5%, but is exposed to some financial risks with the two income stream products. These findings are also illustrated in Figure 7, which shows the distribution of the expected present value of the lender's net payoff.

All of the payout types of reverse mortgages are found to be profitable in the base case scenario. There is a small chance of losses for income stream products and lenders of these products should hold capital against this risk. A reverse mortgage that provides the borrower with an inflation-indexed income, which is often depicted as very expensive for the lender, is found to be sustainable in terms of risk and profitability.

Table 8: Risk and profitability measures for reverse mortgages with lump-sum (LS), fixed income stream (IS) or inflation-indexed income stream (IIS) payments

	Contract	Variable	Payment	NN	$\pi(p.a.)$	EPV	VaR	$CVaR$
A.	Base case: age 75, LTV = 40%, $\varphi = 92\%$, no mortality improvements, VAR(2) model							
	LS		240,000	239	0.011%	51,977	-40,395	-36,336
	IS		8,133	6,404	0.409%	35,829	7,742	14,176
	IIS		6,835	9,714	0.641%	30,859	17,411	23,506
B.	Sensitivity analysis: different borrower ages							
	LS	Age = 65	240,000	2,264	0.061%	88,327	-41,926	-32,780
		Age = 85	240,000	10	0.000%	26,437	-25,507	-24,778
	IS	Age = 65	5,789	7,760	0.328%	59,138	4,238	13,664
		Age = 85	13,656	10,245	1.080%	18,804	13,680	18,614
	IIS	Age = 65	4,534	10,845	0.487%	50,772	17,307	25,869
		Age = 85	12,234	14,728	1.564%	15,908	20,387	24,713
C.	Sensitivity analysis: different loan-to-value ratios (LTV)							
	LS	LTV = 30%	180,000	8	0.000%	39,046	-38,167	-37,230
		LTV = 50%	300,000	2,188	0.076%	64,228	-26,416	-16,853
	IS	LTV = 30%	6,100	716	0.062%	28,306	-13,272	-9,589
		LTV = 50%	10,166	31,779	1.530%	38,775	38,245	46,608
	IIS	LTV = 30%	5,127	1,330	0.120%	25,150	-6,455	-2,254
		LTV = 50%	8,544	42,707	2.090%	31,185	48,972	57,644
D.	Sensitivity analysis: mortality improvements (MI)							
	LS	MI = 10%	240,000	353	0.014%	56,674	-41,363	-36,699
		MI = 20%	240,000	454	0.018%	60,164	-41,881	-36,833
	IS	MI = 10%	8,133	9,932	0.551%	40,348	15,367	21,907
		MI = 20%	8,133	13,139	0.663%	43,530	20,339	27,706
	IIS	MI = 10%	6,835	15,541	0.880%	34,442	26,412	33,461
		MI = 20%	6,835	20,986	1.069%	36,757	33,175	41,011
E.	Sensitivity analysis: different leverage ratios φ							
	LS	$\varphi = 88\%$	240,000	239	0.011%	55,716	-44,256	-40,150
		$\varphi = 84\%$	240,000	239	0.011%	59,455	-48,186	-43,963
	IS	$\varphi = 88\%$	8,133	6,404	0.409%	38,769	4,688	11,095
		$\varphi = 84\%$	8,133	6,404	0.409%	41,709	1,669	8,016
	IIS	$\varphi = 88\%$	6,835	9,714	0.641%	32,457	15,792	21,818
		$\varphi = 84\%$	6,835	9,714	0.641%	34,054	14,093	20,147
F.	Sensitivity analysis: VAR(1) model							
	LS		240,000	42	0.002%	51,912	-48,508	-45,924
	IS		8,133	2,460	0.180%	36,636	-6,798	-512
	IIS		6,835	4,159	0.318%	31,635	4,771	11,405

For the IS the fixed quarterly payment is given. Payments under the IIS increase over time and only the level of the first payment is given. NN is the expected present value of the no negative equity guarantee (NNEG). The insurance premium for the NNEG, π, is reported on an annual basis. EPV is the expected present value of the lender's net payoff. VaR is the 99.5% Value-at-Risk of the present value of the lender's net payoff. $CVaR$ is the corresponding Conditional Value-at-Risk.

