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Developing Equity Release Markets: Risk Analysis for Reverse Mortgages and Home Reversions

Daniel Alai², Hua Chen¹, Daniel Cho², Katja Hanewald², and Michael Sherris²

Abstract: Equity release products are sorely needed in an ageing population with high levels of home ownership. There has been a growing literature analyzing risk components and capital adequacy of reverse mortgages in recent years. However, little research has been done on the risk analysis of other equity release products, such as home reversion contracts. This is partly due to the dominance of reverse mortgage products in equity release markets worldwide. In this paper, we compare cash flows and risk profiles from the provider's perspective for reverse mortgage and home reversion contracts. An at-home/in long-term care split termination model is employed to calculate termination rates, and a vector autoregressive (VAR) model is used to depict the joint dynamics of economic variables including interest rates, house prices and rental yields. We derive stochastic discount factors from the no arbitrage condition and price the no negative equity guarantee in reverse mortgages and the lease for life agreement in the home reversion plan accordingly. We compare expected payoffs and assess riskiness of these two equity release products via commonly used risk measures, i.e., Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR).

Key Words: Reverse Mortgage, Home Reversion, Vector Autoregressive Models, Stochastic Discount Factors, Risk-Based Capital

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1. Introduction

Home equity release products allow retirees to convert a previously illiquid asset into cash payments which can be used for home improvements, regular income, debt repayment, aged care and medical treatments as well as a range of other uses which improve quality of life for retirees. There has been a growing literature addressing risk factors and capital adequacy of reverse mortgage products in recent years, including but not limited to, Boehm and Ehrhardt (1994), Chinloy and Megbolugbe (1994), Szymanoski (1994), Rodda et al. (2004), Ma and Deng (2006), Wang et al. (2008), Chen et al. (2010), Sherris and Sun (2010), and Li et al. (2010). However, little research has been done on risk analysis of other equity release products, such as home reversion contracts. The purpose of this paper is to introduce home reversion schemes to the readers and compare cash flows and risk profiles from the provider's perspective between reverse mortgage and home reversion contracts.

In a reverse mortgage, the provider lends the customer cash and obtains a mortgage charge over the customer's property (or a share of the property). The contract is terminated upon the death or permanent move-out of the customer, at which time the property is sold and the proceeds are used to repay the outstanding loan. Typically, a no negative equity guarantee is included in the contract, which stipulates that the customer is not liable in case the sale proceeds of the property are insufficient to repay the loan. In a home reversion scheme, the provider purchases the ownership right over the customer's property (or a share of the property). The home is sold at discount (typically between 35% and 60% of the market price), and the contract includes a lease for life agreement allowing the customer to reside in the property until death or permanent move-out.

The untouched research area of home reversions is partly due to the underdeveloped market. In the US, reverse mortgage products dominate the equity release market. The Home Equity Conversion Mortgage (HECM) program is considered the safest and the most popular

program of its kind in the US, since it is insured by the US federal government, and accounts for 95% of the market share (Ma and Deng, 2006). The dominance of a single equity release product in the US stands in stark contrast to the dynamics of some foreign markets. In the UK, for example, reverse mortgages, home reversions and other equity release products have been available for 10 to 30 years. Among them, reverse mortgages account for 75% of the equity release products available in the market while home reversions account for most of the remaining 25% (ASIC 2005). The reverse mortgage market in Australia consisted of 42,410 loans with a total market size of \$3.32 billion by the end of 2011. The Australian market saw a 10% growth in the value of new lending in 2011 and a 22.5% growth over the last two years (Deloitte 2012). Home reversion schemes exist in Australia but are relatively new and available commercially through just one outlet, Homesafe Solutions. They are currently available to consumers aged 60 or over living in certain areas in Sydney or Melbourne.

From the provider's perspective, it is important to estimate the probability of termination, as delayed termination results in heavier loan accumulation and increases the chances of negative equity in reverse mortgages, or it causes an unexpected longer term for lease in home reversions resulting in the provider overpaying the customer when the contract originates. The US HECM program initially assumed loan termination rates being equal to 1.3 times the underlying female mortality rates as no termination experience were available. Later on, Chou et al. (2000) use a complimentary log-log regression model to examine how loan termination is affected by key factors based on the actual HECM loan termination data. They find that age, house price appreciation, loan duration, mortality, personal assets, gender and co-borrower status all contribute to explain loan termination. They also report that the initial assumption of 1.3 times the female mortality is too low for younger borrowers and slightly too high for older borrowers. Rodda et al. (2004) find similar results. However, the regression-based termination models used in both studies have several drawbacks. First, they

rely heavily on availability of data. Second, they assume the probability of loan termination remains constant after age 90, which is rather unrealistic. Third, these models do not make explicit allowance for move-outs, health or non-health related (Ji et al. 2012). Szymanoski et al. (2007) suggest that termination of reverse mortgage loans should be modeled based on its key causes: borrower's mortality, long-term care move-out, prepayment and refinancing. In light of this, Ji et al. (2012) develop a semi-Markov model for reverse mortgage terminations for joint borrowers, which incorporates the aforementioned modes of termination. We adapt their model to a single female borrower and consider only two reasons: death and entry to long-term care facility, as prepayment and refinancing are rare for home reversion consumers.

Interest rate risk, house price risk, and rental yield risk are other major risks in equity release products. The previous literature examining the embedded risks in reverse mortgage contracts either focus on analysing the house price dynamics alone (see, for example, Chen et al. 2010 and Li et al. 2010), or modelling the dynamics of house prices and interest rates independently (Chinloy and Megbolugbe 1994, Ma et al. 2007, Wang et al. 2008, etc). This approach neglects correlations among these key variables. In addition, the derived risk-neutral measure fails to represent all sources of uncertainty and the dependency structure among risks. To overcome this, Huang et al. (2011) implement a two-dimensional volatility vector linking the house price and interest rate dynamics. Chang et al. (2012) propose a multidimensional linear regression model that captures the relationship between house prices and key macroeconomic factors. Sherris and Sun (2010) fit a vector autoregressive (VAR) model to examine risks embedded in reverse mortgage insurance policies. Despite its simplicity, a VAR model is sophisticated enough to capture the linear interdependencies among multiple time series. We adopt a VAR process to jointly model the dynamics of interest rates, house prices, rental yields and GDP. Our approach is different from Sherris and Sun (2010) in two major ways. First, GDP is added to the model to acknowledge the impact

of macroeconomic factors on other economic variables of interest. Second, we derive stochastic discount factors based on the VAR model that can capture uncertainty arising from a range of sources: interest rate, house price and rental yield. This approach has not been used in Sherris and Sun (2010) or in any other studies in equity release markets before.¹

Our methodology is closely related to Ang and Piazzie (2003), who use stochastic discount factors, or pricing kernels, to extend their VAR model with an affine term structure of interest rates. In this manner they are able to value all assets and cash flows. Cochrane and Piazzesi (2005) study time variation in expected excess bond returns. They construct an affine model, i.e., prices are linear functions of state variables of the VAR model, that generates the bond yield returns. Hoevenaars (2008) also combines the VAR model with an affine term structure model of interest rates in such a way that there are no arbitrage opportunities. He uses the model to generate macroeconomic scenarios that serve as input for an asset liability management model of a pension fund.

The derived stochastic discount factors are used for pricing the no negative equity guarantee and the lease for life agreement that are fundamental elements in reverse mortgage and home reversion schemes, respectively. We then simulate cash flows and calculate the actuarial present value of net payoffs of the provider. We also quantify risk measures such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) at the 99.5% level to illustrate the amount of solvency capital to be set aside for each type of equity release products. Sensitivity analysis is conducted to investigate the impacts of the loan-to-value ratio (LVR), the initial house price, mortality improvements, and the leverage ratio on the payoffs and risk profiles of reverse mortgage and home reversion contracts.

¹ Following our work, Cho (2012) and Shao et al. (2012) use the VAR model and the stochastic discount factor approach to study other aspects of equity release products. Cho (2012) compares cash flows for reverse mortgages with different payout designs. Shao et al. (2012) quantify the impact of individual house price risk on the pricing of equity release products.

We find that the maximum LVRs offered to customers in the Australian market is set so low that reverse mortgage providers bear almost no risk of capital loss. This suggests that reverse mortgage providers in Australia could increase maximum LVRs to facilitate the expansion of the reverse mortgage market. Compared to reverse mortgage contracts, providers of home reversion schemes obtain a lower payoff and assume a higher risk, which justifies the market dominance of reverse mortgages in Australia. An efficient risk sharing and risk transfer mechanism needs to be developed to stimulate growth of the home reversion market. By providing an appropriate framework of regulation, financial literacy education and by promoting liquidity to investors, governments can encourage private supply of home reversions at modest public expense.

Interestingly, using higher LVRs in the range of those offered under the US HECM program, we find exactly opposite results: reverse mortgage contracts are less profitable and riskier than home reversion contracts. This finding confirms that the insurance of crossover risk in reverse mortgages provided by the Federal Housing Agency (FHA) is an important factor in the US market. The finding also indicates that there is a large potential market for home reversion schemes in the US.

The remaining body of this paper is organized as follows. In Section 2, we review the basic features of reverse mortgage and home reversion contracts, and discuss risks involved in these two products. In Section 3, we present a termination model and use a VAR model to jointly model the dynamics of interest rates, house prices, and rental yields. Stochastic discount factors are derived based on the VAR model. In Section 4, we develop the pricing formula for the no negative equity guarantee in reverse mortgages and the lease for life agreement in home reversions. Cash flow structures are analysed for both contracts. In Section 5, numerical examples are used to compare these two equity release products in terms of payoffs and risks. Section 6 concludes the paper.

2. Product Review in Australia

2.1. The Reverse Mortgage Market

2.1.1. Product Review

The reverse mortgage market has gained considerable momentum in Australia in recent years. According to the media release by Deloitte (2012), the market size of reverse mortgages climbed from \$0.9 billion in 2005 to \$3.32 billion in 2011. There were 42,410 loans in the market as of the end of 2011 while this number in 2005 was 16,584. The average loan size was \$78,249 in 2011, compared to \$51,148 in 2005. While the market is Australia-wide, three states make up more than 70% of the national market: NSW 35%, QLD 20% and VIC 18%. The main features of a typical reverse mortgage contract in Australia are reviewed as follows.