4.3 Sensitivity to the Borrower's Age

In the base case, we have considered a female borrower aged 75 years, which is currently the average age of borrowers in Australia (Hickey, 2012). There has been a shift in the age profile of reverse mortgage borrowers in Australia. In 2006-2008 40% of the settlements were made with borrowers under the age of 70. This number has gone down to 30% in 2011. By contrast, households in the United States have started taking out reverse mortgage loans at younger and younger ages over the last two decades. There, the median borrower age was 69.5 years in the financial year 2011 (Consumer Financial Protection Bureau, 2012).

Panel B of Table 8 provides results on the risk profiles and profitability of reverse mortgages with different payout designs for initial ages 65 and 85. The average loan duration for these ages is 16.1 and 4.4 years, respectively, and the payments for the income stream products are adjusted accordingly.

All three types of reverse mortgages are substantially more profitable and relatively less risky when offered to younger borrowers. Profits for the lender arise from the lending margin accumulated on the outstanding loan balance. Higher lending margins are accumulated when the average loan duration is longer. Offering a reverse mortgage to a 65-year-old female borrower instead of lending to a 75-year-old increases the lender's expected net payoff by 65-70% depending on the product. This increase in the expected value generally goes along with decreases of the 99.5%-VaR, which indicates that the lender bears less risk. Borrowing to a 85-year-old on the other hand results in substantial reductions in the lender's expected net payoff of -48 to -49% compared to the base case and higher financial risks. Providers of the lump-sum reverse mortgage can expect positive net payoffs with a probability of 99.5% for all three borrower ages, 65, 75 and 85.

For the lump-sum reverse mortgage, the value of the NNEG guarantee and the insurance premium, π, decrease with the borrower's age. At age 85, both values are close to zero. Income stream reverse mortgages with fixed or inflation-adjusted payments show a different pattern. For these products, the value of the guarantee is lowest for a 75-year-old borrower, but the insurance premiums increase with the borrower's age. Different effects come into play here: the premium is spread over a longer time horizon and it is applied to loan balances that grow at different speeds.

4.4 Sensitivity to the Loan-to-Value Ratio

In the base case, we have assumed a LTV of 40% for the 75-year-old borrower, which is higher than the loan amounts currently offered in the Australian market, where the average maximum LTVs range between 14% for 65-year-olds and 34% for borrowers aged 80+ (Hickey, 2012). Much higher LTVs are found in the U.S. markets, where LTVs for HECM products range between 55-80% for ages 65-85 (Oliver Wyman, 2008). Maximum LTVs are typically higher for older borrowers because the period of loan accumulation is shorter, which lowers the chances of negative equity.

To test the impact of the LTV on the risk and profitability of the different reverse mortgage products, we compare the results for LTVs of 30% and 50% for the 75-year-old borrower. Panel C of Table 8 reports these results. The lump-sum and income stream payments are adjusted accordingly.

The NNEG value increases dramatically for all three products when the LTV is raised from 30% to 40% to 50%. Accordingly, a higher premium is charged to the borrower for the guarantee. The lender's expected net payoff also increases with the LTV for all three reverse types mortgages because the lending margin accumulates on larger loan balances. The net payoff for the lump-sum reverse mortgage increases almost proportionally with the LTV: plus 33% for the first step from $LTV = 30\%$ to 40% and plus 24% for the second step from 40% to 50%. The lender's expected net payoff for the income stream reverse mortgages increases less than proportionally with the LTV: plus 23-27% for the first step and plus 1-8% for the second. The lump-sum reverse mortgage is the most profitable of the three reverse mortgage designs for all three LTVs.