Conditions: All lenders set a minimum age for the youngest person on the title of the property that is being mortgaged. In most cases, this is 60 years. Some reverse mortgage providers set the minimum age as 63 or 65 years (Bridges et al. 2010). Although the specific terms and conditions vary across products, most contracts oblige the consumer to (ASIC 2005):

- maintain insurance for the property,
- pay all outgoings,
- maintain the property to the standard required by the provider,
- not leave the property vacant for more than six to 12 months,
- not allow new non-approved residents to reside in the property, and
- not sell, lease or renovate the property without the provider's prior approval.

Initial Loans: The loan amount depends primarily on two factors: age and value of the home.² The borrower's age or the younger borrower's age in case of a couple determines the maximum LVR. The LVR increases as an individual's age increases. For example an individual aged 60 may borrow 15% of the value of their home whereas someone aged 80 or older can borrow up to 35% of the value of their home.

Payout Options: Depending on the contract, the borrower can withdraw the loan as a lump sum, income streams, a line of credit, or a combination of these payment plans. As of 2010, lump sum loans take up 95% of the Australian market and income streams account for 5%. The proportions of lump sums and income streams have been relatively stable since 2008 (Deloitte 2011a).

Termination: Repayments are generally not made until an individual moves out of the house or dies. If the home is jointly owned, the loan is only repayable once the last surviving partner dies or moves out.

Guarantee: In Australia, SEQUAL-accredited members must offer a *no negative equity guarantee* which ensures that no matter how long the loan runs for, the borrower can never owe more than the value of the security, in this case, their house.^{3 4} However, the no negative equity guarantee can be negated through a number of actions or inactions on the part of the borrower, including fraud or misrepresentation, failing to maintain the property in a good condition, failing to insure the property, or not paying the council rates on the property.

Interest Rates: Interest rates can be variable or fixed. Variable rate loans are the most popular product in Australia. Variable rates are on average 1% above the standard variable home loan

² In the US, Federal Housing Administration (FHA) imposes a mortgage limit which is \$625,500 for one-family house. The initial loan amount is determined by the younger borrower's age and the adjusted property value. The adjusted property value is defined as the lesser of the appraised value of your home, the FHA HECM mortgage limit of \$625,500 or the sales price.

³ SEQUAL is the abbreviation of the Senior Australians Equity Release Association. In order to protect the customers, SEQUAL has established a strict Code of Conduct that each SEQUAL-accredited member has to agree its equity release product(s) adhere to.

⁴ The no negative equity guarantee is also called a non-recourse provision in the US reverse mortgage market.

rate. The margin (or mortgage insurance premium) is charged to manage the risk of providing the no negative equity guarantee.⁵ Fixed interest rates can be set for varying terms—generally 5, 10 or 20-years or lifetime. The proportion of fixed interest reverse mortgage loans is negligible 1% in 2010 (Deloitte 2011a). There is now only one SEQUAL-accredited lender (RBS) providing a fixed rate option on their products (Bridge et al. 2010).

Fees: There are typically setup fees, ongoing fees and exit fees associated with reverse mortgages which vary from lender to lender.

2.1.2. Major Risks in Reverse Mortgages

Reverse mortgages differ from traditional forward mortgages in the way that the outstanding loan balance grows due to principal advances, interest accruals, and other loan charges over the life of the loan. The loan balance may grow to exceed the property value at the time of termination because of multiple risks.

Termination Risk: If a borrower lives longer than expected, the principal advances and interest accruals will continue, which may drive the loan balance exceeding the sale proceeds of the property. The mobility rate has the same effect on reverse mortgage products. Borrowers may move out of their homes because of their health condition, marriage, divorce, death of the spouse, disasters, or simply the desire to live in another place.

Interest Rate risk: Most of reverse mortgage products feature adjustable interest rates. Therefore, the variation of interest rates imposes additional uncertainty on reverse mortgage providers. A rise in the interest rate can result in a higher rate of interest accruals on the loan balance than anticipated, which increases the possibility of partial non-repayment when the loan eventually terminates.

⁵ In the US HECM program, mortgage insurance premiums consist of two parts: an up-front charge which is either 2% (HECM Standard) or 0.01% (HECM Saver) of the adjusted property value, and an annual rate of 1.25% of the outstanding loan balance for the life of the loan. FHA collects all the insurance premiums and reverse mortgage lenders are allowed to assign the loan to FHA when the loan balance equals the adjusted property value. FHA takes over the loan and pays an insurance claim to lenders covering their losses. So lenders are effectively shifting the collateral risk to FHA.

House Price Depreciation Risk: The uncertainty in house price depreciation rates is another risk we need to consider. If the home price remains stagnant or grows at a lower rate than anticipated, the outstanding loan balance at maturity may exceed the sale proceeds of the property. Lenders or their insurers may suffer from the losses. As indicated by the recent U.S. housing market downturn, home price depreciation risk is only partially diversifiable: pooling mortgage products nationally only reduces the risk of a downturn in the regional housing market, but cannot diversify the risk of a national economic recession.

2.2. The Home Reversion Market

2.2.1. Product Review

Home reversion schemes allow senior homeowners to sell a proportion of equity in their home while still living there. Homeowners receive a lump sum payment in exchange for a fixed proportion of the future value of their home. There are two main types of home reversion schemes: a sale-and-lease model and a sale-and-mortgage model. In the sale-and-lease model, the title to the property passes to the provider at the time of purchase and the property is leased back to the consumer at a nominal rent. The sale-and-lease product provider in Australia, called Money for Living, went into administration in 2005. The Australian Securities and Investments Commission (ASIC) issued legal proceedings in the Federal Court of Australia alleging that Money for Living advertised its product in a misleading and deceptive manner. A resolution was passed in December 2007, placing the company into liquidation. In the sale-and-mortgage model, the title to the property remains in the consumer's name even after the provider pays. To protect the provider's interest in the property, the consumer is required to give the provider a mortgage over the property (ASIC, 2005). Homesafe Solutions Pty Ltd, a joint venture of Bendigo and Adelaide Bank Ltd and

Athy Pty Ltd, has launched Homesafe Debt Free Equity Release since 2005. We review its features in the following.⁶

Conditions: The homeowner must be aged 60 and over. For a couple, the younger partner must be at least 60. Currently, it is available only to customers residing in certain postcodes within Melbourne and Sydney. As a general rule, the home needs to be free-standing. Other property types are subject to approval from Homesafe. The property is the principal place of residence for at least one homeowner at the time of exchange of contracts. The land value of the property is 60% or greater of the total value determined by an independent panel valuer. The homeowner must own the home outright, or use some of the Homesafe funds received to pay out the existing mortgage.

Funds: Under Homesafe Debt Free Equity Release, it is possible to access any amount between \$25,000 and \$1,000,000. The maximum share that homeowners can sell, so-called acquisition rate, is 65% of the future sale proceeds of the home. Homeowners can enter into additional contracts over time, up to a total share of 65%. There is no restriction as to how the funds should be used.

Payout Option: Homesafe currently offers only a lump sum payout option.

Lease: Homeowners receive a discounted lump sum payment (usually 35% or 60%) in exchange for a fixed proportion of the future value of their home. The discount represents the value of *the lease for life agreement* that allows homeowners to live in the house for life or until voluntarily move-out. Homeowners may be eligible for an early sale rebate if they sell their home earlier than expected.

Termination: The contract terminates when homeowners die or voluntarily vacate the property. Homesafe is entitled to the agreed percentage of the sale proceeds of the house and homeowners retain the share of the sale proceeds that they have not sold to Homesafe.

⁶ More details can be found on the website of Homesafe Solutions Pty Ltd:
<http://www.homesafesolutions.com.au/>

Title: Homeowners remain on the title, so they have the right to use their home for as long as they wish. There is no requirement for homeowners to undertake maintenance of the property after entering into a Homesafe contract. The owners can even rent out the home and keep the rental income. Homesafe will register a mortgage and lodge a caveat on the title, only to secure its share of the sale proceeds.

Fees: Homesafe charges a one-off transaction fee of \$1,690.

2.2.2. Major Risks in Home Reversions

The provider of home reversion contracts faces house price risk. For the lease for life agreement, the uncertainty originates from the rental yield, and the duration of the contract.

Termination Risk: In a home reversion contract, the customer is always better off prolonging the duration of the contract. This is in contrast to a reverse mortgage contract, where early termination may be beneficial for the customer under certain circumstances. Therefore, when valuing the lease for life agreement in an annuity setting, it is realistic to assume that the only modes of termination are death and unavoidable entry into a long-term care facility. It should be noted that some home reversion contracts provide a rent rebate for contracts that terminate much earlier than expected, but the amount is not of the magnitude to induce termination.

Rental Yield Appreciation Risk: In a home reversion contract, the property is sold to the provider at a discounted price. The level of the discount reflects the value of the lease for life agreement. The provider's payoff could be impaired if a low rental yield were assumed when calculating the value of the lease but the actual rental yield would turn out to be much higher.

House Price Depreciation Risk: Lenders of home reversion contracts are entitled to sell the property and secure a part of the sale proceeds when borrowers die or voluntarily move out. Therefore, lenders face the risk of house price depreciation.

2.2.3. Advantages of Home Reversions

From the consumer's point of view, home reversion products have unbeatable advantages over reverse mortgages. Oliver Wyman Financial Services (2005) predicted "though equity solutions have traditionally fared poorly in the US, options such as home reversion products should find a market – especially among owners of higher-value homes, for whom equity release may be intended to diversify a portfolio rather than to free up cash". In addition, reverse mortgages involve the accumulation of debt over the life of the contract while home reversions are debt-free. In order to protect borrowers from negative equity, reverse mortgage programs usually provide a no negative equity guarantee so loan repayment is capped by the sale proceeds of the property. This guarantee is financed via mortgage insurance premiums paid by borrowers. In other words, senior homeowners bear various risks, including longevity risk, interest rate risk and property value risk under a reverse mortgage contract. Nevertheless, these risks are partly remitted to providers under home reversion contracts. Commercial providers are generally better positioned to bear such risks. For example, they can transfer risks to the capital market more efficiently compared with senior homeowners. More importantly, the interests of investors and consumers are aligned under home reversion schemes: both want the value of the home to rise (Oliver Wyman Financial Services, 2005). Therefore, we believe that there remains room for significant growth of a diversified equity release market and we see a great potential for the development of home reversion products.