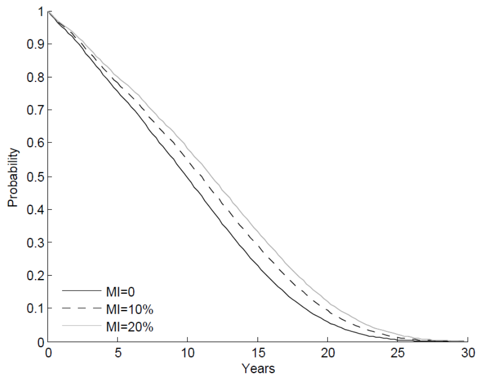
The lowest LTV in the comparison ($LTV = 30\%$) still exceeds the values currently offered in the Australian market. The VaR and CVaR results show that all three reverse mortgages types bear no financial risks for $LTV = 30\%$. The lump-sum reverse mortgage, which is the most common reverse mortgage type in Australia, actually bears virtually no financial risk for LTVs of 40% and 50%. Negative net payoffs can occur for income stream products and their chance and severity increases for higher LTVs. These findings, and similar findings by Alai *et al.* (2013), suggest that Australian reverse mortgage lenders could increase the maximum loan amounts offered to customers to make these products more attractive. In their risk management and solvency capital allocation, lenders need to take into account the products' payoff structure, as well as the age of the borrower and the LTV.

4.5 Sensitivity to Mortality Rate Improvements

Death of the borrower is a main cause for termination of reverse mortgage loans. The risk of concern to a lender is that an individual lives longer than expected, or longevity risk. Other causes of loan termination such as entry into a long-term care facility, early prepayment or refinancing are modeled as occurring at rates proportional to mortality rates. Improvements in mortality rates increase the average duration of the loan, resulting in higher outstanding loan balances at the time of termination. Thus, mortality rate improvements increase the chances and severity of negative equity events.

To assess the impact of unexpected mortality rate improvements on the different reverse mortgage types, we assume that the lender determines quarterly income payments on the base case assumptions for the 75-year-old female borrower with a LTV of 40%. We then assess the impact of an unexpected reduction in mortality rates of 10% or 20%. We implement the mortality improvements by scaling down the force of mortality, $\hat{\mu}_x$, in Equation (17), which determines the probability of the loan being in-force. As a result, the average in-force durations for the 75-year-old borrower increases from 9.3 years in the base case ($MI = 0$), to 10.2 years for $MI = 10\%$ or to 10.9 years for $MI = 20\%$.

Figure 8: Probability of loan in force for a borrower aged initially 75 years.



$MI = 0$ denotes zero mortality improvements. $MI = 10\%$ and $MI = 20\%$ denote mortality improvements of 10% and 20%.

The results given in Panel D of Table 8 show that the NNEG value is much more sensitive to mortality rate improvements for income stream reverse mortgages than for the lump-sum reverse mortgage. The highest relative changes are found for inflation-indexed reverse mortgages: the NNEG values increase by 60% and 116% compared to the base case when mortality rates improve by 10% and 20%, respectively. The insurance premiums for the NNEG also increase, but less so because the payments are spread over a longer time horizon.

All three reverse mortgages yield higher expected net present payoffs to the lender when mortality rates improve and the borrower lives longer. Mortality rate improvements of 10% result in expected payoffs increasing by 9-13%. Mortality rate improvements of 20% result in increases in the payoffs of 16-21%. These findings appear surprising given that the lender based the pricing on a shorter expected loan-duration, resulting in regular payments that are too high. But this is outweighed by the additional accumulation of the lending margin over the longer loan durations.

The risk measures VaR and CVaR show that lenders of lump-sum reverse mortgages do not face financial risks even with mortality improvements, whereas financial risk increases substantially for income stream reverse mortgages. Reverse mortgage lenders need to carefully assess their assumptions with respect to survival rates and other factors of loan termination such as entry into long-term care. Selection effects may also occur. Borrowers with an above-average life expectancy benefit longer from the NNEG, the right to live in the property and continued income stream payments. Including these selection effects would reinforce the main findings of this study: lump-sum reverse mortgages are more profitable and less risky for the lender.

4.6 Sensitivity to the Risk-based Capital

The model assumes that reverse mortgage loans are financed through capital and borrowing. The lender is subject to interest rate charges on borrowing. Lower levels of risk-based capital can expose lenders to greater risk in the case of losses. In the base case, a borrowing ratio of $\varphi = 92\%$ was assumed, since 8% is the standard risk-based capital for mortgages under Basel II. Panel E of Table 8 provides results for alternative borrowing ratios of $\varphi = 88\%$ and $\varphi = 84\%$.