3. Modelling Framework

3.1. The Termination Model

Though a significant proportion of reverse mortgages are issued to couples (around 40% in the US and 50% in Australia, see Deloitte 2012), the study of joint life dependency is

not the focus of this paper.⁷ For simplicity, we assume a single, female policyholder. The joint-life multistate termination model can be readily incorporated in our model framework if necessary. We do not consider voluntary prepayment or refinancing as consumers of home reversion products are always better off by prolonging the duration of their contracts. In other words, contract termination is determined by two major factors: death and entry into long-term care facilities.

We assume a Gompertz structure for the population force of mortality μ_x for females aged x given by

$$\mu_x = \lambda \exp\{\gamma x\}. \quad (1)$$

Equity release products are designed for a policyholder living at home. Therefore, she is susceptible to at-home mortality, which need not equal to female population mortality. Let θ denote the proportionality constant that produces at home mortality from population mortality. That is, the female at-home mortality rates are scaled down by multiplying θ to represent the better health of retirees, who do not move out to long-term care. The possibility of entry into a long-term care facility is represented by a proportionality constant, ρ . These two parameters can be replaced by one contract-mortality loading factor, $\phi = \theta + \rho$. Hence, the contract force of mortality can be written as follows:

$$\mu_x^c = (\theta + \rho) \mu_x = \phi \mu_x, \quad (2)$$

where μ_x^c denotes the contract termination rate.

The parameters λ and γ are estimated using Australian female mortality data for the period 1950-2009 and age 50-105 from the Human Mortality Database.⁸ We fit both an

⁷ Ji et al. (2012) compare the value of the no negative equity guarantee for joint borrowers under the independence assumption and the semi-Markov assumption. Though the assumption of independence generally leads to an overestimation of NNEG prices, the difference is not significant (see Figure 3 in Ji et al. 2012).

⁸ <http://www.mortality.org/>

ordinary linear regression (LR) to the log-transformed mortality rates as well as a Poisson regression (PR) to death counts with an appropriate exposure offset.

$$\text{LR: } \ln m_x = \beta_0 + \beta_1 x + \varepsilon_{x,t} \quad (3)$$

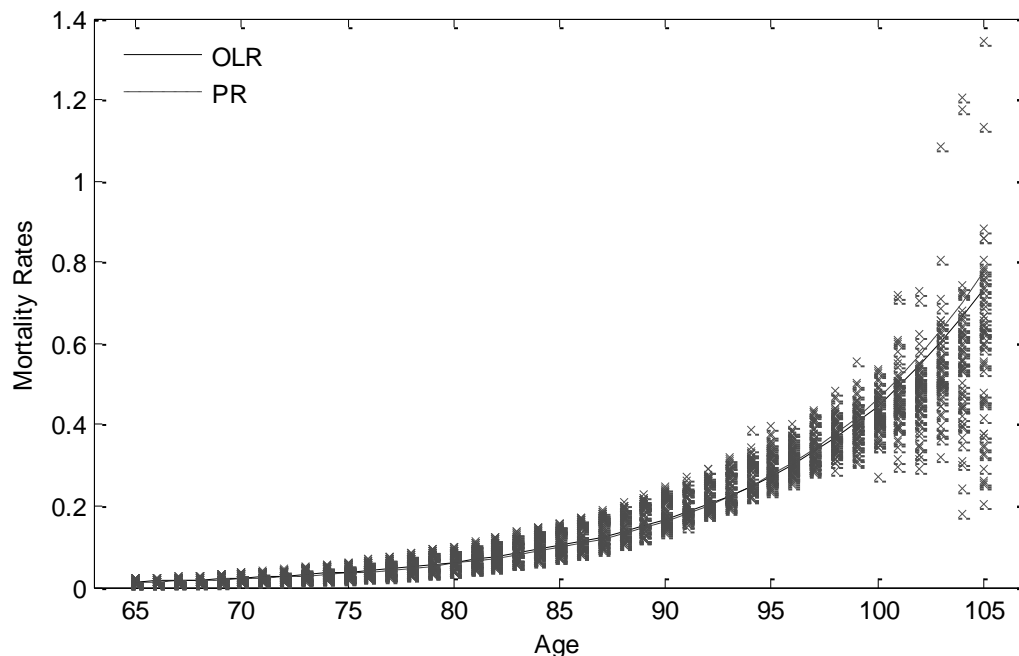
$$\text{PR: } \ln D_x = \ln E_x + \beta_0 + \beta_1 x + \varepsilon_{x,t} \quad (4)$$

where $\ln E_x$ is the offset for the Poisson regression based on the survival counts, E_x . Table 1 reports the estimated parameters and Figure 1 presents the fit graphically. It can be seen that the two regressions produce very similar fits. We use the Poisson regression hereafter due to its intuitive and natural interpretation.

Table 1: Compertz Parameters for the Force of Mortality

	$\hat{\lambda}$	$\hat{\gamma}$
Ordinary Linear Regression (LR)	0.000022	0.099032
Poisson Regression (PR)	0.000014	0.103916

Figure 1: Regression Fit of Log-Mortality Rates



Given estimates for λ and γ , we turn to θ and ρ . Since there is no publicly available contract termination data in Australia, we make use of the parameter estimates

reported by Ji et al. (2012). These authors use the data in the Equity Release Report of the Institute of Actuaries (2005) to estimate the proportional factors for the deviation from an aggregate model to the at-home/in long-term care split model. Table 2 reproduces their estimated proportional factors for females at ages 70, 80, 90, and 100. The proportional factors for ages 71–79, 81–89 and 91–99 are obtained by linear interpolation, while the proportional factors for ages below 70 and ages above 100 are set to the proportional factors for age 70 and age 100, respectively.

Table 2: At home and In Long-Term Care Proportional Factors from Ji et al. (2012)

Age	θ_x	ρ_x	ϕ_x
70-	0.95	0.10	1.05
80	0.90	0.20	1.10
90	0.85	0.33	1.18
100+	0.80	0.46	1.26

Let ${}_tq_x^c = \Pr(t < T < t+1)$ and ${}_tp_x^c = \Pr(T > t)$, for $t = 0, 1, \dots, \omega - x$,

where T is the contract termination time and ω is the maximum attainable age. We have

$${}_tq_x^c = {}_tp_x^c \int_0^1 p_{x+t}^c \mu_{x+t+s}^c ds, \quad (5)$$

which can be solved numerically to yield the desired contract termination probabilities.

We also compute the average contract in-force duration for different age groups (see Table 3). It decreases with the age of the policyholder at loan origination. For individuals aged 65, the average in-force duration is around 18 years. It drops to about 10 years for consumers aged 75 and 5 years for consumers aged 85.

Table 3: Average in-force Duration

Age	65	75	85
Average in-force duration	17.78	9.84	4.80

3.2. The VAR Model

House price modelling itself is a large area of study. Traditionally, house price dynamics are assumed to follow a geometric Brownian motion (see, for example, Cunningham and Hendershott 1984, Kau et al. 1993, Huang et al. 2011). The GBM process is a very popular tool in finance for modelling asset returns, as it provides powerful, yet simple representation of the dynamics. However, the GBM assumption cannot accommodate many stylized facts, for example, conditional heteroskedasticity, serial correlations, and volatility clustering of observed house prices, in real estate markets. Therefore, it is natural to apply time-series analysis to model the housing price dynamics. Chen et al. (2010) and Yang (2011) use the ARMA-GARCH model to fit the house price index in the US and Li et al. (2010) use the ARMA-EGARCH model for the house price growth in the UK.

Another important risk factor in equity release products is interest rate risk. A stochastic interest rate model with a realistic term structure needs to be considered. Furthermore, many empirical studies demonstrate that property returns and interest rates are correlated. Jointly modelling of house price indices and interest rates is particularly important for variable interest rate reverse mortgages, which dominate the US and Australian markets. In light of this, Huang et al. (2011) implement a two-dimensional volatility vector, linking the house price and interest rate dynamics. Sherris and Sun (2010) use a VAR model with two lags to capture the dynamic relationships between a house price index, rental yields, interest rates, and inflation. We adopt the same approach in this paper. A VAR-type model captures the linear correlations embedded in a multivariate time series system. Popularized by Sims (1980), VAR has been extensively used in econometrics and various applications in finance, as it provides flexibility and simplicity over other traditional econometric models.

Macroeconomic variables are likely to affect the dynamics of both house prices and interest rates. Ang et al. (2003) describe the joint dynamics of bond yields and

macroeconomic variables in a VAR model. Previous studies also argue that house prices are affected by macroeconomic factors (see, for example, Abraham and Hendershott 1994; Muellbauer and Murphy 1997). Recent studies have included GDP as a factor in predicting housing prices (Valadez 2010) and the yield curve (Ang and Piazzesi 2003). For this reason, we include GDP in our VAR framework.

The raw data used in this study include zero-coupon interest rates (3-month and 10-year), standard variable mortgage rates (*MR*), a nominal Sydney house price index (*HPI*), a nominal Sydney rental yield index (*RYI*), and nominal Australian GDP (*GDP*). Data is available for the period June 1993 to June 2011. Because the data for GDP is only available on a quarterly basis, other variables are filtered to quarterly frequency. Table 4 describes the variable definitions, sources and frequency of the data.

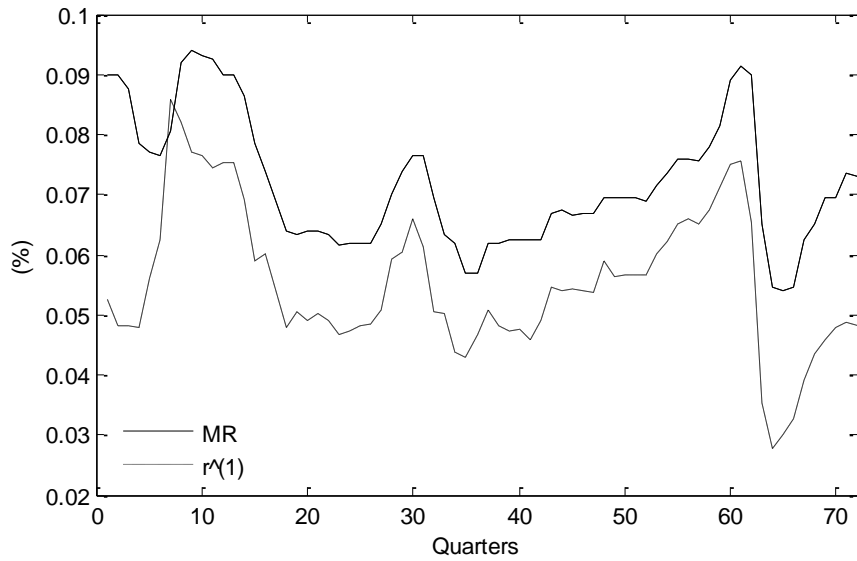
Table 4: Notations, Definitions, Sources and Frequency of Variables

Variables	Definitions	Sources	Frequency
$r^{(1)}$	3 month Zero-coupon yield	Reserve Bank of Australia	Daily
$r^{(40)}$	10 year Zero-coupon yield	Reserve Bank of Australia	Daily
<i>MR</i>	Nominal Mortgage Rates	Reserve Bank of Australia	Monthly
<i>HPI</i>	Nominal Sydney house price index	Residex Pty Ltd.	Monthly
<i>RYI</i>	Normal Sydney rental yield index	Residex Pty Ltd.	Monthly
<i>GDP</i>	Australian Nominal GDP	Australian Bureau of Statistics	Quarterly

Mortgage rates are highly correlated with the three-month zero-coupon rates, as can be seen from Figure 2. A correlation of 77% is found based on historical data of these two time series. To avoid the issue of collinearity, we decide not to include mortgage rate in the VAR model. Instead, mortgage rates in our simulation study are computed as the three-month zero-coupon rate plus a fixed margin 1.648%.⁹

⁹ The margin is calculated based on the average difference between the mortgage rates and the 3-month zero coupon rates for the period June 1993 to June 2011.

Figure 2: Comparison between Mortgage Rate and 3-Month Zero Coupon Yield



Though we would expect that the entire yield curve, not just the arbitrary maturity used to construct the term spread, would have predictive power, it is difficult to use multiple yields in the VAR regression because of collinearity problems. The high correlation between yields with different maturity suggests that we may be able to condense the information contained in many yields down to a parsimonious number of variables (Ang et al. 2006). In this paper, we use two factors from the yield curve, the three-month zero-coupon rates, $r^{(1)}$, to proxy for the level of the yield curve, and the ten-year term spread, $r^{(40)} - r^{(1)}$, to proxy for the slope of the yield curve.

Also note that all the variables are recorded as indices, except for zero-coupon yields and mortgage rates which are given as continuously compounding rates. In order to keep consistency, we transform the index variables into continuously compounding quarterly growth rates by taking the first difference of the logged indices, i.e., $h_t = \log HPI_t - \log HPI_{t-1}$, $y_t = \log RYI_t - \log RYI_{t-1}$, and $g_t = \log GDP_t - \log GDP_{t-1}$. The vector of state variables can be expressed as $z_t = [r_t^{(1)}, r_t^{(40)} - r_t^{(1)}, h_t, y_t, g_t]$. The plots of raw

data and the quarterly housing price growth, rental yield growth, and GDP growth are given in Figure 3 and 4.

Figure 3: Plots of Raw Data

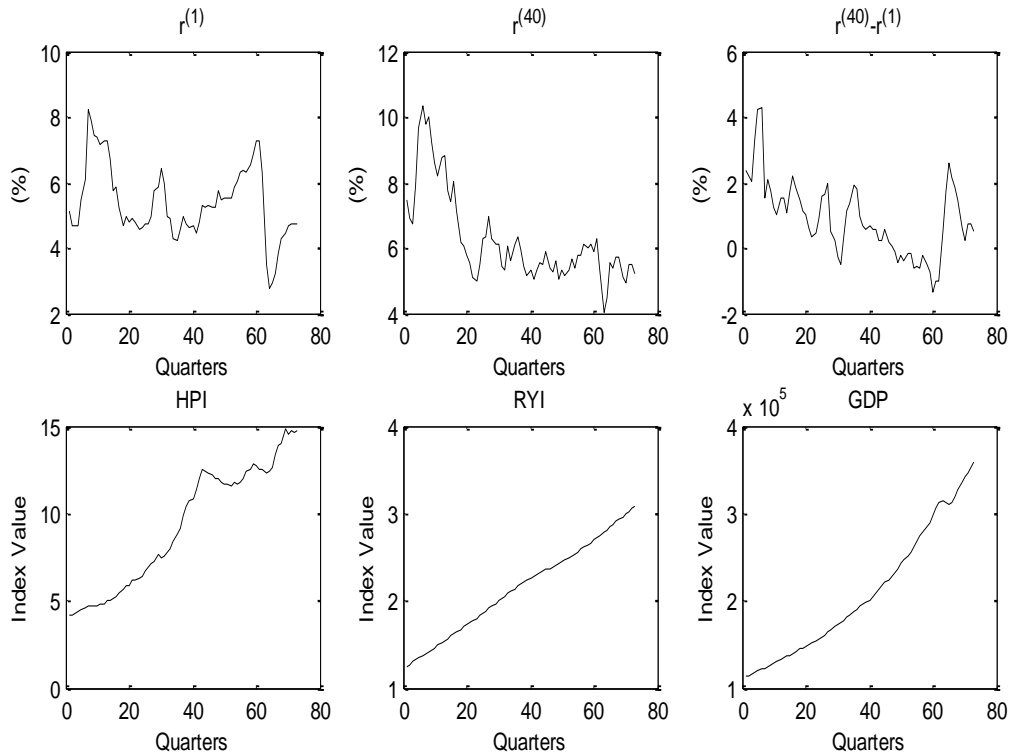
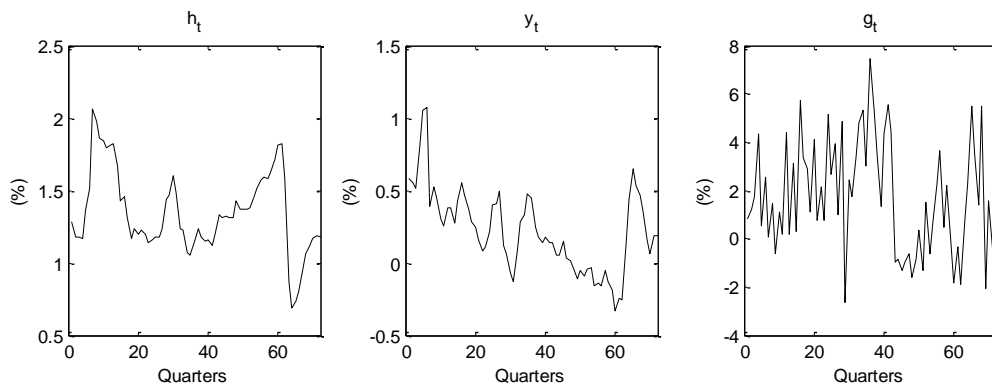


Figure 4: Plots of Transformed Data for House Price Index, Rental Yield Index and GDP



Before estimating the VAR model, we test stationarity of all variables using the augmented Dicky-Fuller (ADF) test and the Phillips-Perron (PP) test, the results of which are reported in Table 5. Both the ADF and PP test results indicate that all variables are stationary at the 10% significance level, except for the quarterly rental yield growth rate, y_t . However, no profound trend is found in the time series plot of this variable. Sims (1990) argues that the

ordinary least square (OLS) estimators of VAR parameters are asymptotically normal distributed, even if some variables are found to be non-stationary and/or cointegrated. Therefore, we proceed to fit the VAR model without any modification on the variable y_t in order to keep consistency and to avoid loss of information.

Table 5: Stationary Test Statistics

Variables	ADF		PP	
	<i>t</i> statistic	<i>t</i> probability	<i>t</i> statistic	<i>t</i> probability
$r^{(1)}$	-3.30786	0.0182	-2.62074	0.0936
$r^{(40)} - r^{(1)}$	-2.95082	0.0447	-2.73857	0.0726
h_t	-3.13597	0.0284	-6.41239	0.0000
y_t	-1.31624	0.6177	-1.22172	0.6608
g_t	-3.50690	0.0107	-2.72914	0.0742

We then proceed to choose the optimal lag length of the VAR model. This step is important as underfitted lag may disregard important dynamics of the multivariate process, whereas overfitted lag may violate parsimony (Kilian 2001). We compare the Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC) and Hannan-Quinn Criterion (HQC) to determine the appropriate lag. Lags of one to six are tested for the above criteria. From Table 6, AIC suggests an optimal lag order of six, whereas both SIC and HQC indicate an optimal lag order of two. Lütkepohl (2005) argues that SIC and HQC are preferred over AIC as they are consistent even if the data series are non-stationary. Ivanov and Kilian (2005) illustrate that the frequency of data series should be taken into account when choosing a lag selection criterion. They suggest that HQC is better when examining monthly or quarterly data. So we choose to fit a lag order of two based on the HQC.

Table 6: Lag Selection Criterion

Lag Order	AIC	SIC	HQC
1	-1.66804	-0.67274	-1.27475
2	-2.93101	-1.10630*	-2.20998*
3	-3.16648	-0.51236	-2.11771
4	-2.89129	-0.59225	-1.51478
5	-3.02817	1.28478	-1.32392
6	-3.18341*	1.95896	-1.15141

* indicates lag order selected by the criterion

The VAR (2) model is given by

$$z_{t+1} = c + \phi_1 z_t + \phi_2 z_{t-1} + \Sigma^{1/2} \varepsilon_{t+1}, \quad (6)$$

where z_t is a $(n \times 1)$ vector of state variables, $\Sigma^{1/2}$ is the Cholesky decomposition of the covariance matrix Σ that captures the dependence structure of the state variables, and $\varepsilon_{t+1} \sim N(0, I)$. The parameter estimates are summarized in Table 7.