The financing structure does not impact the payments made to the borrower, the NNEG value or the level of the insurance premiums for the guarantee. But, the financing structure does impact risk and profitability for the lender. When the borrowing ratio is lowered, the expected present value of the lender's net payoff, EPV, increases for all three reverse mortgage types and the lender's financial risk reduces. For example, lowering φ from 92% to 88% increases EPV by 7% for the lump-sum reverse mortgage and by 8% and 5% for reverse mortgages with fixed and inflation-adjusted payments.

4.7 Sensitivity to the VAR Assumptions

We modeled the dynamics of the economic variables with a VAR(2) model. The VAR(2) model has a large number of parameters and the BIC model selection criterion indicated a VAR(1) model as a viable alternative. Panel F of Table 8 shows the results for the case when a VAR(1) model is assumed. The resulting expected net present payoffs for the lender are very similar to the base case, with differences of less than 3% for all three reverse mortgage types. However, the distribution of the lender's expected payments is changed. The VaR and CVAR values have decreased substantially, indicating that lenders of all three reverse mortgage types would be exposed to substantially lower financial risk if the economic variables would evolve according to a VAR(1) process. Lump-sum reverse mortgages and reverse mortgages with fixed income payments are found to be profitable with a probability of 99.5%. Also, the values of the NNEG and the corresponding insurance premiums are all much lower for the VAR(1) model.

These findings show that the results regarding the financial risks for reverse mortgage lenders are sensitive to model choice. Financial risk is underestimated when a VAR(1) model is adopted instead of a VAR(2) model, which would lead lenders to hold insufficient amounts of capital.

5 Conclusions

Our study compares the profitability and risk profiles of reverse mortgage loans with different payout options from the lender's perspective. We apply a multi-period stochastic framework for simulating and evaluating the cash flows of reverse mortgage contracts with lump-sum payments, fixed income payments or inflation-adjusted income payments. The framework incorporates a multi-state Markov model to derive probabilities of loan termination. A vector autoregressive model is used to project the economic variables and to derive risk-adjusted stochastic discount factors for pricing the no negative equity guarantee typically embedded in reverse mortgage contracts.

Lump-sum reverse mortgages are shown to be more profitable and less risky for the lender than income stream reverse mortgages, reflecting the longevity risk inherent in the income stream products. This finding is robust to several sensitivity tests. A lump-sum reverse mortgages starts with a high loan balance that increases with the interest rate. Income stream reverse mortgages start with a low loan balance, but the loan balance increases with each payment to the borrower and with the interest rate. As a result, income stream reverse mortgages are subject to higher *cross over risk*, which arises when the loan balance exceeds the house value at the time of termination. The risk measure VaR and CVaR calculated at the 99.5% of the distribution of the lender's expected net payoff show that for typical loan conditions lenders do not have to hold capital for lump-sum reverse mortgages, but should hold capital for income stream reverse montages.

We have analyzed the impact of key assumptions on the results. Major effects are found for the borrower's age and for the loan-to-value ratio. All three types of reverse mortgages are substantially more profitable and less risky when offered to younger retirees. Furthermore, all three contract types are more profitable but also more risky for higher loan-to-value ratios. Unexpected improvements in mortality rates increase the lenders' expected net payoffs moderately, but financial risks increase as well. The risk-based capital ratio is also important: a higher risk-based capital ratio increases the profitability and reduces the financial risk exposure of all three contracts. Sensitivity analysis with respect to the economic model shows that very similar expected net present payoff values result along with lower levels of financial risk when a VAR(1) model is assumed instead of a VAR(2) model.

Securing sources of retirement income is one of the most difficult challenges that many countries face today. Reverse mortgage loans can provide flexible borrowing arrangements, enabling retirees to structure cash flows according to their needs. As reverse mortgage markets develop internationally, lenders and regulators need to understand the risks embedded in these products. Our results show that lenders in the Australian market could increase the loan-to-value ratios of lump-sum reverse mortgages. More importantly, lenders could also extend their product range and offer more income stream products, which are found to be profitable in the Australian market.

We have modeled the risks embedded in reverse mortgages on a representative loan basis and using a city-level house price index. In practice reverse mortgage portfolios will be exposed to property values that differ from the market-wide average and this generates basis risk. A recent study shows that the value of the no negative equity guarantee is significantly higher when individual house price is taken into account (Shao *et al.*, 2012).

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