Table 7: Estimated Parameters of VAR (2)

VAR(2): $z_t = c + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \varepsilon_t$										
c (5x1)	ϕ_1 (5x5)					ϕ_2 (5x5)				
0.118	1.147	0.342	0.001	0.654	0.085	-0.234	-0.065	0.001	-0.762	-0.035
0.113	-0.294	0.694	0.001	0.284	-0.052	0.127	-0.051	-0.003	-0.075	0.007
1.956	-1.404	1.286	-0.026	-0.845	0.204	-1.300	-4.224	0.399	4.323	-0.785
-0.018	0.045	-0.025	-0.006	1.091	0.004	-0.039	0.028	-0.002	-0.098	0.010
1.258	0.542	-0.036	0.016	1.112	1.231	-0.350	-0.005	0.007	-1.481	-0.903
Σ (5x5)					ρ (5x5)					
0.013	-0.007	-0.015	0.000	0.021	1.000	-0.444	-0.070	0.025	0.483	
-0.007	0.018	0.030	0.001	-0.003	-0.444	1.000	0.121	0.136	-0.095	
-0.015	0.030	3.486	-0.029	0.009	-0.070	0.121	1.000	-0.399	0.023	
0.000	0.001	-0.029	0.002	-0.001	0.025	0.136	-0.399	1.000	-0.076	
0.012	-0.003	0.009	-0.001	0.048	0.483	-0.095	0.023	-0.076	1.000	

The estimated VAR (2) model is used to simulate the state variables. We simulate 10,000 pseudo random sample paths of the state variables for a period of 40 years. As shown in Figure 5, the cumulative distribution function (CDF) of each of the simulated state variables is found to be comparable to its empirical distribution. We plot the historical data of each variable for the period of June 1992 - June 2011 and the mean simulated paths for the period of September 2011 - September 2051 (as log differences) with the 90% confidence interval in Figure 6. The mean simulated paths look remarkably stable due to the averaging effect of simulated paths. From the visualized confidence interval, we can see that the simulated values of variables span reasonable range of values. We also transform the quarterly growth rates of house price indices, rental yield indices, and GDP back to the index values in Figure 7. The plots clearly show that the mean simulated future paths of the index variables follow the historical dynamics.

Figure 5: CDF of Historical and Simulated State Variables.

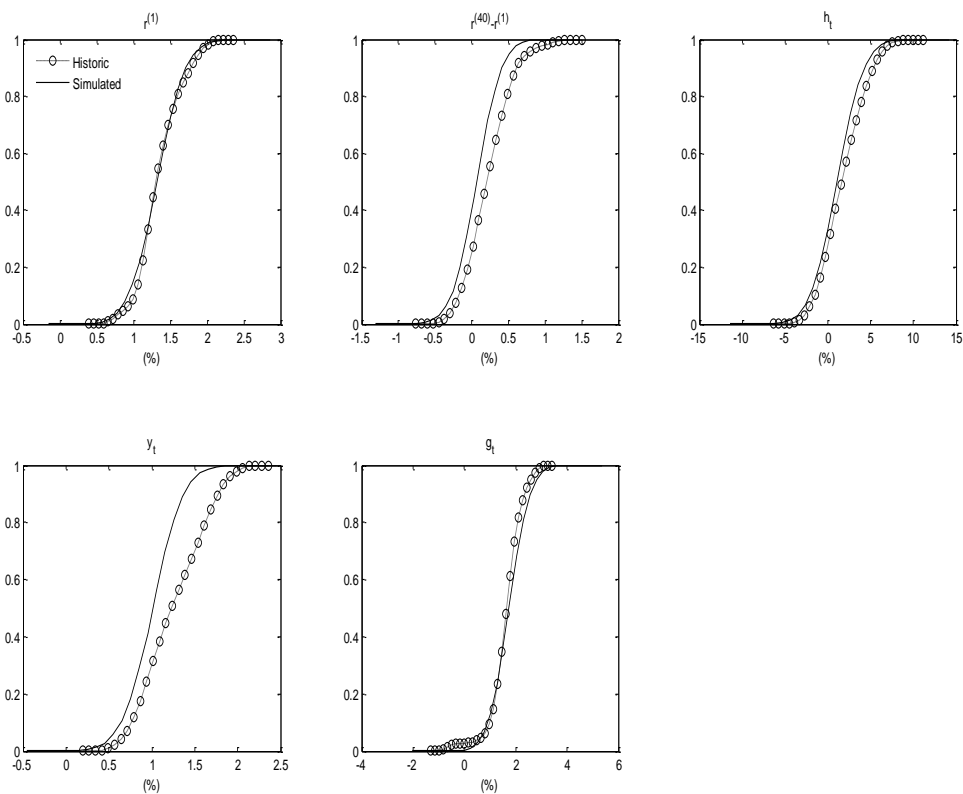


Figure 6: Historical and Mean Simulated Paths of State Variables with 90% CI

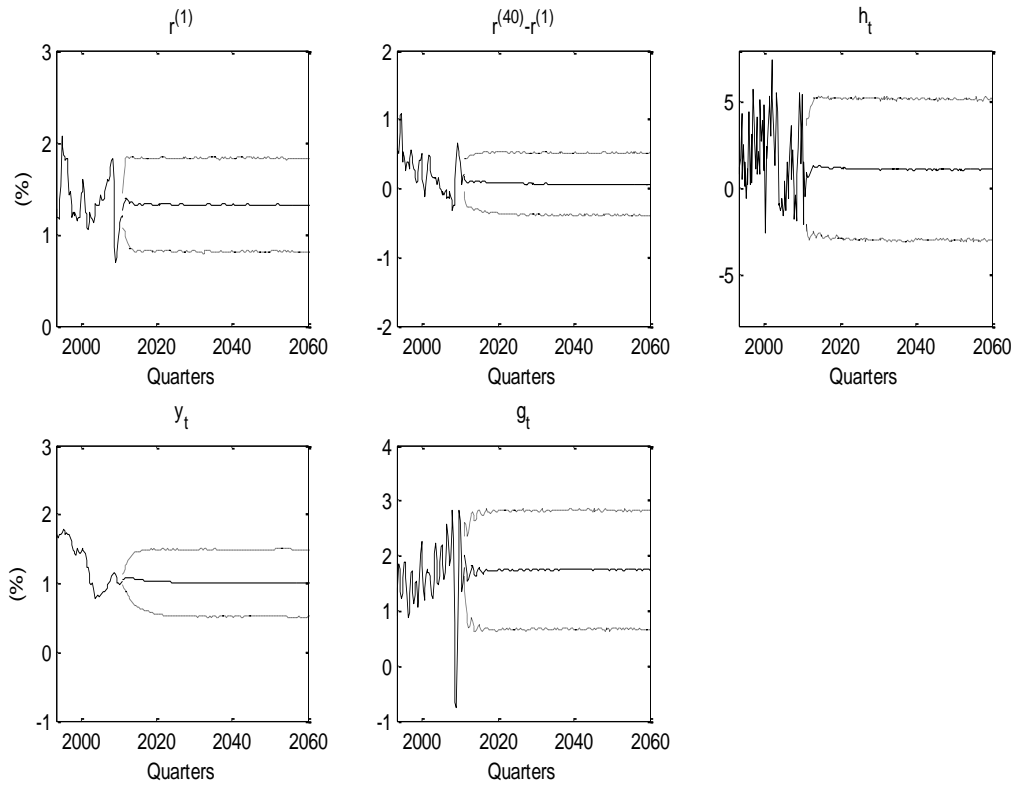
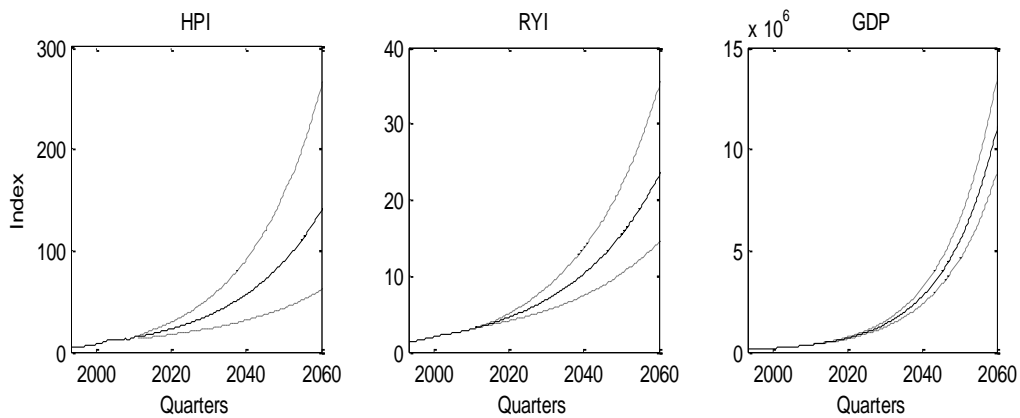


Figure 7: Historical and Mean Simulated Paths of Indices (HPI, RYI and GDP) with 90% CI



3.3. Stochastic Discount Factors

In this paper, we follow Ang and Piazzesi (2003) and Ang et al. (2006) to develop a pricing kernel that can be used to price all nominal assets in the economy.

Denote ξ_{t+1} the Radon-Nikodym derivative that converts the risk-neutral measure to the data-generating measure. Thus, for any $t+1$ variable X_{t+1} we have that

$$E_t^Q [X_{t+1}] = E_t [\zeta_{t+1} X_{t+1}] / \zeta_t, \quad (7)$$

where the expectation is taken under the risk-neutral measure Q .

Assume that ξ_{t+1} follows the log-normal process

$$\zeta_{t+1} = \zeta_t \exp \left\{ -\frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \right\}, \quad (8)$$

where λ_t are the time-varying market prices of risk associated with the sources of uncertainty ε_t . We parameterize λ_t as an affine process of the state variables

$$\lambda_t = \lambda_0 + \lambda_1 z_t, \quad (9)$$

where λ_0 is a n -dimensional vector and λ_1 is a $n \times n$ matrix accounting for time-variation in the risk premia.

The pricing kernel or stochastic discount factor, m_{t+1} , is defined as

$$m_{t+1} = \exp \{ -r_t \} \zeta_{t+1} / \zeta_t = \exp \left\{ -e_1' z_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \right\}. \quad (10)$$

where $e_1' = (1, 0, 0, 0, 0)$.

For an asset having a payoff X_{t+1} at time $t+1$, the price of the asset, P_t , is given by

$$P_t = E_t [m_{t+1} X_{t+1}]. \quad (11)$$

Particularly, the price of an n -period nominal bond at time t can be solved recursively by the following formula

$$P_t^{(n)} = E_t [m_{t+1} P_{t+1}^{(n-1)}], \quad (12)$$

with the termination condition $P_t^{(0)} = 1$. The resulting bond prices are exponential linear function of the state variables in the VAR, that is,

$$P_t^{(n)} = \exp \left\{ A_n + B_n' z_t + C_n' z_{t-1} \right\} \quad (13)$$

where A_n , B_n and C_n follow the difference equations:

$$\begin{aligned}
A_{n+1} &= -\delta_0 + A_n + B_n' (c - \Sigma^{1/2} \lambda_0) + \frac{1}{2} B_n' \Sigma B_n \\
B_{n+1} &= -\delta_1 + (\phi_1 - \Sigma^{1/2} \lambda_1)' B_n + C_n \\
C_{n+1} &= \phi_2' B_n
\end{aligned} \tag{14}$$

with the starting values $A_1 = 0$ and $B_1 = -e_1$ and $C_1 = 0$.¹⁰

Given the nominal bond price $P_t^{(n)}$, the continuously compounded yield $r_t^{(n)}$ on an n -period zero-coupon bond is given by

$$r_t^{(n)} = -\frac{\log P_t^{(n)}}{n} = -\frac{A_n}{n} - \frac{B_n'}{n} z_t - \frac{C_n'}{n} z_{t-1}. \tag{15}$$

From the above equation, it is clear that the parameter λ_0 only impacts average term spreads and average expected bond returns, while λ_1 controls the time variation in term spreads and expected returns. The risk parameters (i.e., λ_0 and λ_1) can be estimated conditional on the VAR parameters. This is done by minimizing the sum of the squared differences between the fitted yields of the term structure model and historical zero-coupon yields, i.e.,

$$\min_{\{\lambda_0, \lambda_1\}} \sum_{t=1}^T \sum_{n=1}^N (\hat{r}_t^{(n)} - r_t^{(n)})^2. \tag{16}$$

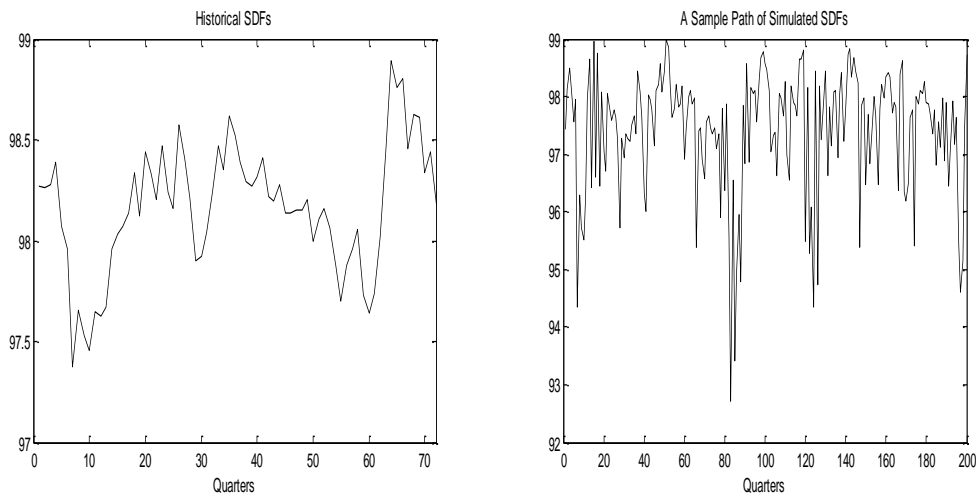
Besides the 3-month and the 10-year zero-coupon yield rates, we calibrate the model to 1-year, 2-year, and 5-year zero-coupon yields. The estimated parameters in the market price of risk are reported in Table 8.

¹⁰ Please refer to Shao et al. (2012) for detailed proof.

Table 8: Estimated Parameters in the Market Price of Risk

Variables	λ_0 (5x1)	λ_1 (5x5)				
$r^{(1)}$	0.9204	-2.3931	-0.2864	-0.1300	0.8383	1.0222
$r^{(40)} - r^{(1)}$	0.2199	0.1342	-0.0219	-1.1957	1.1733	-1.6144
h_t	6.5198	2.2503	1.0482	0.2543	-1.1396	-0.0363
y_t	-1.3300	-1.4757	-1.1437	4.2605	-4.2072	4.3283
g_t	1.7039	2.2737	-3.5390	0.6715	0.7444	-0.3215

Based on the fitted market price of risk, we calculate the stochastic discount factors and show its plot in Figure 8. We also show a sample path of simulated stochastic discount factors in the same figure. The correlations between the fitted stochastic discount factor and state variables are reported in Table 9. It can be seen that the stochastic discount factor has a high negative correlation with the short rate, which is intuitive. In addition, the house price growth positively contributes to the stochastic discount factor.

Figure 8: Stochastic Discount Factors and Bond Risk Premiums**Table 9: Correlations between Stochastic Discount Factors and State Variables**

Correlation	$r^{(1)}$	$r^{(40)} - r^{(1)}$	h_t	y_t	g_t
SDF	-0.94	0.26	0.38	-0.31	-0.23

4. Risk Analysis

In the previous section, we have described a termination model and a VAR model for economic variables. We use these models to simulate the input variables and calculate the provider's capital at some future dates. We estimate an empirical distribution of the capital amount by running the simulation procedure a large number of times. The capital distribution is then used to calculate the target solvency capital level. This simulation-based approach was also used in Daykin et al. (1994), Lee (2000) and Tsai et al. (2001). Various measures can be used to decide risk-based capital level for solvency requirement and there is no general consensus as to which one is the most appropriate. We consider two commonly used risk measures, VaR and CVaR, to calculate the solvency capital in this paper.

4.1. Payoff Structure of Reverse Mortgages

4.1.1. Pricing the No Negative Equity Guarantee

In a reverse mortgage contract, borrowers are typically protected by the provision of the no negative equity guarantee. When the loan terminates, if the net proceeds from the sale of the property are sufficient to pay the outstanding loan balance, the remaining cash usually is paid out to the borrower or his/her beneficiaries. If the proceeds are insufficient to cover the loan balance, the no negative equity guarantee prevents the lender from pursuing other assets belonging to the borrower. Denote L_t and H_t the loan outstanding balance and the value of the property at time t , respectively. Suppose there is a transaction cost of selling the house, γ , given by a percentage of the house value. The payoff of the no negative equity guarantee at loan termination time t is

$$NN_t = \max(L_t - (1 - \gamma)H_t, 0). \quad (17)$$

In our analysis, we consider a lump sum payout option, which is most popular payout in Australia. The maximum initial loan amount is determined by the LVR that is set as a proportion of the value of the property. LVRs increase with the age at which the loan is taken

out. Suppose the borrower always takes out 100% of the allowable limit, i.e., $L_0 = H_0 \times LVR$. The loan accrues quarterly with interests and mortgage insurance premiums. As aforementioned, the variable mortgage rate is computed by adding a fixed margin on top of the short rate (3-month zero coupon rate). Thus, L_t is given by

$$L_t = L_0 \exp \left\{ \sum_{i=0}^t (r_i^s + \kappa + \pi) \right\}, \quad (18)$$

where r_i^s denotes the three-month zero-coupon rate, κ is the lending margin and π is the mortgage insurance premium rate.

As the termination time t is random, we use the probability of contract termination, ${}_t q_x^c$, to model the randomness of loan termination. We then use stochastic discount factors, m_t , to discount the value of the no negative equity guarantee at an arbitrary termination time t to the time of loan origination, taking into account the uncertainty in the future development of house prices, rental yields, and interest rates. Hence, the value of the no negative equity guarantee, NN , is given by

$$NN = \sum_{t=0}^{\omega-x-1} E \left[\prod_{s=0}^t (m_s) {}_t q_x^c \max(L_t - (1-\gamma)H_t, 0) \right]. \quad (19)$$

The no negative equity guarantee is usually financed by mortgage insurance premiums paid by the borrowers. There is no clear mortgage insurance structure in Australia, but previous studies usually assume a zero up-front premium and a fixed premium rate each period. The actuarial present value of mortgage insurance premiums, MIP , is then given by

$$MIP = \pi \sum_{t=1}^{\omega-x-1} E \left[\prod_{s=1}^t (m_s) {}_t p_x^c L_t \right]. \quad (20)$$

The actuarially fair quarterly premium rate π can be calculated by equating the value of mortgage insurance premiums with the value of the no negative equity guarantee.

4.1.2. Cash Flows of the Reverse Mortgage Contract

We assume that the provider of a reverse mortgage contract finances the payout through its existing capital and leveraging. The proportion of borrowed capital, or the leverage ratio (LR), is denoted by φ . The borrowed capital accrues with the short rate. Therefore, the total financing cost at time t can be written as

$$C_t^{RM} = \varphi L_0 \exp\left\{\sum_{i=0}^t r_i^s\right\} + (1-\varphi)L_0. \quad (21)$$

The provider receives $\min(L_t, (1-\gamma)H_t)$ from the sale proceeds of the property when the loan terminates. Its net payoff discounted back to time zero can be calculated as

$$RM = \sum_{t=0}^{\omega-x-1} {}_tq_x^c \exp\left\{-\sum_{i=0}^t r_i^s\right\} \left[\min(L_t, (1-\gamma)H_t) - C_t^{RM}\right]. \quad (22)$$

4.2. Payoff Structure of Home Reversions

4.2.1. Pricing the Lease for Life Agreement

Under a home reversion contact, the provider buys a share of the property at a discounted price and offers the customers a lease for life agreement. The agreement can be valued using annuity pricing techniques, where the annuity is indexed to the property's rental yield rate. For the purpose of comparison, we assume that the acquisition ratio is the same as the LVR in the reverse mortgage. For a certain lifespan, the value of the lease for life agreement at time 0 can be expressed as a function of the termination time T ,

$$LL_0 = \sum_{t=0}^T E\left[\prod_{s=0}^t (m_s) H_t R_t \times LVR\right], \quad (23)$$

where R_t denotes the rental yield rate in year t .

Again, the termination time T is random. Therefore, the actuarial present value of the lease for life agreement can be written as

$$LL = \sum_{t=0}^{\omega-x-1} E\left[\prod_{s=0}^t (m_s) {}_t p_x^c H_t R_t \times LVR\right]. \quad (24)$$

4.2.2. Cash Flows of the Home Reversion Contract

In a home reversion contract, the provider purchases a share of the equity that is worth $H_0 \times LVR$ and discounts it by the value of the lease for life, LL . The resulting lump-sum payment at contract origination is $H_0 \times LVR - LL$. Again, the provider is assumed to finance the payout by borrowing φ % of the required capital. At the time of loan termination t , the property is sold and the provider receives a share of the sale proceeds, which is $H_t \times LVR$. Thus the provider's net present value of payoffs at time zero is given by

$$HR = \sum_{t=0}^{\omega-x-1} {}_tq_x^c \exp\left\{-\sum_{i=0}^t r_i^s\right\} (H_t \times LVR - C_t^{HR}), \quad (25)$$

where the total cost $C_t^{HR} = \varphi(H_0 \times LVR - LL) \exp\left\{\sum_{i=0}^t r_i^s\right\} + (1 - \varphi)(H_0 \times LVR - LL)$.

5. Numerical Illustration

In this section, we compute the value of the no negative equity guarantee in the reverse mortgage contract and the value of the lease for life in the home reversion contract. We then compare these two equity release products with respect to profitability and risk under various scenarios. We conduct sensitivity analyses to identify the impacts of key factors, such as age at contract origination, the initial house value, mortality improvement and the leverage ratio, on cash flows and risk profiles of both equity release products.

5.1. The Base Case Scenario

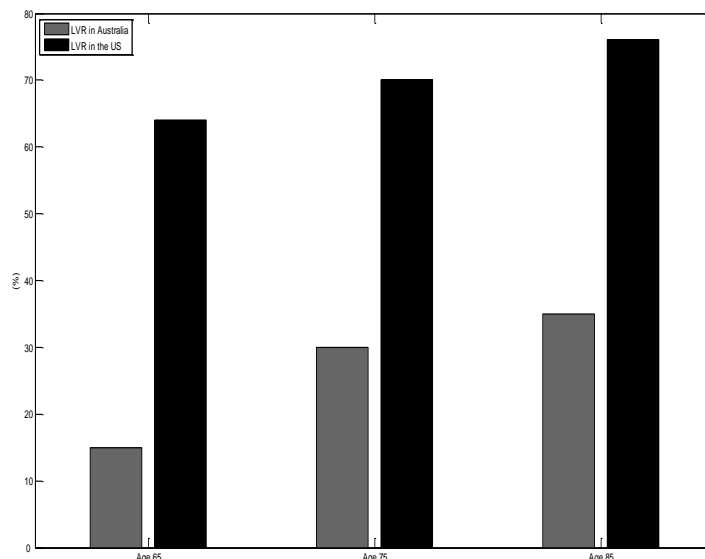
In the base case scenario, we assume a single female aged 65 residing in Sydney, Australia, with an initial house value of \$600,000.¹¹ To finance her retirement consumption and/or aged care, she can either enter a reverse mortgage contract or sell a share of the equity by entering a home reversion contract. If she decides to participate in the home reversion

¹¹ Median Price and Number of Established House Transfer, Australian Bureau of Statistics.

scheme, the acquisition ratio is set to be the same as the LVR for the purpose of comparison. We assume that the equity release provider finances the lump-sum payout to the homeowner completely through borrowed capital, i.e., the leverage ratio is 100%.

Note that the prevalent maximum LVRs in Australia are much lower than those used in the US. Figure 9 compares typical maximum LVRs for different borrower ages in Australia and in the US HECM program. The maximum LVR increases with age because the time horizon for the loan accumulation is shorter. The US market is overwhelmingly led by HECM products, which offer significantly more generous LVRs than comparable products in foreign markets. For example, the typical US LVR is more than quadruple that of Australia for borrowers aged 65 and more than double for age 75 and 85. Many lenders have recently reduced their HECM interest rate margins to attract additional sales, which has produced even higher LVRs. We will show later that this distinction makes the Australian equity release products carry a quite different payoff and risk structure compared to the US products.

Figure 9: LVRs in Australia v.s. LVRs in the U.S.



We project the probability of loan termination based on the termination model presented above and simulate 10,000 paths of the economic variables based on the VAR(2)

model for 40 years. We assume the provider of the reverse mortgage charges a zero up-front premium and annual premiums with an actuarially fair rate π . We then calculate the value of no negative equity guarantee. For the home reversion, we calculate the value of the lease for life agreement. We obtain the distribution of the actuarial present value of payoffs of the provider for both products. Given the payoff distributions, we assess riskiness of each program by computing VaR and CVaR at the 99.5% level. Table 10 summarizes the results in the base case scenario.

Table 10: Payoffs and Risks in the Base Case Scenario

Assumption: Age=65, H0=\$600,000, LR =100%, No mortality improvement								
LVR	Reverse Mortgage				Home Reversion			
	NN	E[RM]	VaR	CVaR	LL	E[HR]	VaR	CVaR
15%	0	29,623	0	0	35,764	25,906	-3,873	-6,564
64%	39,280	82,155	-78,849	-93,941	152,593	110,533	-16,524	-28,005

Note: NN is the value of the no negative equity guarantee and LL is the value of the lease for life agreement. E[RM] (or E[HR]) denotes the average actuarial present value of the reverse mortgage (or home reversion) contract. VaR and CVaR are calculated at the 99.5% level.

When we use the maximum LVR typically found in Australia (15% for age 65), the no negative equity guarantee has no values, which shows the reverse mortgage loans has virtually no likelihood of losses. As a result, the actuarially fair premium for the guarantee is zero. However, the fact is that reverse mortgage providers in Australia charge more than 1% insurance premiums to protect themselves from crossover risk (Bridge et al. 2010). Our results show that there is a possibility of reducing interest rates for reverse mortgage loans to be closer to those for standard home loans. The VaR and CVaR at the 99.5% level are both zero, implying that reverse mortgage providers do not need to set aside risk-based capital. This finding is consistent with the comments from many brokers that LVRs in Australia are set too conservative and that the premium or fees could be lowered given the very low risk of default or even of negative equity being reached (Bridge et al. 2010). On the contrary, our

results show that home reversion providers do bear some risks and need to reserve some solvency capital. The risk mainly comes from the housing price depreciation.¹²

Figure 10: Loan Outstanding Balance L_t and the Sale Proceeds of the Property $(1-\gamma)H_t$ (LVR=64%)

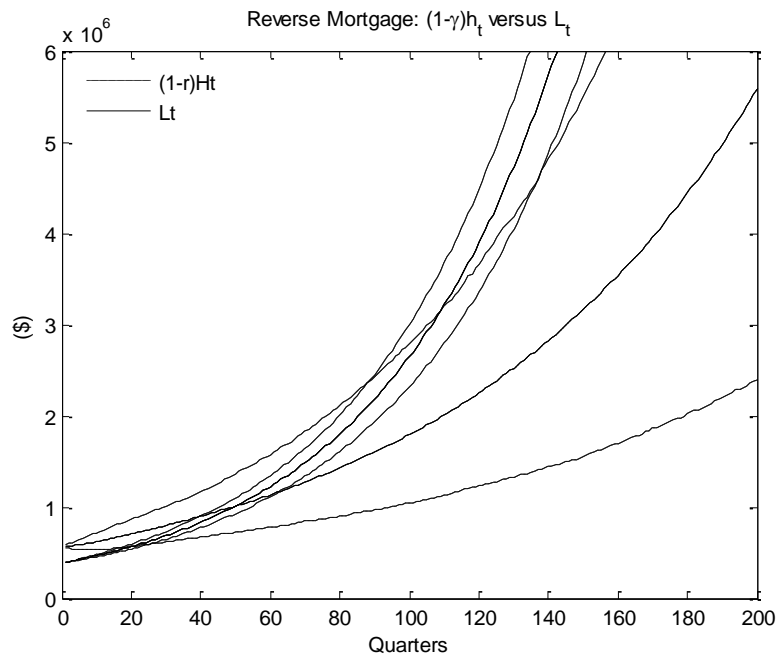
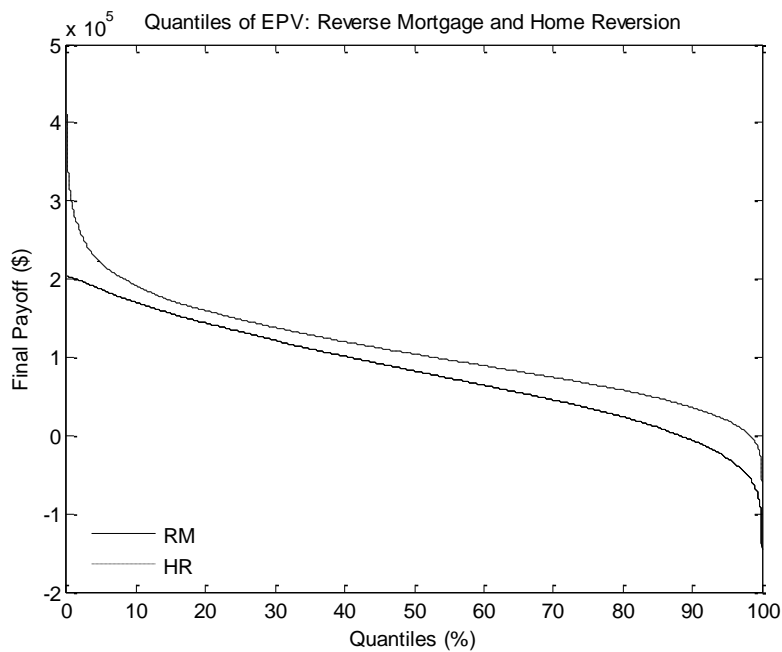


Figure 11: Distributions of the Actuarial Present Value of Net Payoffs (LVR=64%)



¹² The results are similar when we change the age to 75 and 85 and use the corresponding maximum LVRs in Australia (i.e., 30% and 35%)

We also produce results assuming a high LVR that can be found in the US HECM program (64% for age 65). The LVRs are substantially higher in the US and this has a significant impact on the risk profiles of equity release products. The simulation results show that negative equity results in several scenarios, which suggests that the reverse mortgage providers offering a high LVR would face crossover risks. In order to better understand the development of negative equity in a high LVR case, we plot the loan outstanding balance, L_t , versus the sale proceeds of the property, $(1-\gamma)H_t$, over time in Figure 10. Compared with the variability of house price outcomes, the loan balance (driven by interest rate fluctuations) is much less volatile. Negative equity arises when the accumulated loan balance crosses over the sale proceeds of the property. Crossover risk occurs after 12 years of the loan duration. If we consider a severe housing market downturn (represented by the lower 5% quantile of the house price distribution), negative equity occurs after circa 5 years. Figure 11 gives the quantile distribution of the actuarial present value of net payoffs for both equity release products. The graph shows that the home reversion contract is more profitable and less risky than the reverse mortgage when a LVR of 64% is assumed as found in the US market.

The comparison between reverse mortgages and home reversions yields contradicting results when using the LVR found in Australia versus that typical of the US. The appropriate setting of LVRs is a key issue. In order to further investigate how the LVR affects the payoff and risk structure of these two products, we fix the initial age to be 65 and the initial house value to be \$600,000 and vary the LVR from 15% to 64%. The results are shown in Table 11.

Table 11: The Impact of the LVR

Assumption: Age=65, H0=\$600,000, LR =100%, No mortality improvement								
LVR	Reverse Mortgage				Home Reversion			
	NN	E[RM]	VaR	CVaR	LL	E[HR]	VaR	CVaR
15%	0	29,623	0	0	35,764	25,906	-3,873	-6,564
25%	0	49,210	0	0	59,607	43,177	-6,454	-10,939
35%	614	68,023	0	0	83,449	60,447	-9,037	-15,316
40%	1,616	76,262	0	0	95,370	69,082	-10,328	-17,504
45%	3,636	83,052	0	-7,293	107,292	77,719	-11,618	-19,691
50%	7,456	88,131	-12,840	-27,451	119,213	86,354	-12,909	-21,879
55%	14,178	90,087	-34,914	-49,778	131,134	94,989	-14,201	-24,067
64%	39,280;	82,155	-78,849	-93,941	152,593	110,533	-16,524	-28,005

Note: NN is the value of the no negative equity guarantee and LL is the value of the lease for life agreement. E[RM] (or E[HR]) denotes the average actuarial present value of the reverse mortgage (or home reversion) contract. VaR and CVaR are calculated at the 99.5% level.

The change in payoff and risk for home reversion schemes has a clear trend, i.e., the average payoff increases with the LVR and so does the risk. This is intuitive since with a higher LVR, both the payoff and risk are magnified. We need to take a closer look at reverse mortgages since LVRs play a more important role in reverse mortgages and cause some trend changes. The value of the no negative equity guarantee increases with the LVR since a larger LVR reduces the gap between the house price and the loan balance, resulting in a higher crossover risk. When the LVR is low, the guarantee has a zero or a small value, indicating no or low crossover risk. In this case, the provider would receive the outstanding loan balance at loan termination. So the provider's payoff is mainly the accumulation of the lender's margin based on the initial loan amount. As a result, a larger LVR leads to a higher payoff for the provider. However, when the LVR increases above a critical level, negative equity can occur and reduce the payoff. For the same reason, the risk measure starts at zero but increases when the LVR is higher than 50%. We conclude that reverse mortgage providers receive higher average payoffs than home reversion providers and bear nearly no risk for LVR levels lower

than 50%. For higher LVR levels, expected payoffs from reverse mortgages become less and the risk turns out to be higher than home reversions.

5.2. Sensitivity Analysis

In the following analysis, we use LVRs set by the US HECM program in order to avoid zero risk in reverse mortgages and observe clear trends on comparative results.

5.2.1. Sensitivity to the Initial Age

The borrower's age has two competing effects on the risk/payoff structure: an increase in age reduces the average time of in-force duration and thus lowers the crossover risk; at the same time the resulting increase in LVR raises the initial loan amount and leads to higher crossover risk. We find that the value of the no negative equity guarantee is lower for reverse mortgage loans with a higher borrower age, showing that the age's effect on loan termination dominates the age's effect on LVRs. For the same reason, the risk (measured by VaR and CVaR) decreases with age. As to the expected payoff, the provider has less time to accumulate profits when the loan is issued to an older borrower, whereas the increase in the LVR, or a larger initial loan amount, results in a higher margin accumulation until loan termination. The dominant effect of loan duration results in the payoff decreasing with age.

The same logic applies equally to home reversion schemes, but we should keep in mind that the age effect on loan termination takes over. The value of the lease for life decreases with age because an older age means a shorter time period that rents are payable. Home reversion providers gain from the future house price appreciation. Nevertheless, a higher age at contract origination allows less time for the property value to appreciate. So the payoff decreases with age. The risk increases with age for a similar reason. Compared with the reverse mortgage provider, the home reversion provider receives a higher payoff on average and bears a lower risk.

Table 12: Sensitivity to the Initial Age

Assumptions: H0=\$600,000, LR=100%, No mortality improvement									
Age	LVR	Reverse Mortgage				Home Reversion			
		NN	E[RM]	VaR	CVaR	LL	E[HR]	VaR	CVaR
65	64%	39,280	82,155	-78,849	-93,941	152,593	110,533	-16,524	-28,005
75	70%	29,523	59,254	-56,010	-71,390	116,934	85,663	-25,350	-36,086
85	76%	18,131	33,583	-42,686	-51,783	72,186	46,588	-40,972	-48,700

Note: NN is the value of the no negative equity guarantee and LL is the value of the lease for life agreement. E[RM] (or E[HR]) denotes the average actuarial present value of the reverse mortgage (or home reversion) contract. VaR and CVaR are calculated at the 99.5% level.

5.2.2. Sensitivity to the Initial House Value

Changing the initial house price has a monotonic effect on the payoff and risk structure. It is evident that the value of the no negative equity guarantee and that of the lease for life decrease proportionally with the initial property value. The average payoff and the tail risk decrease with the house price for both products, but payoffs from the home contract are higher for the provider and this contract bears less risk than the reverse mortgage.

Table 13: Sensitivity to the Initial House Value

Assumptions: Age=65, LVR=64, LR=100%, No Mortality Improvement									
H0	Reverse Mortgage				Home Reversion				
	NN	E[RM]	VaR	CVaR	LL	E[HR]	VaR	CVaR	
600,000	39,280	82,155	-78,849	-93,941	152,593	110,533	-16,524	-28,005	
540,000	35,352	73,940	-70,964	-84,547	137,333	99,479	-14,872	-25,205	
480,000	31,424	65,724	-63,079	-75,153	122,074	88,426	-13,219	-22,404	

Note: NN is the value of the no negative equity guarantee and LL is the value of the lease for life agreement. E[RM] (or E[HR]) denotes the average actuarial present value of the reverse mortgage (or home reversion) contract. VaR and CVaR are calculated at the 99.5% level.

5.2.3. Sensitivity to Mortality Improvements

Table 14 illustrates the effect of mortality improvement on payoff and risk. The termination model used to determine contract termination probabilities is based on population mortality rates. Mortality improvements can lengthen the contract duration and therefore

increase the value of the no negative equity guarantee and that of the lease for life agreement. Mortality improvement has a relatively small impact on the average payoff and the risk embedded in the equity lease products.

Table 14: Sensitivity to Mortality Improvement

Assumptions: Age=65, LVR=64%, H0=\$600,000, LR=100%								
Mortality Improvement	Reverse Mortgage				Home Reversion			
	NN	E[RM]	VaR	CVaR	LL	E[HR]	VaR	CVaR
0%	39,280	82,155	-78,849	-93,941	152,593	110,533	-16,524	-28,005
10%	43,367	82,376	-84,832	-100,979	158,169	113,609	-15,765	-27,610
20%	46,523	82,594	-90,338	-106,769	162,558	116,128	-15,479	-27,582

Note: NN is the value of the no negative equity guarantee and LL is the value of the lease for life agreement. E[RM] (or E[HR]) denotes the average actuarial present value of the reverse mortgage (or home reversion) contract. VaR and CVaR are calculated at the 99.5% level.

5.2.4. Sensitivity to the Leverage Ratio

Lastly, we change the leverage ratio given by the percentage of the payout that the equity release provider finances through external sources. The decrease in the leverage ratio has no impact on the value of the no negative equity guarantee and that of the lease for life (which one would expect and we do not report in Table 15), but results in an increase in average payoffs and a decrease in risk for both products.

Table 15: Sensitivity to the Leverage Ratio

Assumptions: Age=65, LVR=64%, H0=600,000, No mortality improvement						
Leverage Ratio	Reverse Mortgage			Home Reversion		
	E[RM]	VaR	CVaR	E[HR]	VaR	CVaR
100%	82,155	-78,849	-93,941	110,533	-16,524	-28,005
90%	103,172	-57,183	-72,434	123,257	-3,791	-15,003
80%	124,286	-35,427	-50,920	135,981	0	-2,017

Note: NN is the value of the no negative equity guarantee and LL is the value of the lease for life agreement. E[RM] (or E[HR]) denotes the average actuarial present value of the reverse mortgage (or home reversion) contract. VaR and CVaR are calculated at the 99.5% level.

6. Conclusions and Discussions

The actuarial literature on pricing of equity release products is still rather limited. In this paper, we analyse cash flows and risk profiles for equity release products from the provider's perspective. We assume a single female policyholder who intends to make use of either the reverse mortgage or the home reversion scheme to liquidate her equity and finance her retirement consumption and care costs. We find that with a low LVR, reverse mortgages provide a higher payoff and deliver less risk to the provider than home reversions. This finding justifies the dominant market share of reverse mortgage schemes in Australia and many other countries, such as the UK. When we use a high LVR, as found in the US HECM program, we find that home reversions are better in terms of the payoff and risk structure for the provider than reverse mortgages. The appropriate setting of LVRs plays an important role in the product risks.

Our results indicate that reverse mortgage providers in Australia could consider increasing maximum LVRs and decreasing insurance premium rates or on-going fees, in order to expand the reverse mortgage market. Usually, the LVR depends on the age of the borrower at loan origination. Our sensitivity analysis indicates that among all the factors that we consider, the initial age of homeowners has a profound and significant impact on payoffs and risks of equity release product providers. It affects both the contract termination time and the LVR (thus the initial payout to consumers) and results in two competing effects on the risk and payoff profile. Caution has to be used when determining the LVR based on age.

Our results have important implications to policymakers and regulators in many other countries that face the issue of aging population and underfunded pensions. For example, the UK has a similar, conservative pattern of LVRs as in Australia. UK providers have the potential to increase LVRs to stimulate the reverse mortgage market. Though our results indicates a high LVR as found in the US makes reverse mortgage products less profitable and

riskier than home reversion schemes, this has been based on economic scenarios from Australia experience. The US housing market and economic conditions have been quite different in recent years and this has to be considered when assessing the US markets. In addition, in the US, the HECM providers are insured by the federal government and can transfer the risk to FHA.

As a newly developed equity release product, the home reversion scheme has advantages to both homeowners and investors. It usually sets a limit on the share of equity that can be sold to a home reversion company, leaving a remainder to consumers which can be used to fund aged care after the property is sold. As an asset class, much of the risk attached to 'traditional' property investment is either irrelevant in home reversion contracts such as tenancy or default risk, or can be diversified in a 'pooled' residential property pool, for example, duration risk and location risk (Deloitte 2011).

However, the private market for home reversions has been developing slowly. Lack of awareness and low financial literacy among consumers are the main reasons on the demand side. In particular, the implicit lease for life agreement in the home reversion contract may be poorly understood. On the supply side, liquidity is the major concern of investors. In addition to providing an appropriate framework of regulation and education, governments should consider policies to support the development of the equity release market such as providing liquidity for providers.

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